Selected Topics in Decision Modeling
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Lecture - 06
Machine Allocation and Cargo Loading Problem

So, welcome to all of you. In this particular lecture, we are going to discuss the Machine Allocations and the Cargo Loading Problems right. So, machine allocation and cargo loading problems in the context of dynamic programming right.

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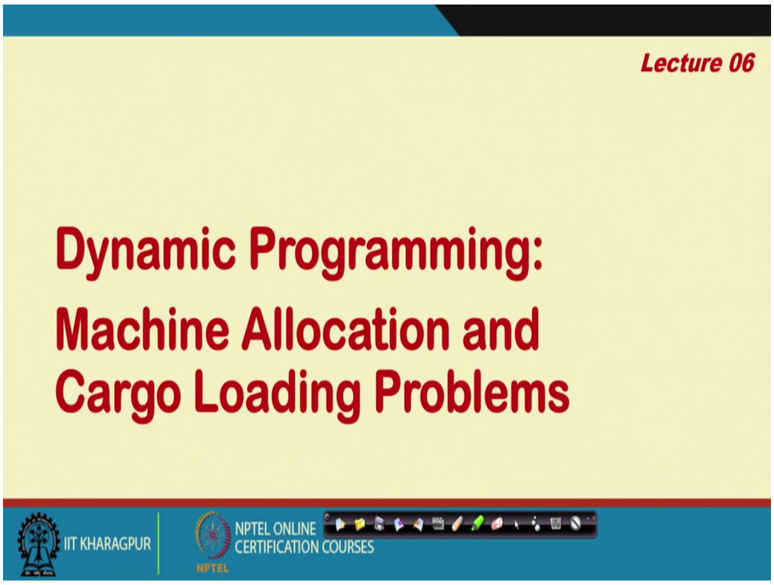

So, that will be our topic Dynamic Programming: Machine Allocation and Cargo Loading Problems. So, two problems we shall discuss; one is a machine allocation problem, another is a cargo loading problem.

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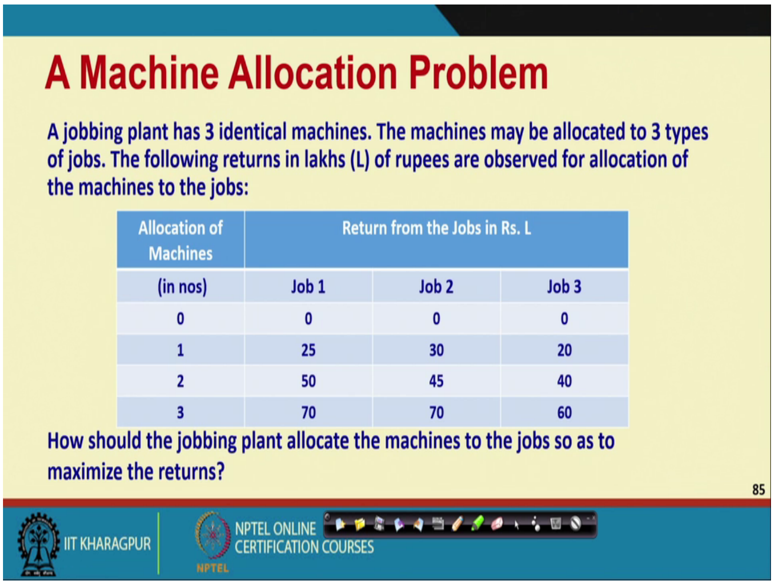

So, let us see the first one the machine allocation problem. So, the machine allocation problem is like this. There is a jobbing plant which has 3 identical machines. You see what is jobbing plant does, it does a set of jobs which are not fixed right. So, it gets all types of jobs.

And you know it would like to do obviously, those jobs which gives them maximum return, is it alright. So, those jobs are coming to them, but they would like to they cannot do all possible jobs. So, they would like to do those type of jobs that gives them maximum return, so that is assumption under which we take up this problem.

So, the particular plant has got three identical machines. The machines may be allocated to three types of jobs the following returns in lakhs of rupees are observed for allocation of the machines to the jobs right. So, we can allocate you know zero machines. So, in that case, this job gives no return.

But if you allocate only one machine to a job 1, then we get a return of 25; but if we locate 2, we get a return of 50. And if we locate all the 3 machines, then 70 would be the return. So, return will be 70 for 3 machines. For job 2, the similar returns are 0, 30, 45 and 70; and for job 3, the returns are 0, 20, 40 and 60.

So, how should the jobbing plant allocate the machines to the job so as to maximize the return, is it all right? So, you can a lot all the 3 machines to job 1 or all the 3 machines to job 2, or all the 3 machines to the 3rd job or any combination there off. So, really to the question is what is that combination which gives us the optimal return; right, maximum possible return.

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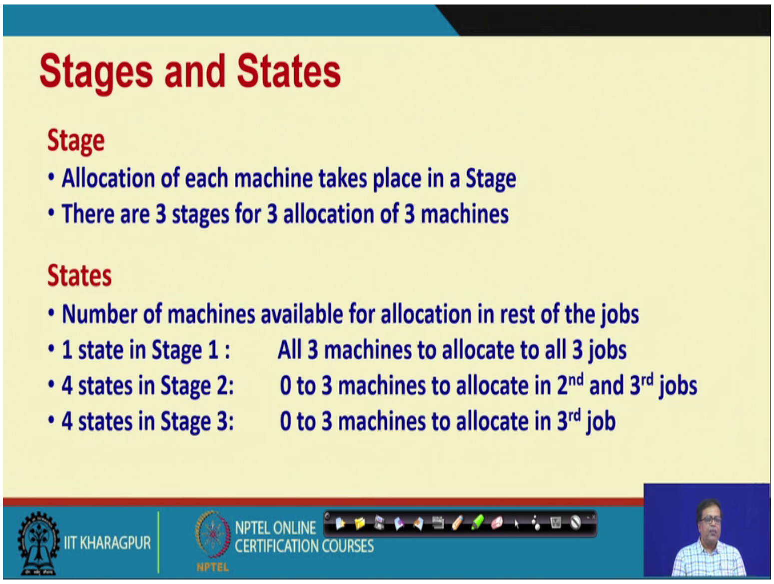

So, if that is so then you know we have again we have to define the stages. So, each allocation can be called as a particular stage. So, we have 3 machines. So, really speaking we have 3 allocations right. So, there are 3 stages for allocation of the 3 machines; The states the number of machines available for allocation in rest of the jobs, right.

At the stage 1, so it is like you know all the possible number of machines that we have for the rest of the jobs. In the first stage, we have all 3 machines to allocate to all the three jobs right. In the second stage, we have 0 to 3 machines to allocate in 2nd to 3rd job. If we allocate 0 only then that chance comes otherwise it will be less.

So, they are four possible states. And there are again four possible states in stage 3 that is 0 to 3 machines to allocate in the 3rd job.

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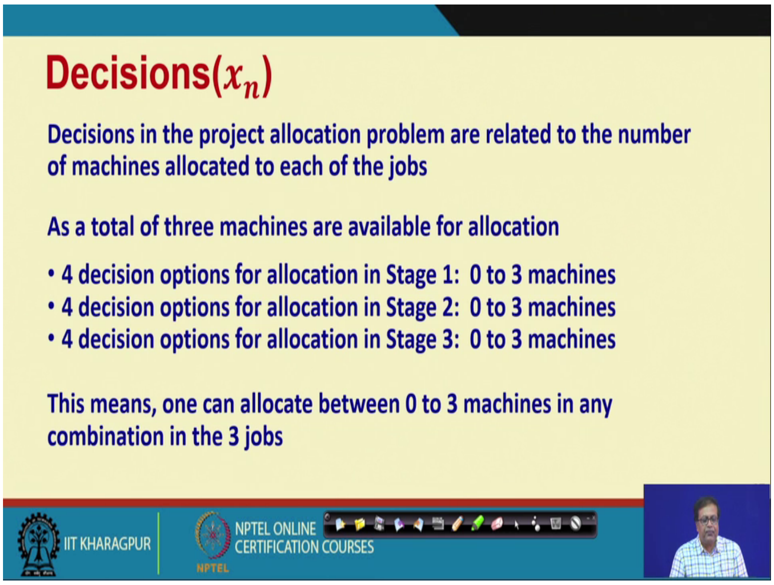

And finally, the decisions; In this case, the decisions in the project allocation are related to the number of machines that are actually allocated to each job, is it all right. So, as total of 3 machines are available so obviously that could be four decision options in every possible stage. So, this means one can allocate between 0 to 3 machines in any combination of the three jobs right.

So, once again for every allocation of a machine to the job is a stage the possible number of machines to be allocated at every stage is the state. So, initially all 3 machines are available for all three jobs; then 0 to 3 machines available to allocate in 2nd and 3rd job; and in the final stage, 0 to 3 machines to allocate in the 3rd job.

And the decision is that you know the number of machines allocated to each of the jobs. So, one can allocate therefore between 0 to 3 machines in any combination in the three jobs, so that is how the decisions are.

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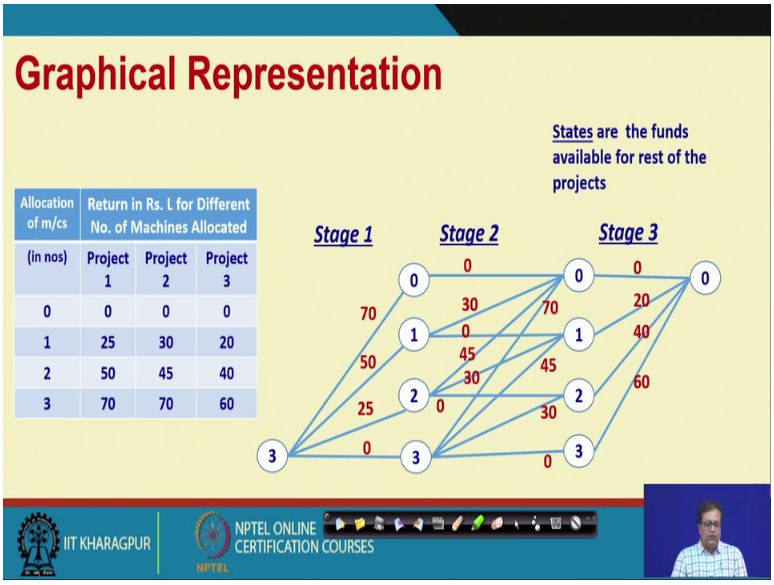

So, you know these problem can again be represented graphically, and these graphical representation really goes like this. In the stage one, you see we have 3 machines that is the only stage because 3 machines left to allocate, all 3 machines left to allocate. Depending on how many you allocate, if you allocate none then you have 3 machines to allocate in stage 2 for rest of the jobs. But if you have already allocated one in stage 1, then you have two left; if you have allocated 2, then have 1 left; and you have allocated all 3, then you have 0 left. Similarly, from one of these stages that is 0, 1, 2, 3 at in the beginning of stage 2, after stage 2, for stage 3, you have 0, 1, 2 or 3 possible number of machines to allocate for the last stage. And finally, if you have allocated all the machines, then obviously you will have nothing left.

So, these are the different stages. And as you move from a stage to another stage, the returns and 0, 25, 50, 70 at the first stage; 0, 30, 45, 70 depending on how many transitions are taking place at the second stage. And the at the third stage, you know the 0, 20, 40, 60 at the third stage. Here they are not projects really, there all jobs so that is how we have at the graphical representation.

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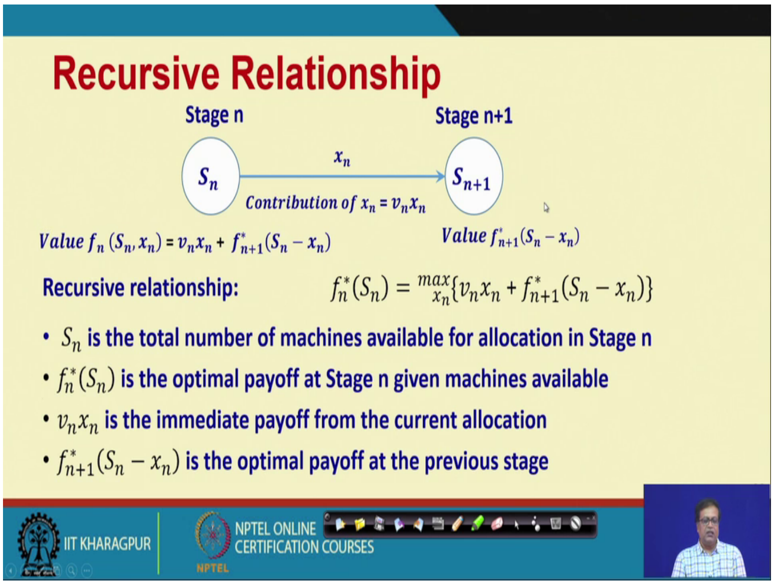

Now, after that we have the recursive relation. These recursive relation will be very similar to the kind of problems that we have already discussed for investment problems. So, it should be a f n star S n equal to maximum of x n that is the you know allocation value v n x n the current contribution plus f n star n plus 1 S n minus x n that is the optimal value at the previous stage.

So, this is how you can really work out the recursive relationship. So, we have the recursive relationship. And we have the stages, we have the states and we have the decision. So, once we have that let us see how we combine all of them into our stage 3 calculation.

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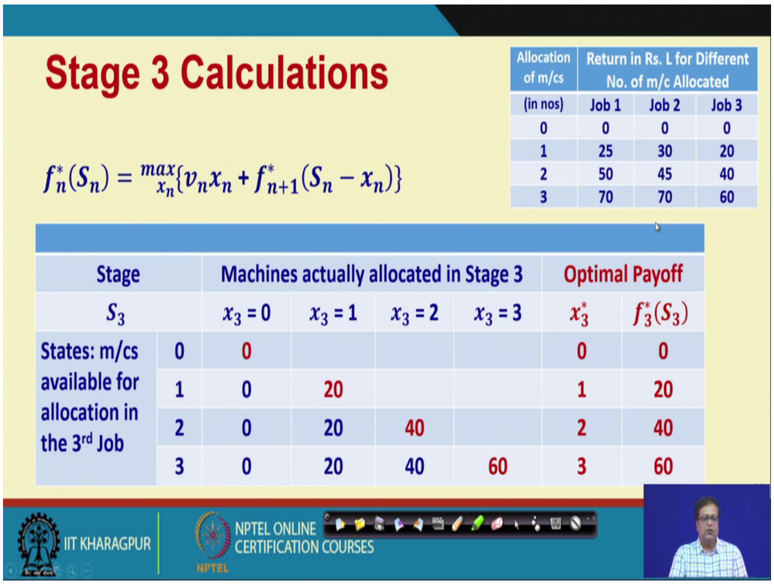

So, this is our you know stage 3 calculations and these are our original data. And how we go about? Now, look here that number of states the machines that are available for allocation in the third job, it could be anything between 0, 1, 2 or 3 because if you look at these diagram, we had four possible states that is 0, 1, 2 or 3 to be allocated for machines in the third stage to the different jobs the machines that could be allocated. Now, the machines actually allocated in stage 3, those are our decisions, so there are four possible decisions x 3 equal to 0, x 3 equal to 1, x 3 equal to 2 and x 3 equal to 3.

Now, assume your x 3 equal to 0, and you had 0 machines available. So, if you have 0 machines available, and you have allotted only 0, obviously your return will be 0 right. But even if you had more machines available, the return will be 0 only because you have not allotted any machine.

Therefore, there will be not profit either. But if you have allotted only one machine out of whatever number you might have had, if you had 0 then obviously, you cannot get any profit that means, that should have been 0 that case does not arise. It starts only when you had at least one machine available for allocating. So, at that stage your profit or your return could be 20 only because look here at the 3rd stage the 3rd job you can get 20 if you allocate one machine.

By similar logic if you had 2, then your return will be 40; and if you had 3, your return will be 60. So, using this figures, you can at this stage 3 level find out the optimal payoff would be you know it could be 1 of 0, 20, 40 or 60 depending on whether your x 3 star is 0, 1, 2 or 3 right, so that is your the 3rd stage calculations.

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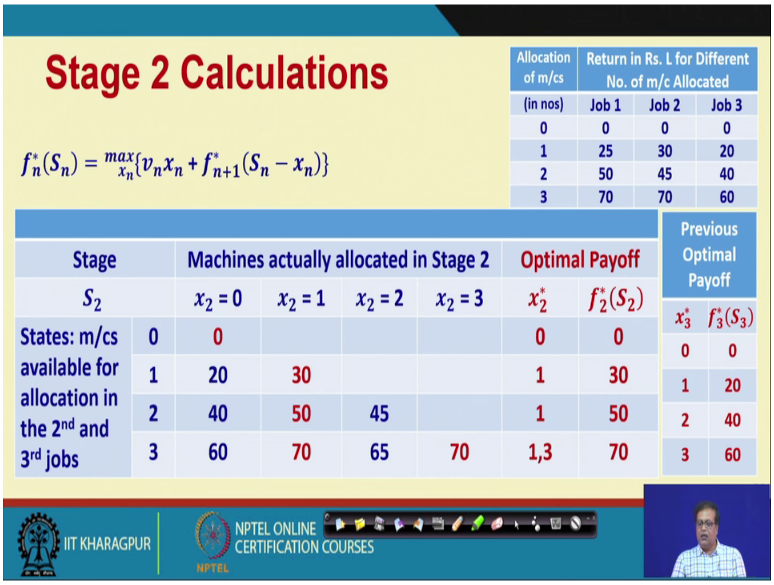

Now, using this 3rd stage calculations, let us move over to stage 2. What we have done here, these 3rd stage optimal calculations I have kept here that is 0, 1, 2, 3 leads to 0, 20, 40 or 60. Now, at the 2nd stage, you know again you can allocate machines either 0 or 1 or 2 or 3, because all four choices could be available. And the machines available for allocation in the 2nd and 3rd jobs that is in the beginning of stage 2. In the beginning of stage 2, suppose you had 0 machines available obviously at the second stage you can allocate only 0 in that case you get 0.

But if you had one machine available, then what will happen, see then you have to look at the previous optimal payoff. See, you had one machine available at the beginning of second stage, you have actually allocated in the second stage how many machines 0.

So, you had that one machine available, so that means, that one machine you must have allocated at the 3rd or onward stage, you know not need not be just 3rd stage. In this case there are three stages, but assume there are more stages. So, 3rd onward if you have allocated one machine, then you would have got a return of 20 that is how the optimal think says, so that means, it should be 0 plus 20 that will be 20 that will be your optimal profit.

By the same logic, if you have 2, it should be 40; it should be 60 if there are 3. But if you have one machine available, and you allocate it, then you get 30 profit, because that is what you see from these original table. But if you had 2 machines available, then for the second machine which you have allocated in the 3rd stage, you have got 20, so 30 plus 20, 50.

And if you have 3, then 2 you allocate at the 3rd stage and your profit will be 70. So, going by the same logic you had 45 and 65 for x 2 equal to 2; and 70 for x 2 equal to 3. Now, you have to see row wise what is the optimal so that means, if a 0 machines available maximum return will be 0. If you have one machine available, maximum return will be 30. If you had two machines available, maximum return will be 50. And if you had 3 machines available, maximum return will be 70.

But in the 3rd stage, you see optimal is both 1 and 3, and optimal payoff is 70. So, using this we have already then calculated the optimal pay off at the second stage. Let us look at the first stage that is the final stage now. When we come to the first stage, again we have written the previous optimal pay offs 0, 30, 50 and 70. And at these stages, you had all the 3 machines available. So, if you had allocate only 0 to in the first stage, you get 0 for first stage, and 70 for the third stage. So, 0 plus 70 that is 70.

But if you had one machine allocated, then 25 from stage 1 and further stages you have 50 because that is what is the optimal for two allocations that is the 2 0 1, 2, 3. So, for two allocations, you get 50, so it is 70. But if you had 2, then 50 from here, and 30 from here, so it is 80. And at the 3 stage, if you had 70 from here and 0 from here so that means 70. Now, out of all these the maximum is 80 and therefore, x 1 star is 2 and total pay off will be 80.

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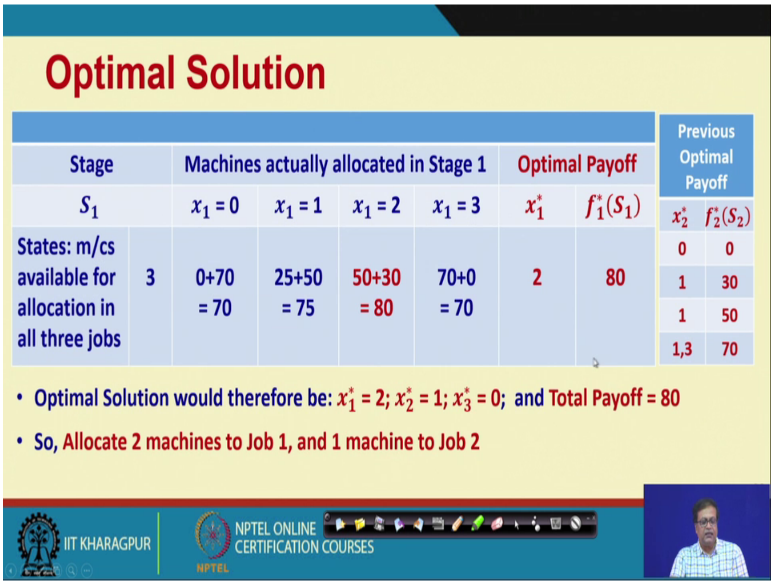

So, all of these are combined here. And when you combine then you see x 1 star is 2 and x you know the x 1 is 2 that means 1 is left and that 1 is here, so that 1 x 2 star for 1 is 30, so that 30 you have added.

So, that means, x 2 star is 1 and if you really go back to x 2 star, then you see that if you have x 2 star equal to 1, then really that it is that x 3 is 0 the really that calculation 30 came from x 2 equal to 1 and x 0 equal to x 3 equal to 0. So, even you combine them all, you get x 1 star equal to 2, x 2 star equal to 1, and x 3 star equal to 0 and the total pay off equal to 80. So, the final answer will be allocate two machines to job 1 and one machine to job 2 right.

So, that is what is shown here in the diagram that you know two machines get a profit of 50, return of 50; 1 machine get a return of 30 nothing at the stars stage 3 that is 0 and therefore, 80 which is the total maximum possible payoff or return is it all right. So, this is an extension of our investment problems you know similar logic we could use. And using similar logic we have been able to find out answer for a machine allocation problem.

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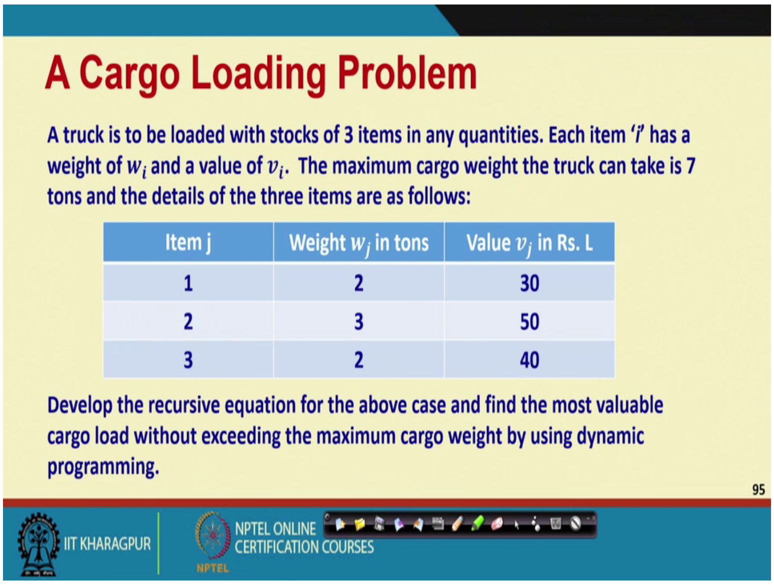

Now, go a little beyond. And see a special class of problem which can be called a cargo loading problem. This problem is slightly different from the kind of problems that we have taken up so far right. So, so far we have taken essentially two types of problem, one is the distance problems, another is the investment problems. Several other problems like project allocation problem, the machine allocation problem or similar such allocation problems will follow the board logic of the investment problem, is it alright.

The distance problems again the shortest path problem, the longest path problem etcetera. The cargo loading problem you will very soon see has got certain special characteristics. So, what are they? See, essentially a cargo loading problem is like this there is a truck which is to be loaded let us say with a stock of three items in any quantities. So, we have basically pool of three items right, any number you can have large number and you have to load the track. But you know that truck has got a maximum weight limit. The maximum possible weight limit in the truck is you know 7 tons in this particular example. The maximum cargo weight the truck can carry is 7 tons.

Now, the item 1 has a weight of 2 tons; item 2 a weight of 3 tons; and item 3 an weight of 2 tons. But the values in rupees lakhs are different. For item 1, it is 30; for item 2, it is 50; item 3, it is 40. So, the question here is how many number of different numbers of item 1, item 2 and item 3, you can pack inside the truck, so that you carry maximum possible value, is it all right.

So, basically develop the recursive equation for the above case, and find the most valuable cargo load without exceeding the maximum cargo weight by using dynamic programming. So, what has been different here. Earlier you know we used to know how much because in the machine allocation problem we have precisely 3 machines. So, we could allocate between 0 to 3 machines to give n jobs, but here how many items do I really have you know we really do not know, we do not know. We like to pack as many items that we can actually have so as to maximize value.

So, therefore, we should do really the states you know how do we define, can you think of what is that item I mean what is it that should give us the consideration of the stages, the consideration of the states, and the consideration of the decision variables, think over a little bit.

See, the value it cannot be value, it cannot be value because value is something which is our objective function. And based on the value we have to maximize the value at a different given cargo that is the truck should have a maximum possible cargo value. It cannot be the item because we really do not know how many items we really have. So, it has to be then it has to be weight right so that how it should be done these cargo loading problem.

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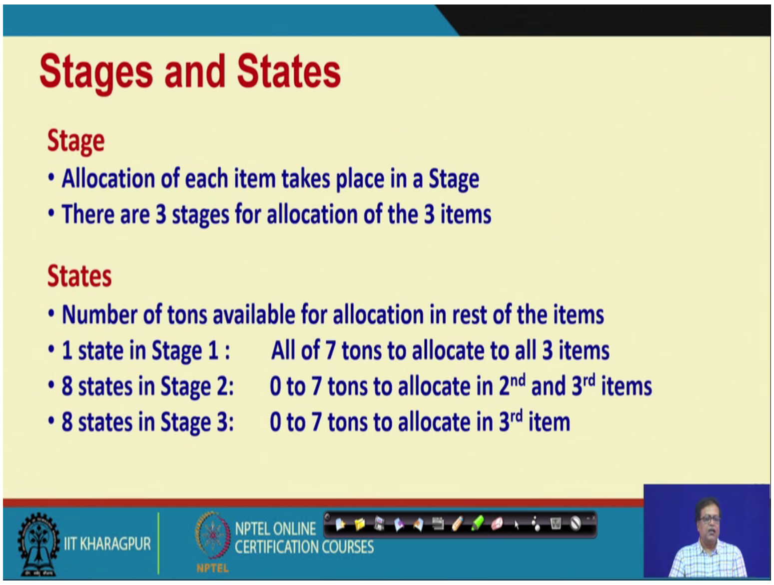

That is the stage the allocation of each item takes place in a stage, and there are 3 stages for allocation of the 3 items. The number of tons available for allocation in rest of the items that should be the state right; So, as I said that we have 7 tons that is available; and our state should be defined in terms of the weights is all right; The weight that we have available to allocate to the item right.

So, in the stage 1, we have only one state and all the 7 tons to allocate to all the 3 items; in the stage 2, we have again 8 possible states that is 0 to 7 tons to allocate in the 2nd and 3rd item. And we have 8 possible states in the stage 3 that means, 0 to 7 tons to allocate in the 3rd item.

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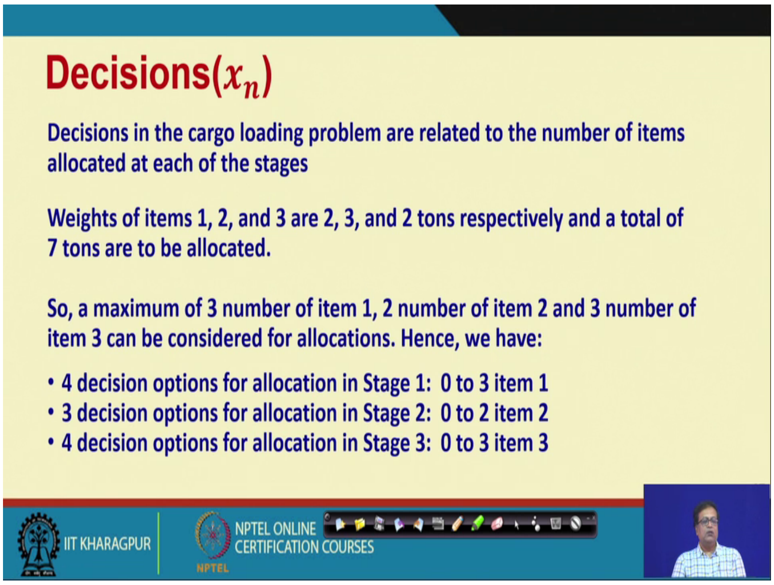

So, what should be sum of our decisions? The decisions in the cargo loading problem are related to the number of items that you allocate. Now, here comes the interesting thing, how many items can you really allocate.

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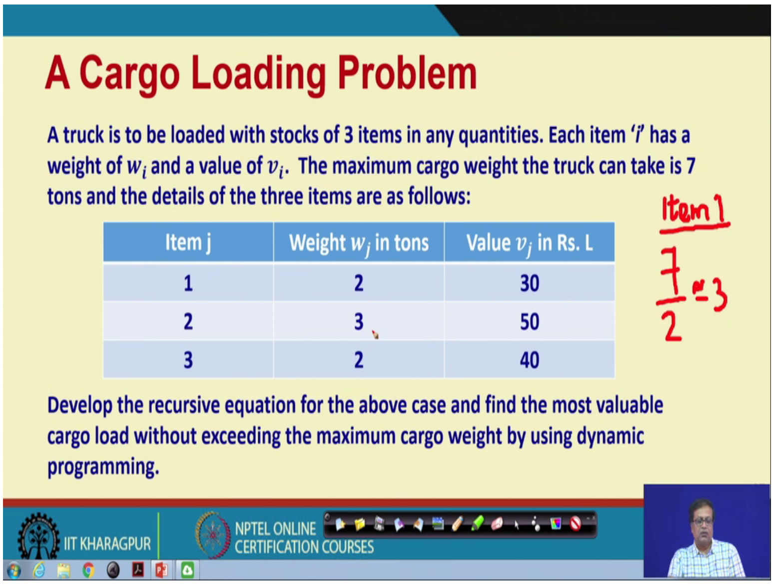

So, before seeing the slide let us look at these once again; You see total we have 7 tons. So, if we have 7 tons, how many of item 1 can you really load into the truck. If the item 1 has got 2 tons weight, you know what should be it, it should be 7 by 2 , but 7 by 2 is 3.5, but you cannot have you know approximate that is fractional number of items. So, item 1 will be only 3 numbers.

So, can you see that that item one all that we can have only 3 numbers because total we have 7 tons, and weight is 2. So, how many of item two can we have, item 2 we can have will be again 7 by 3 that is 2 numbers. And again item 3, 7 by 2, fractional, but we cannot have fractional. So, we can have that is again 3 items so that means 3 items of item 1, 2 items of item 2 number of item 2, and 3 number of item 3 that is the maximum number that we can have is it all right. So, you must remember this.

So, if you remember this then that will give us our decisions. Let us look at our decisions. The decisions in the cargo loading problem are related to the number of items allocated at each of the stages. Now, weights of items 1, 2 and 3 are 2, 3 and 2 tons respectively. And a total of 7 tons are to be allotted. So, a maximum of 3 number of item 1, 2 number of item 2, and 3 number of item 3 can be considered for allocations right. So, we have seen how why it is.

So, therefore, 4 decision options for allocation in stage 1 that is 0 to 3 items of 1; 3 decision options for allocation in stage 2 that is 0 to 2 items of 2; and 4 decision options for allocation in stage 3 that is 0 to 3 item 4 this is all right. So, once again since total was 7, and weight of the items was 2 tons that is item 1, so maximum 3 number of item 1, you can have.

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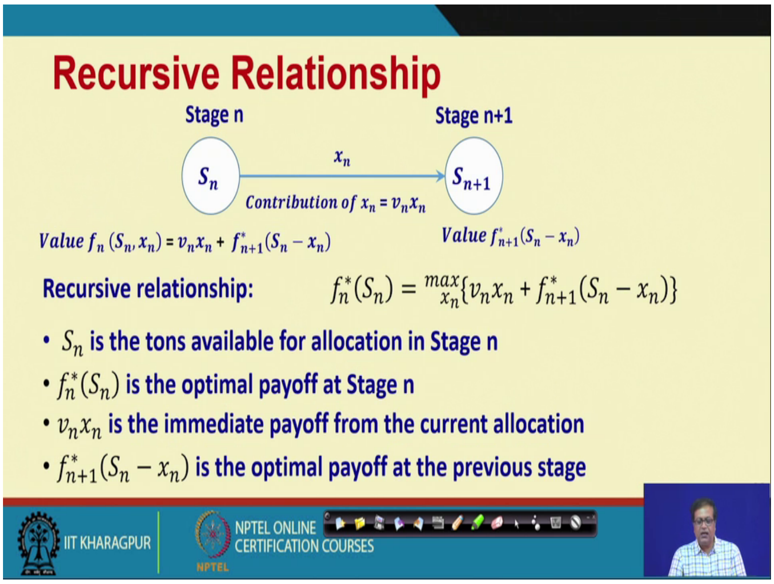

So, let us look at the recursive relationship. Fortunately for us the recursive relationship is very similar to the recursive relationship that we discussed so far that is f n star S n equal to maximum of x n v n x n that is the value current value plus f ns plus 1 f ns n plus 1 star S n minus x n that is optimal at the previous stage.

So, optimal at the previous stage and the current contribution, and then you maximize. Why S n minus x n, because S n was available, x n you have allocated in these particular stage, so S n minus x n is available at the previous stage. So, optimal value at the previous stage S n minus x n f star n plus 1 and plus the current contribution that should have given you the optimal value at this stage.

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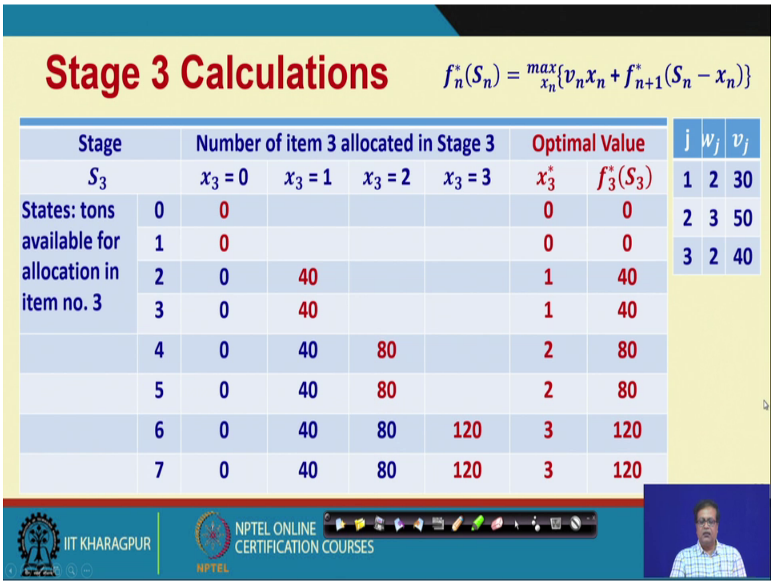

So, using this recursive relationship, let us do calculation. So, let us go to the stage 3. Now, in the stage 3, the stage can be defined as the tons available for allocation in item number 3 right. So, how many tons are available the totally anything between 0 to 7 so, that is what is written here 0 to 7.

And if you allocate only 0 items say how many items can be allocated, see this is the values written that the third item weight is 2, value is 40. So, if you allocate none, then irrespective of how many you have you get 0 value. If you allocate 1, it will be 40; if you allocate 2, it will be 80; and if you allocate 3, it will be 120. So, these are straight forward calculations.

And only thing to remember is that if you allocate 1, since its weight is 2 tons, therefore you know only at 2 tons you know you can start getting value. But if you had 3 tons available obviously, since the weight of the item three is 2 tons, you cannot make more than 1. But only 4 tons are available you can make you know more than that, but then you have done only 1 so you know nothing changes.

So, that is why it is that you know only if you have 4 available only then you can make 2; if you 6 six available, only then you make 3, and therefore, those are the optimal ones. For example for 80, you know the optimal number is 2; and for these 80 also optimal number is 2. So, these are the optimal values at the last stage.

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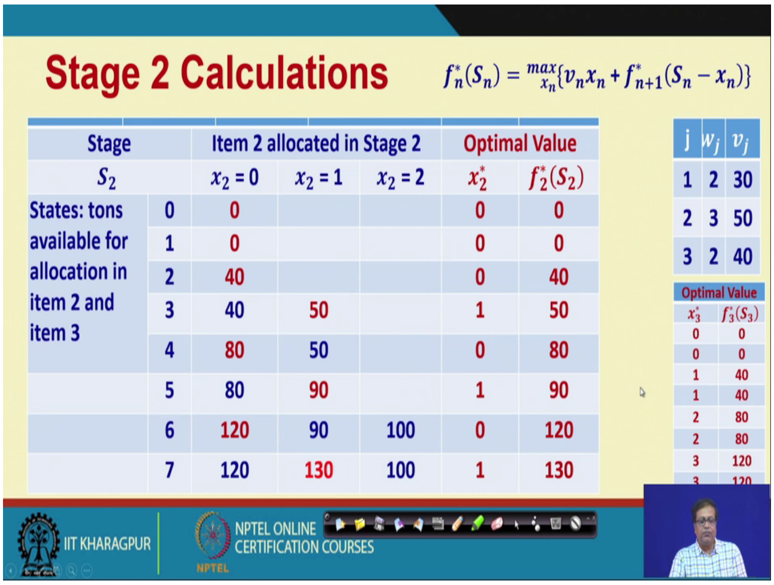

Now, let us see what is the optimal value at the second stage. At the second stage, we are considering the item 2; and at this stage that states are terms available for allocation in items 2 and item 3. So, you see again anything between 0 to 7 that is available, and since x 2 weight is 3 only up to 2 can be made.

So, it could be either 0 or it could be 1 or it could be 2. If it is 0 and 0 is available then obviously, you do not get anything. But if you have two available, then what will happen at the 3rd stage you know you make you know one number which is giving you a profit of 40, so that is that is a previous stage values are given here.

So, at the 3 level, when 3 are available you know two or more are available. So, 0, 1, 2 is available you start getting 40, and then 40 and then 80 and then 120. So, these are the optimal value. Since, for S 2 equal to 0, at this stage the v n x n is actually 0, the f n star n plus 1 S n minus x n these values. So, those values are replicated at x 2 equal to 0. At x 2 equal to 1, if you have 3, available then you get a profit of 50 right.

But if you have 4, then you have only 50 and but if you have 5, then with 2 you add a 40 that is from previous stage so 90. And if you have 7, then you had 2. So, you get a profit of 130 is it all right. And if x is equal to 2, then you cannot have anything else, so it will be only 6 or 7, and then 100. Why you cannot make anything else because if x 2 is equal to 2 that would require that will take 6 tons and have only 1 ton available; with 1 ton, you cannot add another item right

So, with all these values, we now see the optimal x 2 star those are 0 0 0 then 1 then 0 then 1 then 0 and then 1, and these are the optimal values at the second stage. Now, let us go to the last stage that is the stage 1. So, at the stage 1, we have all the seven available, and these are the optimal values. If you allocate nothing, then for 0 the optimal value was you know 130 that was from the previous stage.

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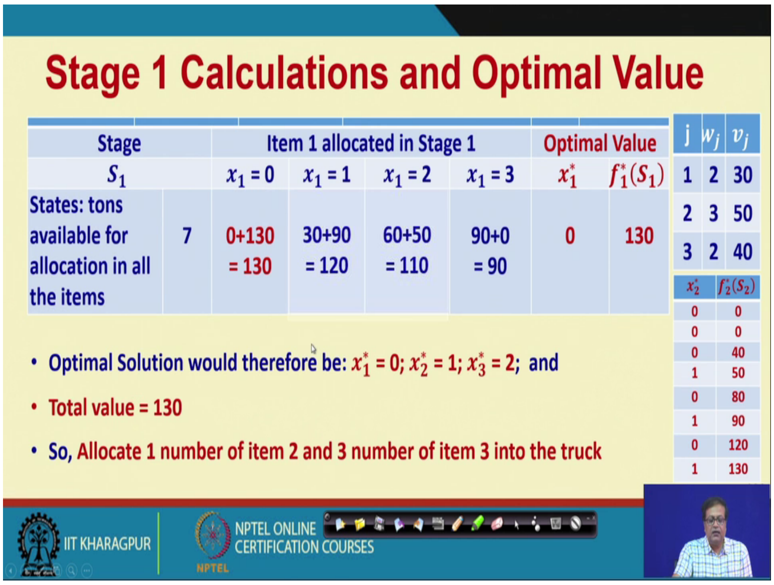

So, if you allocate none that is x 1 equal to 0 that means, the full 7 is available for the next stage which is given here. The full 7 if it is available then you get an optimum of 130, so that these portion is 130 in the previous calculation that previous stage optimal decision is 130, v n x n is 0. So, 0 plus 130 that is 130. But if you have 1, then you make a profit of 30 that is what is 1, but you have already taken over 2 that means, you have 5 left; So, with 5, 1, 0, 1, 2, 3, 4, 5 so that would be this.

So, like this you can actually calculate. And you can see that I know you can therefore, make different profits, and finally the optimal one is the first one that is 130. So, the optimal solution would therefore, be x 1 equal to 0, x 2 equal to 1 and x 3 equal to 2 with a total value of 130. So, the final solution is allocate 1 number of item 2, and 3 number of item 3 into the truck right so that is how you solve the cargo loading problem. We shall see more of these problems in the next set of lectures.

So, thank you very much.