Selected Topics in Decision Modeling
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Lecture – 37
Tabu Search

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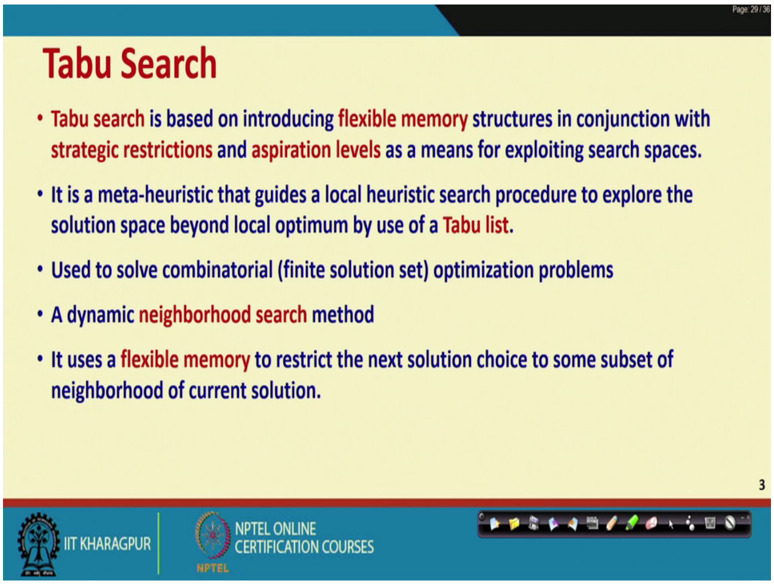

So in our course Selected Topics in Decision Modeling, we are in our 37th lecture that is on Tabu search. Now, Tabu search is meta-heuristic you know technique which guides a local heuristic search procedure and it explores the solution space beyond local optimum by using a Tabu list. So, what is a Tabu list, the Tabu list is set of solutions which are not to be used anymore you know in a sense forbidden.

And you know it uses the flexible memory structure in conjunction with strategic restrictions and aspiration levels. What is aspiration level, we shall come little later. And kind of problem, that it solves are all combinatorial in nature and obviously with a finite solution space; and it is a neighborhood search method. So that means, once we have a point its searches around its neighbors, and there is a flexible memory structure.

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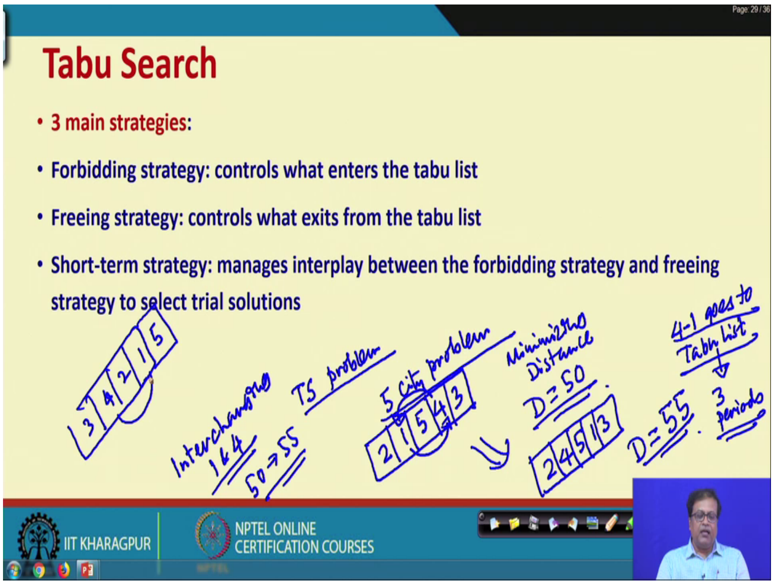

So, there are three broad strategies in Tabu search. This is the forbidding strategy that what goes into the Tabu list; and freeing strategy under which condition that we take out from the Tabu list; and then there is also short-term strategy that manages the interplay between the forbidding and freeing strategies for select trial solutions. Now, the question is that what goes into the Tabu list and what comes out of the Tabu list.

So, let us have an example. Supposing, you know we have a problem let us say the problem on travelling salesman problem TS problem, where a person goes from one city to all the other cities and comes back to the same city. Supposing, there is a 5 city problem right. There are 5 cities, and the person has to go to one city to another city, and then come back to the same city.

So, assume that 2 1 5 4 3 is a current solution that means 2 to 1 to 1 to 5 to 5 to 4 to 4 to 3, and 3 to 2, so that is say our code. Now, what happens let us say that supposing we are minimizing the distance minimizing distance and the current distance is D equal to 50. Now, assume I interchange 1 and 4, the location 1 and 4, and then we get a new solution 24513. So, 24513 is the solution which is a neighboring solution I may call it neighboring solution.

So, this neighboring solution that we have got and let us say it improves the solution value from 50 to 55. So, what has happened by interchanging 1 and 4, we got an improvement from 50 to 55, is it all right. The question is that should I again allow these

1 and 4 to be interchanged. Look here at the solution 1 and 4 are interchanged, and we have got something better; but if we interchange again, then it will become worse, is it not.

So, what we do we then put a restriction that four to one goes to Tabu list 4 to 1 goes to Tabu list. Why 4 to 1 goes to Tabu list, and let us say for certain period say for example, say for 3 periods. So, for the coming 3 periods, we shall not be doing 4 to 1 switch, because by 1 to 4 switch we have come here. If we do again, then we go back to the old solution with a loss of objective function value. So, we should not do that is it alright. So, this is what is actually the Tabu list and, but at a later stage it may so happen that we have may be another you know current solution which may be very different.

Say for example, it could be 3 and then 4 and then 2 and then 1 and then 5. Look here these 3 4 2 1 5 is a very different kind of solution. So, now these 4 1, which was originally in the Tabu that is that 4 to 1 interchange which may not be meaningful anymore; So, by these interchange, we may get actually even better solution. So, these are the things that we need to think about, so that would be our aspiration level, is it all right. So, to really get something better by removing the Tabu, is it all right. So, this is precisely what these different strategies actually mean right.

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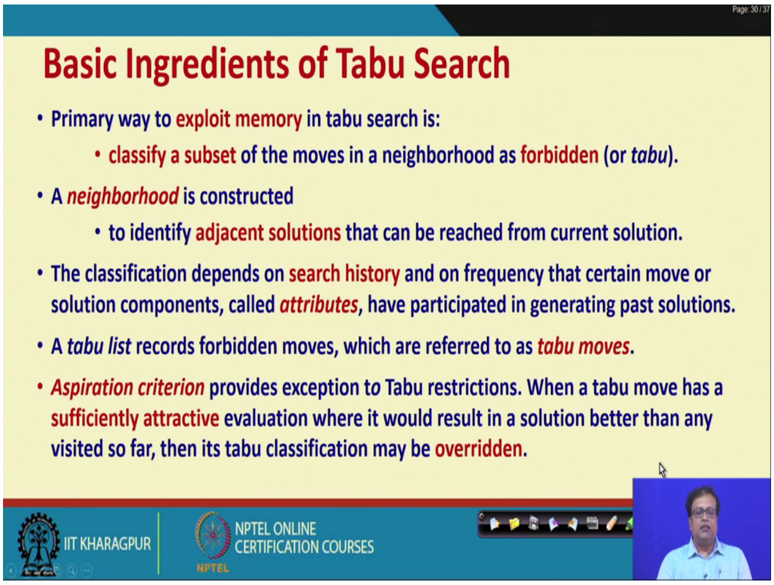

So, having known this let us look some of the basic ingredients of the Tabu search method; So, first of all it is a primary way to exploit memory in Tabu search by classifying a subset of the moves in a neighborhood as forbidden or Tabu. And a neighborhood is constructed to identify adjacent solutions that can be reach from the current solution. The classification based on the search history and the frequency that certain move or solution component called attributes have participated in generating past solutions.

And then Tabu list records forbidden moves which are referred to as Tabu moves. Then this is exactly what I was telling that about that freeing strategy the aspiration criteria provide exception to Tabu restriction. When a Tabu move has a sufficiency sufficiently attractive evaluation where it would result in a solution better than any visited so far, then its Tabu classification may be overridden right. The example I was giving about the travelling salesman problem; one move that was under Tabu list earlier you know it is really going to give better solutions in the next set of iterations. And in that kind of situation, we have what is known as the aspiration criteria, fine.

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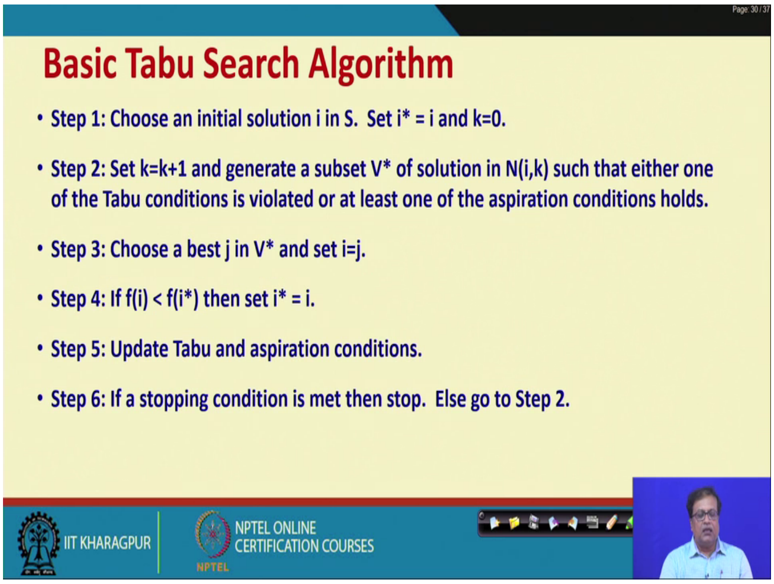

So, having known this let us move over and look at the basic Tabu search algorithm. The first choose an initial solution i in the solution space and then set i star equal to 0 and a parameter number of iterations k equal to 0. Now, set k equal to k 1 and generate a subset V star of the solutions in N i, k, N i k is the neighboring solution of that initial solution i, is it alright; and such that either one of the Tabu conditions is violated or at least one of the aspiration conditions holds.

So, after that choose a best j in v star and set i equal to j. What does it mean? So, in the neighborhood, we have say found out some solutions which are better than the current one, is it all right. So, better or you know they are evaluated with the best of their, what is called functional value, and they are listed as sorted. Now, at least suppose one such solution in the neighborhood is found for which these condition is satisfied. What is that condition that is f i star equal is greater than f i. Obviously in a maximization problem, the value will be higher the new value is higher; in a minimization problem, the new value is lower, so that means, that we have a certain value objective function value the new value is better than the previous value. Then what happens that becomes our current solution so that is set i star equal to i.

So, now, we should you know really look from that point of view. Now, update Tabu and aspiration conditions aspiration conditions could be that we know increases objective function value by a certain amount. And if a stopping condition is met, then stop, else continue is it all right. This is how the basic Tabu search algorithm works, fine.

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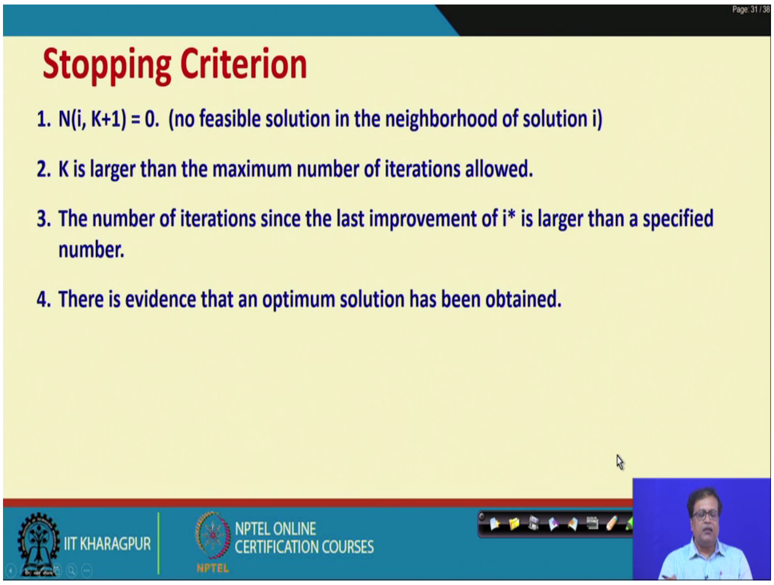

So, having known that let us see some stopping criteria. The stopping criteria is that there is no feasible solution in the neighborhood of the solution i right that means, we cannot find a neighborhood. Then K is larger than the maximum number of iterations allowed that means, total number of iterations are exhausted. And then number of iterations since the last improvement of i star is larger than the specified number that means, we are iterating for quite some time, but we are not getting any further improvement that means convergence is reached. And there is a evidence that an optimum solution has been obtained.

Suppose, we have otherwise some method to know what is the vicinity of the optimum solution. Suppose, we have some kind of the what you call as you know this control limits you know something like the bounds like upper bounds or lower bounds and we have estimated those bounds and the bound solution is reached, then obviously, there is no point continuing anymore.

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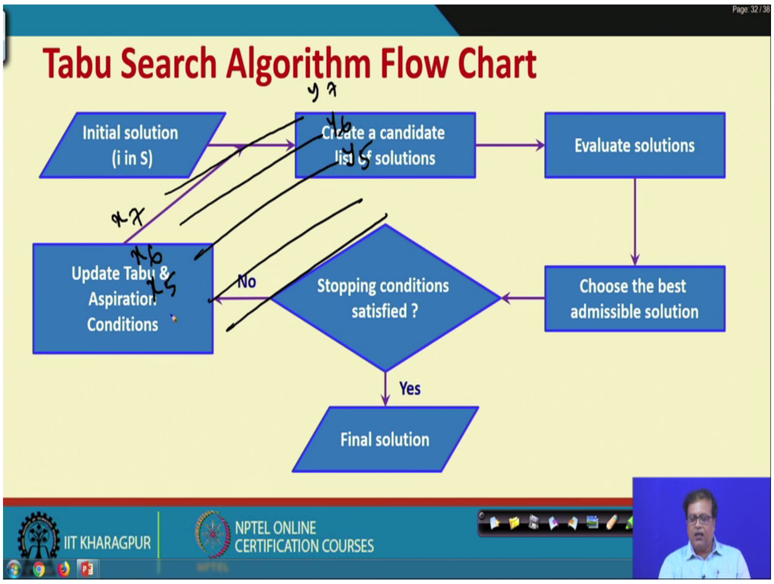

So, this is how it you know in a flow chart kind of thing that you know we have an initial solution. So, from the initial solution, create a condition candidate list of solutions in the neighborhood, then evaluate the solution, choose the best one, then stopping condition satisfied, then final solution otherwise update Tabu and aspiration conditions and continue right. So, that is the Tabu search algorithm flow list.

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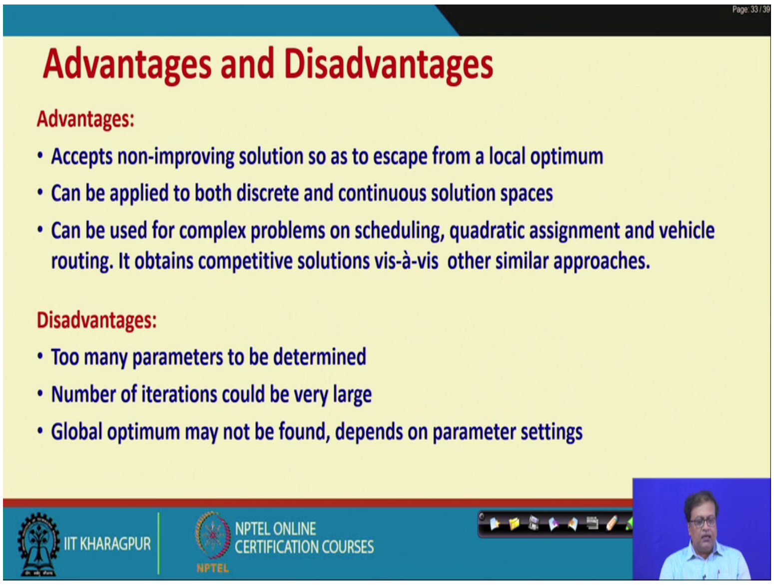

So, having seen that next we continue and see what are some advantages and disadvantages before taking up a practical problem. So, advantages – accepts non-improving solution so as to escape from a local optimum; can be applied to both discrete and continuous solution spaces; can be used for complex problems like scheduling, quadratic assignment vehicle routing; and also obtains competitive solutions vis-a-vis other similar approaches.

Disadvantages – too many parameters large number of iterations, and like any meta heuristic method global optimum is not guaranteed, is it alright we may or may not get the global optimum solution, so that is some of the disadvantages of the Tabu search method right.

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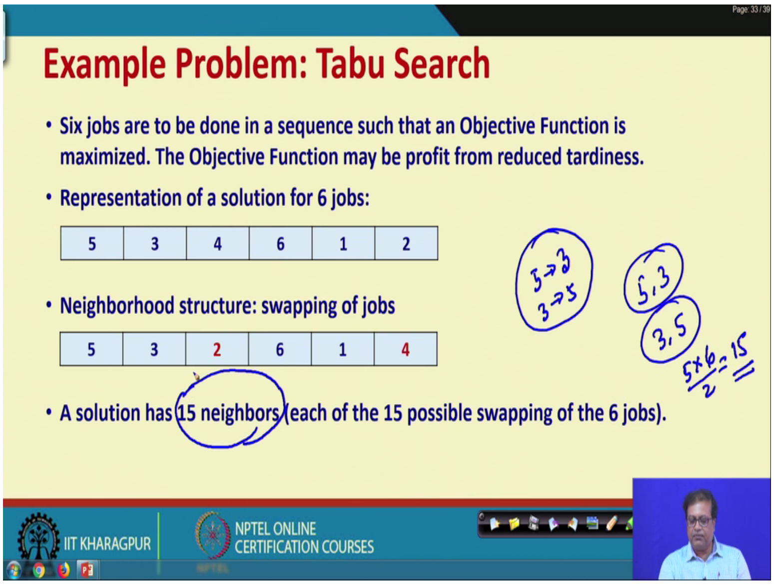

Now, let us look at a particular problem. Suppose, we have a 6 jobs that is to be done in a sequence that an objective function is maximized. The objective function may be profit from the reduced tardiness. The tardiness is basically lateness. So, usually what happens that in a shop condition, there are several jobs to be done and may be they all requires certain crew or certain people or certain machines certain facilities, so that at a time only one job can be done.

Now, each job has a due date, is it not, and each job require certain number of days to be performed. So, depending on suppose job number 5 you do first, and then 4, then 4 then 6, then 1, then 2, you get one set of lateness of jobs may be 5 is not late, but 3 is late, 4 is late also, 1 is not late, because ones due date was pretty far ahead and 2 is also not late. So, what happens what is the total lateness of all these jobs and is it not. So, like these we can compute the tardiness. And if you can reduce the tardiness, obviously, the profit is going to go up.

So, assume that we are maximizing some objective function. What is that objective function we are not going into the detail. So, a h this is one solution, is it not, 5, 3, 4, 6, 1, 2. So, what could be a neighbor, a neighbor could be by swapping these jobs we can get the neighborhood structure. So, how many such neighborhood structures are possible, see there are six jobs. So, we can have 15 possible swapping of these 6 jobs, is it not, because these 5 could be going to any of these 5 positions.

So, similarly all these 6 can go into any of these I mean 3 can go to rest of the 5 positions; 2 can go to rest of the 5 positions. So, 6 into 5 - 30 possible combinations, but then each combination has a double. What are does it mean it means say 5 3 2 and 3 2 5. So, basically 5 3 swap and 3 5 swap essentially same, because after all 5 is here and 3 is here, so that is how that 5 into 6 by 2 into 15 possible swaps right.

So, 15 possible jobs swap are possible, so that is how we have the solution has got 15 neighbors. So, I hope it is understood that fifteen possible way the jobs can be swapped, so that means the neighborhood structure has 15 possible solutions, is it all right. So, the question is that we should evaluate them, and we should see which one is giving us a better possible solution right.

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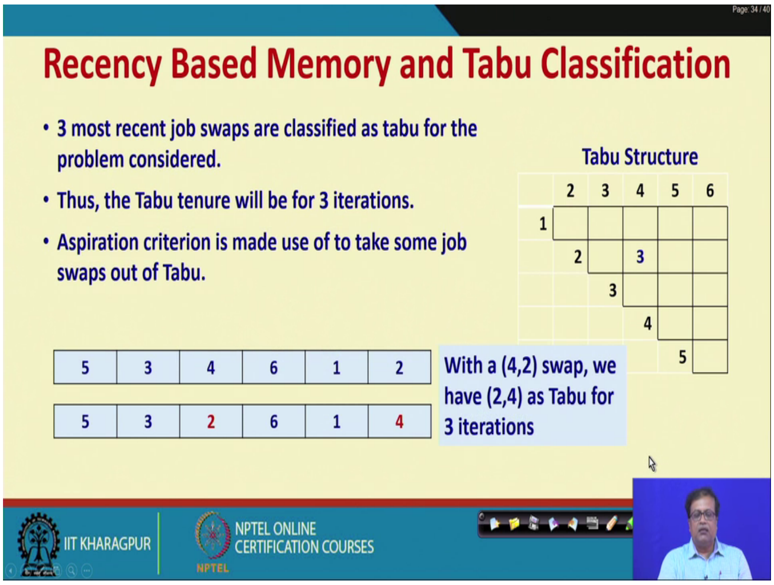

So, let us see further. Then we have let us say these recency based memory and Tabu classification for this problem. Then how do we go about it. So, a 3 most recent jobs works are classified as Tabu for the problem considered. Thus the tab tenure will be for 3 iterations. And aspiration criteria is made use of to take some jobs swaps out of Tabu. So, let us see that 5 3 4 6 1 2 with a 4, 2 swap, we have 2, 4 as Tabu for 3 iterations right.

So, look here this is where the we have put Tabu structure because you see once a 4 and 2 are swap, then we have 2 to 4. Obviously, we can mention it at 2 to 4, there is only 1 square between 2 and 4 in this Tabu structure, so that means, since 2 to 4 is already here automatically it means that Tabu is the reverse direction that is 4 to 2.

So, we just need these you know this kind of triangular structure should be sufficient to hold all the Tabu structures so, that is that is the thing. So, now 2, 4 is Tabu you know and Tabu for 3 iterations. So, this is our Tabu list so, right So, if this is our starting solution, and this is our new solution with a 4 to 2 swap, then 2 to 4 should be Tabu for 3 next iterations. So, this is how the recency based memory and Tabu classification can be carried out.

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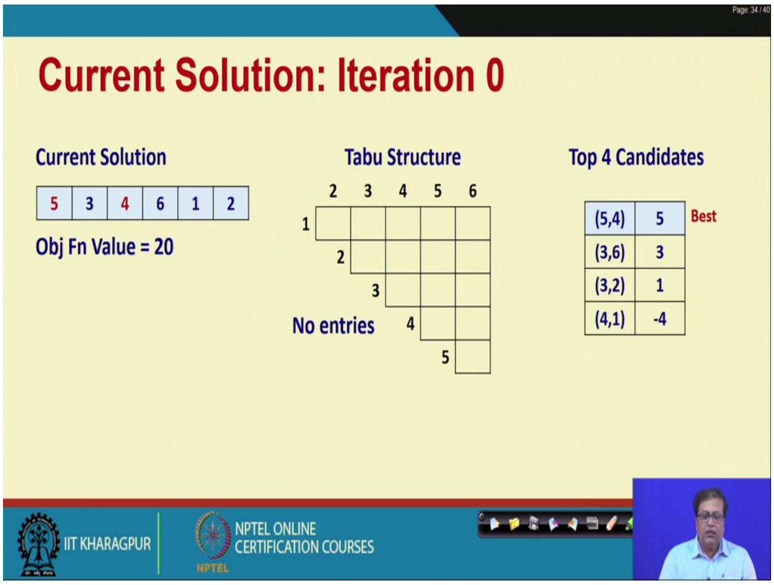

Now, let us see what happens in the 0th iteration. So, let us say our original current solution is 5 3 4 6 1 2, and objective function value is 20. Question is how do you get it? So, you know I have not shown details. So, I just assume that this much is available for the given objective function. And now there is nothing in the Tabu structure, so that is how where the problem you have started about 6 jobs. And let us say that top 4 candidates in the nearby are a 5, 4 what you call swap; a 3, 6 swap; a 3, 2 swap; and a 4, 1 swap.

Now, what are the result, the result is if you do a 5, 4 swap, your objective function value improves by 5; second one improves by 3; third one improves by 1; but fourth one it objective function value goes from 20 to 16 that means, it is not an improvement, it is a decrease. So, out of all these the best possible solution is found for a 5 to 4 swap. So, you can see that that this one is the best possible swap that we can have. So, obviously, we choose it; that means, we choose a 5 to 4 swap after the iteration 0. Now, as we do that what is resulted let us see that. So, at the next iteration this is what we have. So, please see our previous iteration, we have 5 3 4 6 1 2.

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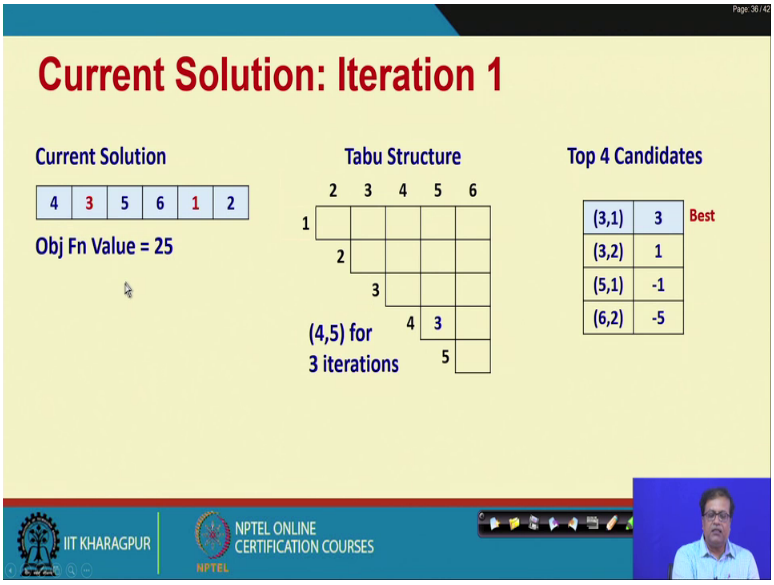

And in the next iteration 5 and 4 are swap. So, we have 4 3 5 6 1 2. And objective function value becomes 25, why 25, because it improves by 5. So, 5 3 4 6 1 2 becomes 4 3 5 6 1 2 with an objective function value of 25. And 4, 5 will be in Tabu for 3 iterations. So, you see because 5 to 4 swap we had done; now 4 to 5 swap, we should not do for 3 iterations. So, it is in the Tabu list, is it all right.

Now, again we have to evaluate in the neighborhood what are the top candidates. So, we have decided to see top four candidates. So, these are our top four candidates. So, we see that 3, 1 gives us an increase of 3, so that is the best possible solution that we can get at these point right. So, in the previously we have 5 and 4 we have swapped; and now is 3 to 2 swap is going to give us further improvement by 3, is alright. So, should we go for it? Yes, we should go for that.

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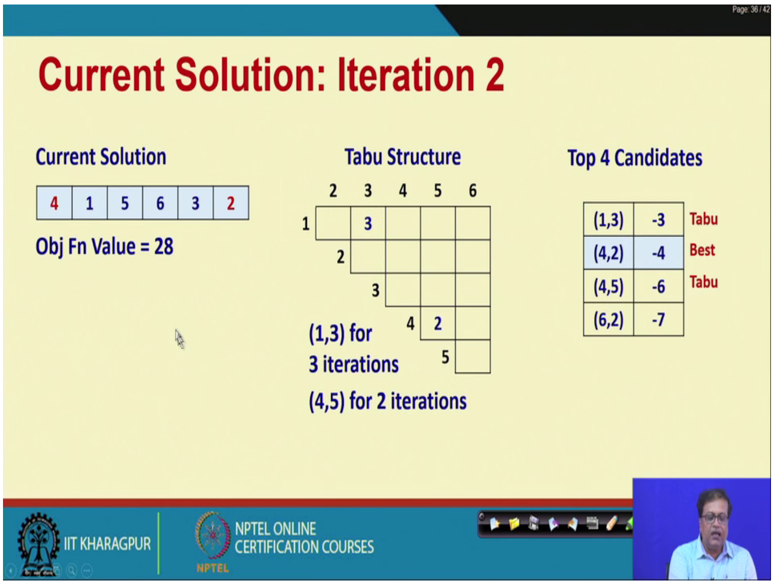

So, we have gone for that and then look here we have made 3 and 1 swap. And now we have 1 and 3. So, our new job sequence is 4 1 5 6 3 2 and objective function value has improved from 25 to 28, is it all right. So, it has gone from 25 to 28. Now, the Tabu structure, so you see in the earlier one, the 4 5 was only under Tabu. And since 3 to 1 swap, we have done previously, so 1 2 3 should actually be Tabu also, is it all right. So, 1 2 3 should be in the Tabu now. So, 1 2 3 you know will be Tabu for 3 iterations, and 4 to 5 already one iteration gone. So, this is also in the Tabu for 2 more iterations. And objective function value is 28.

So, again we evaluate top 4 candidates. And then what do we find 1, 3; 4, 2; 4, 5; 6, 2. But look here none of them improve the objective function value none of them. So, actually there minus 3, minus 4, minus 6 and minus 7, and minus 4 is the best. The question is should we do it or should not, we do it do it. So, it depends on the particular aspiration level or particular way we conduct the Tabu. In these particular one we decide to go for it, because sometimes even though the objective function value is reduced may be in future it might get give us some better points. Suppose, that search yields nothing then we can come back here and we can say look here that is the best that we have got so far, is it all right.

So, with that thinking we decide to explore 4 to 2 further, why 4 to 2, why not 1 by 3, because 1 to 3 swap is a Tabu that is in the Tabu list, so that cannot be considered, so understood that 1 to 3 cannot be considered because it is Tabu. The 4, 2, 5 cannot be considered, because it is in Tabu. And 4 to 2 is then best possible thing; although the best is also not that good because it reduces the objective function by 4. We decide to still go for it right, so that we may explore around or in the neighborhood of that point and who knows we might get something better in the next iteration, so that is what we do.

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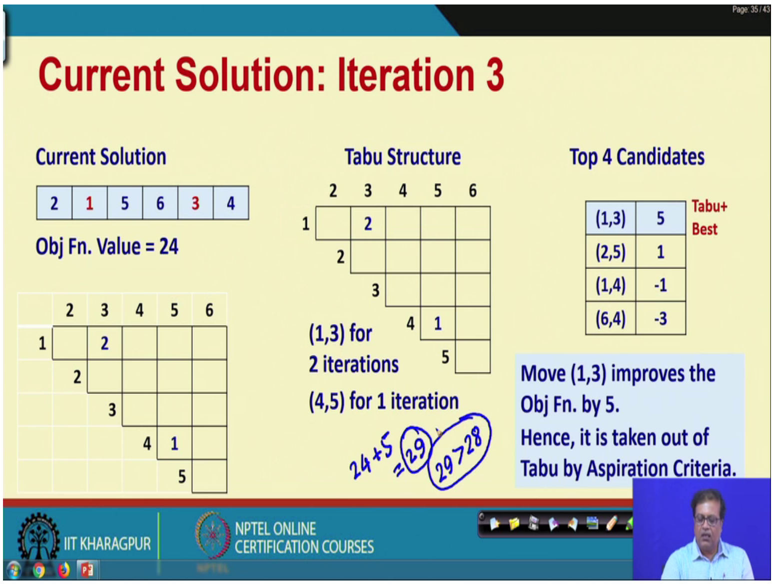

And after that swap that means, 4 to 2 swap, now we got 2 to 4, 2 1 5 6 3 4 and objective function value you see has come down from 28 to 24. So, as we have that you know what we get then the Tabu structure, see look here earlier our Tabu structure was 1, 3 and 4, 2 that was 3 and 2 iterations. So, it has now come down by one more; so, 1 3 by 2 more iterations and 4 to 5 as just 1 more iteration. And what are my top 4 candidates now.

Look here now, 1, 3 has come out as best followed by 2, 5; 1, 4 and 6 4. So, 1, 3 that it is in Tabu, but it is best. And it is going to give us improvement of objective function by 5. So, actually it satisfies our aspiration criteria, how it satisfies let us look at that. So, look here so far our best solution is our best solution is 28 right.

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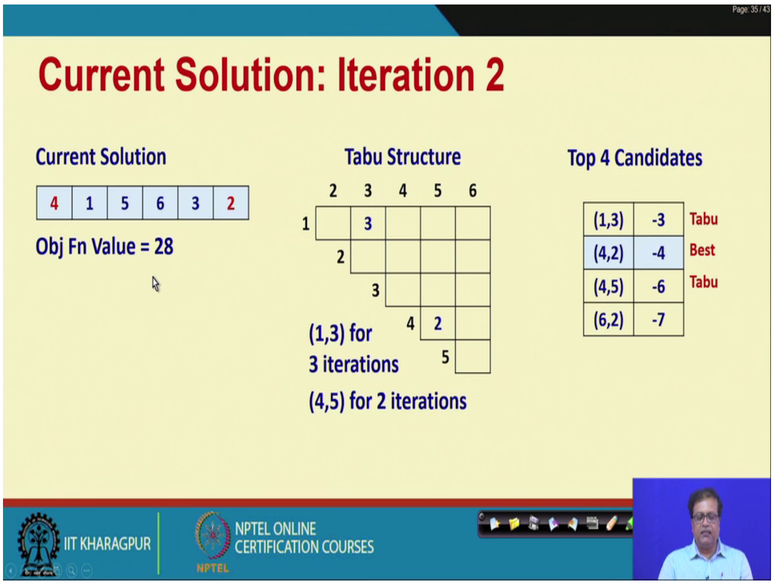

And current solution is 24, 1 by 3 swap is giving us 5 extra. Now, question is that same 1, 3 which was negative earlier how come it has become positive now, it is very clear because now the stream has changed. And we are now searching in a different neighborhood. So, how much we are going to get 24 plus 5 that is equal to 29. And 29 is better than 28 right. So, 29 is better than 28, since 29 is better than 28, it really satisfies the our aspiration criteria that means, the aspiration criteria could be the resulting solution is better than the best obtained so far right, so that is sufficient really to take 1, 3 out of the Tabu.

So, if I take 1, 3 out of the Tabu you know it gives sufficient thing to take 1, 3 out of the Tabu. So, we decide to do that; that means, we although 1, 3 was Tabu, but now we decide to take it out from the Tabu list that means, that is a freeing strategy right. So, this is exactly what is that through the aspiration criteria, we take something out of the Tabu list.

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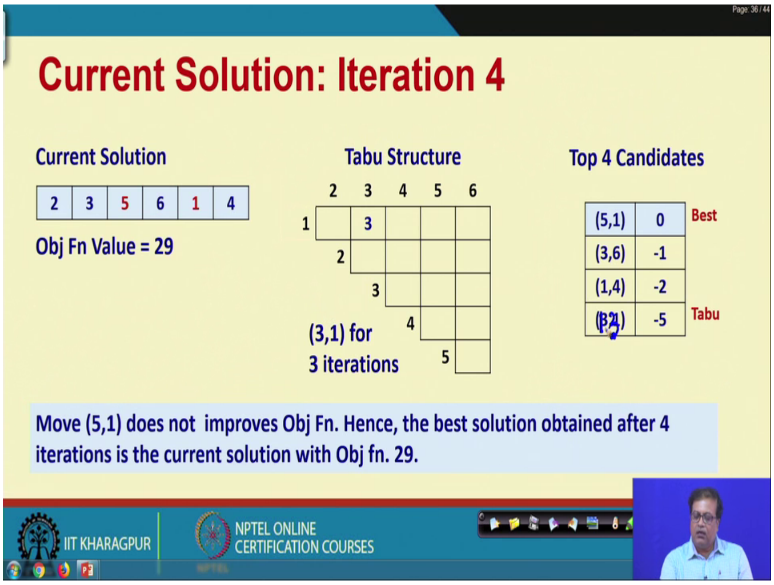

So, we do that. And we get our fourth iteration that is see look here 1, 3 is now become 3, 1. So, 2 3 5 6 1 4 and the objective function value has improved by 5, it has become 29. So, since you know our new list would be then you know see last movement was our 1, 3. So, now 3, 1 should be Tabu right and so this is the new top 4 candidates; 5, 1 is 0 that is best; 3, 6 minus 1; 1 4 minus 2 and 3 1 minus 5. So, this will be now Tabu that is 3 to 1 and because you know the order is 3 to 1, so automatic it should be should be the other way should be movement, is it not.

So, here so this is that 3, 1 is that. So, this Tabu should be then again 1, 3 right not 3, 1, it should be then 1, 3 right, so that is how it is that is that we have our new list. So, 5, 1 does not improve objective function hence after 4 iterations the current solution objective function value comes out to be 29. So, this is how we really solve the Tabu search problem. So, before leaving let us very quickly go through another problem although we do not solve it, but just try to give a general idea of that problem very quickly. So, this problem is something like these that you know as retailer allocation.

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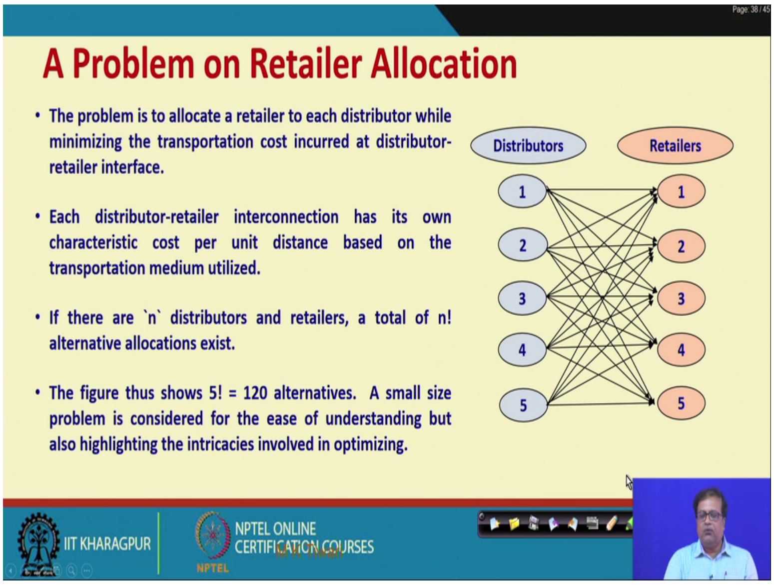

So, supposing that are some distributors and there are some retailers. So, there are 5 distributors and 5 retailers. Now, each distributor has to be allotted a retailer. So, look here there are 120 such alternatives even such a small problem of allocating 5 distributors to 5 retailers has got 120 different alternatives, is all right.

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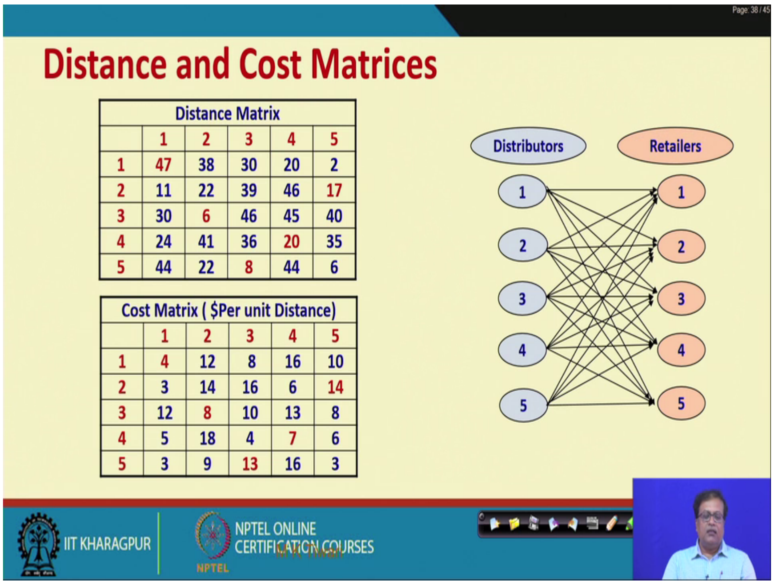

And there is a distance matrix and there is a cost matrix right. So, there are distances between those distributors and retailer; and there is also certain cost that is considered between them.

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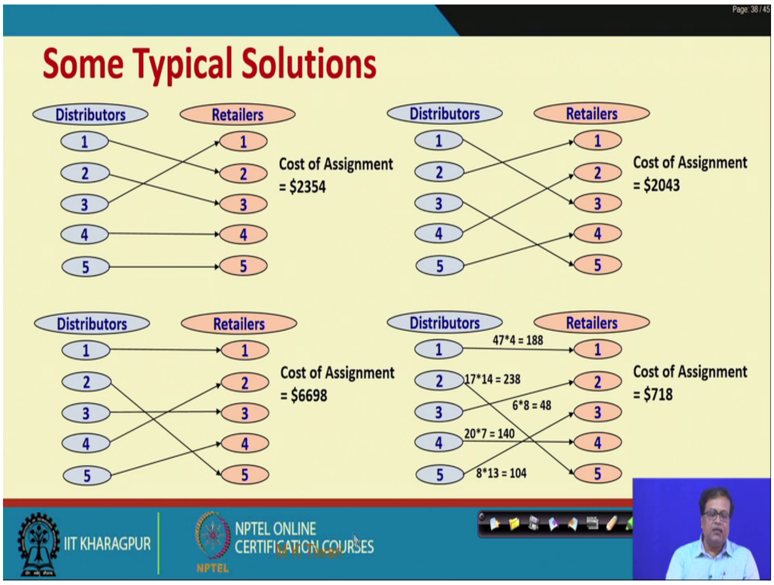

So, supposing some options, suppose 1 is allotted to 2, 2 is allotted to 3, 3 is allotted to 1, 4 is to 4, 5 is to 5, the cost is 2 3 5 4. Similarly, different other combinations are shown here with different cost of assignments, is it all right. So, these are maybe some typical solutions right.

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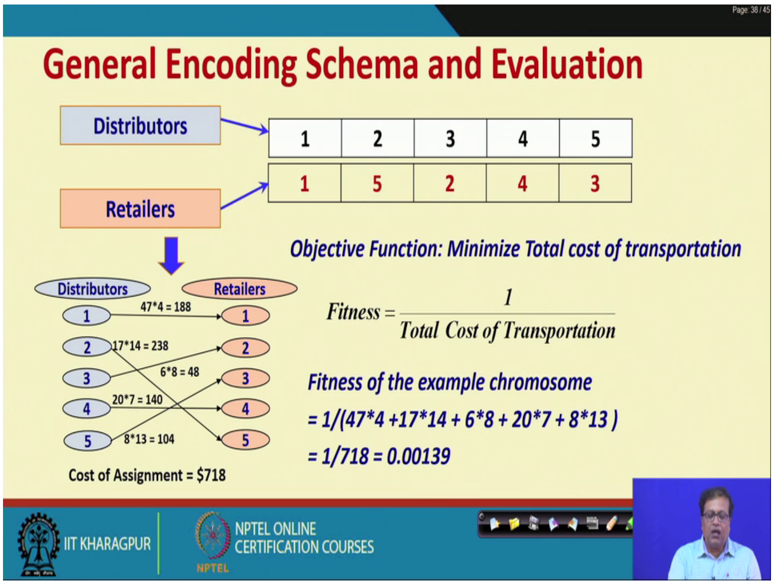

So, how we can encode that we can encode that distributors 1 2 3 4 5 and retailers maybe 1 5 2 4 3; that means, 1 to 1, 2 to 5, 3 to 2, 4 to 4, 5 to 3 actually that first row is not even required second is sufficient automatically means this is going to these. So, suppose the fitness is 1 by total cost of transportation. And total cost of transportation is the distance multiplied by unit cost. So, therefore, the fitness for these particular configuration could we say 0.00139; and our job is to minimize total cost of transportation or maximize fitness, is it all right.

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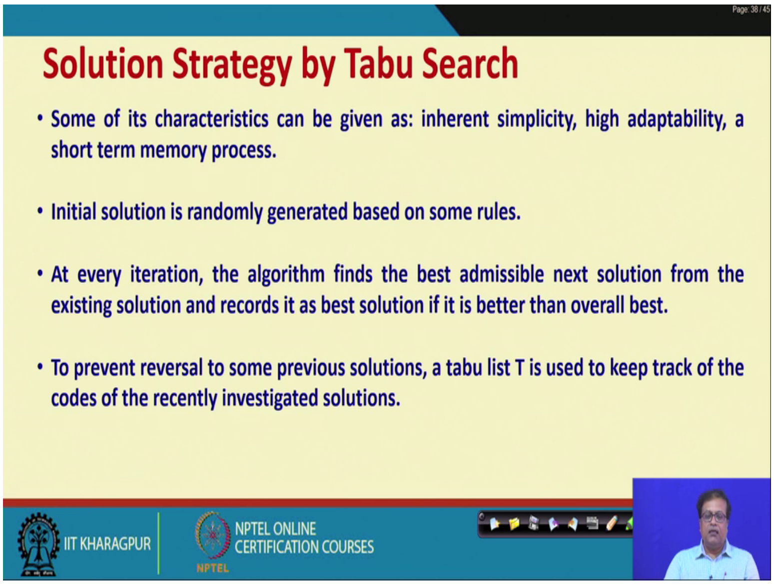

So, that is how it is and you know that is when you try to solve this by Tabu search you know inherent simplicity and all that that initial solution is randomly generated. Algorithm, find the best admissible next solution at each thing and Tabu list is obtained.

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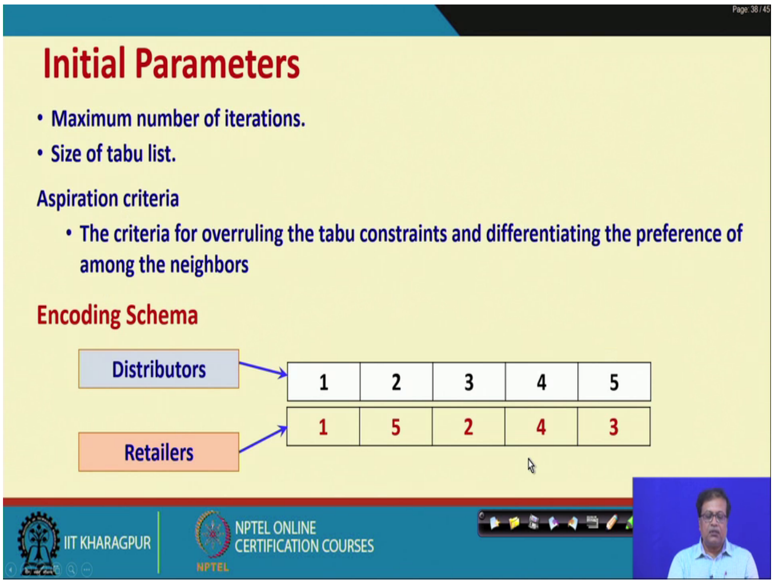

So, some of this, this is how already I have shown how it should be done.

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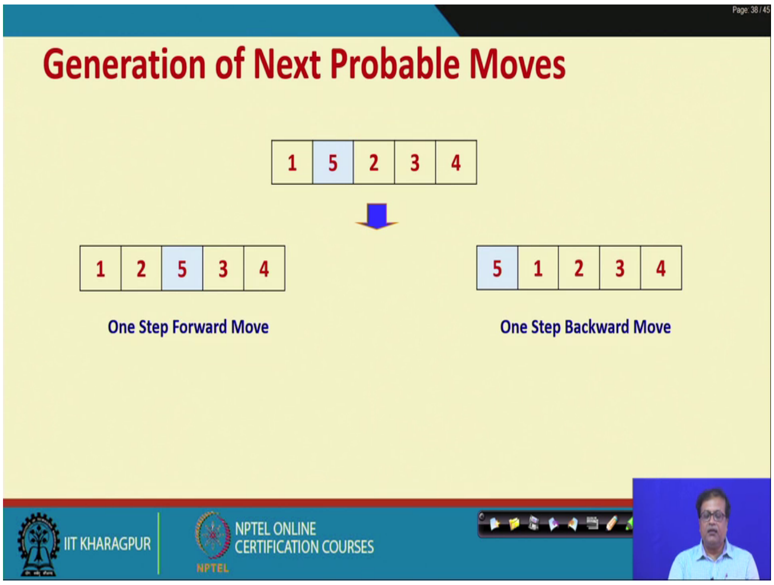

And say from one such you know coding, we can do either one step forward move or one step backward move. And by the kind of problem that we have shown earlier in the similar way we can solve this problem. So, how it is one step forward 5 has moved here and 5 that means, you know as 5 has come forward you know this two has actually come and 5 has gone back. And here one step backward that means you know the rest of the that one has so 5 as if 5 was gone ahead of 1, and 1 has gone you know that is how that is called the backward move.

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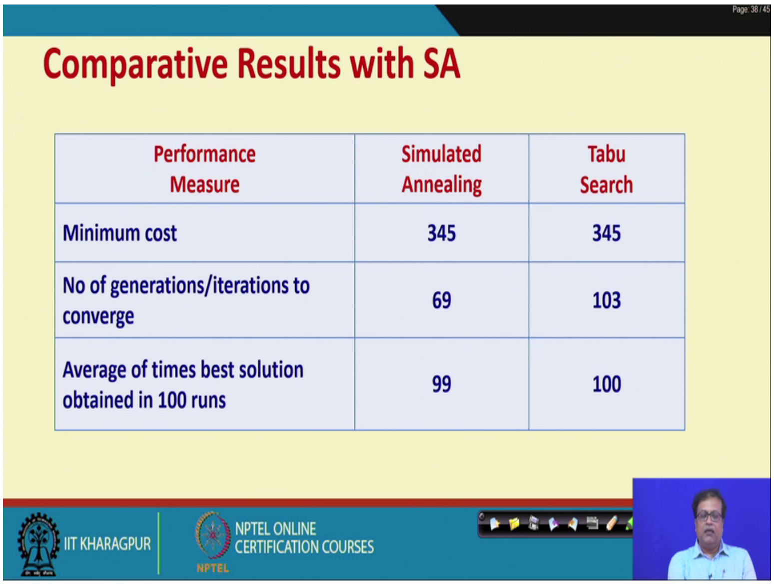

And when solve the comparative result shows against simulated annealing, same solution is found although Tabu search took more generations or iterations. And average time best solution obtained in 100 runs Tabu search is slightly better, is it all right.

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So, that is how it is. So, these are some of the remarks about Tabu research. And I hope you got some basic idea of the Tabu search technique; although it is not sufficient to solve realistic problems, but you know our purpose is to give a general idea.

Thank you very much.