Selected Topics In Decision Modeling
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Lecture – 05
Further Examples

So, today let us revisit some examples, which we have already done similar type of problems earlier, but let us do some of the problems again so that our understanding becomes clearer. So, that is what is our topic today; that is further examples for dynamic programming.

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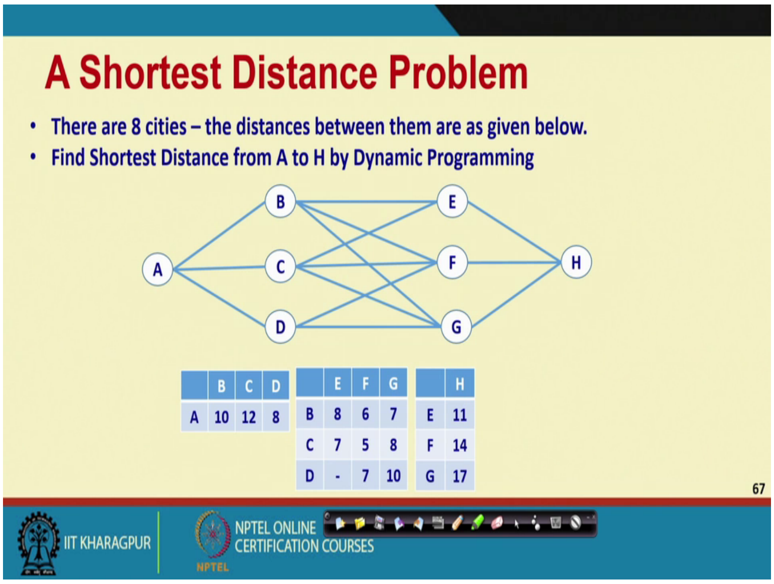

	B	C	D
A	10	12	8

	E	F	G
B	8	6	7
C	7	5	8
D	-	7	10

	H
E	11
F	14
G	17

Two problems we shall be taking up in this particular exercise; one is a shortest distance problem and the other is the longest distance or the longest path problem. The problems are very similar, but you know the conceptually one where you try to find out the minimum possible path from the source to the destination, and the other one the maximum possible distance from the source to the destination.

So, like the stagecoach problem that we have already discussed, let us see we have a number of cities to be precise in this particular example let us take suppose there are 8 cities; A that is at the first level which may be called the source, then B C D at the next level, then E F G and finally, H. And the distances between these cities are given, if you just see that while all possible paths are available, but there is no path between D to E.

So, precisely there is no path between D to E. So, you can understand that you cannot go from D to E.

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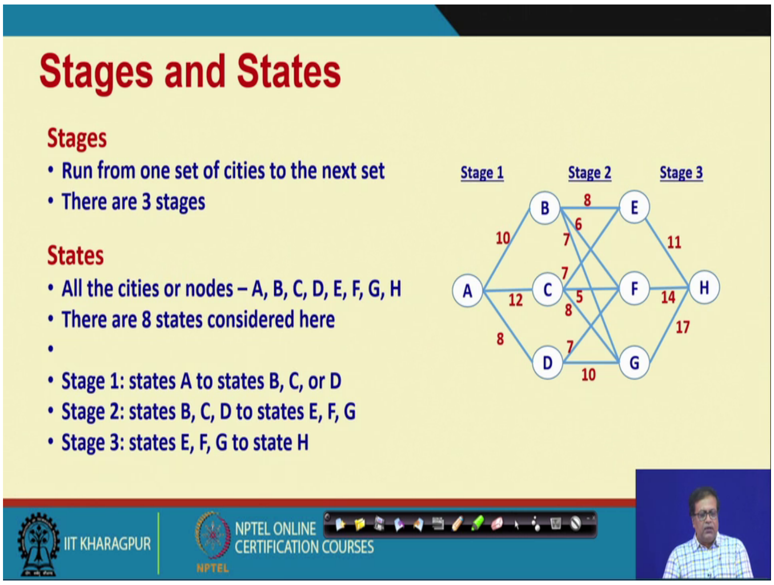

Now, in any dynamic programming setup, what we need to really find out I mean really first obtain the stages, the states and the decisions that we are going to have for such kind of problems.

So, the 3 stages are essentially is the run say precisely, if you are moving by stagecoach then the stagecoach run from the first set of cities to second set of cities, that we can call as first stage. From the second stage of cities to the third stage of cities that will make all the stage 2, and the third stage of cities to the destination as the stage 3. So, like A is the source. So, when you move from A to B or C or D that is our stage 1 and B C D to E F G H is our stage 2, and E F G to H that is our stage 3. So, these are our 3 stages the states the all the cities are nodes that is A B C D E F G H they all represent an individual state.

So, as you know you are moving from one set of cities to you know you are only moving from one city to the other, but those are the choices, the alternatives that are available. So, as you move from a city in the first level to the city at the next level, you know you are making a change of state. So, you make one change of state in every stage. So, since there are 3 stages, you make 3 state changes is it alright, but then the possible states that you can have from A you can go to one of the 3 possible states.

So, that is how the states could be defined. So, as you can see in stage one you can go from A to B or C or D. A stage to you can go either from B to E F G or C to E F G or d to E F G and finally, from E F G to H. So, these are the possible states. Obviously, there could be a solution from you know by really have been kind of a brute search or a total search kind of things, but dynamic programming will reduce the number of iterations, that is what we are attempting here.

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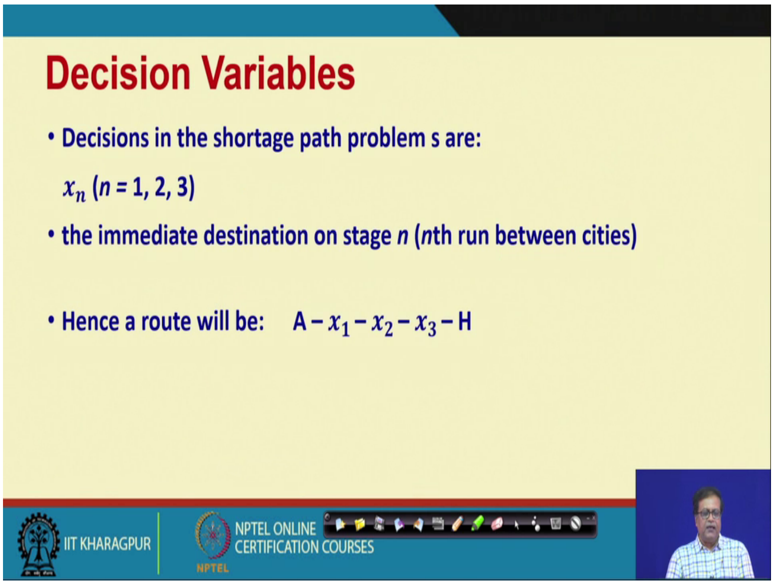

Now, the decision variables for such a problem that is a kind of a shortest path problem, can be defined as you know x n equal to you know where n could be 1 2 or 3 depending on the stage, the immediate destination is it alright.

So, supposing from A if our immediate destination is B, then we know A B is our decision variable is it alright that is our destination. So, it really represents that from A we move to B; then if E and then finally, H is our next set of distinctions then final path will be A B E H. So, that is what is shown here that could be called A route the route will be A to x 1, to x 2 to x 3 to H. So, once again we have defined a set of stages there are 3 stages between movement from a city to the next city, the states are the cities of the nodes all right. So, there are 8 of them and the decisions are the destination on every stage; So, that if we know the destinations on every stage, we can really know the route or the path.

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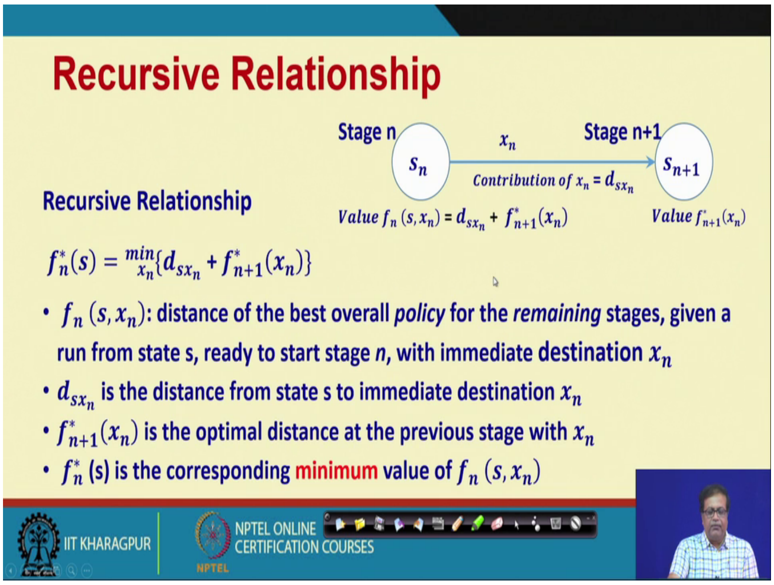

So, how do we move about? The first thing that we must have, we must have a recursive relationship that is the essential idea of a dynamic programming. So, like before and by applying Bellman's principle, the optimum value will be a candidate which will be the a value that is f n s, x n is it not that value which will really the contribution for the current lake that is the decision that you make that is our x n plus the optimal decision at the previous stage, that is n plus 1, if you are really going for the backward dynamic programming.

So, since we are coming from back that is why the S n can be calculated based on S n plus 1. So, if f star n plus 1 x n is our optimum value at the next stage, then if you add that to the current contribution of a particular destination chosen by you that is d s x n that will give us a given value. And the optimal value at this current stage s will be f n star S minimum of this x n d x n plus f star n plus 1 x n. So, that will be our the recursive relationship.

So, these recursive relationship we have we shall actually apply to such kind of problems and you know use them to find out our minimum value. So, already we have discussed this in previous problems.

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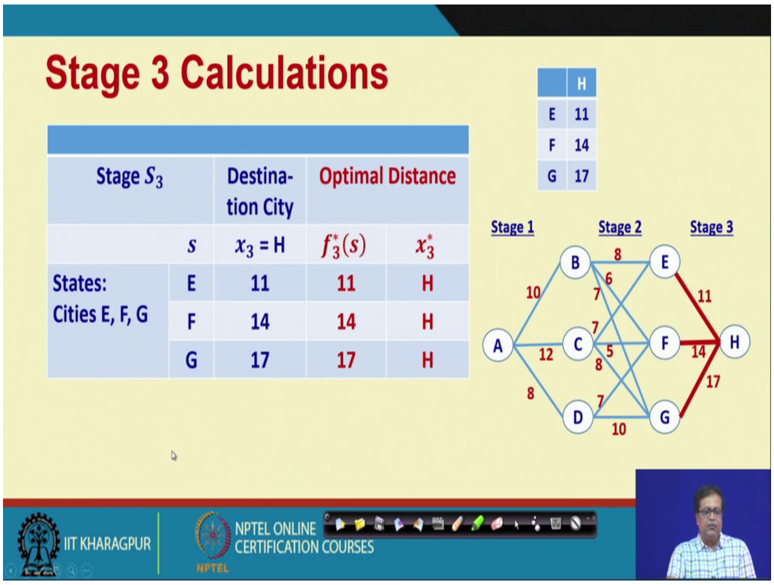

So, without wasting any more time let us move over to the stage 3 calculations. So, at the stage 3 if you if you look at this particular diagram, the destination is only 1. So, since there is only one destination we do not have much choice you know at every you know situation, we have to choose that particular destination.

And the distances are already given that is from E it is 11, from F it is 14 from G it is 17. So, therefore, you know from E these are the I mean from E the distance is 11 and there is only one choice that is x 3 equal to H. The x 3 is not going to vary that is that the decision that we make at the third stage.

So, therefore, you know the optimum decisions will be 11, 14 or 17 and that can be really the x 3 star will be H. So, you know it has been marked here. So, you know you what you really got from here that at the third stage you know the for reaching the destination H from S that is let us say E, the optimal distance is 11 right.

So, we already got that from E the optimal distance will be 11, if we have to reach H. So, that is what we have at the stage 3 calculations. Now, using this stage 3 calculations by using the recursive relationship, we can really then find out the stage 2 calculations.

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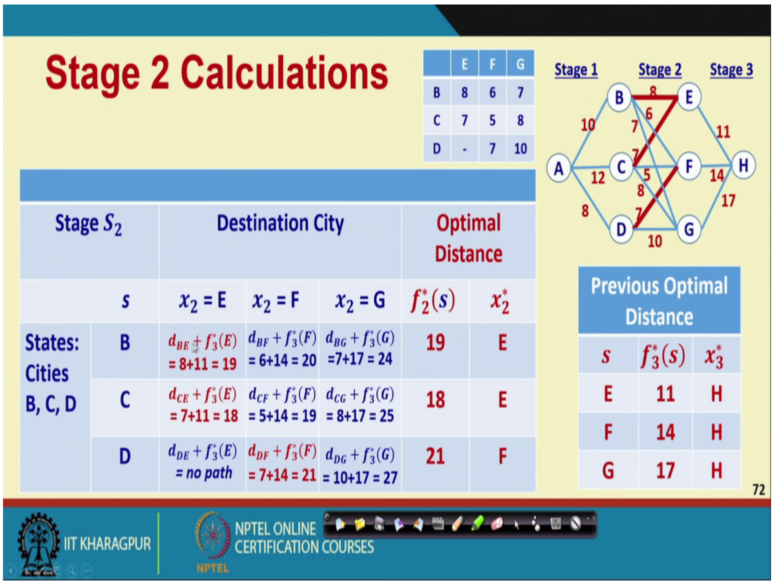

	E	F	G
B	8	6	7
C	7	5	8
D	-	7	10

Stage S_2		Destination City			Optimal Distance	
	s	x_2 = E	x_2 = F	x_2 = G	$f_2^*(s)$	x_2^*
States: Cities B, C, D	B	$d_{BE}+f_3^*(E)$ = 8+11 = 19	$d_{BF}+f_3^*(F)$ = 6+14 = 20	$d_{BG}+f_3^*(G)$ =7+17 = 24	19	E
	C	$d_{CE}+f_3^*(E)$ = 7+11 = 18	$d_{CF}+f_3^*(F)$ = 5+14 = 19	$d_{CG}+f_3^*(G)$ = 8+17 = 25	18	E
	D	$d_{DE}+f_3^*(E)$ = no path	$d_{DF}+f_3^*(F)$ = 7+14 = 21	$d_{DG}+f_3^*(G)$ = 10+17 = 27	21	F

s	$f_3^*(s)$	x_3^*
E	11	H
F	14	H
G	17	H

So, let us come to stage 2 calculations. Now here in the stage 2 calculations we are really seeing from set of cities B C D to a set of cities E F G. Now please look that B C D this set of cities to these are our state the states are cities B as B C D and the destination city which are decision is E or F or G and we have to see which one is the optimum you know optimal distance from B from C or from D.

So, to calculate we calculate the optimum value from E to H, now look here the if the distance is from E to H, the optimal value is 11. The from F the optimal value was 14, and from G the optimal value was 17. So, these are our previous optimal distances, which we have already obtained from our previous calculations. So, these previous calculations we will use in the recursive relationship that is from B to E it should be the distance between B to E plus the optimal decision from f 3 star E. So, that would be 8 plus 11 because B to E is 8 and 11 is optimal decision here.

So, these 8 and 11 will give us 19; that means, B to E and optimal decision will be 19 then B to F. The B to F will be the optimal at F what is the optimal at F 14 and what is the distance from B to F it is 6. So, that gives us 6 plus 14 equal to 20 right. So, similarly we can calculate from a given city to a destination depending on what we have. So, if you see that for B to G the calculation is d B G plus f 3 star g which is 24 from C, C to E d C E plus f 3 star E that is 18 and C to F 19, C to G 25 and D to E there is no path right D to E we have no path then D to F 21 and D to G 27.

So, in order to find the optimal distances we have to then see row wise what are our best possible paths. So, if I see from B the best possible destination to choose is E because calculation show here it is 19, here it is 20 and here it is 24. So, we get B to E as the optimal distance. So, B to E, E is our optimal destination and 19 is the optimal value. Similarly from C the optimal destination will be E again and that particular value the optimal distance value will be 18 because this comes out to be minimum out of 18, 19 and 25.

Finally from D there is no path from E up to E, there is 21 value for F and 27 since 21 is minimum. So, d D F will then become 21. So, these gives us the optimal distances for our next set of calculations. So, you know and these paths are indicated here also that from B the optimal distance is E, from C the you know the optimal distance is again E and from D the optimal is F is alright.

So, now we have the optimal distance from B to H, C to H and D to H, which are you know given at the you know here that is 19, 18 and 21.

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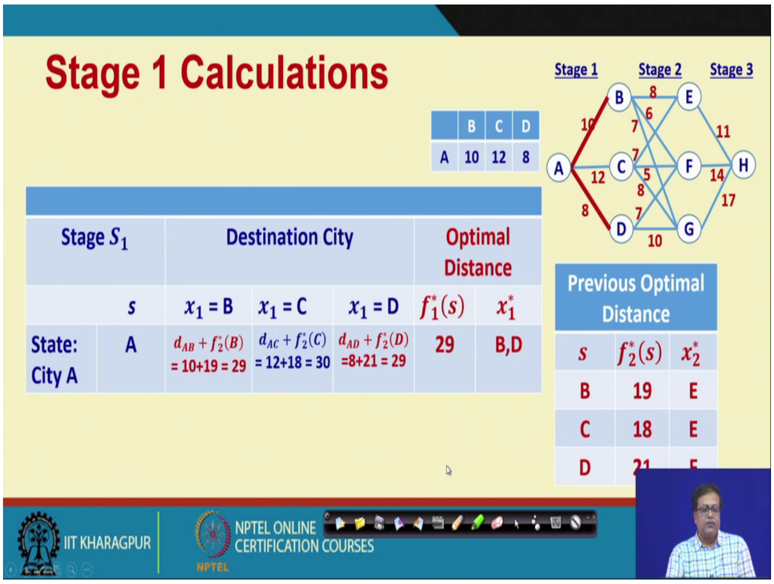

These facts we can make use in our final stage that is stage 1 calculations. In the stage 1 we are really from calculating from city A. And from city A you know again we can go to one of the destinations that is B or C or D and if I have go for to B, then d A B plus f 2 star B and f 2 star B from our previous optimal calculation we found it is 19. So, it should be A to B the distance is 10. So, 10 plus 19, 29 for C it is 12 plus 18 30, for D it is 8 plus 21, 29 and then since 29 is the is the shortest.

But it can be both for destinations B and D. So, therefore, you know the x 1 star will be B or D and f 1 star S will be 29. So, if you look at these calculations carefully you could see how the Bellmans principle has operated at each of the calculations. See really speaking although there are. So, many combinations of cities all that we had done at a given stage is only seen the distances from a given city to its possible destinations is it alright.

So, at the last stage we had seen 3 combinations at the second stage we have seen 9 combinations one was not available, and at the last stage we have seen another 3 combinations. So, 3 plus 9 plus 3 around 15 combinations only we have seen.

And the good thing is you know if we have more stages just assume another two-three stages, all we have to add is you know as you add more number of stages it does not increase in an exponential manner, it actually increases in a linear manner that is where the advantage of dynamic programming. Because it is a very small 3 stage problem, the advantage is not really seen. But just assume you know we have something like a 7 stage or 8 stage or even 20 stages.

So, assume a set of cities with 20 different stages, all that is happening from the 3 stage from 3 stage to fourth stage to fourth stage to fifth stage and so on we simply add linear number of you know combinations and we can still solve you know a 20 stage problem of shortest distance problem by dynamic programming in finite time without much difficulty.

That is where the real advantage of dynamic programming comes in. While it happens because once we find out the optimal path from you know the previous what you call stage, we can really utilize those optimal values with the current calculations or the current distances real advantage really comes from there.

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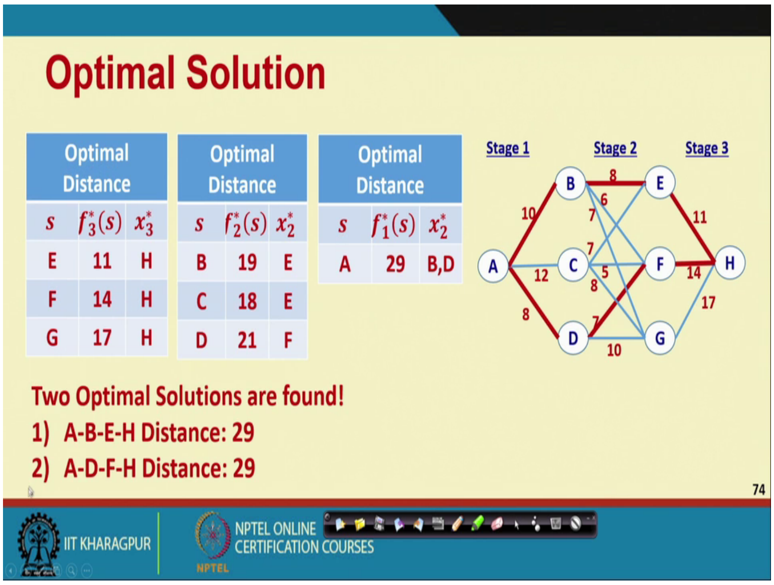

So, what we do here that we then combine all the optimal distances at the different stages. So, this is our stage 3, stage 2 and stage 1. So, if I really combine them all then you see from A the optimal is B or D from B the optimal is E, from E the optimal is H. So, it is A to B to E to H and A to D because from D optimal is F. So, F to H. So, A to D to F to H and the distance will be 29.

So, that is how you can combine them all and really get the optimal solution from for such a shortest distance problems and here it is shown in the graph you go from A to B from B to E and from E to H and the distance will be 29.

And the other optimal there are 2 optimal paths that is A to D to D to F and from F to H, which will be also 29 right. So, we earlier have seen the stagecoach problem and now another example of similar type we have seen where how do we really find out the shortest distance between a source city to a destination city, and I am sure that using these examples you will be able to solve all such shortest distance problem.

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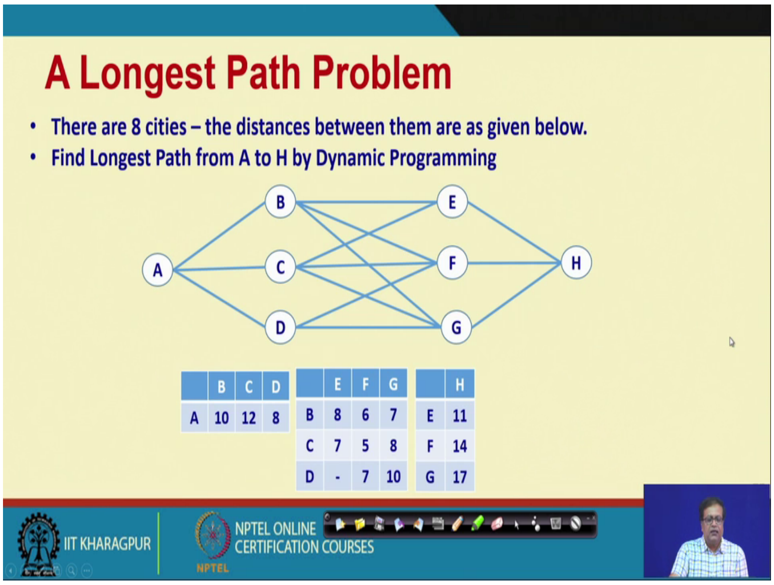

But now let us look at the same problem once again, but from a different angle. Now instead of the shortest distance, we try to find out the longest distance or the longest path between the source city to the destination city. So, if you really have to find out the longest path from the source city to the destination city, what would really change. Will there be any change in the stages the way we have defined? Answer is no. because problem structure is the same. Will there be a change in the states? Again that will be no change in the states.

States will again become the cities and the stages will become every run right could be stagecoach run or vehicle run truck run or any such thing. From a given city I mean a set of city to the next you know level of cities. The destinations I mean that is the decision variables that will be also going to be the same. So, really speaking therefore, the if we really find out the longest path in state of the shortest path, you know all that we really change is the way we calculate our optimal distances is it alright.

The change here is going to come in the way the optimality is calculated, rest of the calculations will be very similar let us see then how they combine.

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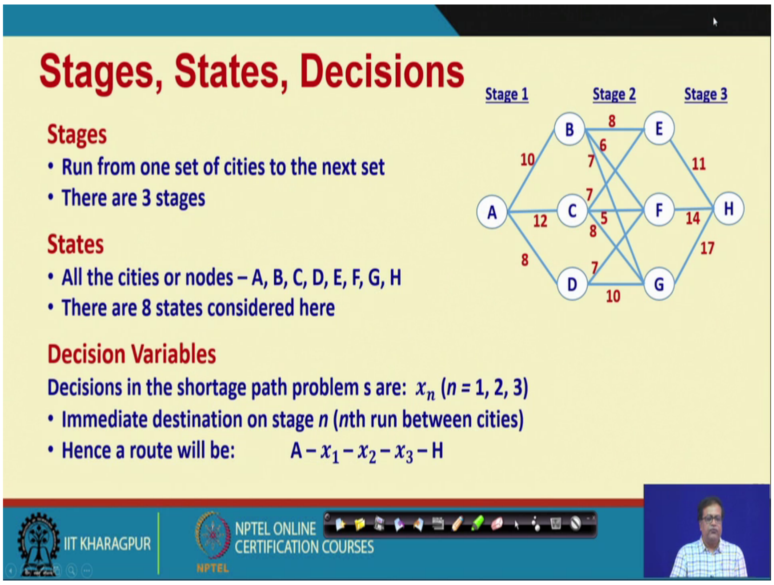

So, first of all as I have really said that stages are the run from one set of cities to the next set and there are 3 stages. The states all the cities are notes that is A B C D E F G H and there are 8 states considered here. The decision variables the decisions in the shortest path problem s will be again x n where n equal to 1 2 3 right.

So, x 1, x 2 or x 3; So, these decisions will be nothing ah, but you know the destination city the immediate destination on stage n so; that means, for A the candidates could be B C or D and the second stage the candidates could be E F or G, and at the third stage the candidate could be H only and therefore, the final route will become A to x 1 to x 2 to x 3 to H.

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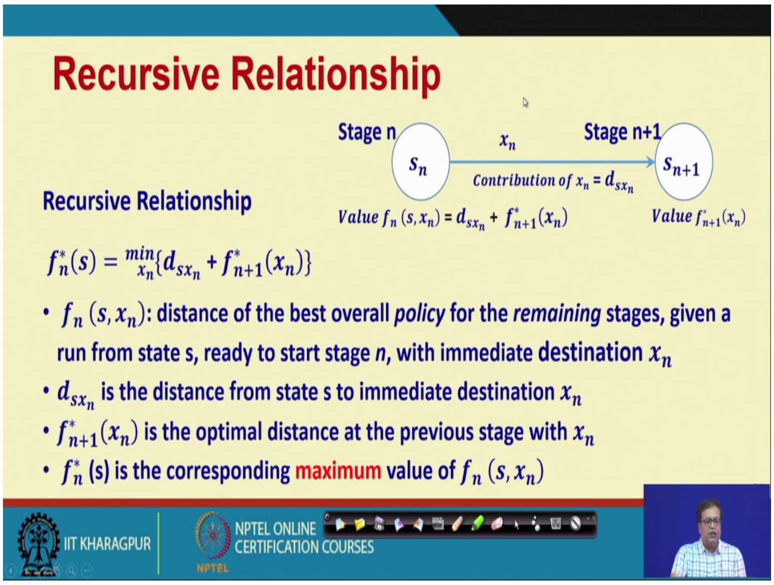

Now, let us see the recursive relationship, here we have made a small mistake the mistake is like this that, this one should not be min it should be max right.

Because that is the change really we have that this one should be maximizing rather than minimizing. So, we really have to maximize. So, let me write it once again that it should not be minimizing, it should be maximizing. So, you see that mistake if we correct then automatically we get the longest path. So, at any point that in fact it highlights, this mistake highlights the change from the previous problem that is the shortest path problem. In shortest path problem we were minimizing in the longest distance problem the longest path problem, we are really maximizing the distances.

So, but then rest of the things are all same that f star n S will be equal to maximum of that x n, which is d s x n the distances the immediate distances or the contribution of x n plus f star n plus 1 x n that is the optimal at the previous stage, rest of the things are as we have seen earlier right.

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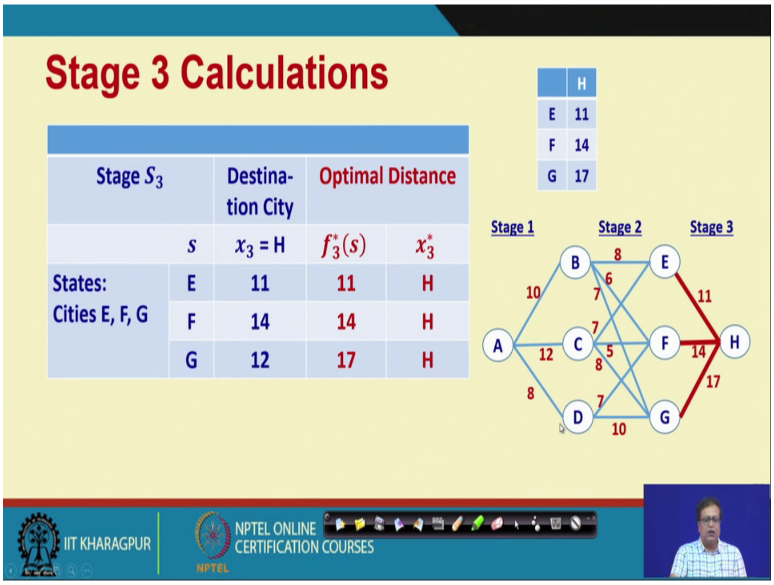

So, the next one will be our. So, if we really do the stage 3 calculations now, in the stage 3 calculations we can see that again the destination city is only 1 that is only H because from E F G in the last lec we can go only to H because that is our final destination.

So, you know here that S is E or F or G then your H there is one more mistake here that is this value should be 17 right. So, this value is not yes 17 that is fine. So, this you know if you really look at this then from E to F to E to H is 11 from F to H is 14 and G to H is 17.

So, there is no other distances that is available so; obviously, they must be optimal; that means, what is the longest path from E to H? It has to be 11 because there are no other paths right similarly from F to H should be 14 and G to H it should be 17. So, that really gives us the stage 3 calculations and the optimal distances the longest distances.

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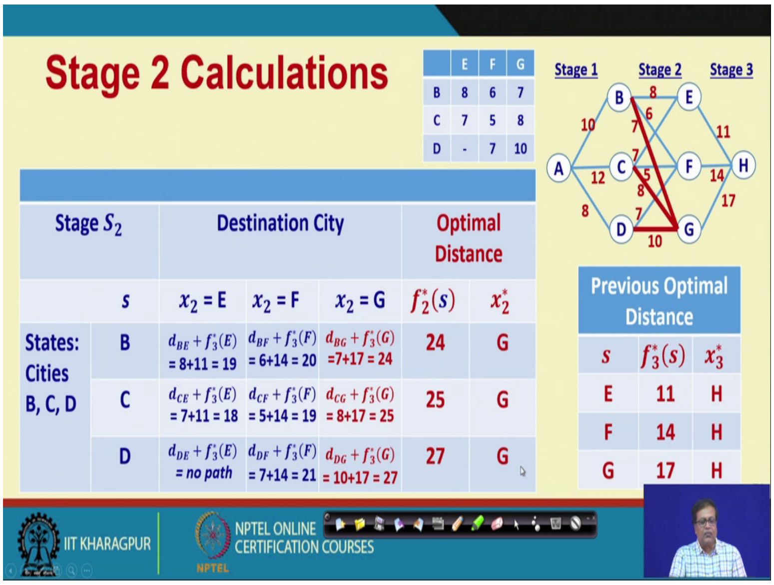

So, if we then move further then we see the at the stage 2 calculation, that is the distances are to be calculated from B C and D to E F and G. So, by similar method the calculations are done once again; that means, see 11, 14 and 17 they were our optimal distances from E F and G. So, we use them and use the immediate distance. So, let us say from B to E the immediate distance is 8. So, if I add this 8 to 11 then I get 19 right.

Similarly, for B to F the d B F is 6 and if I add this 6 to the you know optimal distance from F to H that is 14 we get 20. So, using them you know similar kind of formula, we got all the distance from B to E, B to F and B to G and since B to G 24 is the maximum, that should be the optimal as far as B is concerned. So, really what is the optimal distance from B to H? It should be 24 is it alright so; that means, the optimal that is the maximum possible distance from B to H has to be 24 and it is actually through G. So, therefore, look here in this diagram if I see from B to G is 7 and G to H is 17. So, that is 24.

So, by similar kind of calculations we can do for C for D as well and in all these cases we find that is through you know really happening through the point G right. So, what we really get that from C to H the optimal distance should be 25 and it has to be through G; that means, the immediate destination of C should be G and immediate destination of D should also be G, but the longest durations longest distance from D to H will be 27 from C to H will be 25 and B to H is 24.

So, look here when we talk about optimal distances, these optimal distances are from that given point to the final destination, but the immediate destination is our x 2 star which at G in this particular stage.

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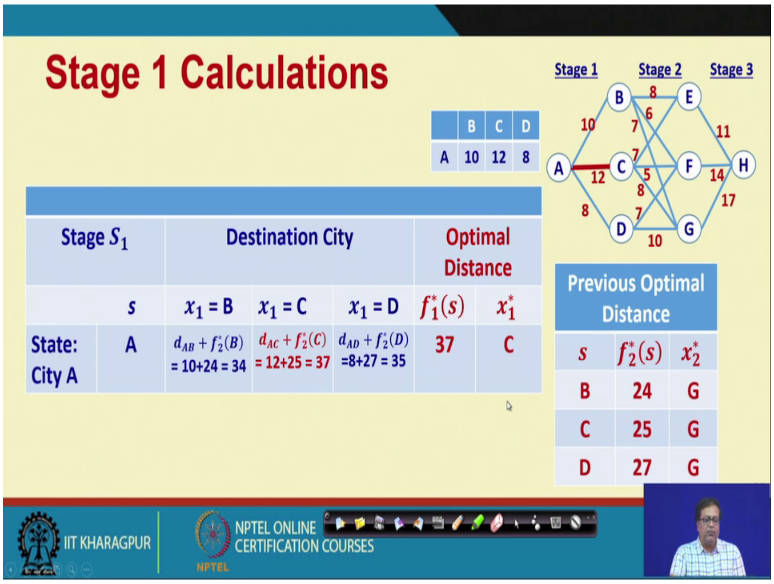

Now, using them we can combine the result in the stage 1, in the stage 1 this (Refer Time: 28:17) the distances are calculated from A to B, A to C and A to D and the immediate destinations are B C and D the optimal distances from B to the last that is final destination was 24, from C it was 25 and from D it was 27.

So, those values are utilized along with the current distance and we got 34, 35 and since 37 is the maximum; obviously, the optimal distance therefore, should be 37 is it alright.

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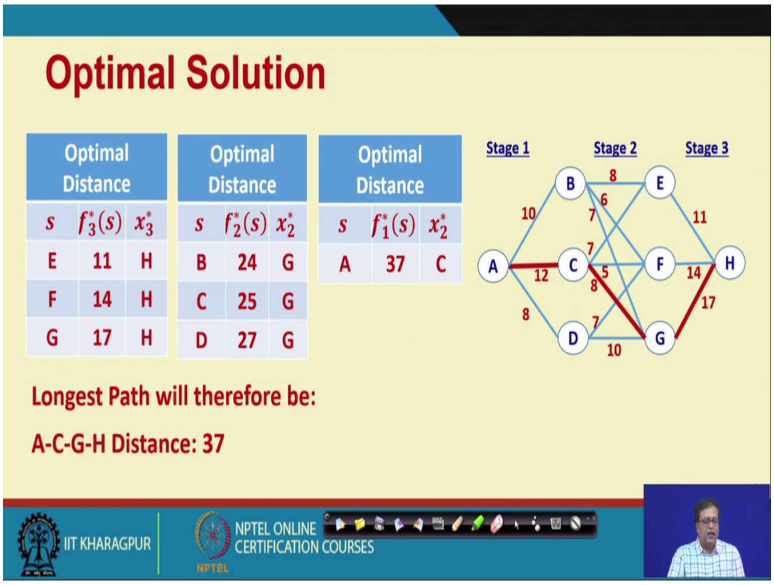

So, that how we should really calculate and these gives us if I compare all of these the longest path will be A C G H that is 37, 12 plus 8 plus 17 right.

So, you see what you have done in this particular lecture, we have really given a look back into our stagecoach problem and we have seen not only how to calculate shortest path, but also how to really calculate longest path right. So, that is how we have done in this particular,

Thank you.