Six Sigma
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Lecture – 23
Probability Theory

Hello friends. I welcome you to the lecture 24 of our Six Sigma journey. And we will discuss probability theory as a part of this lecture, so I would like to remind you that we are discussing the major phase of DMAIC, Six sigma cycle and we are discussing the various topics specific to this.

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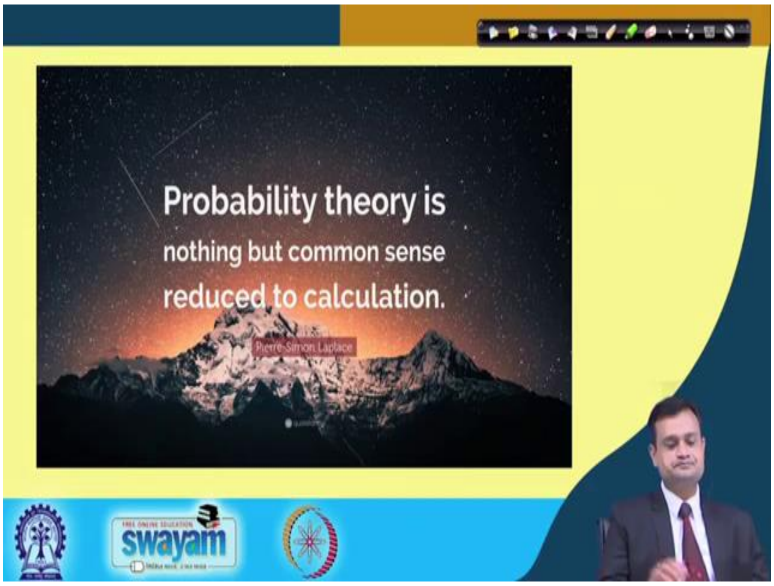

So, let us begin this lecture with a beautiful quote given by a great mathematician and researcher Laplace. So, he says probability theory is nothing, but common sense reduced to calculation.

So, if it is a common sense, is it really required. I would say yes. If you are exposed with a phenomena, which is occurring randomly and it is difficult for you to converge to a particular decision then definitely you need a tool or a technique like statistics to make better decisions. Suppose you make the forecast about the rain season you make forecast about the demand and you try to say control the quality of your process through certain statistical measures. So, in all the cases statistics has a vital role to play and it enables the intuitive judgmental capability of the managers, executives and they can take better informed decisions.

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So, we can see that as a part of recap, we have discussed in the previous lecture basics of statistic, descriptive versus inferential, measures of central tendency, relationship, covariance, correlation and numerical descriptive measures and the central limit theorem. I have also introduced the concept of random number.

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Now, in this particular lecture we will talk about basic concepts of the probability, complementary rule of probability, conditional probability, mutually exclusive event and probability distributions.

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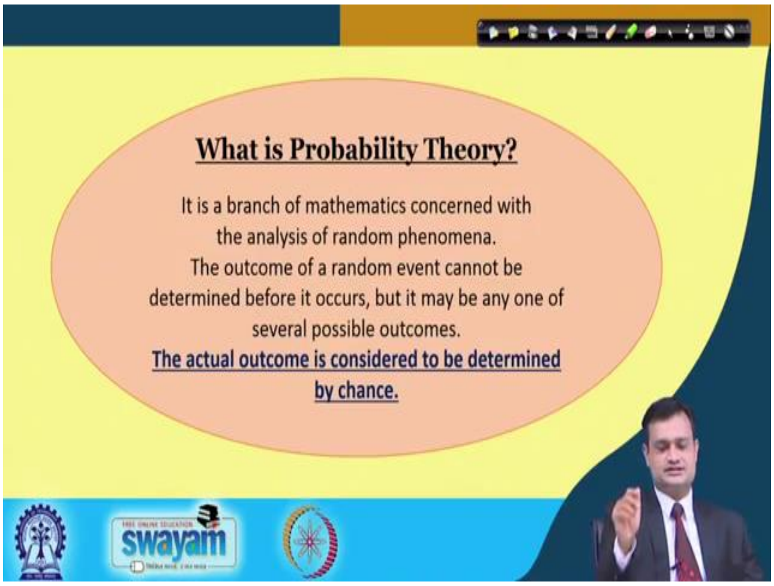

So, the question is that what is probability theory. It is important, but what exactly we mean by probability theory. So, many phenomena they are random in nature and they occur by chance. So, typically statistics is a branch of mathematics which is concerned with the analysis of random phenomena and that can help us to draw some conclusion about the problem under investigation or that phenomena itself.

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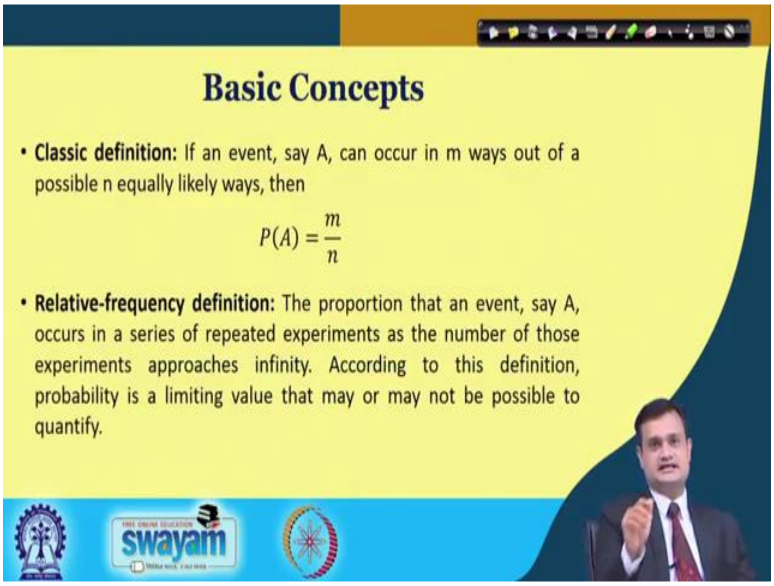

Now, just see the basic concepts that what is the classic definition of a probability. So, when I say probability of some event A then P(A) is m/n and this is typically m ways out of n equally likely ways. So, there are n equal possible opportunities, out of that m times let us say something is happening then P(A)=m/n. In the context of our six sigma let us say I am getting say 10 defectives out of may be 1000, then 10/1000 is the probability of getting a defective when I select a particular sample.

Now, there is a relative frequency definition also. So, it says that the proportion that an event, say A, occurs in a series of repeated experiments as the number of those experiments approaches infinity and according to this definition probability is a limiting value that may or may not be possible to quantify. So, it is little bit the extension of the classical definition, and typically it considers the repeated experiment, number of times and we keep the window open up to infinity.

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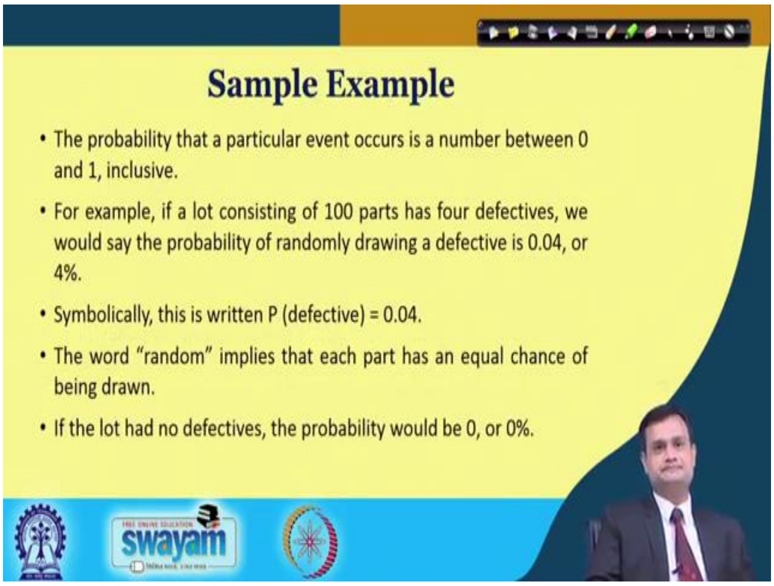

Now, just see the example, sample example, that the probability that a particular event occur is obviously, between 0 to and 1 and your total probability will be 1.

Now, let us say for example, if a lot consisting of 100 parts has 4 defectives we would say that probability of randomly drawing a defective is 0.04. So, 4/100 will give me the probability of getting or drawing a defective part. Now, symbolically I would write P in bracket defective is equal to 0.04, and the word random implies that each part has a equal opportunity for say chance of being drawn. If the lot had no defective the probability of getting a defective will be 0 or 0 %.

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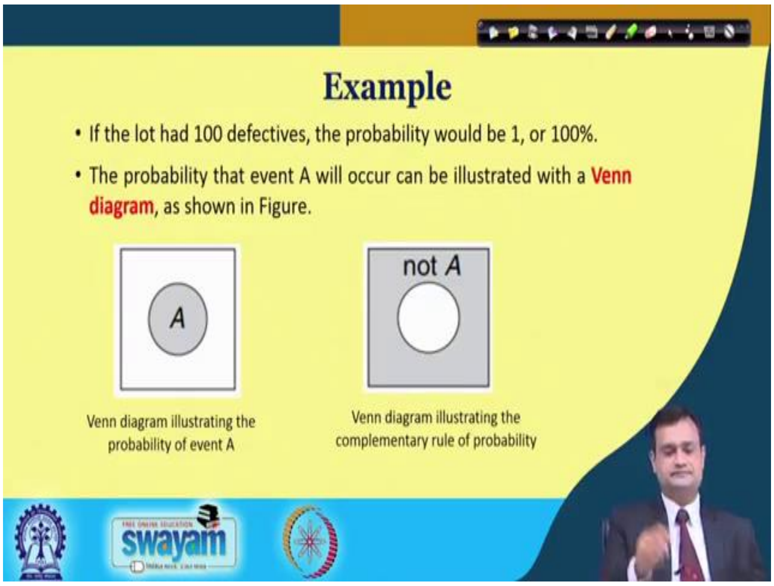

Then, if you see some basic concepts on probability then here if a lot of 100 defective the probability would be 1, it means entire lot is defective. So, probability is 100 percent and the probability that event A will occur you can illustrate by a Venn diagram. You can see the first Venn diagram here.

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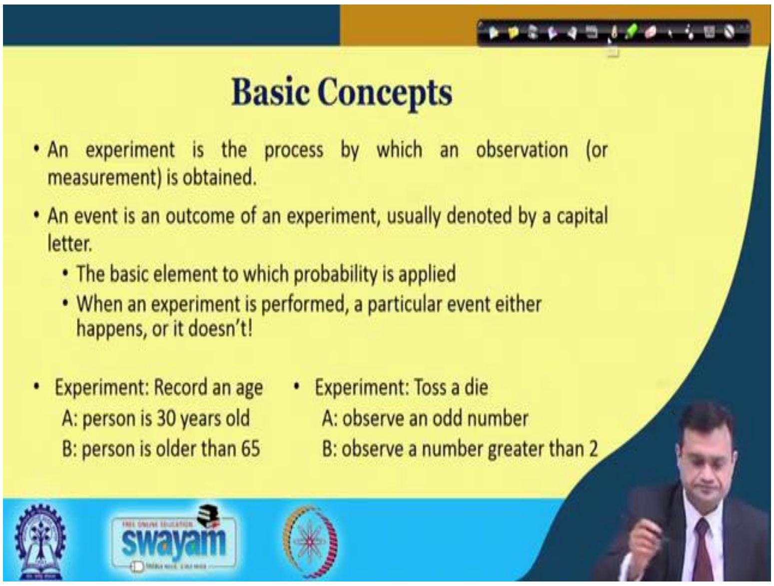

You can see the first Venn diagram here, and this illustrates the probability of event A. Now, if you see the second Venn diagram here it typically illustrates the complementary rule of probability. So, the area shaded in grey, grey color says that not A, it means that this is the probability that A will not occur and the circle if I consider that as A than that is the probability that A will occur.

Now, suppose you are conducting an experiment and typically you are analyzing a particular manufacturing process which an observation or measurement you are trying to take. And an event is an outcome of this particular experiment, so the event may be the defective parts or defects produced, so the basic element to which probability is applied. So, this particular event is typically denoted by a capital letter. And when an experiment is performed a particular event either happens or it does not happen. So, it is a complementary. It happens or it does not happen.

So, for experiment like record an age. So, A, person is 30 years old; B, person is older than 65, I want to find the probability. Toss a die, so this is a coin also or toss a die then observe an odd number observe a number greater than 2. So, I would be interested in finding the probability of occurrences that how many times I can observe the odd number or I can observe the number greater than 2.

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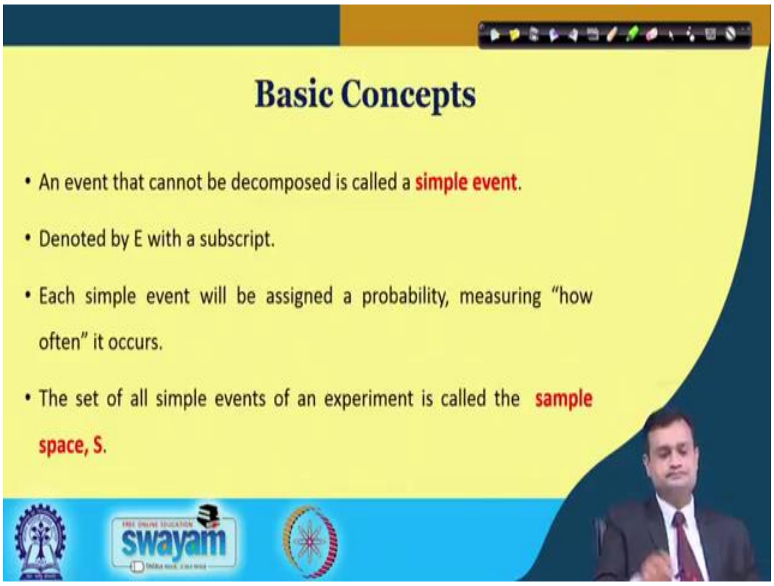

So, an event that cannot be decomposed is called simple event. It is typically denoted by E with a subscript. Each simple event will be assigned a probability measuring how often it occurs. And the set of all simple event of an experiment is called the sample space denoted as capital S.

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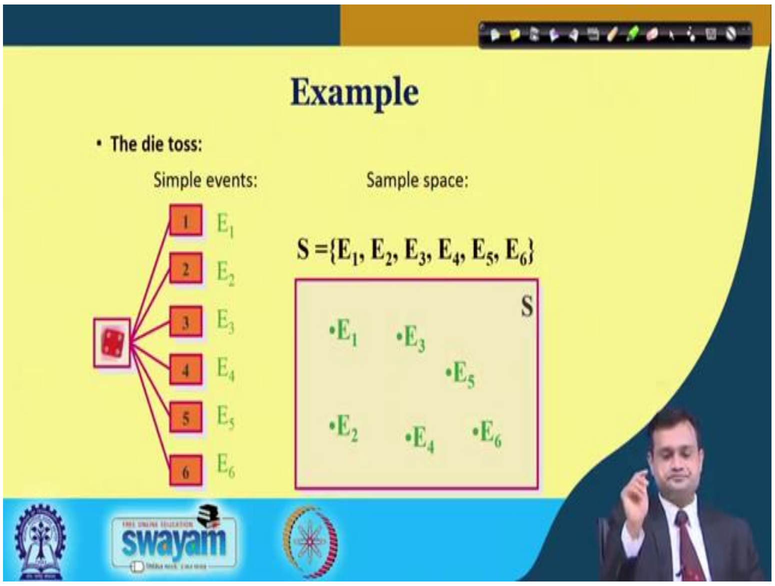

Just see the example that would be make the concept better clear. Suppose I am tossing the die, rolling the die then my simple event I may get say 1 as a number that is E_1 event 1; E_2, 2 ; E_3 number 3 and so on. So, my sample space would include all the possible outcomes E_1, E_2, E_3, E_4, E_5 and E_6. And I am just representing this in the box so this becomes my sample space. So, all the events comprised in a space typical space is my sample space.

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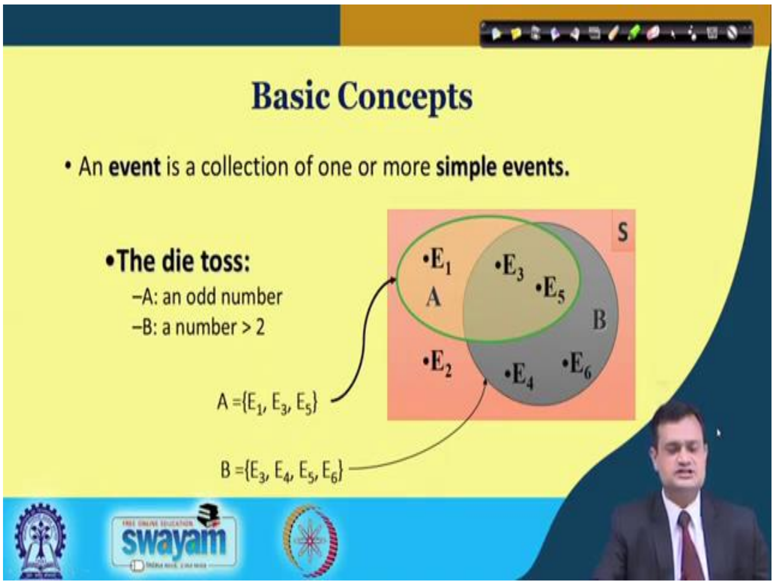

Now, just see the little bit extended version of the example. Suppose I consider event A as odd number and event B a number greater than 2. Now, in your sample space which is typically shown as pink color you have all the possible events E_1, E_2, E_3, E_4, E_5, E_6, but when I put a boundary with a condition that A would have only odd number than E_1 , E_3 and E_5. So, my subset of the sample space A would be E_1, E_3 , E_5. Similar way B would comprise all the number greater than 2, so it would be E_3, E_4, E_5, and E_6.

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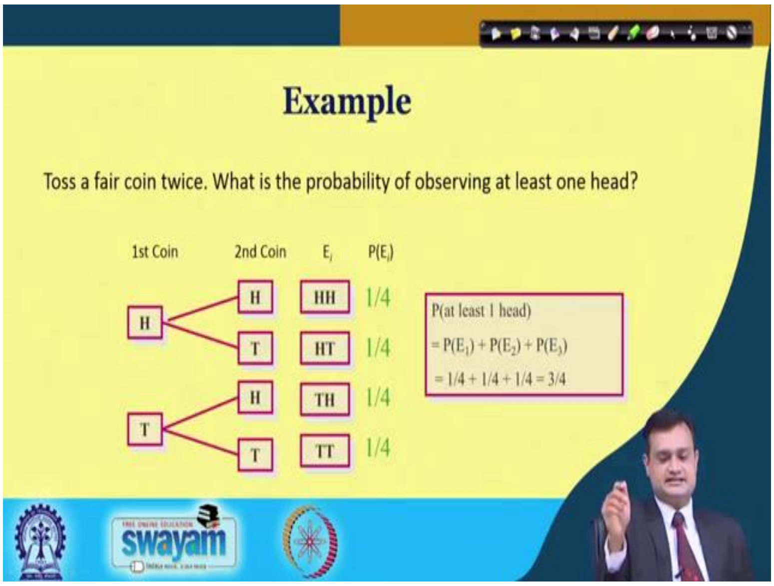

Now, I will just explain the other example that is tossing a coin I may get head or tail. So, if you toss the coin, first coin then I will get head or tail. So, if there is a head and if I coin second time then I will get either head or tail. Similar way first time if I got tail and I coin it second time I may get head or tail. So, the probability I would be interested to find that how many times or two times I will get head 1 time head 1 time tail, 1 time tail 1 time head and 2 times tail.

So, you can easily find it that probability of at least one head is $P(E_1) + P(E_2) + P(E_3)$ and each individual will have equal probability 1/4, 1/4, 1/4, 1/4 and when I sum it up this $P(E_1)$, $P(E_2)$, $P(E_3)$ I cannot consider the 4th one that is TT because there is no head, so my probability of getting at least one head would be 3/4.

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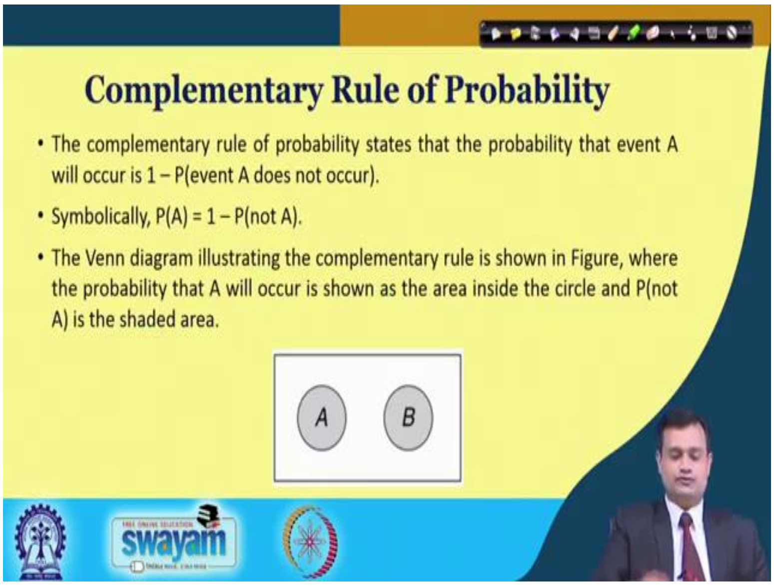

Now, there is a complementary rule of probability which is very important and this says that the probability that event A will occur is 1 minus probability that event A does not occur. So, simply if I ask what is the probability that today it will rain, you will say if I know the probability that today it will not rain then total probability of any event cannot be more than 1. So, 1 minus the probability of not happening, the event will be your probability of happening the event.

So, typically it is written as P(A) = 1 – P(not A) and if you see the Venn diagram then complementary rule is shown in this figure where the probability that A will occur is shown as the area inside the circle and probability not A is shaded area. So, this can help us to appreciate the complementary rule.

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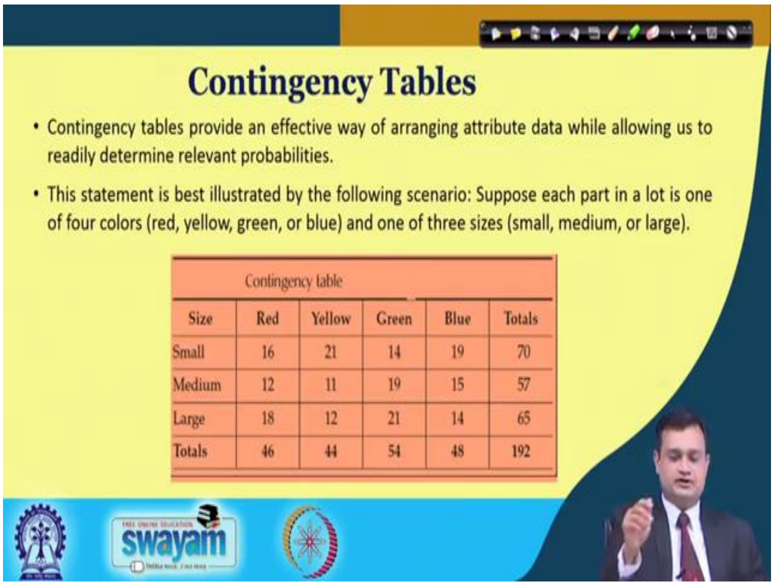

Now, there is another important representation of the data in the form of contingency table, and contingency table will basically represent the number or the frequency of some phenomena occurring.

So, for example, here I have a table and I say that I am trying to create the different scenario and suppose I have a lot which contains the 4 color and it is a 3 size. So, you may consider that it is a lot of ball and there are 3 different colored sizes of ball and 4 different colors. I have small, medium and large, red, yellow, green, blue and I can find the total in the last column as well as in the last row.

Now, once you have represented the data in the form of contingency you can very well compute the probabilities of various events.

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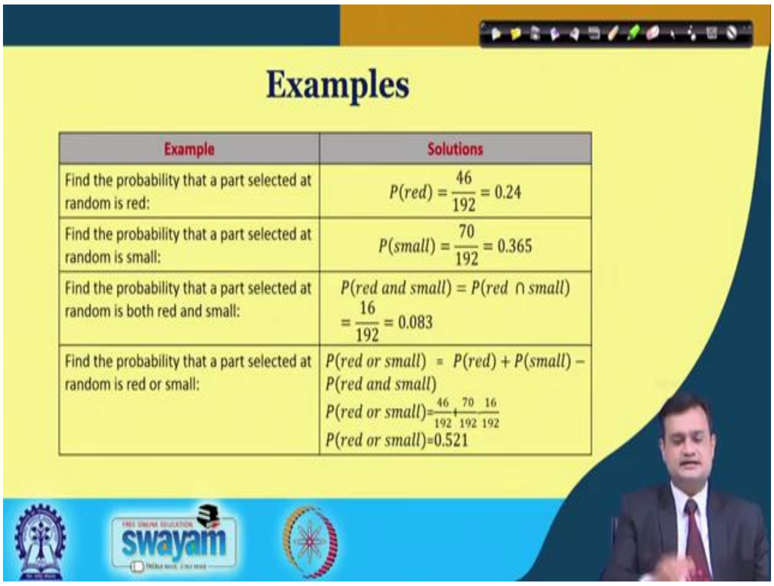

So, let us see that suppose I am interested to find the probability of red. It means find the probability that a part selected at random is red, everything is mixed. So, probability of red is equal to I can go back you just see this particular number that is 46. So, this my total red. So, I want to find what is the probability that red can occur. So, this is 46 and if you see the computation I have 46/192, so 192 is the total number. So, what is the probability that I will get a red is 46/192.

Similar way what is the probability that a part selected at random is small respective of the color. So, I will say small is basically 70 here and my total is 192. So, I can take the ratio of this two and it would be 0.365. You can further analyze that find the probability that part selected at random is both red and small, fine, this is my interest particular phenomena of interest and I would say that probability red intersection small means both should occur red and small, so 16/192 and same way you can find the probability that part selected at random may be red or may be small. So, you will say probability of red or small basically expressed as $P(red) + P(small) - P(red\ and\ small)$. So, you can just say plug in the values and find the probability of red or small.

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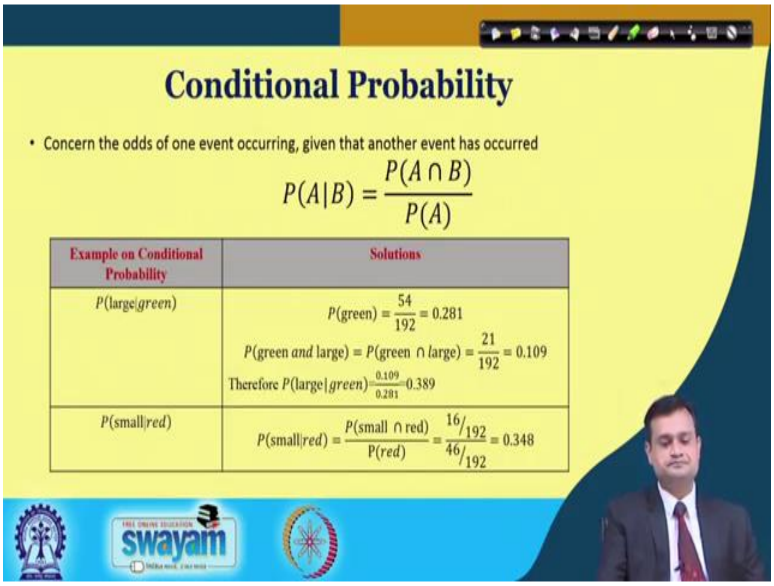

Now, many a times say events are not occurring independently and we may have some conditions. So, under this conditions when we are analyzing and we want to find the probability we basically take the help of conditional probability rules.

So, let us say I say that probability that event A will occur given that event B will occur, so this can be expressed as $\frac{P(A \cap B)}{P(A)}$. So, I can just continue with my example of say 3 different colored balls and 4 may be 3 different sizes and 4 different colored balls. So, probability that I get a larger ball given that it is green, then I can plug in the values in this particular expression. So, I know what is the probability of green. Total green is 54, total number of say balls or parts are 192. So, this probability of green is 0.281.

I can find probability of green and say large. So, probability green intersection large that would be 21/192, so it would be 0.109 when I put it in the expression so my numerator will be 0.109, denominator is 0.281, I will get 0.389. Similar way you can find the probability of small given red and you will get the conditional probability.

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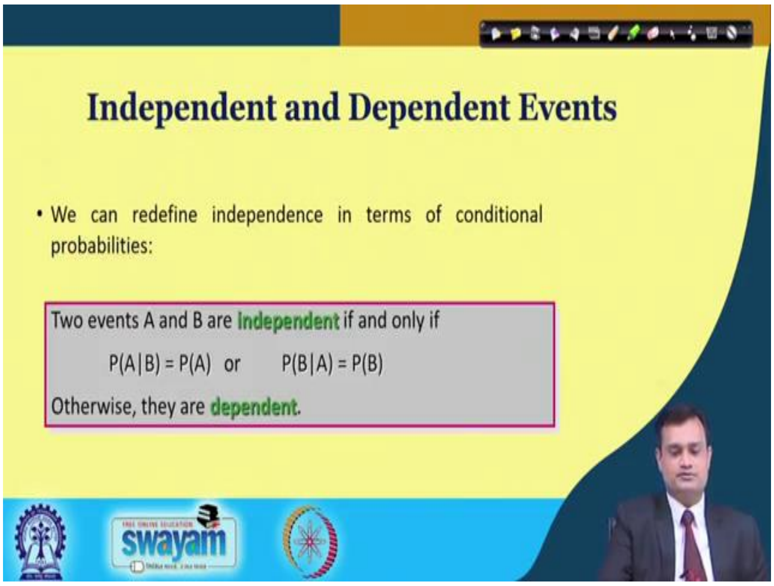

Now, in probability you cannot always have dependent event, independent event both the possibilities exist. So, how to express it? So, suppose event A and B are independent and if I have to express it then I would say they are only independent if the conditional probability, $P(A|B) = P(A)$, means there is no effect of B when I am finding the probability of A, so this is my say case of independent events. Similar way you can say $P(B|A) = P(B)$, otherwise if this is not true, they are dependent.

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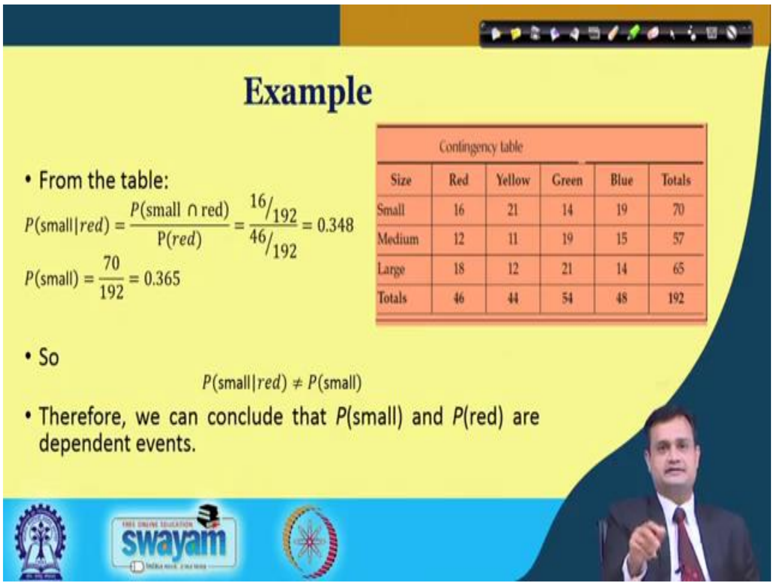

We can just see the example and that would be say interesting to clarify our understanding and strengthen it. So, let us say probability of small ball as I am considering a component or item as a ball given it is red I can find and the probability is 0.348. Suppose, I find probability of simply small I am not considering any given red as I did in the first part, so it would be 0.365.

Now, what is the conclusion that $P(\text{small}|red)$ is not equal to probability of small and we can say that they are dependent events. So, we can conclude that probability of small and probability of red are dependent events and we cannot just consider them independent of each other.

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Now, another dimension to express my event is to exclusiveness. So, you can have mutually exclusive event. You can see it very well in the first particular say figure that mutually exclusive event there is no common area and they are exclusive if both events cannot occur at the same time. So, P(A or B)=P(A)+A(B), and if you see the second one then in this case these are non-mutually exclusive event. So, P(A or B) is given by P(A)+A(B)-P(A and B).

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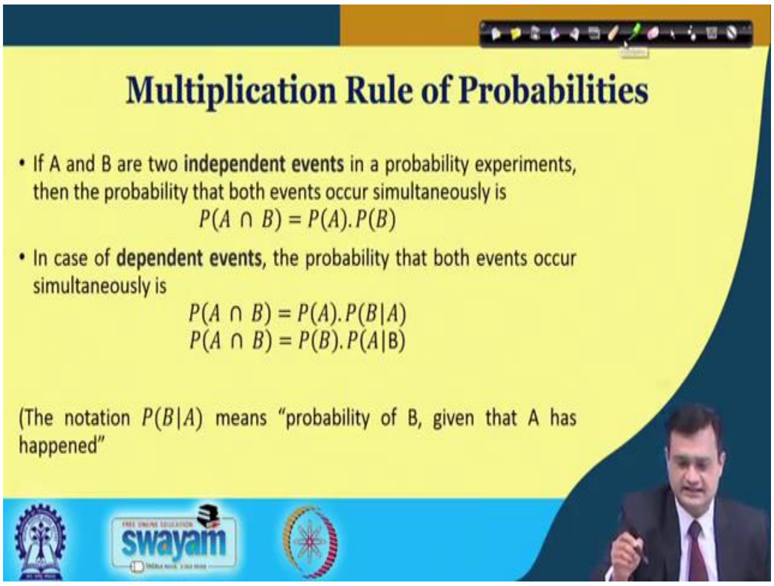

Now, there is a multiplication rule of probabilities and if I say probability of two independent events it means probability if I want to find A intersection B then it would be $P(A).P(B)$ and here my consideration is that two events are independent of each other. When I say dependent event then the probability it depends on occurrence of both the events and $P\big(A \cap B\big) = P(A).P(B|A)$, and same way it can be expressed as $P(B).P(A|\mathrm{B})$. So, B by A or B given A means probability of B given that A has happened because they are dependent events.

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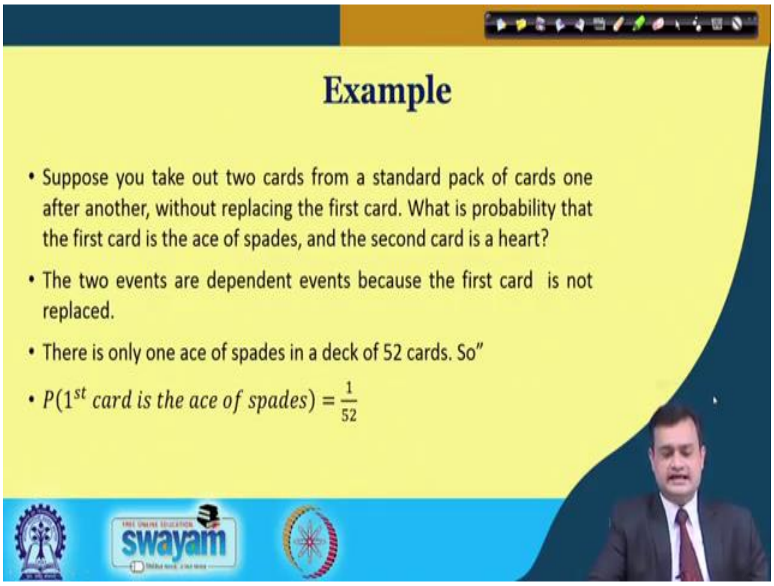

We can see the example to appreciate the concept better. Suppose you take out two cards from a standard pack of cards one after another without replacing first card means I am not putting it back. So, what is the probability that the first card is ace of spades and the second card is heart?

So, your two events are dependent events because the first card is not replaced, please remember. So, there is only one ace of spades in a deck of 52 cards, so probability of first card is ace of spades is equal to 1/52.

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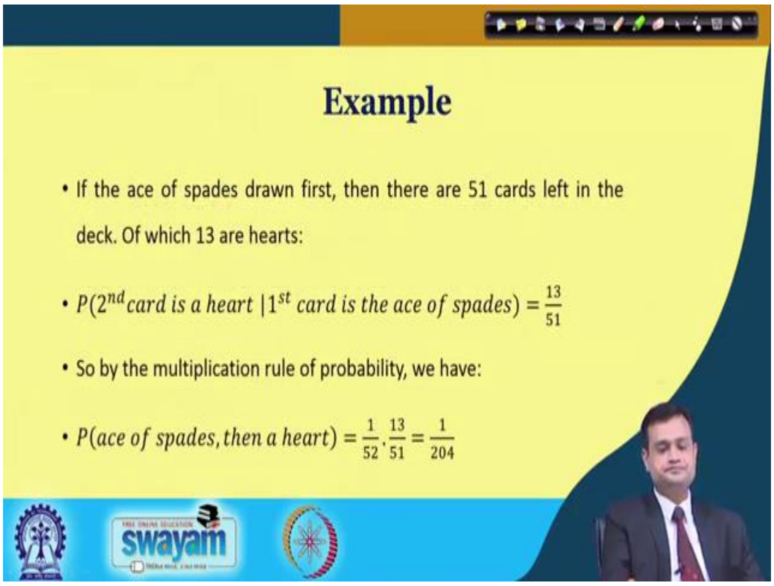

Now, if I say if the ace of spades drawn first then there are 51 cards left because I am not putting it back. Now, of which 13 are hearts, so probability that second card is a heart given that first card ace of spades 13.51, and I can say that the multiplication rule probability of ace of spade then heart is 1/52 into 13/51, so it would be 1/204.

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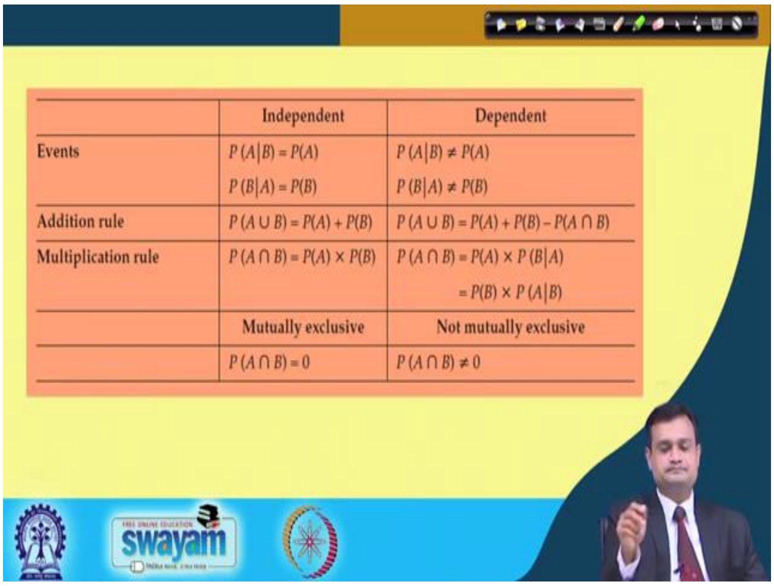

So, I would just like to present some summary of the various rules. So, if you have independent event and if you have dependent event and suppose you are talking about the event $P(A|\mathrm{B}) = P(A)$, $P(\mathrm{B}|A)$ when it is independent it is $P(B)$ but it is just reverse not equal to when it is dependent.

Addition rule, $P(A \cup B) = P(A) + P(B)$ when you look at the dependent event you have to subtract the $P(A \cap B)$ and there are multiplication rules for independent event as well as dependent event we have seen. Mutually exclusive event $P(A \cap B) = 0$ there is no commonality. $P(A \cap B) \neq 0$ they are not mutually exclusive.

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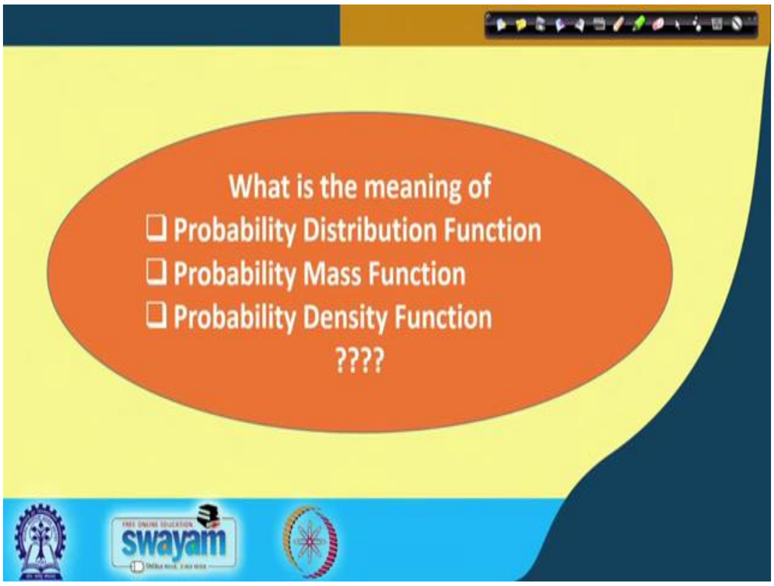

Now, let us try to appreciate the couple of things. What is the meaning of probability distribution function? Probability mass function? Probability density function? And then subsequently we can have a quick overview of couple of distributions.

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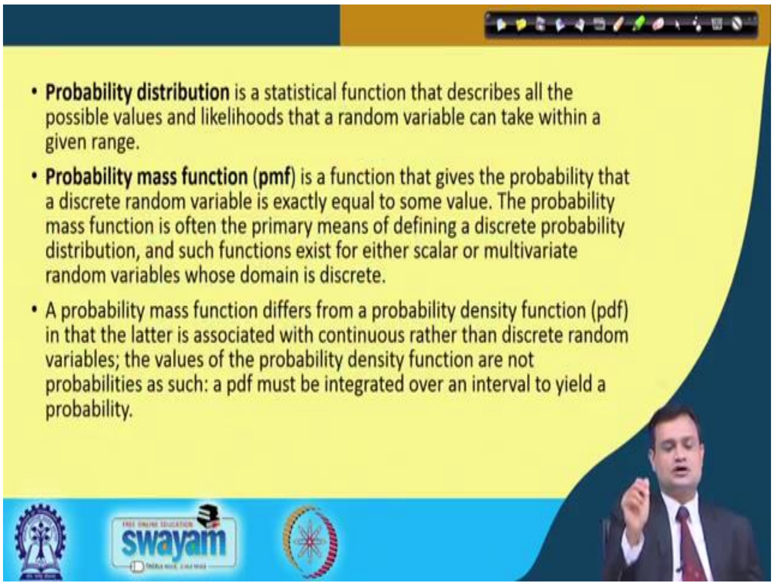

So, before that let us try to appreciate these 3 terms. So, probability distribution typically it is a statistical function that describes all the possible events and likelihood that a random variable can take within a given range. For example, suppose you want to see that what is the distribution of waiting time at particular bank ATM? For a specific period of time you collect the data, plot the data with respect to time and then you will get an idea that how my data is distributed; in what way typically entire range of data is represented so, we have already seen that you can also get an idea when you put a histogram and skewness, kurtosis, those things can also help you. So, typically it is a mathematical function, statistical function that describes how a particular phenomena is occurring.

We have probability mass function, this gives a probability that a discrete. So, you have two types of variables one is discrete, other is continuous. If I have discrete random variable then it gives the probability that a discrete random variable is exactly equal to some value, and the probability mass function is often the primary means of defining the discrete probability distribution.

So, such function exist either scalar or multivariate random variable whose domain is discrete and typically probability mass function differs from probability density function. When I say a probability density function again it is for a specific value, but density function pertains to the continuous random variable.

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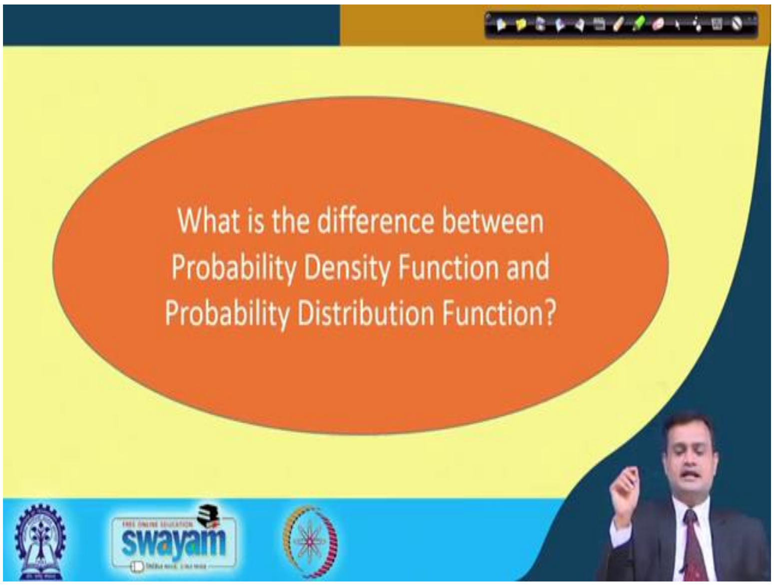

So, typically what is the difference between probability distribution function and probability density function if I talk about a continuous random variable?

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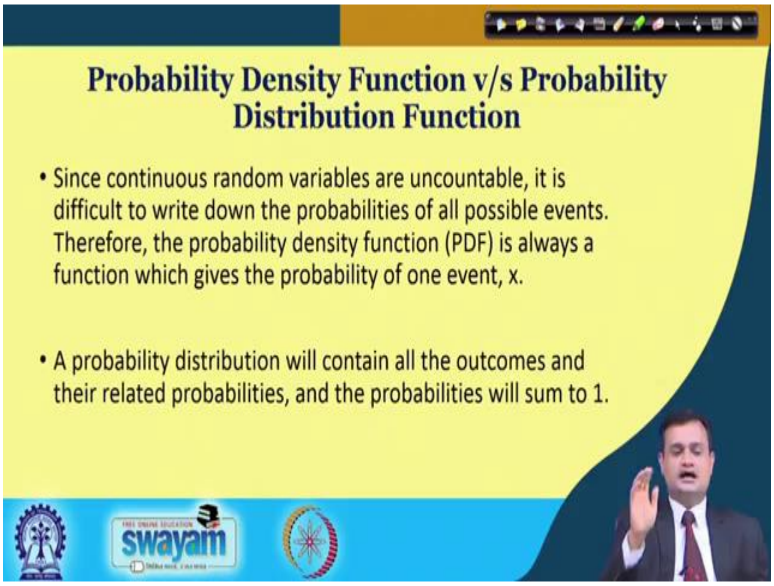

Now, the difference is like this, that since you have the continuous random variable and they are uncountable it is difficult to write down the probabilities of all the possible events and therefore, you take the help of probability density function, typically expressed in the form of mathematical expression and it is a function which gives the probability of a particular event, x. So, a probability distribution function will contain all the outcomes and their related probabilities and the probability will sum up to 2.

There are commonly used distribution I would quickly like to give you the idea. Broadly because we have two types of random variables, continuous and discrete, you have two types of distributions. Distribution again I am saying it is a way mathematical way of describing a particular phenomena which is occurring in a random fashion, and we have a family of distribution both under continuous and discrete. There is a huge say number of distributions available. We cannot discuss all, we are just trying to appreciate some of the widely used distributions here as continuous distribution and discrete.

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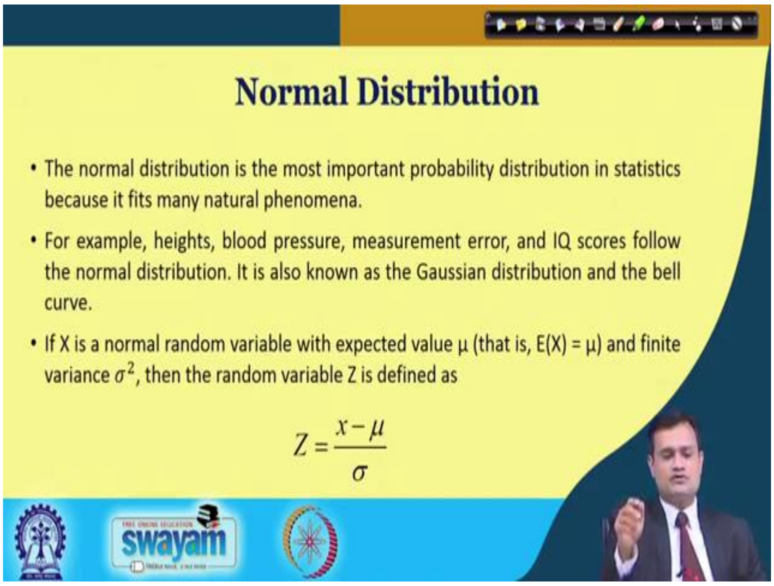

So, you have continuous distribution the most widely used one is the bell shaped distribution which is a normal distribution, and typically a standard normal distribution the critical value Z is expressed as $\frac{x-\mu}{\sigma}$ So, x is the individual value, μ is your population mean, σ is your population standard deviation.

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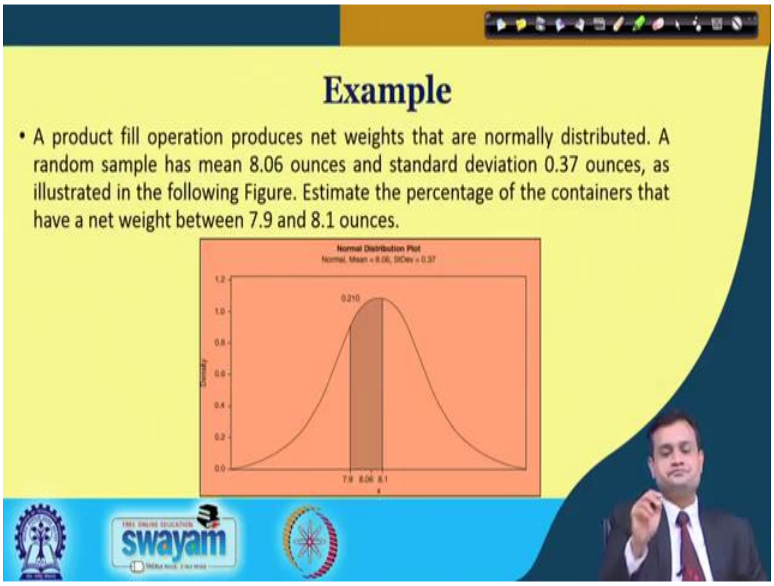

Just see the example to appreciate the normal distribution. You have a product feel operation produces net weight that are normally distributed in a random sample has mean

8.06 ounces and the standard deviation 0.37 ounces as illustrated in the given figure. And this bell shaped curve also shows that your data has some variability.

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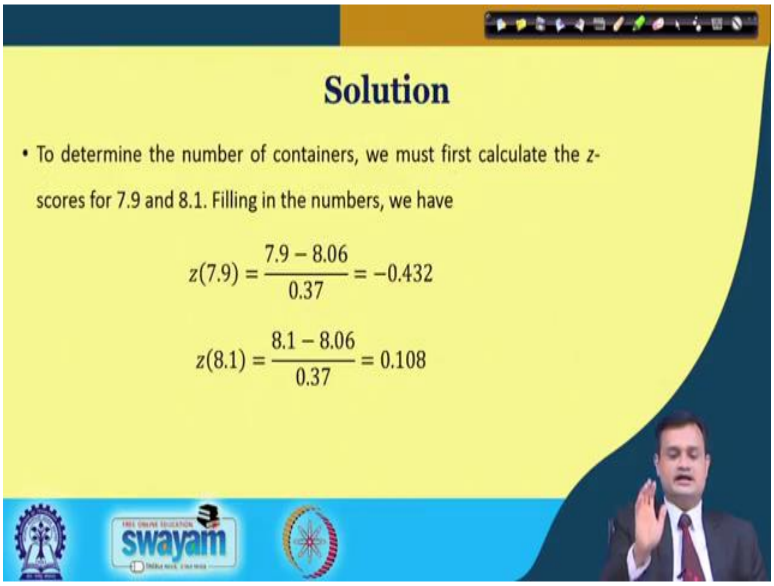

So, now estimate the percentage of containers that have a net weight between 7.9 to 8.1 ounces.

So, I am using the expression of standard normal distribution z (7.9), so $\frac{7.9-8.06}{0.37}$, this will give me the value minus 0.432. Similar way z (8.1), I want to find the score between these two. So, I will get Z value 0.108. We have the tables for all the distributions typically called statistical tables, these tables are derived using the standard mathematical expression of the distribution typically called as probability distribution function. And this table documents the values of various parameters as well as the critical values, and it gives us the critical values as well the probability values for decision making.

So, these tables are available in any suggested book by me and you can easily find the probability values or critical values which can help you to make the inferences or the decisions.

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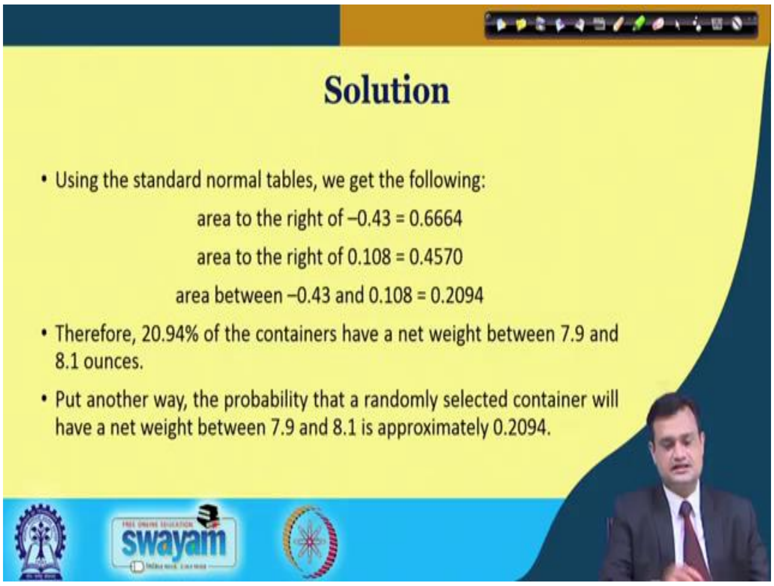

So, now in case of my typical normal distribution I would like to find the area to the, right of minus 0.43 and area to the right of 0.108, and from the table, I get this area 0.6664 with respect to the critical value and 0.4570. So, I will say that 20.94 percent of the containers have a net weight between 7.9 and 8.1 ounces.

So, another way I can say that the probability randomly selected container will have a net weight 7.9, 8.1 is approximately 20.94 percent or 0.2094.

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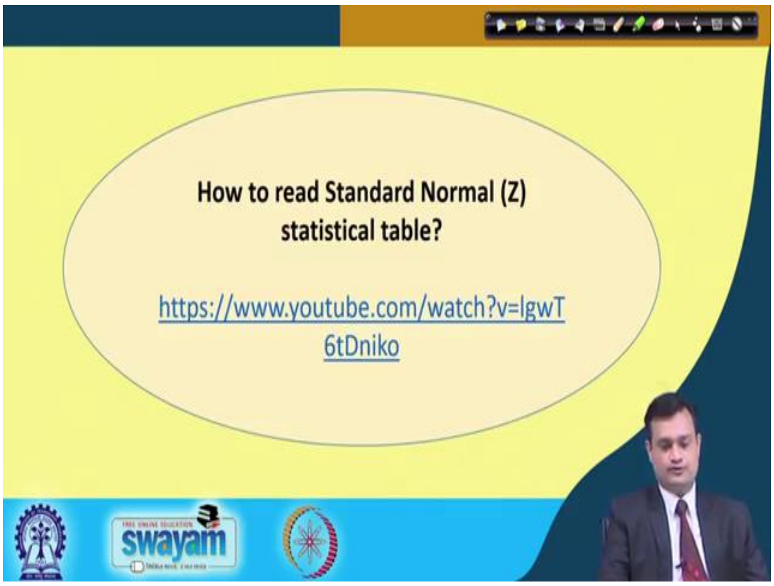

You only need little bit practice to use the statistical table. I am suggesting here a very small video link. You can refer and just try to get acquainted with the use of statistical table.

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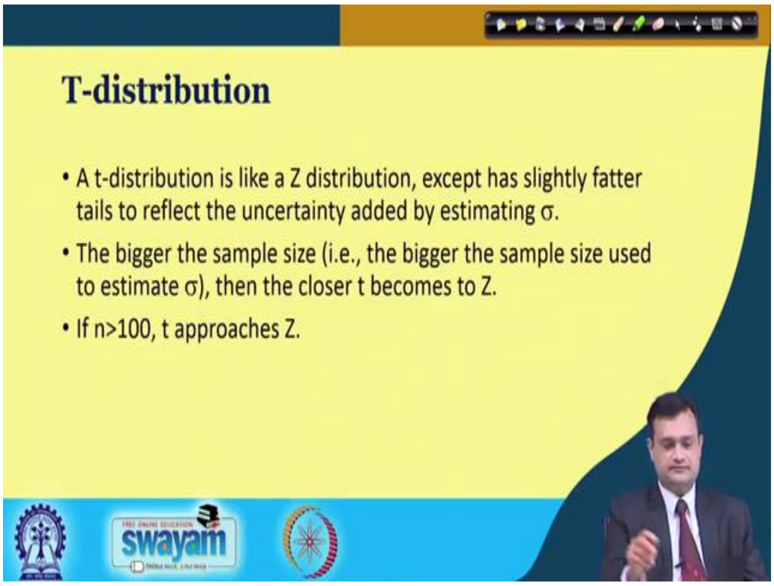

Another important distribution T distribution or also called as the student T distribution, very much similar to Z distribution, but typically when the sample size is small you make use of T distribution when your n sample size is greater than 100 it almost approaches to the normal distribution.

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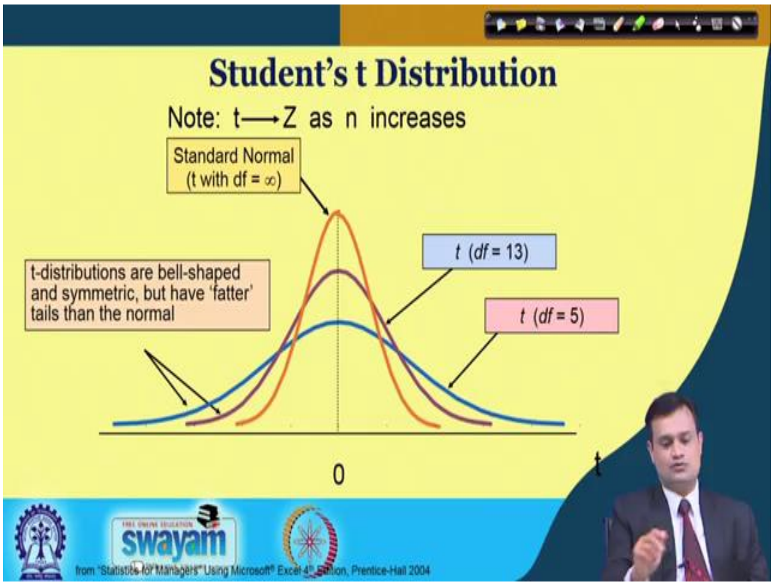

So, you can see it very well that you have t with degree of freedom infinity, t with degree of freedom 13, t with degree of freedom 5. So, when you will look at the t distribution table you need to find the critical value with the help of degree of freedom, and already I explained what is degree of freedom. So, how much opportunity? How many opportunity you have to maneuver in the data set? Typically that is your degree of freedom.

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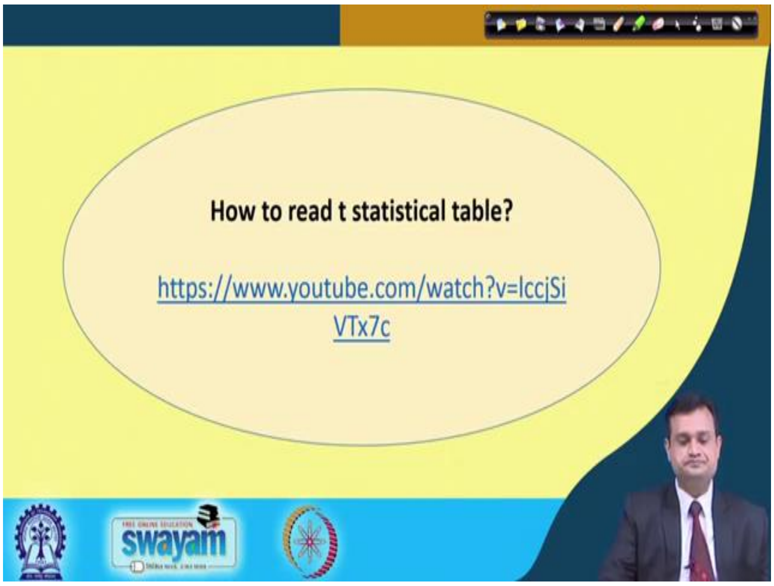

So, again I am suggesting a link to refer the T statistical table and this you can do very easily just by going this particular reference.

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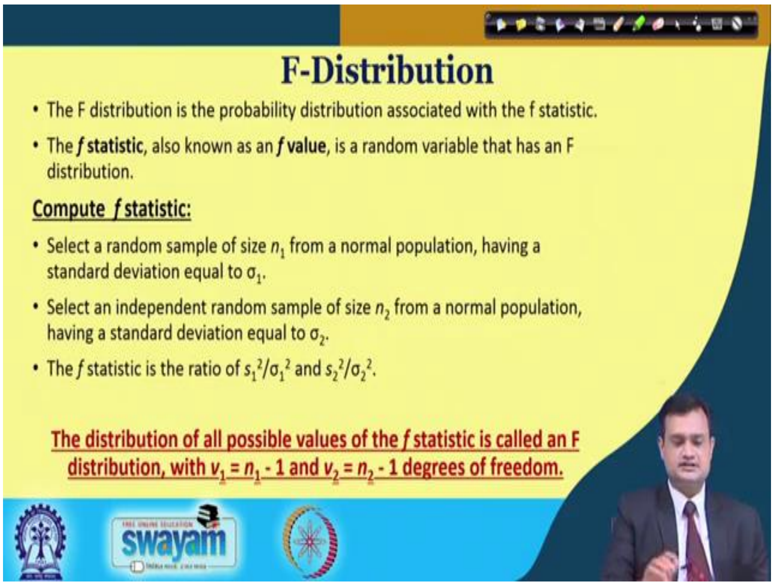

Very important distribution F distribution after your normal and T distribution, and then we will discuss the concept of ANOVA, F distribution would be widely used.

So, typically it is a probability distribution associated with F statistics and F statistics is basically the ratio of your sample variance and the population variance. So, you basically select a random sample of size n_1 from a normal population having a standard deviation σ_1. Select an independent random sample again of n_2 having a population standard deviation σ_2, and F statistic basically is a ratio of s_1^2/σ_1^2 and s_2^2/σ_2^2 and this distribution will have two degrees of freedom ϑ_1 and ϑ_2, which is n_1 - 1 because you are taking two sample $n_2 - 1$.

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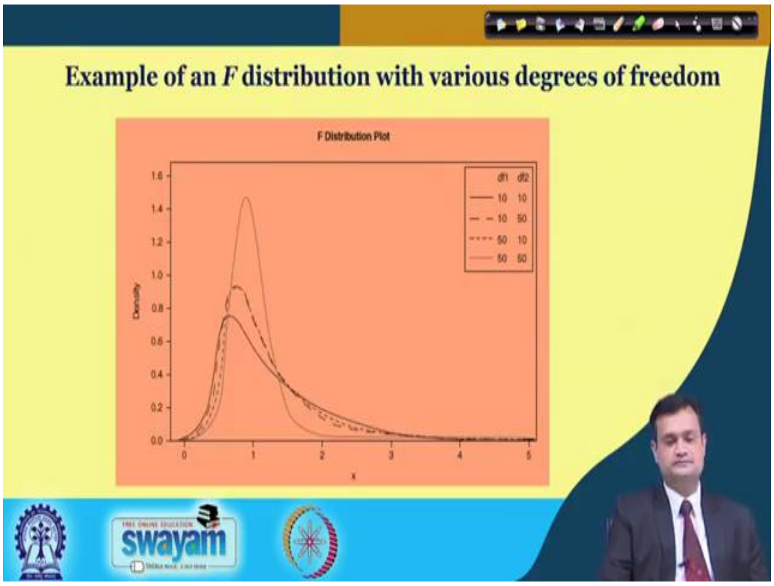

So, you can just appreciate the shape of the F distribution, and it is not like minus infinity to infinity which we have in normal and for different degree of freedom you can see that the shape of F distribution changes.

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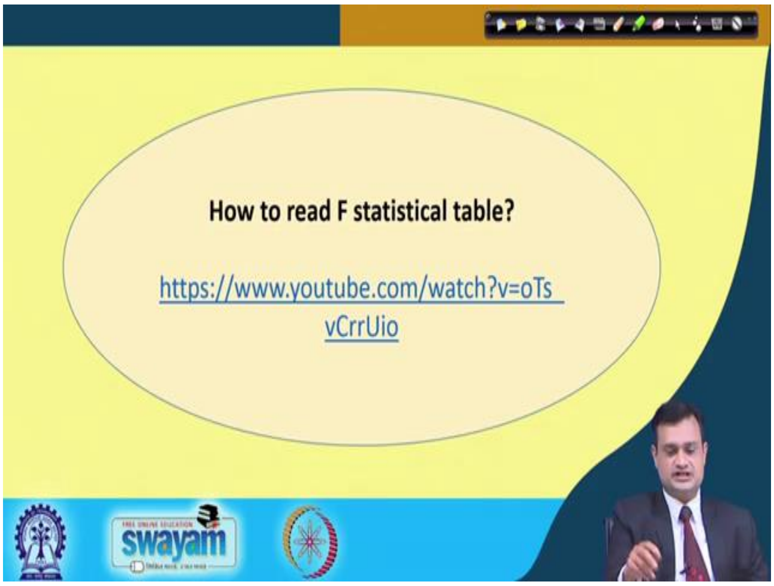

Further, you can just refer how to read the F distribution please open the appendix of the suggested book or download the statistical table and use the guidelines to understand how to read the statistical table.

Now, let us try to see quickly couple of discrete distribution.

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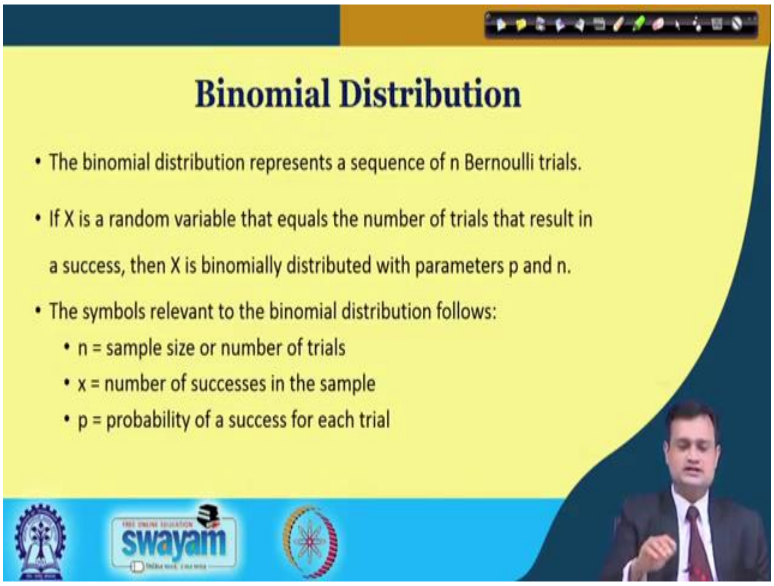

So, in quality domain one distribution which is widely used is the binomial distribution and typically it has two outcome success or failure, head or tail, and when you want to capture this kind of phenomena you try to use binomial distribution. The nomenclature it has n is sample size, x is the number of successes in the sample and p is the probability of a success of each trail.

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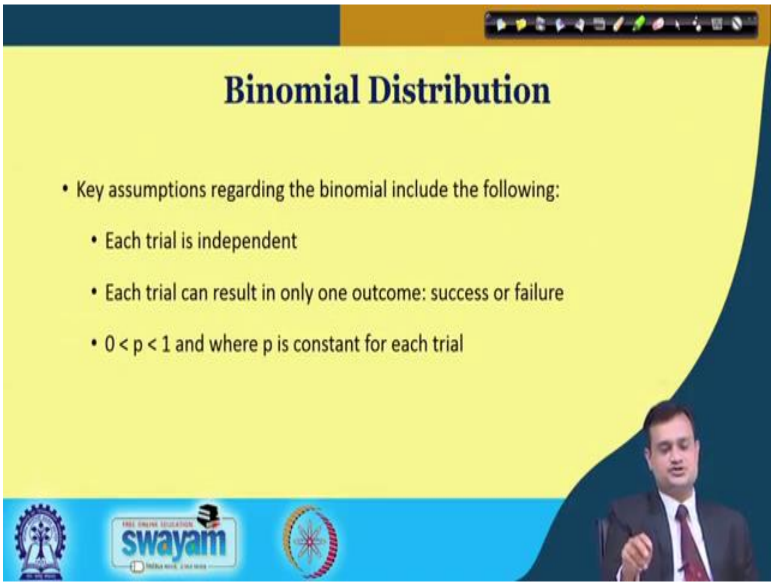

So, there are certain key assumptions in binomial distribution, each trial is independent, each trial can result in only outcome success or failure and obviously, your probability between 0 to 1.

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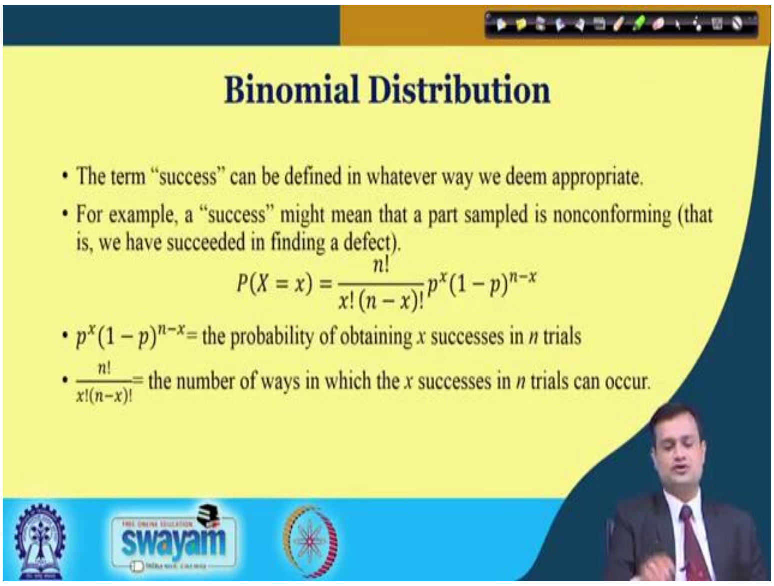

So, typically the mathematical function describes the binomial distribution is $P(X = x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$

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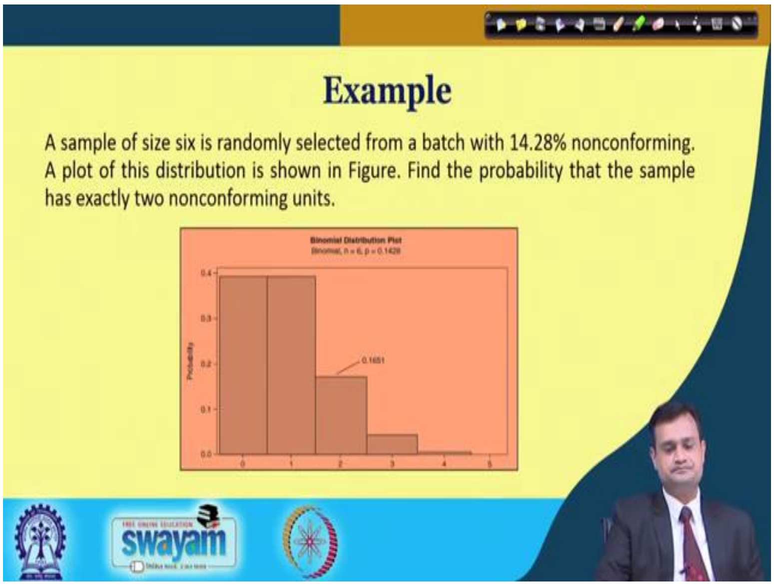

So, you have a sample size 6 is randomly selected from a batch with 14.28 percent nonconforming.

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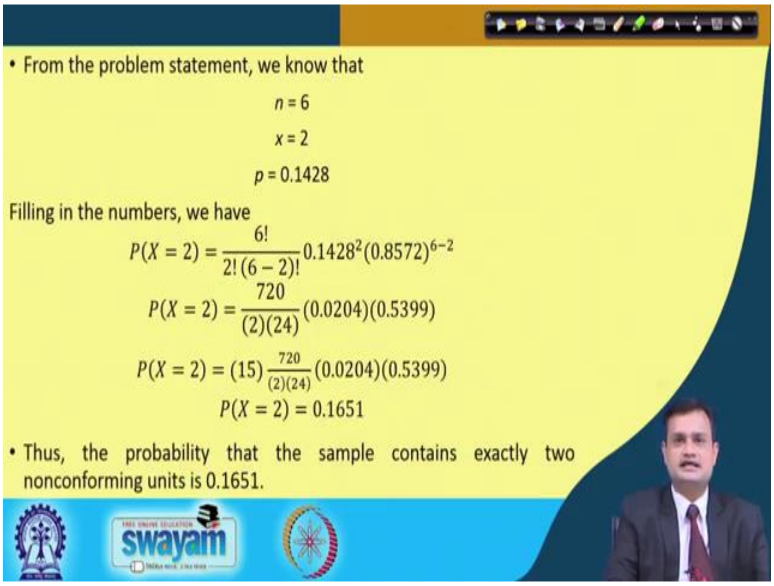

And a plot of this distribution you can see here, and I am just trying to plug in the values in the distribution to give you an idea and what I get as a result for $P(X = 2) = 0.1651$. So, so probability that sample contains exactly two non-conforming units is 16.51.

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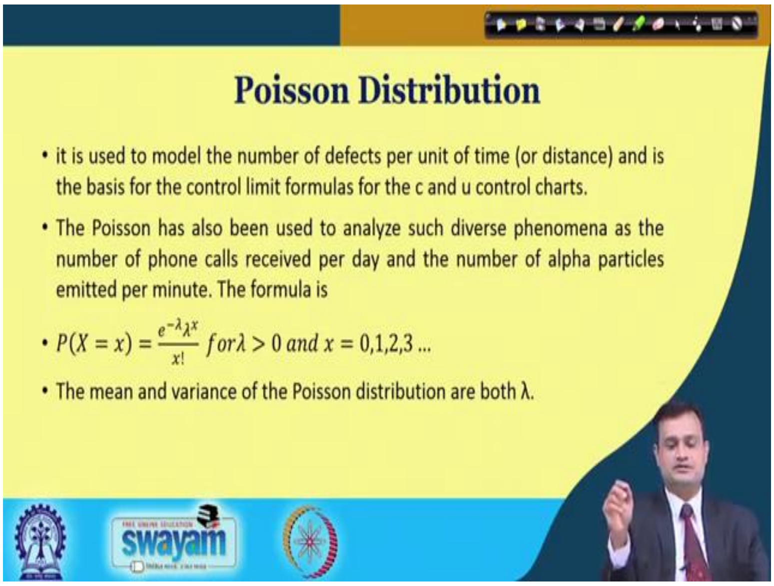

Another distribution in the family of discrete distribution is your Poisson distribution and typically it is used when you have to model the number of defects per unit of time and the basis for the control limit formulas for the c and u control chart which we will discuss later on.

So, Poisson has been used to analyze such diverse phenomena as the number of phone calls received per day and the number of may be alpha particles emitted per minute or sometimes let us say very simple day to day example that what is the occurrence of mutter in your vegetable pulao. So, then also it follows the Poisson distribution. So, $P(X = x)$ mathematical function which describes the Poisson distribution is $\frac{e^{-\lambda}\lambda^x}{x!}$.

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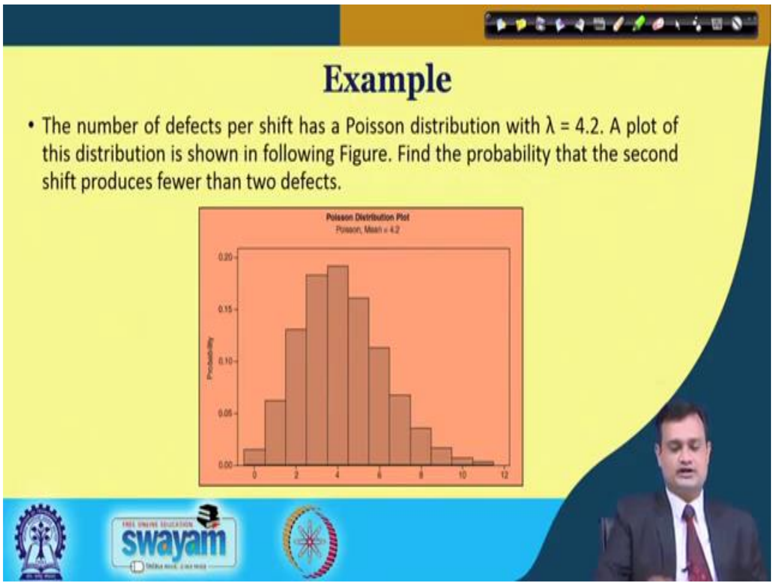

So, we can see the example here lambda is given 4.2, it is a typical histogram depicting.

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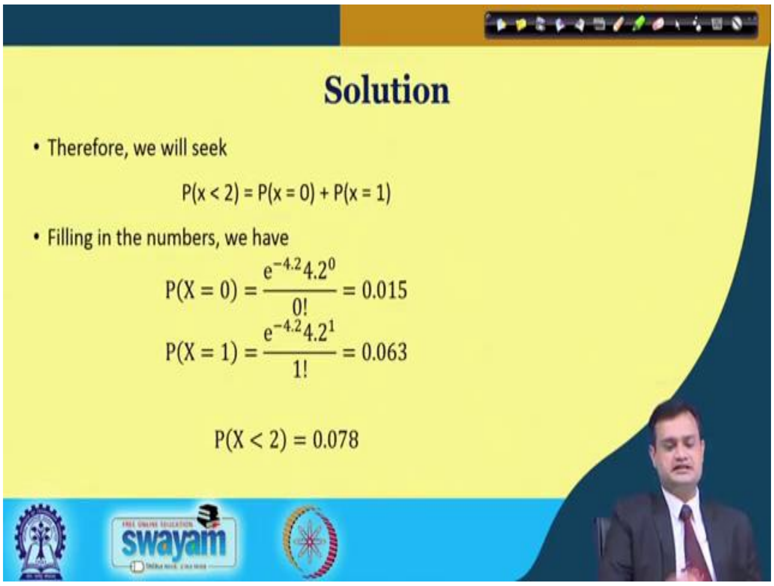

The Poisson distribution. I can just plug in the values to find the $P(X < 2)$ defective and I find that it comes out to be; so less than 2 means it could be 0 it could be 1, so when I sum it up 0.015 and 0.063, it is 0.078

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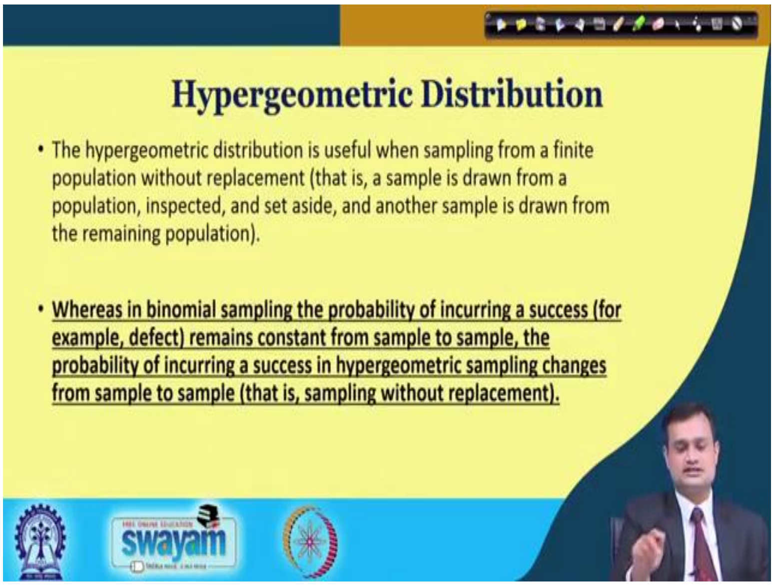

Hypergeometric distribution, typically is another additional important distribution in the domain of quality and in binomial sampling the probability of incurring a success. For example, defect remains constant from sample to sample. The probability of incurring a success in Hypergeometric changes from sample to sample because this distribution is without replacement. So, I am taking a sample from a lot, I will not replace it and then once again I am taking the sample and checking. So, because of this without replacement phenomena here your probability of getting defective changes every time.

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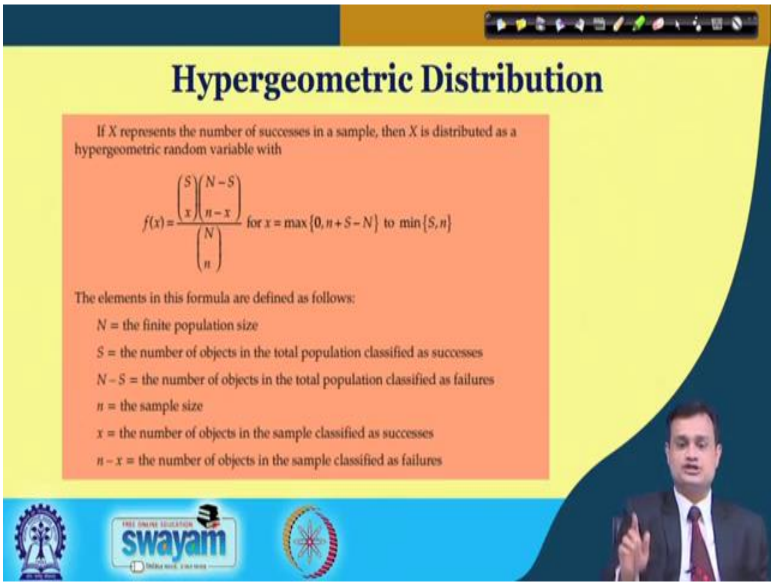

So, you have the mathematical function to describe $f(x) = \dfrac{\binom{s}{x}\binom{N-S}{n-x}}{\binom{N}{n}}$ and you can go through n capital N is lot size, capital S is number of objects in the total population, N minus S is the number of objects in the total population classified as failures, small n is sample size and so on.

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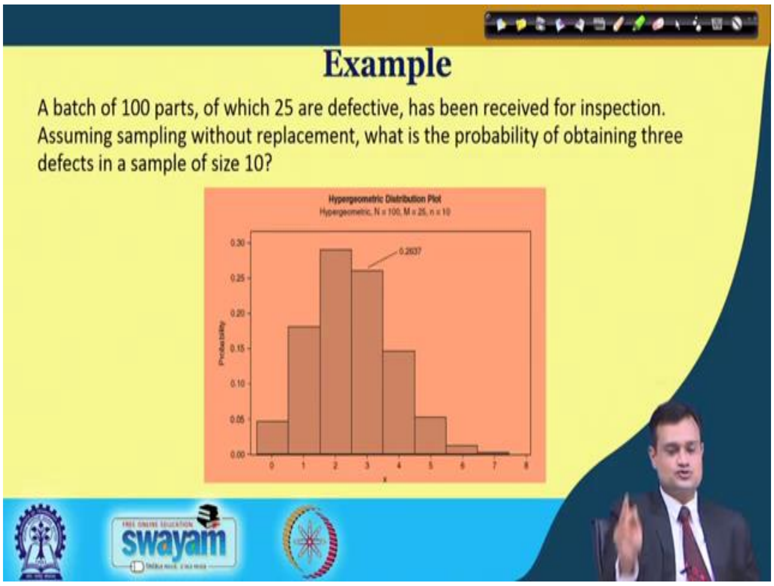

So, we can take the example in which we have 25 defectives and I want to find the probability of 3 defect in a sample of size 10.

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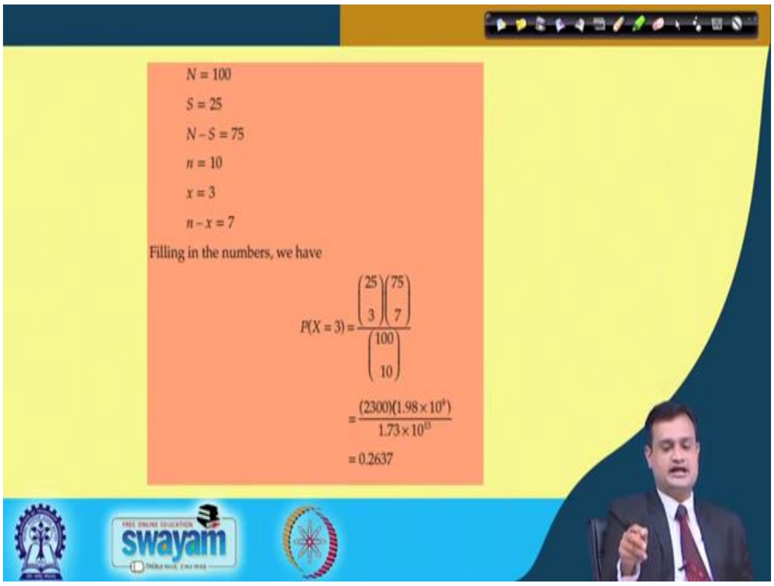

So, you can just plug in the values in the expression, capital N is 100, s is 25, N minus S is 75, n is 10, x is 3, n minus x is 7,. So, P (X) is equal to 3 is 0.2637.

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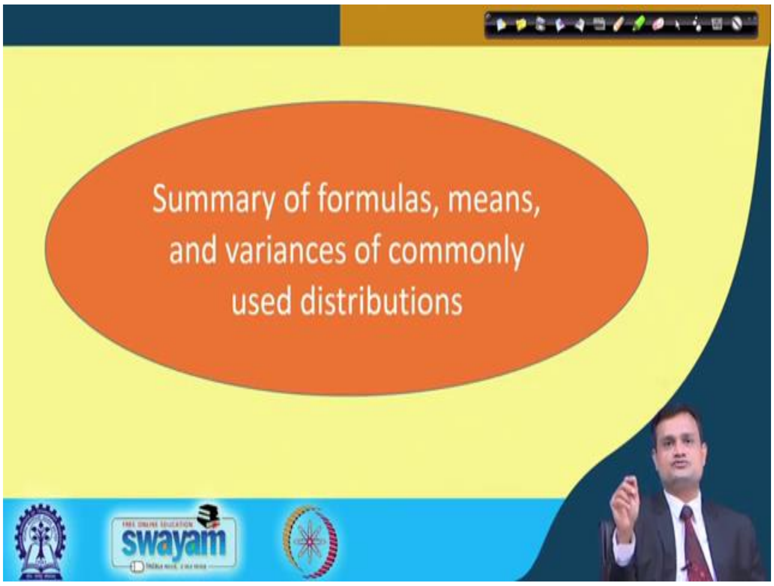

So, summary of formula, mean and variance you can see here.

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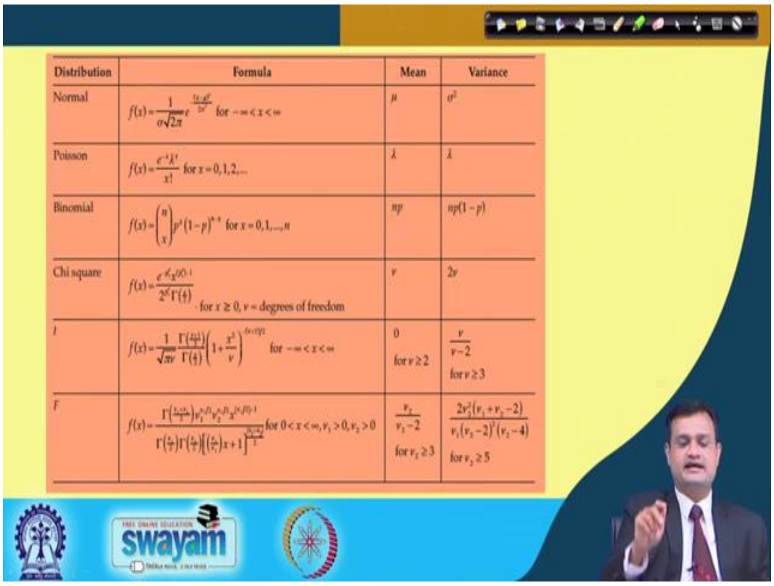

Distribution	Formula	Mean	Variance
Normal	$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ for $-\infty<x<\infty$	μ	σ^2
Poisson	$f(x)=\frac{e^{-\lambda}\lambda^x}{x!}$ for $x=0,1,2,\ldots$	λ	λ
Binomial	$f(x)=\binom{n}{x}p^x(1-p)^{n-x}$ for $x=0,1,\ldots,n$	np	$np(1-p)$
Chi square	$f(x)=\frac{e^{-x/2}x^{(\nu/2)-1}}{2^{\nu/2}\Gamma(\frac{\nu}{2})}$ for $x\geq 0$, ν = degrees of freedom	ν	2ν
t	$f(x)=\frac{1}{\sqrt{\pi\nu}}\frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})}\left(1+\frac{x^2}{\nu}\right)^{-(\nu+1)/2}$ for $-\infty<x<\infty$	0 for $\nu\geq 2$	$\frac{\nu}{\nu-2}$ for $\nu\geq 3$
F	$f(x)=\frac{\Gamma(\frac{\nu_1+\nu_2}{2})\nu_1^{\nu_1/2}\nu_2^{\nu_2/2}x^{(\nu_1/2)-1}}{\Gamma(\frac{\nu_1}{2})\Gamma(\frac{\nu_2}{2})\left[(\frac{\nu_1}{\nu_2})x+1\right]^{\frac{\nu_1+\nu_2}{2}}}$ for $0<x<\infty, \nu_1>0, \nu_2>0$	$\frac{\nu_2}{\nu_2-2}$ for $\nu_2\geq 3$	$\frac{2\nu_2^2(\nu_1+\nu_2-2)}{\nu_1(\nu_2-2)^2(\nu_2-4)}$ for $\nu_2\geq 5$

I have put all normal, Poisson, binomial, chi-square, T and F, and their mathematical expressions typically describes the nature of a particular distribution mean and variance that are the parameter of interest and their particular expressions.

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Some other distributions we are not discussing, but they are also used in many other domain. We have focused on couple of distributions which are very important from six sigma quality point of view, but other distributions may be chi-square, exponential, uniform, lognormal and Weibull distribution.

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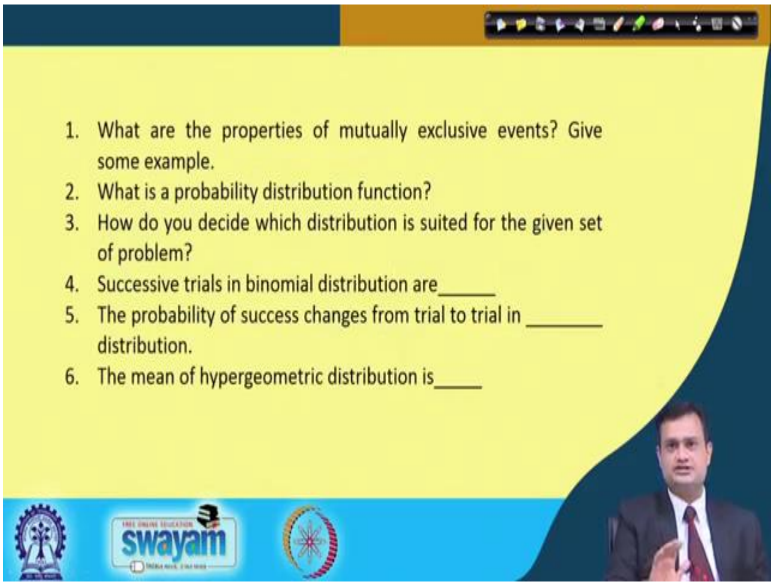

Think it. What are the properties of mutually exclusive events? Give some example from your day to day life. What is the probability distribution function? We have seen PMF probability mass function, probability density function, and there is another term which is called probability distribution function. How do you decide which distribution is most suitable for a given set of problem? Successive trails in binomial distributions are, fill in the blank. The probability of success changes from trial to trial, fill in the blank just now we discussed. And the mean of Hypergeometric distribution, fill in the blank.

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So, with this you refer this couple of references to deepen your understanding.

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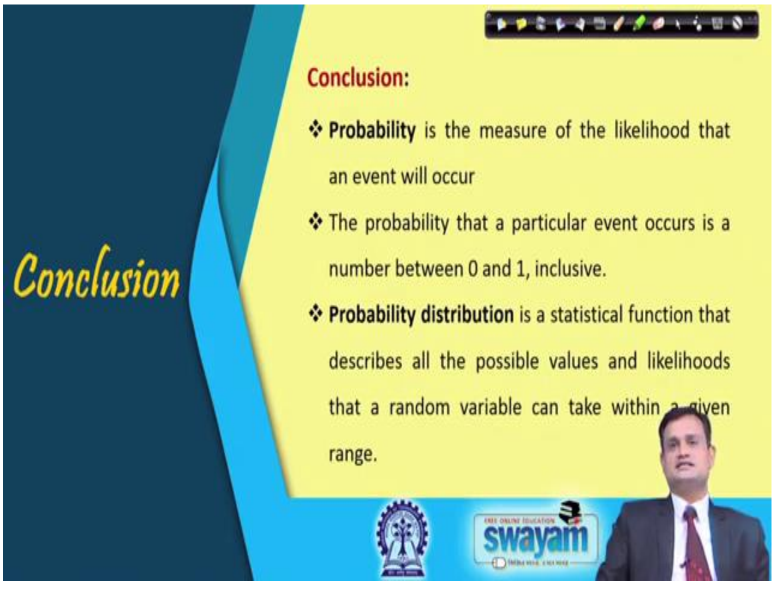

And we can conclude that probability is the measure of the likelihood that an event will occur. So, its chance and the probability is always between 0 to 1, and probability distribution is a statistical mathematical function that describes all possible values and likelihoods that a random variable can take within a given range.

So, thank you very much. Keep revising. Enjoy.