Six Sigma
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Lecture – 21
Measurement Systems: Gage R & R Study

Hello friends, I welcome you to the journey of Six Sigma and typically we are in the measure phase of Six Sigma and discussing various topics. So, today as a part of lecture 21, we will continue our discussion on Measurement System, with a special focus on Gage R & R Study, we have seen some of the concepts and importance of measurement system in the previous lecture. And we will strengthen our understanding on one of the very important aspect that is gage R & R study in this lecture.

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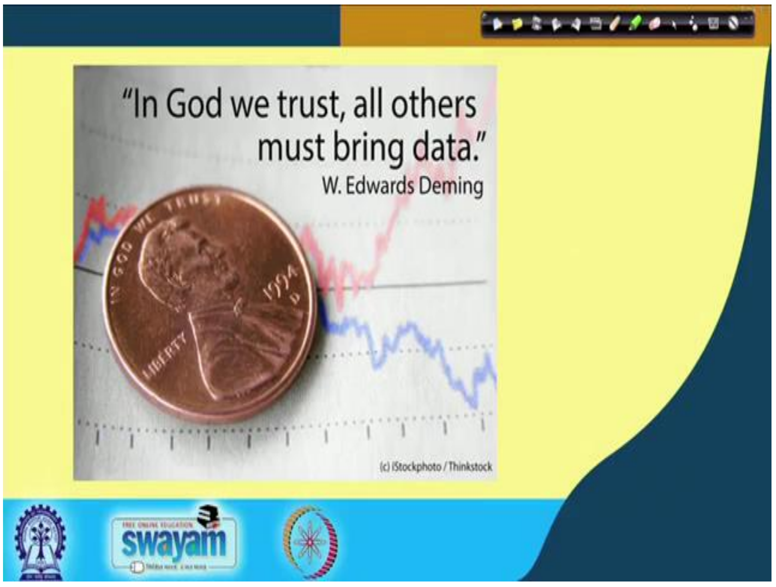

So, before we begin let us see a beautiful quote given by W Edward Deming. "In god we trust all other must bring data". So, we can have our intuitions guts and feeling, but this will not work every time and six sigma approach typically advocates the use of scientific, inquiry and use of data. I will go one step ahead and say that not only the quality not only the data, but quality of data is very important.

So, there is a principal called GIGO garbage in garbage out and if your data quality is poor, then whatever analysis you will conduct that will draw the faulty conclusions and misguide you in the in taking the corrective actions.

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So, we need data and we need quality data and hence the importance of measurement system is crucial, we have seen couple of issues like importance of measurement characteristics of measurement system, gage R & R and its importance in brief in the last lecture.

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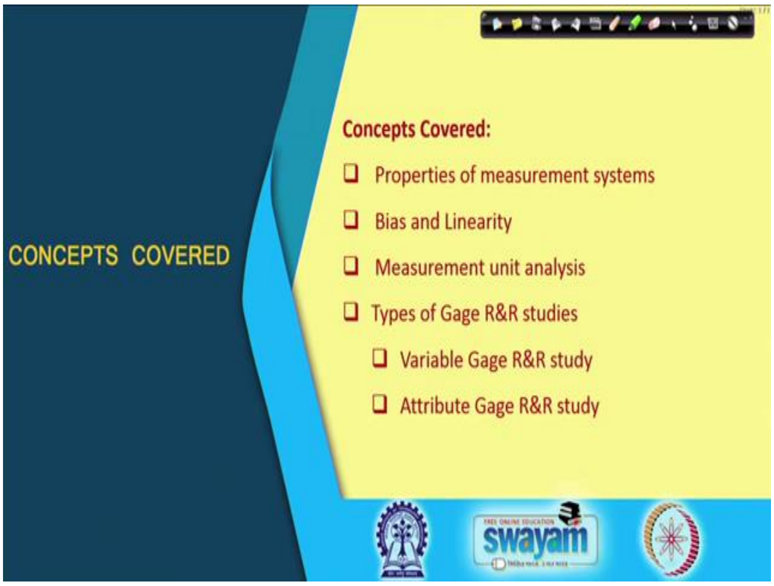

This lecture 21, we will focus on some of the important properties of measurement system, bias and linearity, measurement unit analysis, types of gage R & R typically variable gage R & R study, attribute gage R & R study and so on.

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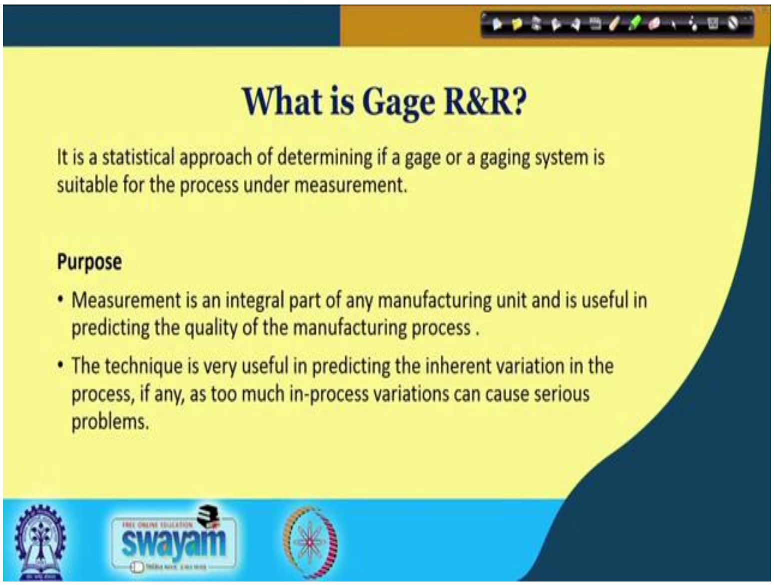

So, let us see what is gage R & R? So, typically it is a statistical approach of determining, if a gage or gaging system is suitable for the process under measurement. So, if you recall I mentioned that typically when I am concerned about precision of my gage, I focus on gage R & R study and whether it is really useful for the measurement or not that I need to investigate.

So, there are two broad purpose is one is measurement is an integral part of any manufacturing unit and is useful in predicting the quality of the manufacturing process, to what extent my manufacturing process is performing well and this investigation is possible through a proper measurement. Second is the technique is useful in predicting the inherent variation in the process if any as too much in process variation can cause serious problems.

So, there are two things one is the process specific variations, other is instrument or gage specific variation and when I am trying to judge the inherent variations in the process, then I must have a very precise gage or measuring instrument so, that my quality of measurement is not affected.

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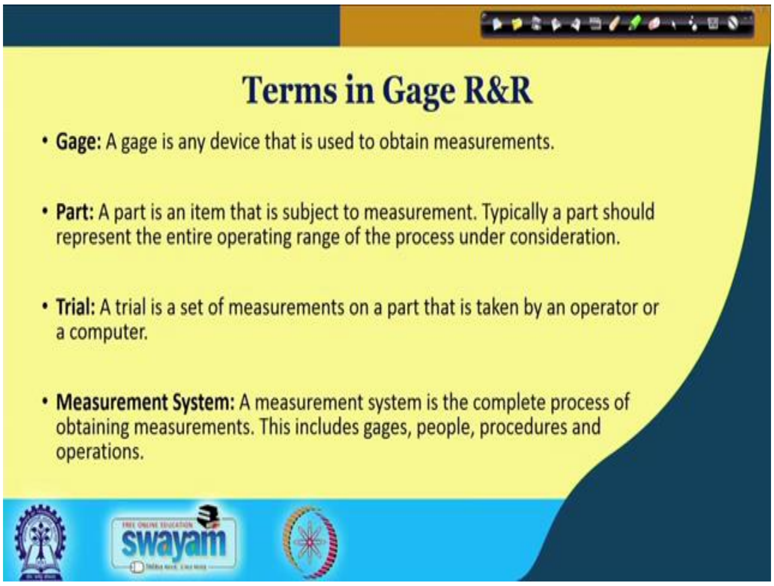

Now, there are some important terms in gage R & R. Gage is any device that is used to obtain the measurement. Part; a part is an item that is subject to measurement you want to measure it and typically a part should represent the entire operating range of the process under consideration.

So, when you want to conduct gage R & R study you must see that you should have good as well as bad, quality parts or products in the measurement system and then you will really get an idea when you see it over a range that whether your gage or instrument is performing well or not. Trial; a trail is a set of measurements on a part that is taken by an operator or a computer and measurement system we have seen in detail in the last lecture, it is a complete process of obtaining measurement this includes gage instrument, people, procedure, operations environment and everything.

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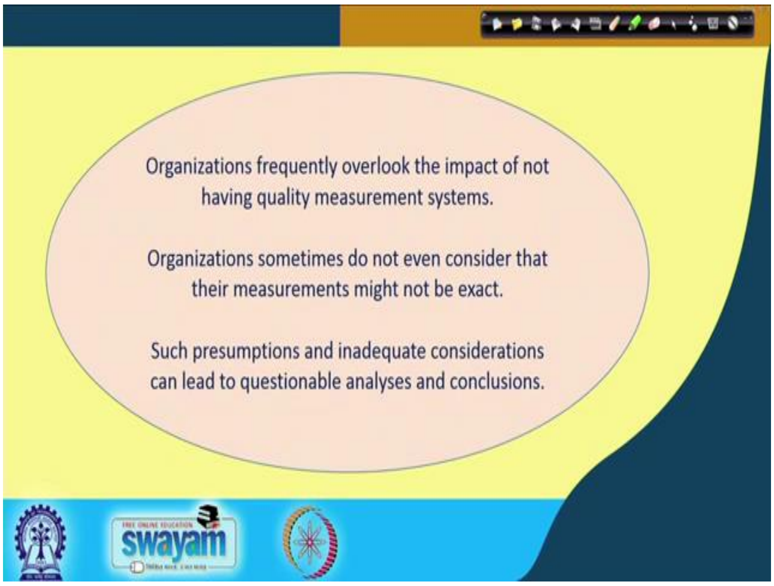

So, typically organizations frequently they overlook the importance of measurement and sometimes do not even consider that they are measuring may be wrong and such kind of presumptions and inadequate consideration can lead to questionable analysis and conclusions. And over a period of time you will lose faith in your results, in your recommendations and then you will stop may be taking the corrective actions.

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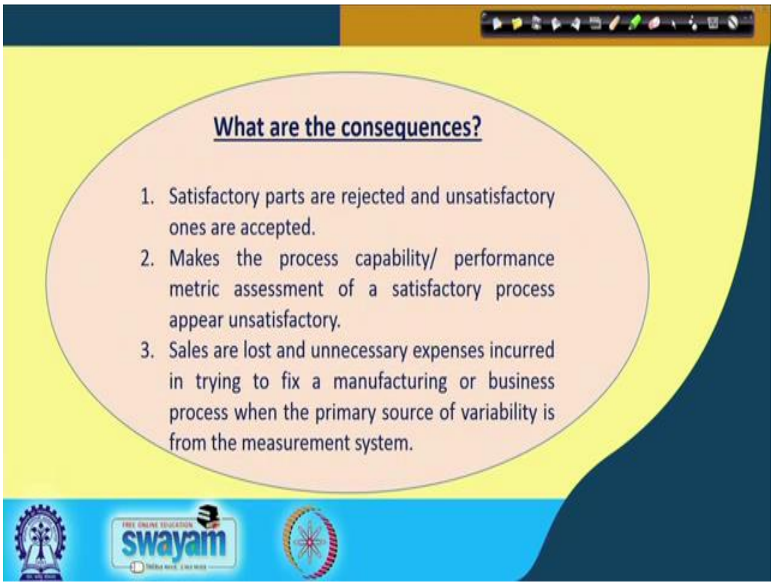

What are the consequences? So, it is not just that I am getting the good or bad quality data, but there are many consequences and one of them that satisfactory parts are rejected and unsatisfactory factory parts are accepted. You just think if this happens wherever in the process and if unsatisfactory part will go to the next stage or may be finally, to the customer then it will have a huge impact. And the really good quality parts they are rejected where you are losing the value of entire processing.

Now, second makes the process capability performance matrix assessment of a satisfactory process appear on satisfactory. As a whole you will all together reject the process saying that, this process is not capable enough to meet the desired specifications. And hence the process should be entirely changed, revamped or this process cannot produce.

Third, sales are lost unnecessary say expense will be incurred in trying to fix a manufacturing or business process, when the primary source of variability is from the measurement system. So, you will do all the effort and you will put all the energy in investigating, but actually the problem is with the measurement otherwise your product is ok.

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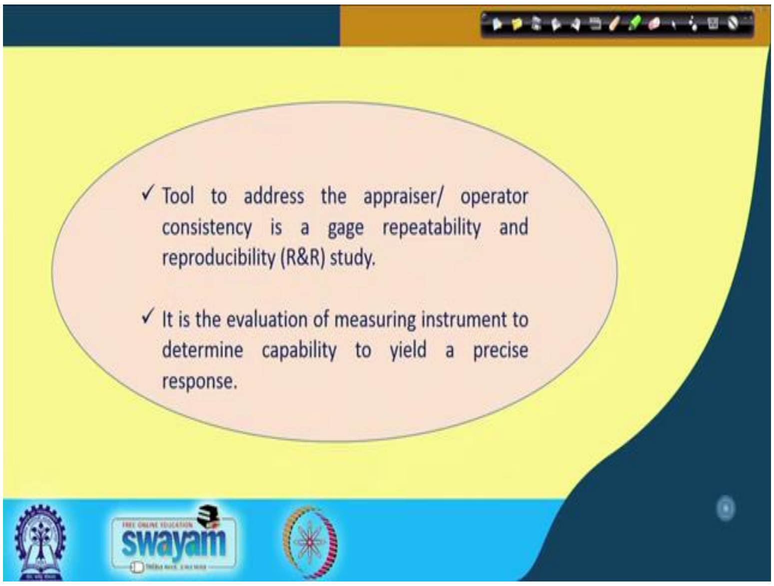

So, typically measurement system gage R & R is a tool to address appraiser operator consistency and this is very much addressed by gage R & R study, it is the evaluation of measuring instrument to determine capability to yield a precise response. So, the word here is precision and to what extent my instrument is precise that basically I want to check.

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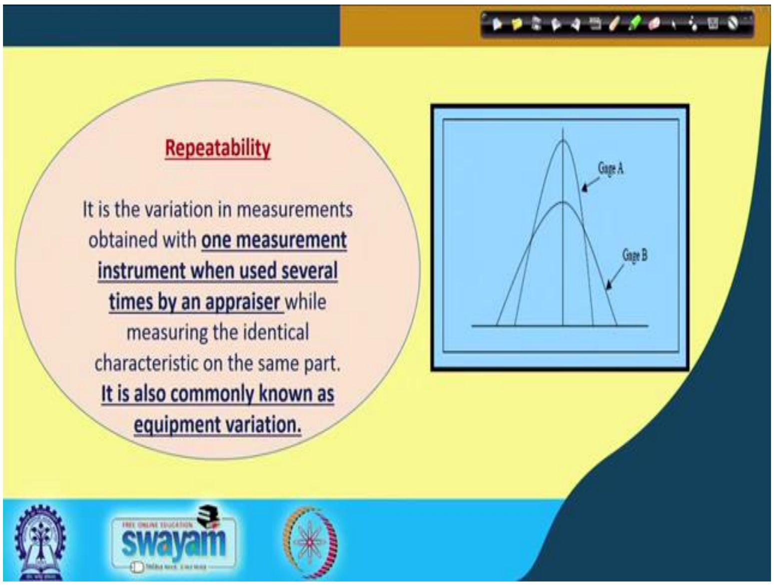

Now, let us quickly go through some of the terms associated with my gage R & R study. So, first one is repeatability and it is also called say equipment variation. So, typically this is the variation in the measurement, obtained with one measurement instrument when used several times by an appraisal while measuring the identical characteristics of the same part.

So, here I have one measurement instrument which is used several time, and I am expecting that it should give me the repeatabilty. So, if you see the graph a figure put here there are two bell shaped curve one is gage A and gage B, you can easily see that gage A is closer to the target value gage B has a wider spread and hence your repeatability is affected.

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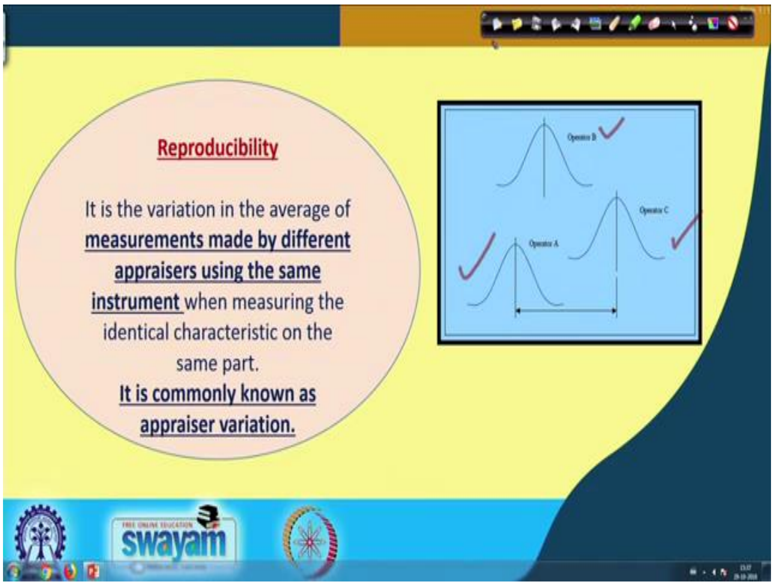

If you see the reproducibility then this is also called appraiser variation, or the inspector variation operator variation and typically it is the variation in the average of measurements made by different appraisers using the same instrument. I have same instrument, but I am just checking the component using the different appraiser and trying to see that to what extent their readings align with each there. You can see in the figure that operator A typically this, operator B and operator C, there is a difference even if I take the mean value of this 3 and this difference basically indicates that there is a concern with the reproducibility.

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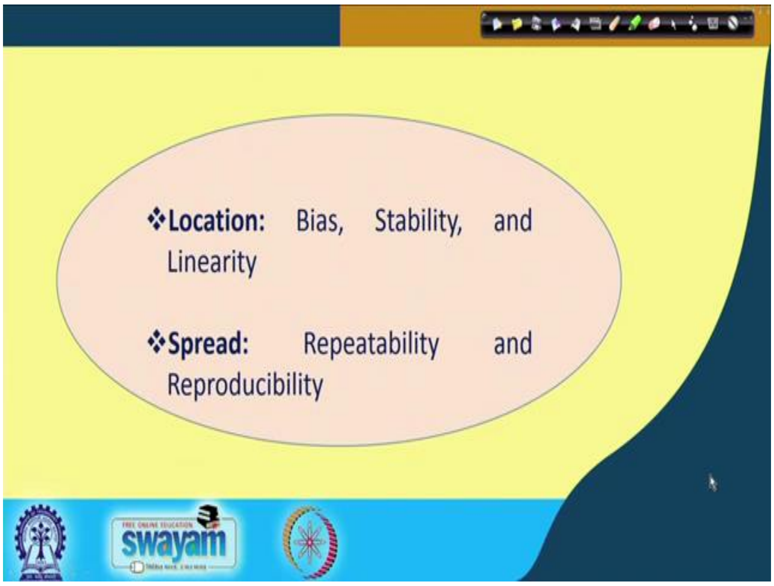

So, if we go ahead then, there are broadly two issues one is the location another is the spread, when you are talking about the location bias stability and linearity are three important issues. When you are talking about spread repeatability and reproducibility are the important issues. So, first I would like to check for the location by a stability and linearity once this is done, then my interest would lie in checking the spread that is the repeatability and reproducibility through gage R & R study.

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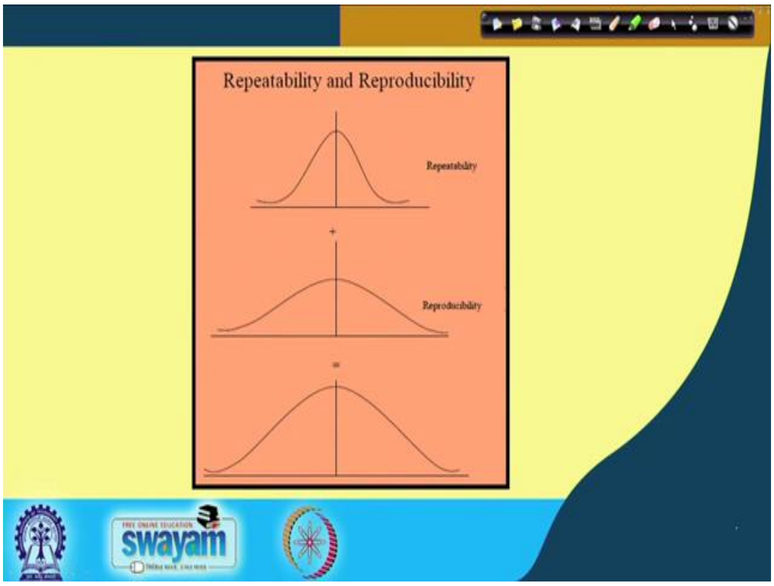

So, just see that what happens when I put repeatability and reproducibility together. So, you have a repeatability reproducibility merging together, will give you quite a wider spread of your measurement system which is basically undesirable.

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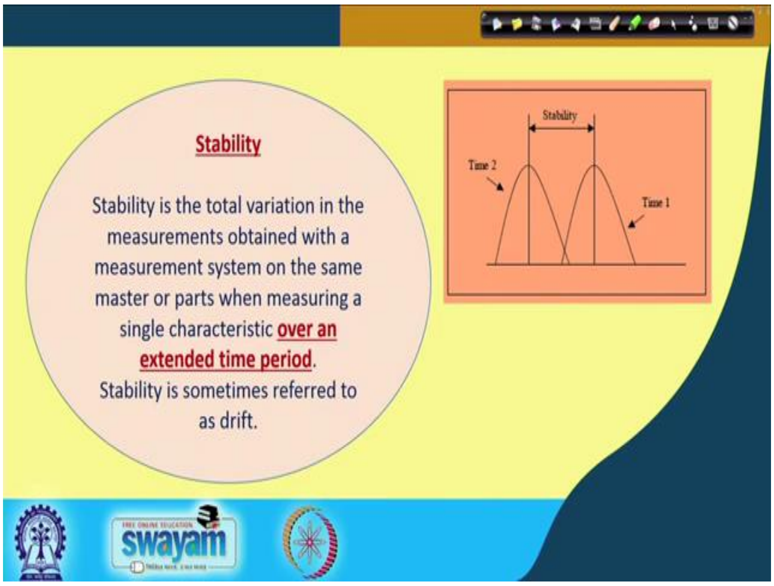

So, stability you can see in the figure that I have time 1 measurement, I have time 2 measurement and something which is shown here is the stability. So, typically stability refers to the total variation in the measurements, obtained with a measurement system, on the same master or parts when measuring a single characteristic over an extended period of time. Suppose I take the measurement at 7 O'clock in the morning and then I take the measurement may be at 4 O'clock in the afternoon and then in the evening and then, I can just try to see that whether they are matching with each other or there is an issue with the stability of the measuring instrument.

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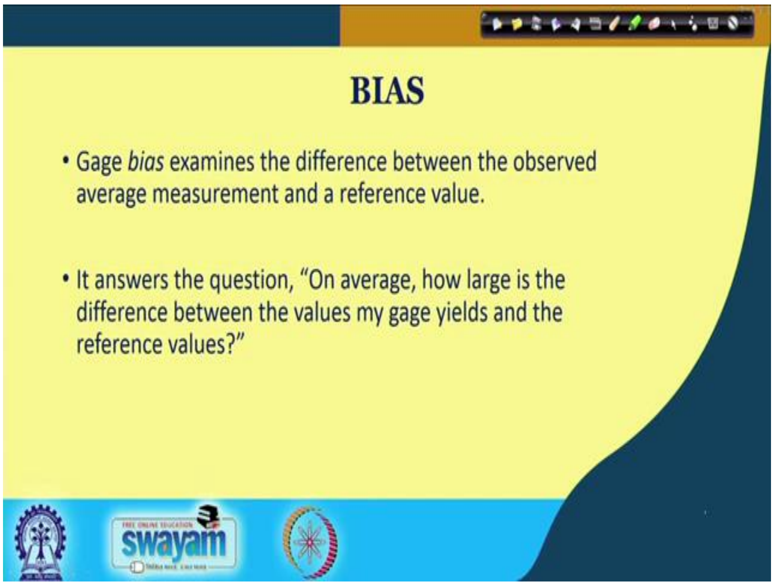

The other term which is very important as a part of location is bias. So, gage bias examines the difference between the observed average measurement and the reference value. So, you have the benchmark standard reference value taken by a highly precise instrument or an instrument a part of say a high grade laboratory. And here I am interested to ask a question on an average, how large is the difference between the values my gage yields and the reference values.

So, the difference between these two would be a bias of my instrument measuring instrument and higher the bias poor quality measurement I will be taking.

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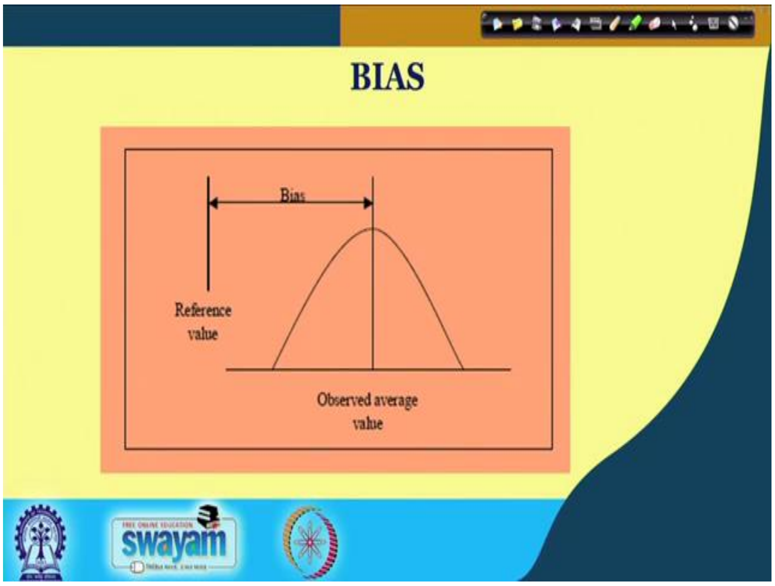

So, just see this you have here a reference value and you have the observed average value; obviously, there is some spread and something which you can see here is the bias that is the difference between reference and the observed average value.

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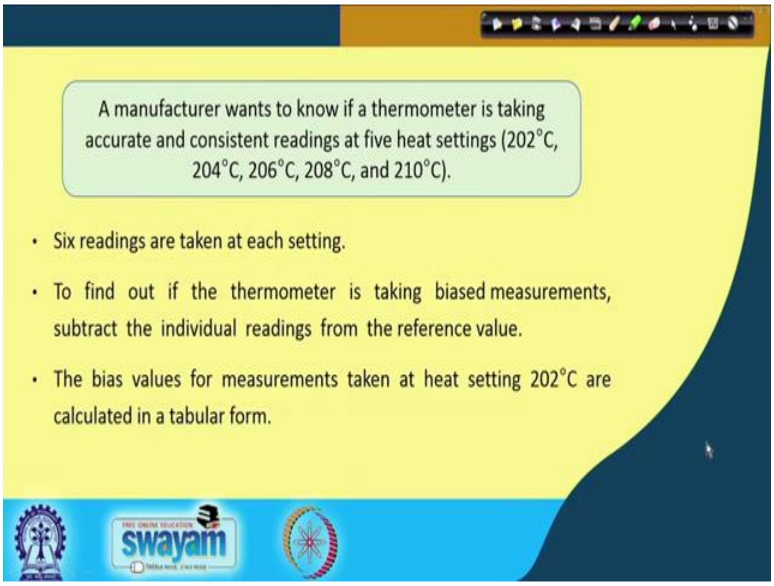

Let us see a some example to clarify the bias, suppose a manufacturer wants to know if a thermometer is taking, accurate and consistent reading at 5 heat settings 202 degree centigrade, 204 degree centigrade, 206, 208 and 210 and he has taken six readings at each setting. To find out if the thermometer is taking bias measurements subtract the individual reading from the reference value and the bais values of the measurement is taken, suppose let us say at 202 degree centigrade which are now tabulated like this.

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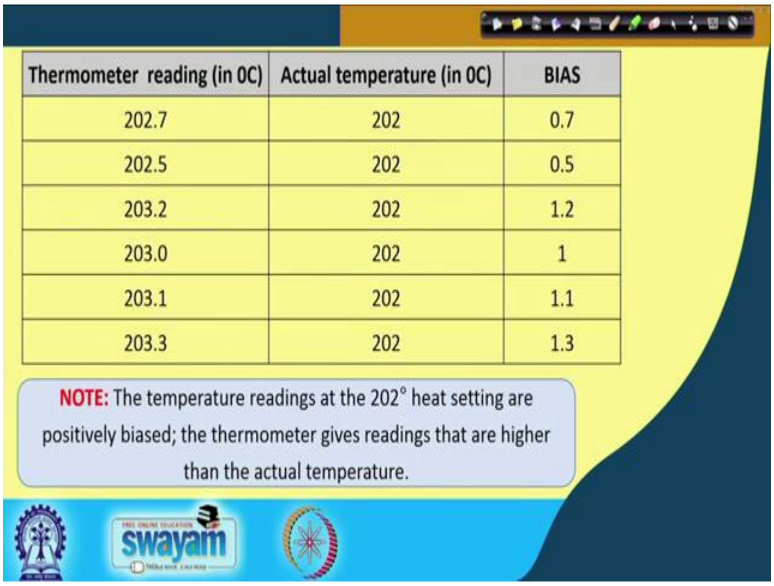

So, you can see that second column actual temperature in degree centigrade, I have all the 202 these are my reference reading and thermometer reading which are at say my degree centigrade. For a particular heat setting 202, I can see that first time my thermometer is taking 202.7, then 202.5 203.2 and so on. So, every time I can find the bias by subtracting column 1 from column 2 and you can see that 0.7, 0.5 1.2, 1, 1.1, 1.3.

So, the temperature reading at the 202 degree heat setting are positively biased, it means say subsequently there is increase in bias and thermometer gives reading that are higher than the actual temperature.

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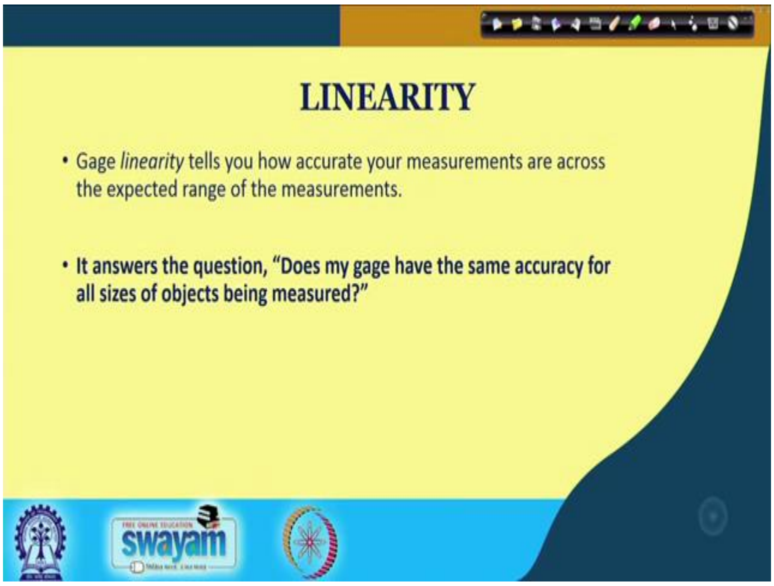

Now, there is another important concern as a part of location, which is linearity. Gage linearity tells you how accurate your measurements are across the expected range of the measurement. So, you do not want your gage to be utilized for a very very narrow range. Suppose if I talk about the diameter and let us say, I am using micrometer, then say I do not want diameter of may be 10.2 mm and 10.5 mm only to be measured and in between value I would like my gage to be operated for a larger range.

So, here my question is that does my gage have the same accuracy for all sizes of objects being measured. I want to check the diameter of an object having 20 millimeter diameter or may be 2 millimeter, I should have I should get the same accuracy.

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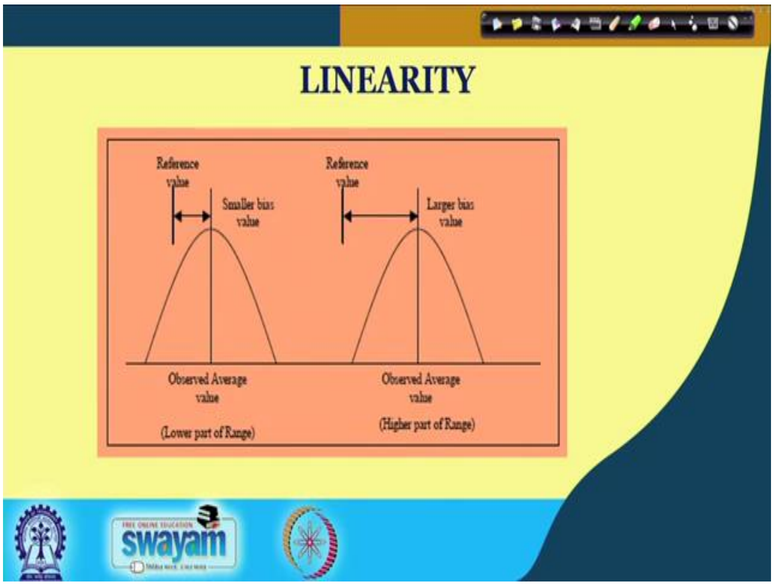

So, now you see in figure that linearity in the first figure, you will see that lower part of the range is considered and here higher part of range is considered. So, lower part of range you will see that when I compare with the reference value, there is basically a smaller bias and when I look at the higher part of the range, when I compare it with the reference value there is a larger bias. It means yes there is an evidence that when I operate my gage, for a higher part of the range and lower part of the range, there is some concern regarding the measurement and the linearity.

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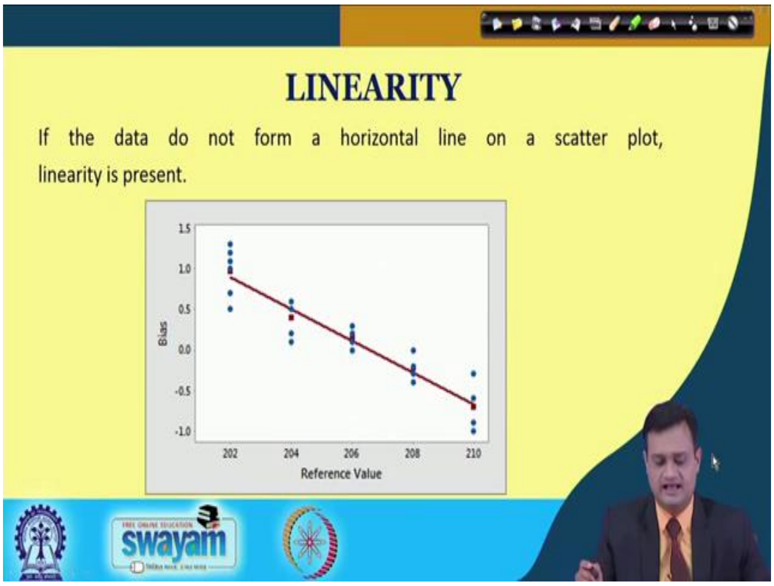

So, this dimension is very important to see that our gage can be operated for a larger range. And just see here for the example of this heat setting 202, 204, 206, 208 210, I am plotting the various values taken by the thermometer at a particular heat setting.

Suppose I have taken 1 2 3 4 5 6 readings you can see here, that I have taken ok, I have taken 6 readings here these are the 6 readings at particular 202. Similar way I have taken some reading here at 204, I have taken some reading here at 206 and some readings here at 208 and 210 I have taken some readings.

Now, when I can see that this particular straight line can pass through this readings or I do not find any horizontal line on the scatter plot, then I would say that yes linearity is present.

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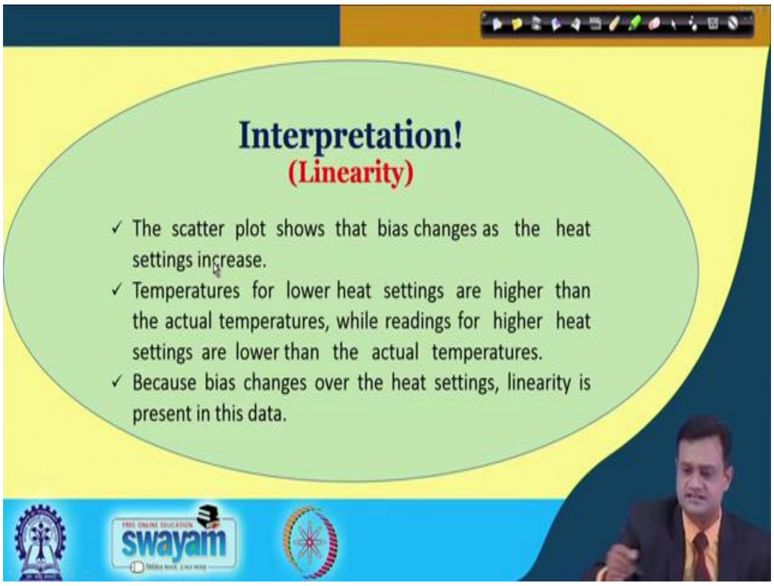

So, interpretation of linearity is that the scatter plot shows, that bias changes as the heat setting increases. And temperature for lower heat setting are higher than the actual temperature while reading for higher, its settings are lower than the actual temperature and because bias changes over the heat setting linearity is present in this data.

Now, gage R & R study which is the focus of this particular lecture 21, let us try to see and the location measures bias stability and linearity whatever I discuss that are specific to accuracy of the gage. Now, I am talking about repeatability and reproducibility of the gage it is specific to the precision of the gage.

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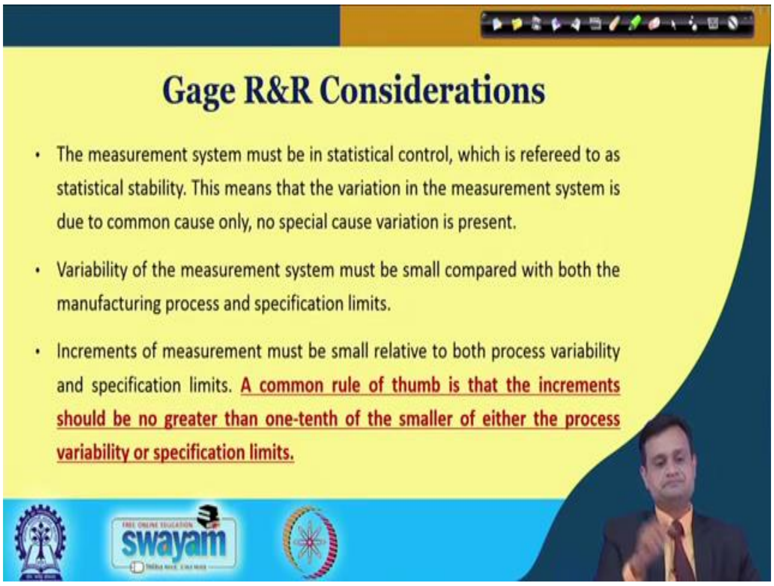

So, gage R & R there are certain considerations that measurement system must be in a statistical control and this refer to as the statistical stability. It means whatever variation I observe that should be because of only some random causes chance causes some variability would be there and no assignment cause or special cause should be present.

So, a variability of the measurement system must be small compared to or compared with both the manufacturing process and specification limits, if my measurement system itself has a larger variability how will you predict the health of the process? So, this is very important that if I want to analyze my process as a part of Six Sigma, I must have accurate and precise measurement system.

So, a common rule of thumb is that the increment should be no greater than one tenth of the smaller value of either the process variability or the specification limit.

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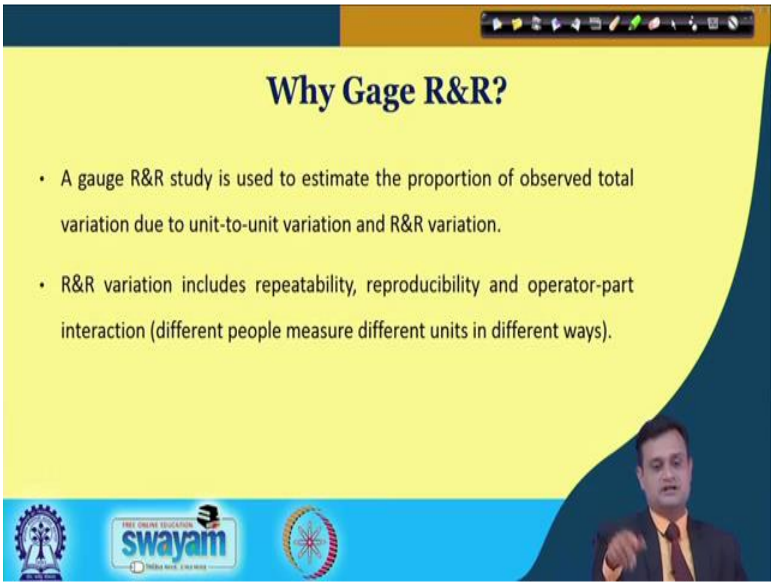

So, the issue is very clear why gage R & R. So, this is typically to estimate the proportion of observe total, variation due to unit to unit variation and R & R variation. So, if there is a process variability there will be part to part variation, if there is a variability because of gage there would be say some repeatability and reproducibility issues.

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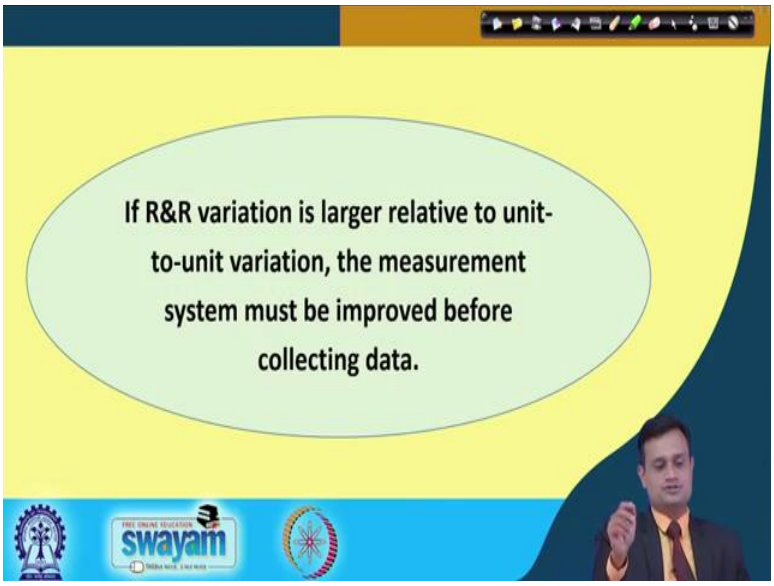

So, if R & R variation is larger relative to unit to unit variation, the measurement system must be improved before collecting the data, otherwise you will simply be collecting the useless information which will definitely lead you to the wrong decision making.

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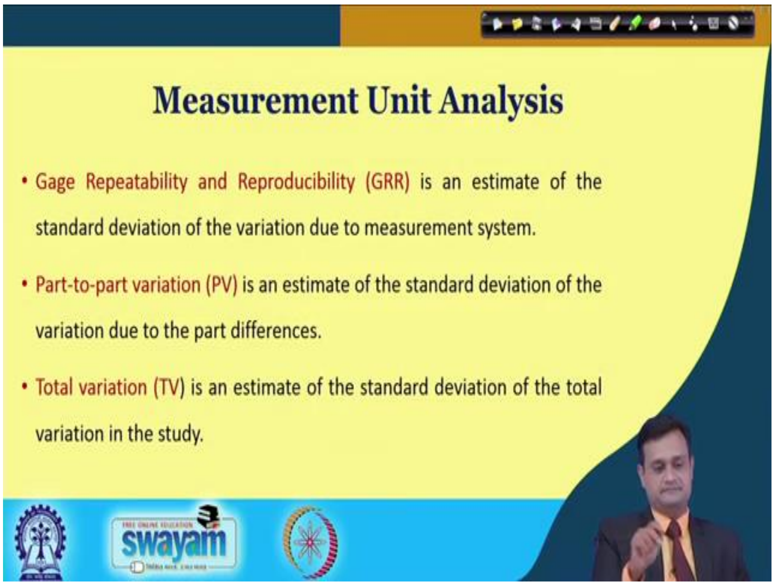

So, just try to see some of the terms associated with gage R & R measurement unit analysis. So, repeatability and reproducibility is an estimate of the standard deviation of the variation due to measurement system. Part to part variation, it is mainly because of your process variation. Total variation is an estimate of the standard deviation of the total variation in the study.

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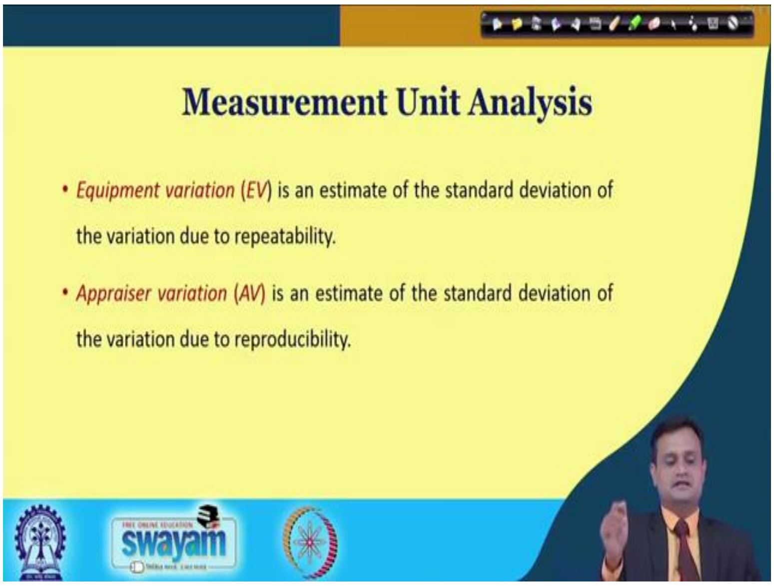

You have equipment variation so, estimate of the standard deviation of the variation due to repeatability and appraisal variation as I mentioned, this is due to reproducibility and this is because of my operator or the appraiser.

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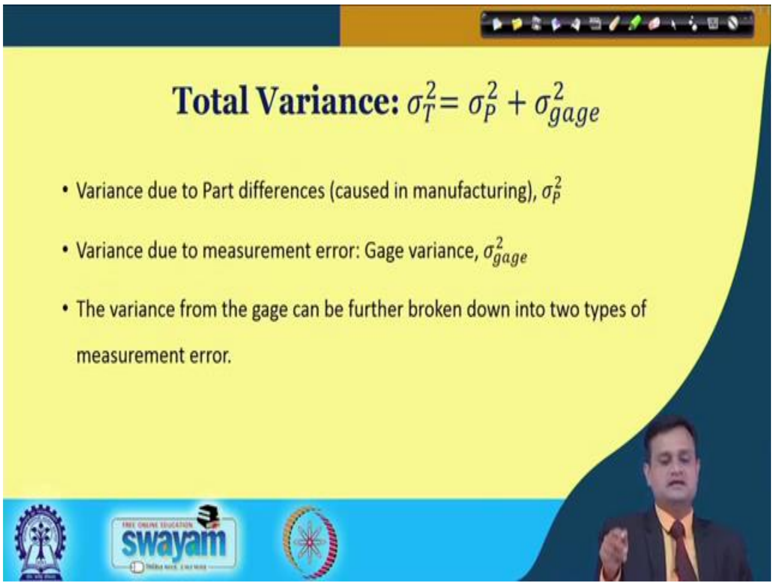

So, typically you have $\sigma_T^2 = \sigma_P^2 + \sigma_{gage}^2$ and variance due to part difference caused in manufacturing this is called σ_P^2, variance due to measurement error gage variance this is called σ_{gage}^2. And the variance from the gage can further be divided into say types of measurement error.

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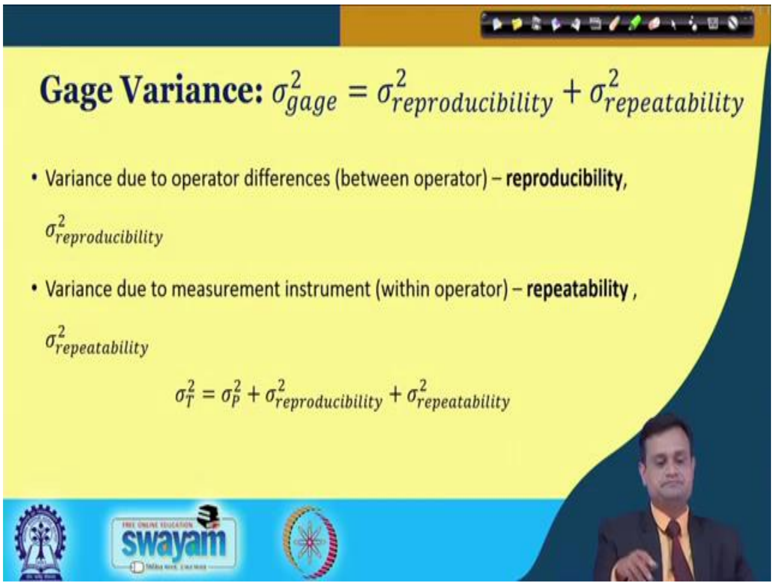

So, when I talk about specifically focus on σ^2_{gage}, I have $\sigma^2_{reproducibility} + \sigma^2_{repeatability}$ and if total I want then I have to add σ^2_P.

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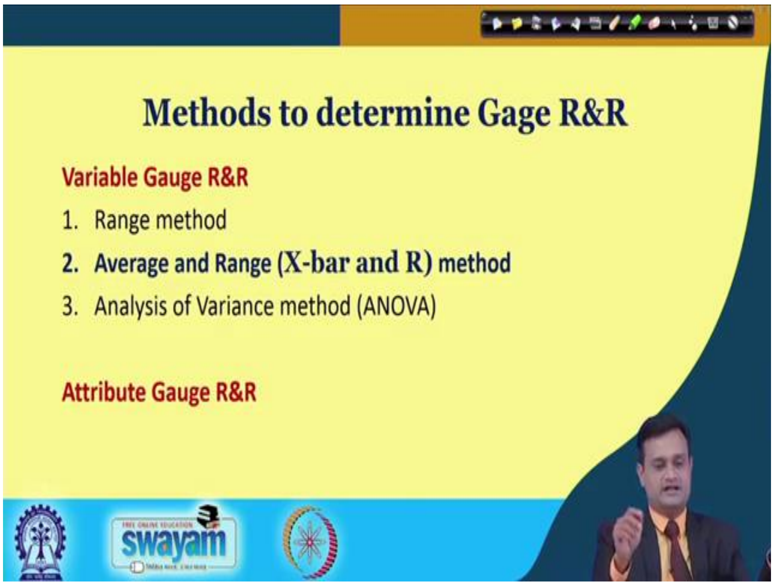

So, let me try to clarify that broadly there are three methods for conducting gage R & R study, range method average and range X-bar and R method, widely used method and another widely used method is ANOVA. As the part of this lecture, we will focus on the most appealing easy to use, widely used method say average and range method ANOVA study we will do or we will try to understand in the subsequent phases of the statistical analysis.

So, this is all three part of variable gage R & R, if you have attribute kind of characteristic simply accept or reject go or no go, then you go for attribute gage R & R, is in this lecture we will focus only on variable and on average and range method.

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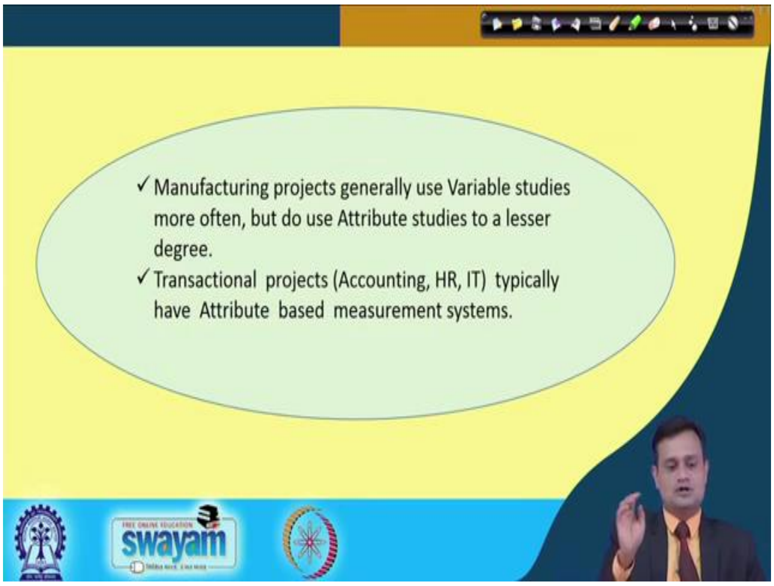

So, typically if you see the example from manufacturing as well as transactional then manufacturing projects generally use variable studies, but sometimes they do use attribute study. Transactional process like HR, IT, accounting typically, they go by the attribute study.

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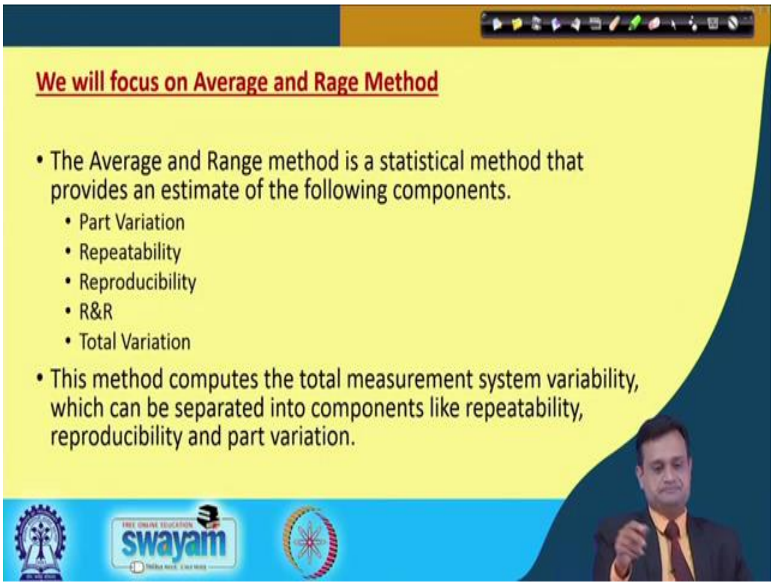

So, we will focus as I mentioned average and range method and the couple of terms already I introduced associated with this is part variation, repeatability, reproducibility, R & R and total variation. And this method captures the total measurement system variability which can be separated into the components like repeatability, reproducibility and part variation. So, let us see the step by step procedure.

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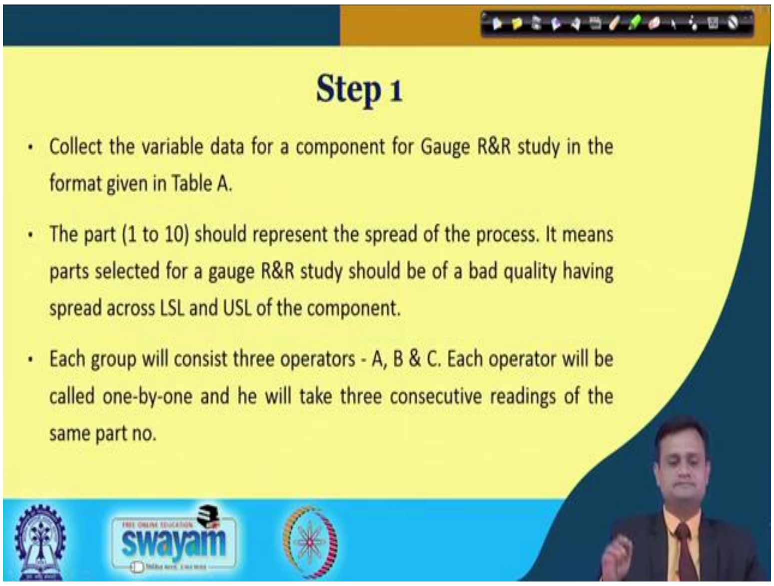

So, step 1 collect the variable data for a component for gage R & R study, then the part 1 to 10 should represent the spread of the process it means parts you select for gage R & R study should have bad quality as well as good quality and they should lie on the either side of the specification limit.

Each group will consist three operators, I am just giving you a template that can be extended for more number of operators also, here I am considering three operators each operator will be called one by one and he will check 3 consecutive readings of the same part number.

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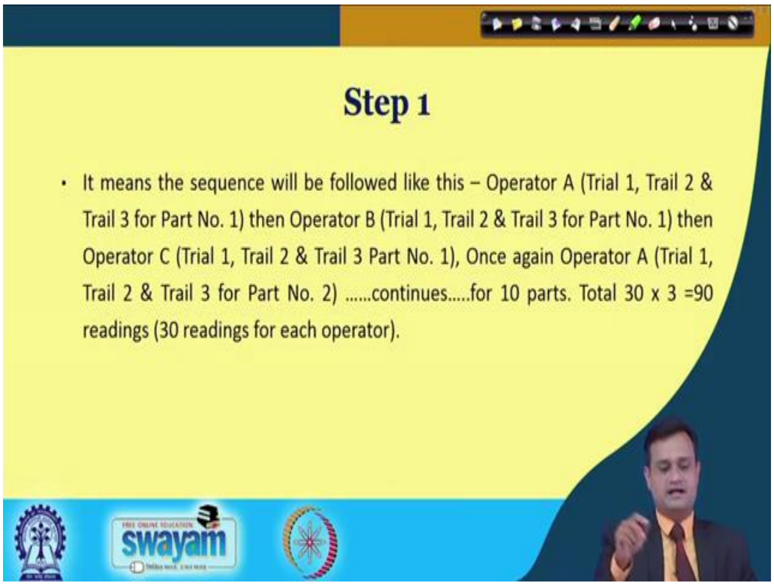

So, step 1 the sequence will be followed like this operator a will come trial 1, trial 2, trial 3 for part number 1, then operator B will come trial 1, trial 2, trial 3 for part number 1, then operator C will come trial 1, 2, 3 for part number 1. Once again you will call the operator a now again you will take three trials for part number 2 and same way the process will continue so, for the given template here you will check 30 into 3, because you have 3 operators and total 30 say trials so, 30 into 3 you will have 90 readings 30 readings for each operator.

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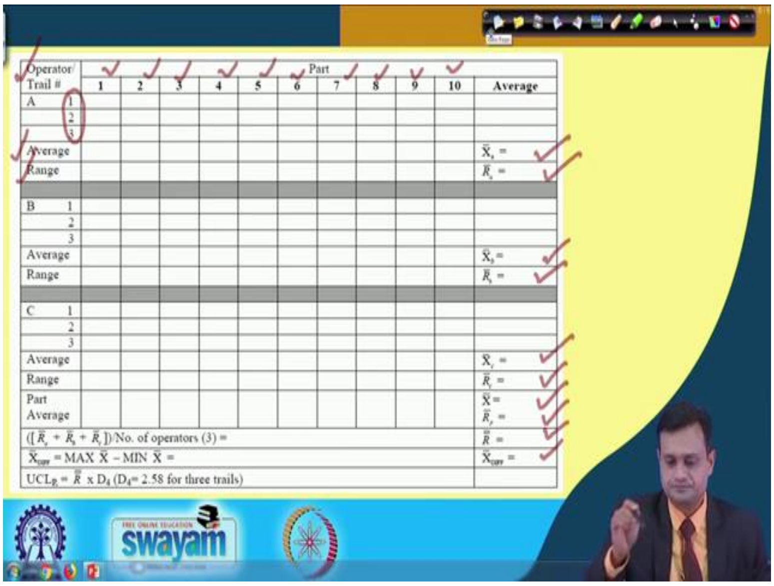

So, table A will look like this and what you will have you can just see here, that you have operator and trial and you are taking 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 parts and you will see that each operator is taking 3 trials, you find average and range and here you find for operator A, $\bar{X}_a$ and $\bar{R}_a$.

So, this is basically the average value for the operator A, similar way you find the average value for operator B, range for operator B, average of the range of the operator B, same way average value for operator C, average range for operator C, you find $\bar{X}$, $\bar{R}_p$. and then you find $\bar{\bar{R}}$ which is the average of the average of the range is and then you find $\bar{X}_{Diff}$ that is MAX $\bar{X} - MIN\ \bar{X}$. So, once you complete this particular table, then you can go for the subsequent analysis.

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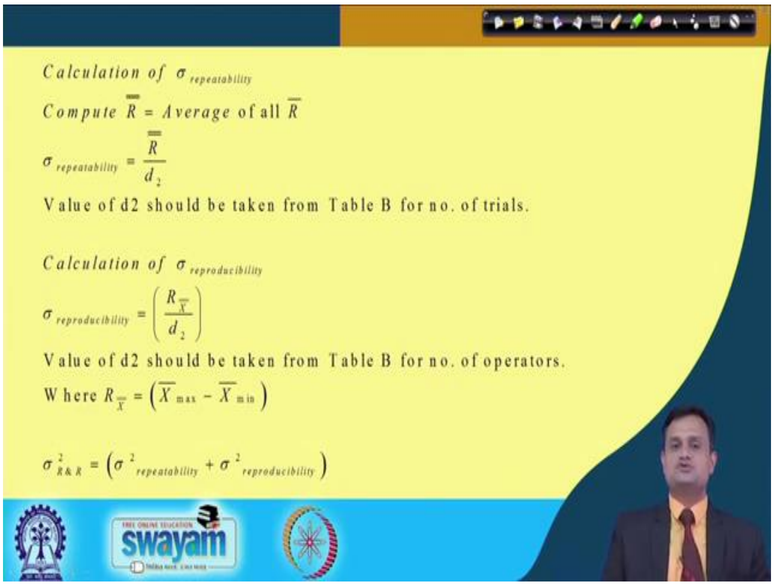

So, step 2 gage R & R calculation which is very simple, calculation of sigma repeatability and calculation of sigma reproducibility, calculation of sigma repeatability you compute $\bar{\bar{R}}$, which is average of all $\bar{R}$. Hence $\sigma_{\text{repeatability}}$ is basically $\frac{\bar{\bar{R}}}{d_2}$, d_2 is a standard value I will give you the table and typically for statistical process control, this standards are determined and we make use of this standard values for the purpose of calculation.

So, value of d_2 should be taken from table B. I will show you the table B, similar way you have calculation for $\sigma_{\text{reproducibility}} = \frac{R_{\bar{\bar{X}}}}{d_2}$. And here $R_{\bar{\bar{X}}} = \bar{X}_{MAX} - \bar{X}_{MIN}$ so, $\sigma^2_{R\&R} = \left(\sigma^2_{repeatability} + \sigma^2_{reproducibility}\right)$.

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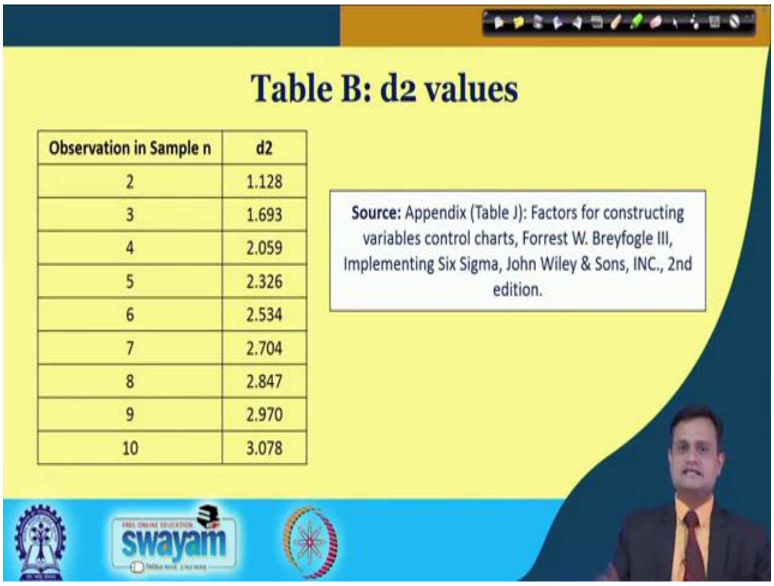

So, just see table B d_2 values for number of observations and you have various standardized d_2 values. For example, for 2 observation it is 1.128, 3 observation it is 1.693. And you can see the full table from this particular reference given as a part of appendix table J and factor for constructing variable control chart, basically the book on implementing Six Sigma by Forrest.

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So, once you have done this there is a decision rule, guide lines step 3, if your gage R & R is 10 percent or less ideal, you select accept your measurement instrument gage and go ahead with the measurement, you will not find your measurement and the subsequent analysis miss leading. If it is 10 percent to 20 percent acceptable, 20 percent to 30 percent marginal, but there is a caution.

Look at the severity of making wrong decision on the final outcome what is its impact, and if it is not accepted, then you even do not accept the gage when the G R & R is between 20 to 30 percent. And if it is greater than 30 percent not at all acceptable first correct the gage change the gage measuring instrument and then only go ahead with the measurement.

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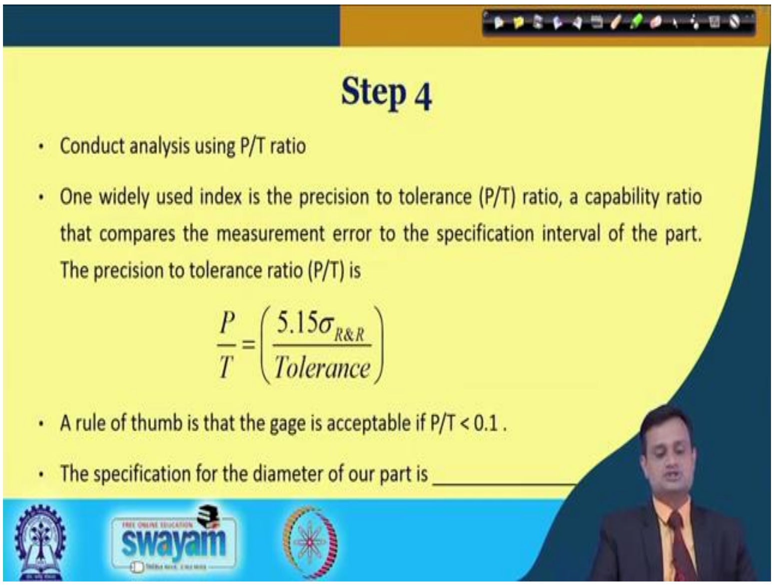

Step 4, there is one very important ratio which is called P/T ratio, so, it is your precision of tolerance ratio and this standard equation for this is $5.15\sigma_{R\&R}$ which you have already determined divided by tolerance and as a rule of thumb, if P/T is less than 0.1 your gage is acceptable. So, here I am considering my tolerance part also with respect to my precision and if it is less than 0.1, I will accept.

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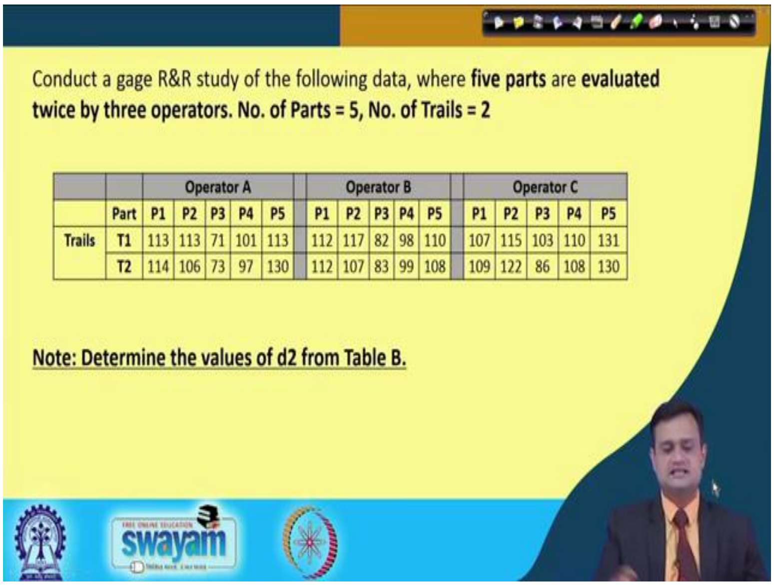

Let us see the worked out example on gage R & R and the example is like this, say I have parts P1, P2, P3, P4, P5 and trail T1, T2 each operator as I mentioned previously, here he is measuring each part two times so, there are 5 parts each operator will measure each part two times there are 2 trails and I have basically 3 operators. So, once again I mentioned that determine the value of d_2 from the given table B as I mentioned previously.

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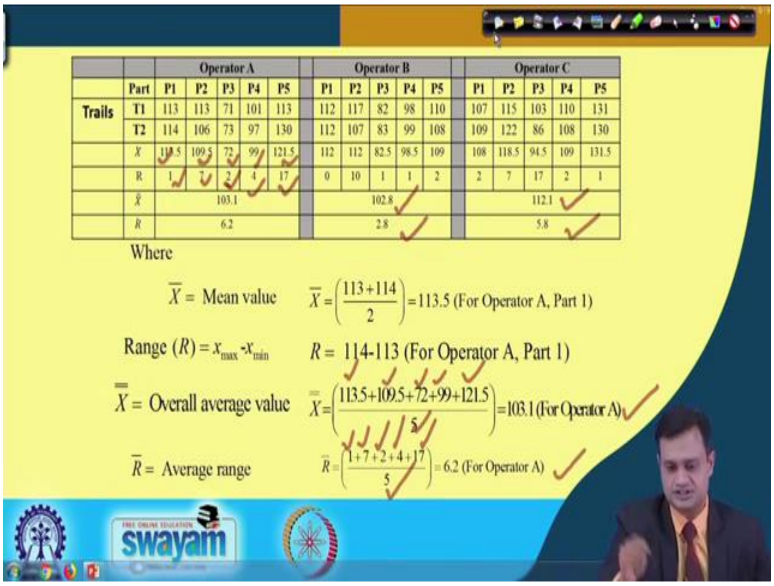

So, now I am finding some of the statistics like $\bar{X}$, R for operator A operator B and operator C and then you can find the range $\bar{X}$, $\bar{\bar{X}}$, $\bar{\bar{R}}$ so, you can see the calculation which is very easy to follow. So, I have the $\bar{X}$ value for operator A and you can see that 113.5. So, here it is 113.5, 109.5, 72, 99, 121.5, and because there are 5 total parts I am divided it by 5.

So, I am getting the average of the average or overall average $\bar{\bar{X}}$ 103.5. Similar way I can take $\bar{R}$ value is 1 so, 1, 7 so 7, 2 so, 2, 4 so 4 and 17, 17 dividing by 5, I will get $\bar{R}$, average range for operator A 6.2, similar way you can compute for operator B and operator C.

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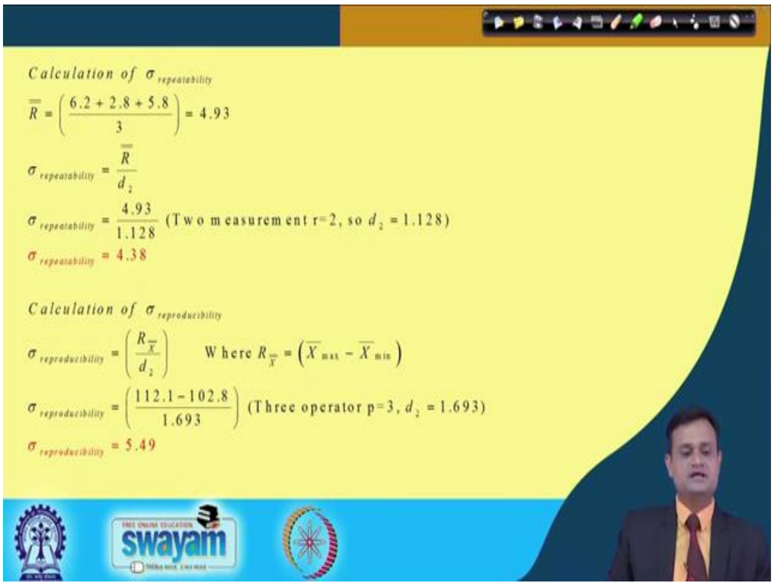

So, my calculation goes like this I am finding $\bar{\bar{R}}$, $\sigma_{repeatability}$, $\frac{\bar{\bar{R}}}{d_2}$.

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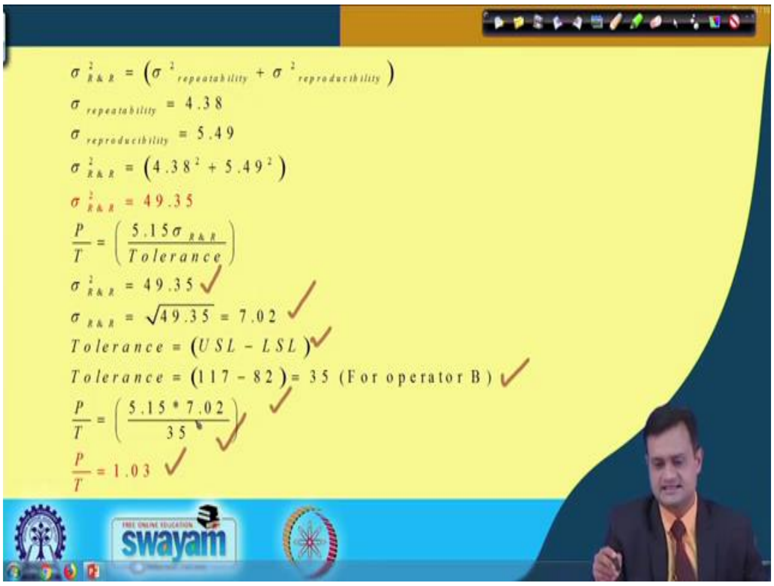

So, my repeatability value comes out to be 4.38, reproducibility $R_{\bar{\bar{X}}} = \frac{\left(\bar{X}_{\max} - \bar{X}_{\min}\right)}{d_2}$, it comes out to be 5.49 and then I can just add it to find the $\sigma^2_{R\&R}$, I will take the square root I will get 4.38 $\sigma_{reproducibility}$ 5.49. So, I am putting the values and $\sigma^2_{R\&R}$ is 49.35.

So, now I can take the square root and put this values into P/T equation, so, I have $\sigma^2_{R\&R}$ 49.35 square root is 7.02 I have USL minus LSL and this is for let us say, operator B I have considered I am putting the values here P/T comes out to be 1.03.

So, if it is less than you can see the guiding principle and decide. So, if it is less than 0.1 it is acceptable here it is larger than these so, I have to think and check my measurement system, correct it or adapt the new one and then only I should take the correct readings.

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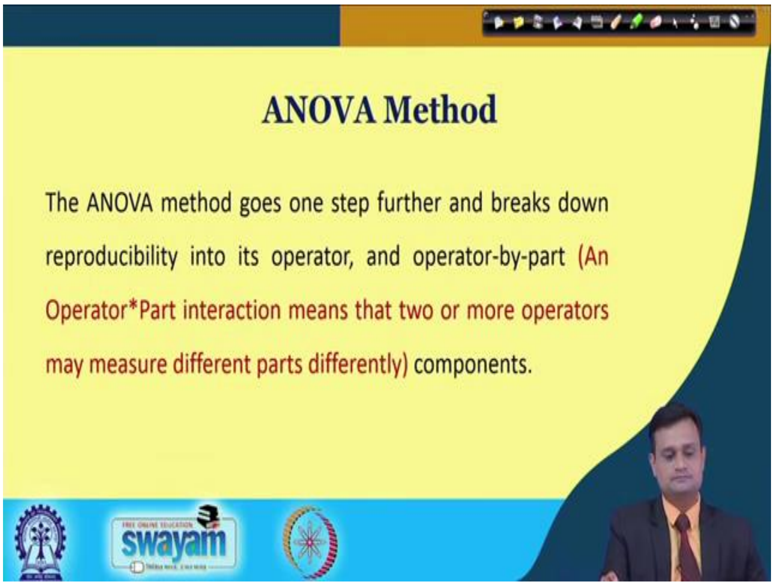

So, there is another method which is ANOVA and typically this method has an advantage, because it considers within and between variation in brief, we will discuss it in detail later on. And the advantage of this method is that, it considers the interaction between operator and the part interaction.

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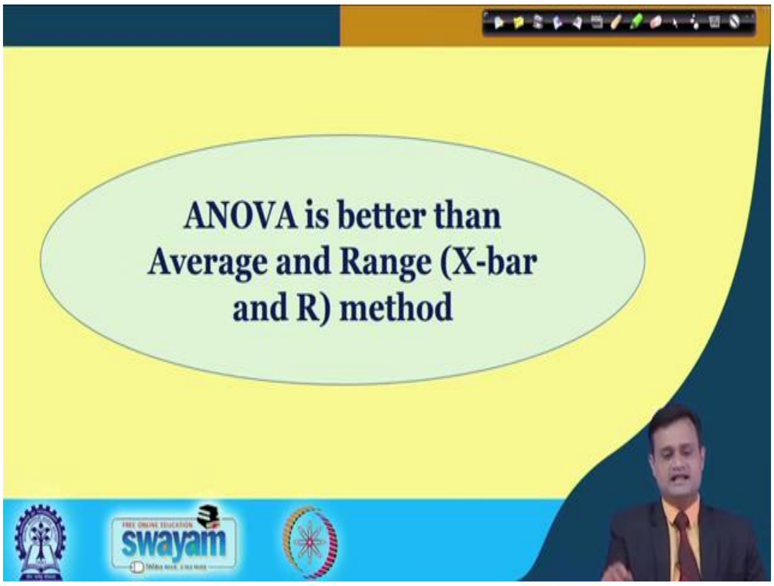

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So, many a times you have an operator and part and depending upon their skill of measurement, this dimension may have influence on the measurement system, which is not actually considered as a part of X-bar and R.

So, many a times when this issue is of concern, ANOVA is a better method, then X-bar and R. To mention AIAG automotive industry action group method, they are the well established body and they are recommending many best practices for the measurement system. And this particular body has published the manual which can also be referred.

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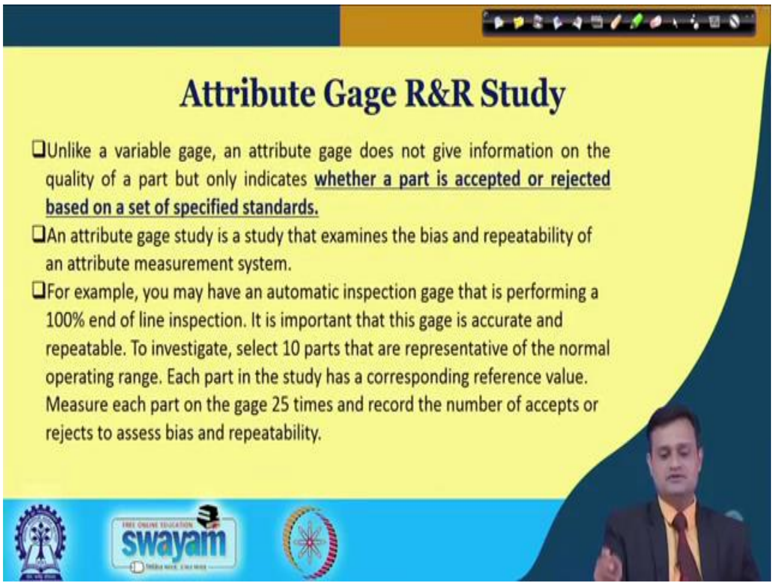

So, finally, as I mentioned if your quality characteristic is of acceptable or rejectable type, you cannot conduct variable gage R & R you have to go for say attribute gage R & R. And just for an example you can see that you may have an automatic inspection gage, performing 100 percent end of line inspection an important that this gage is accurate and repeatable. So, to investigate you select 10 parts that are representative of smaller operating range and each part in the study has a corresponding reference value, measure each part on gage 25 times and record the number of accepts or rejects to assess the bias and the repeatability.

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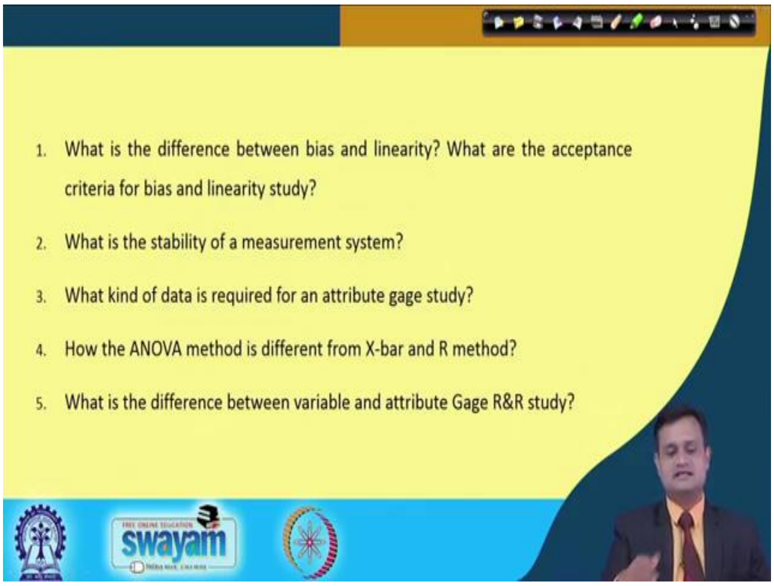

So, I would like to end this session with couple of questions for you to introspect and revise the lecture, what is the difference between bias and linearity, what are the acceptance criteria for bias and linearity study, what is the stability of the measurement system? What kind of data is required for an attribute gage study? And how the ANOVA method is different from X-bar and R method and what is the difference between variable and attribute type of gage R & R study.

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So, you can use this reference for further understanding.

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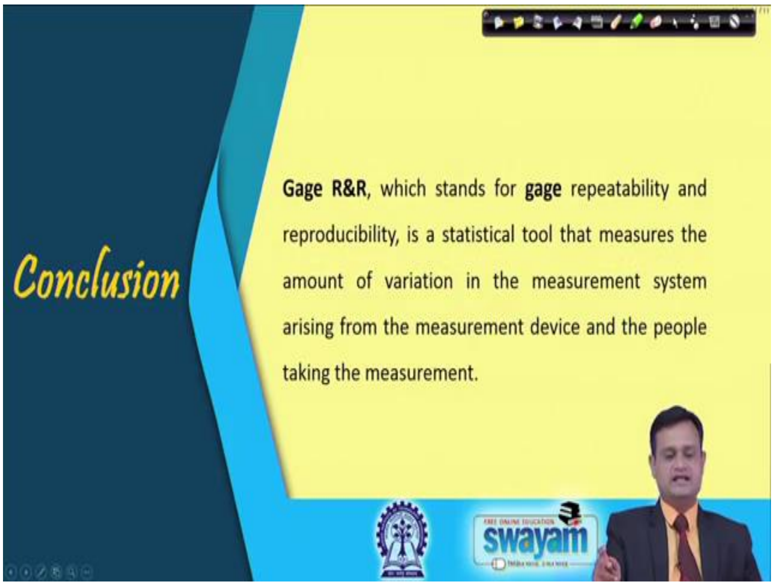

So, finally, I will conclude that gage R & R which stands for gage repeatability and reproducibility is the statistical tool that measures the amount of variation in the measurement system, arising from the measurement device and people taking the measurement. So, in totality it checks the measurement system as well as the people repeatability and reproducibility.

So, with this thank you very much, revise the lecture and say keep revising the contents we are exactly going, in our journey as per DMAIC cycle so, that you can undertake any project after completing this particular course and systematically say execute various phases of six sigma be with me, keep learning enjoy.