Selected Topics In Decision Modeling
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Lecture – 02
Stagecoach Problem

Right, so, in this particular lecture that is lecture 2, we are going to discuss the stagecoach problem, how to solve stagecoach problems which is a shortest distance problem by making use of dynamic programming, right.

So, that is going to be our topic for this particular lecture.

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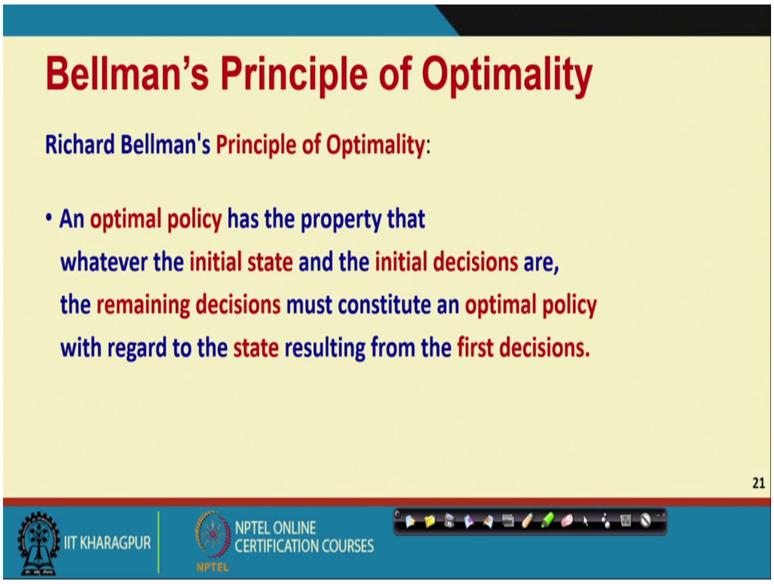

Now, again let us start with Bellman's principle of optimality. It basically states and optimal policy has the property that whatever the initial state and the initial decisions are the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decisions installed right.

So, that is Bellman's principle of optimality and we will come back to this once again the stagecoach problem is essentially.

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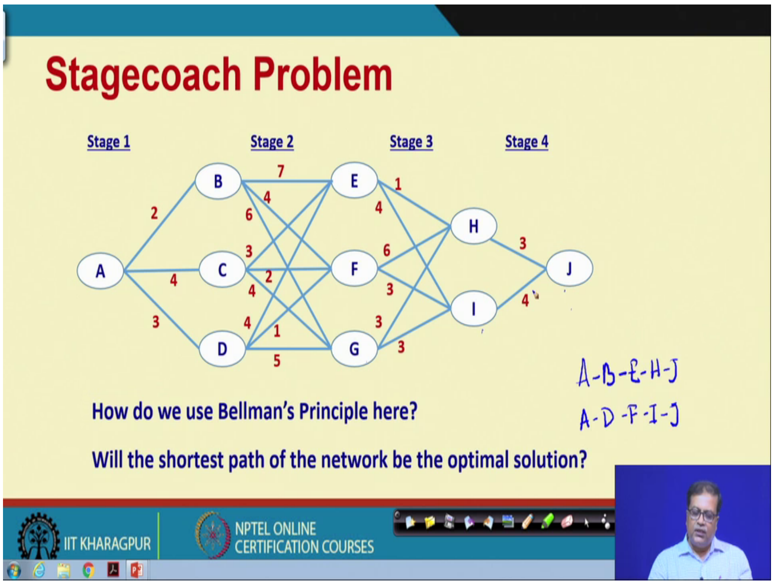

You know let us say a stage coach is moving from a city A, city A to city J, is all right; the stagecoach has to move through number of cities and you know initially to a state set of cities B, C, D from where to another set of city E, F, G to another set of cities H and I and finally to J; obviously, it can take a given path for example, it can go from A to B to E to H to J or it can move to a totally different path say A to D to F to I to J.

So, you can see that you know the stagecoach need not go to all the cities you know, it can choose as particular set of cities from the source to the destination and in order to fulfill some objective function, right, may be the shortest path may be the minimum cost, is it ok, maybe maximize profit and things of that sort. So, if you recall in our previous class, what exactly I said is that assuming you know we have somehow come to a given city G. So, having come to G irrespective of how we have come to G, we must now optimally complete the our journey from G to J in the optimal manner, right.

So, if we find the optimal path from G to J, is it all right; that optimal path we will help us in finding the total optimal path from A to J by combining. So, you see supposing we start our journey from G itself, then how about you know what is the optimal path from I to J, you know you can see very clearly the optimal path from I to J is 4 only because there is no other path. Obviously, you can go from I to E, go back and then come to H and then J, but then those parts will be much longer. So, I to J you know that is the optimal distance from I city I to city J.

So, if we know that that in order to go to J, I either you have to come to I or to H and optimal distance from I to J is 4 and H to J is 3. So, you know if we know this facts, can we find out what is the optimal distance from G to J look here G to H the least distance is 3 and G to I, the least distance to is 3 again and I to J the optimal path is 4. So, what is the optimal path from G to J is it through I or is it through H, you can see that G to J through I is 7 and through H is 6, right.

So, therefore, the G to J the optimal path is really 6, right. So, this is how as I said in my previous lecture, we can create a dynamic programming methodology through which we can find the optimal solution , but before even that let us see how this you know how this optimal path can be actually you know the problem how the stagecoach problem was has originally I mean how it originated.

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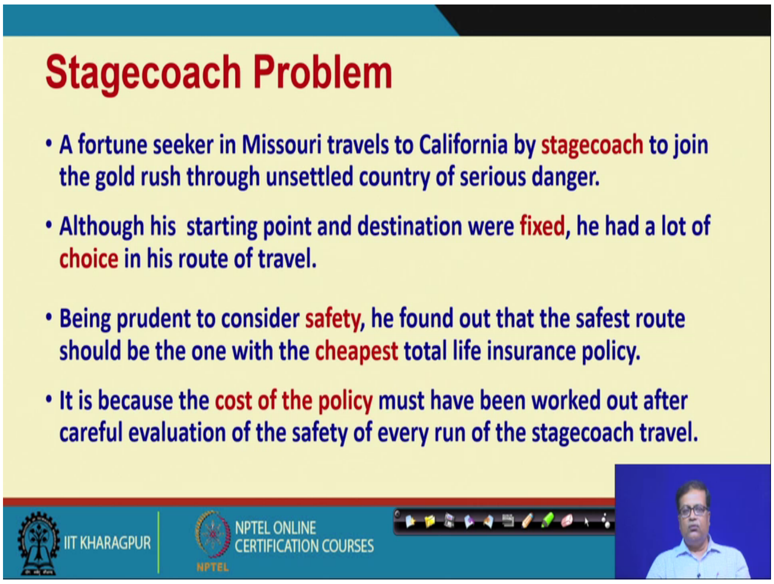

The story goes like this somewhere around 1900s, a fortune seeker in Missouri travels to California by Stagecoach to join the gold rush through unsettled country of serious danger; is all right. So, all though his starting point and destinations were fixed he, here the lot of choice in his root of travel; so, being prudent to consider safety, he found out that the safest route should be the one with the cheapest total life insurance policy.

Because in those days you know as the stagecoach is used to travel through these unsettled countries of serious danger the they use to go for life insurance and this life insurance companies use to ask for certain premium depending on you know what is the risk in that given path, right and it is possible for a person to find out those life insurance policy premiums, is it all right.

So, if you really count all these policy premiums to move from a given city to another city let us say from A to B, A to C, A to D, etcetera. So, you know all those policy premiums instead of the distances you know, if you look at those policy premiums because this policy premiums apart from the stagecoach travel costs you know this policy premiums, we will actually determine the you know because a stagecoach travel would depend on let us say that number of days and we know the number of days assuming from a given city to the next set of cities the number of days are more or less the same.

So, it is the insurance policy premiums which are going to decide the total cost you know of the of the path or maybe you can even say the safest path will be that where the cost of the policy is minimum not only you see you pay less, but also your risk is the least.

So, that is how the stagecoach problem was originally formulated and you know this model.

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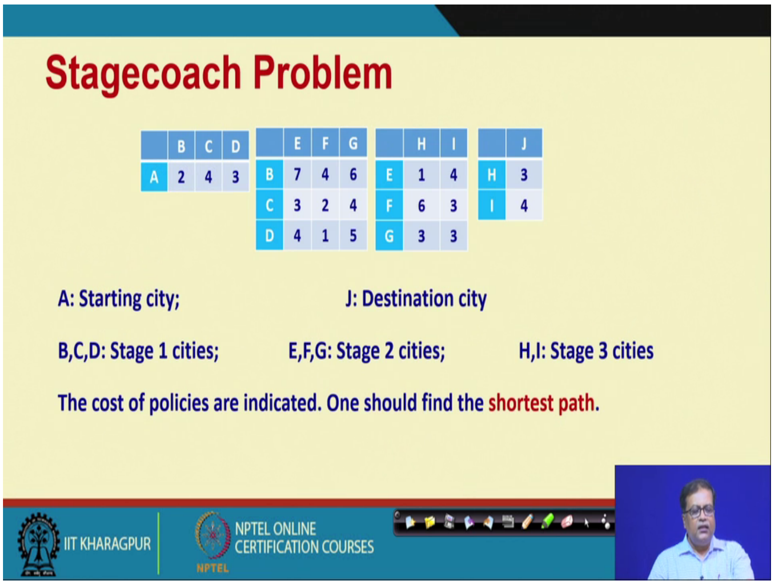

Therefore would look something like this that from A to B, let us say cost is 2, A to C cost is 4, A to D cost is 3 and like that; then what is the path from A to J; that one should take. So, that we get the least possible cost, right, sometimes we can actually make a matrix of this form, you know very easily we can find out the costs you know let us say A to B is 2, A to C is 4 and in this matrices they can be called as distance matrices the distance matrices gives us the cost figures in a very you know summarized way. So, what we need to find out we need to find out the shortest path. So, what should we do now if we have to solve this problem from dynamic programming point of view?

What are the different things that we should fix up, right in the beginning? Please recall our previous lecture; can you tell me what are the things that we should fix now number one is the stages different stages right, number 2 the different states, right, number 3 the decision decisions that we should make at every stage, is it all right and forth the recursive relationship which connects a given stage to the next stage right. So, these are the things that we should fix up to solve these problem with dynamic programming.

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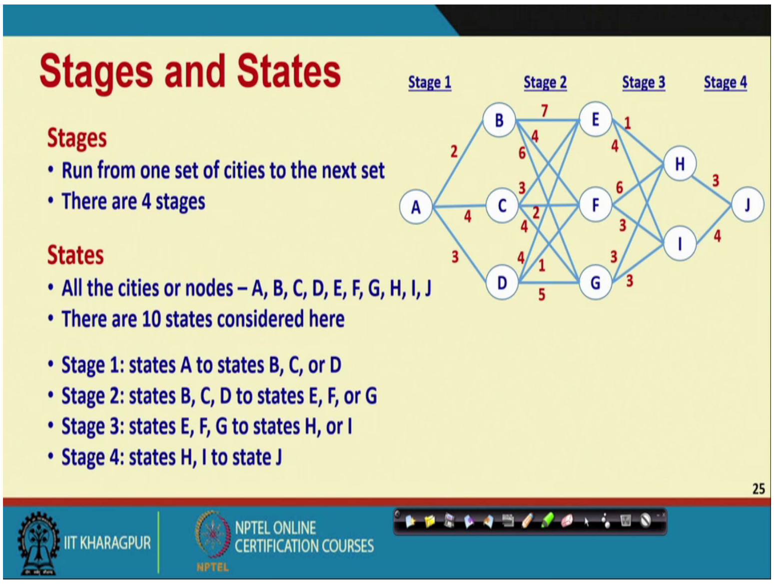

So, what are some stages and states, is it ok. So, as you can understand very clearly that every leg of the stagecoach run can be called as stage, is it all right.

So, from the first set of cities let us say A to the next set of cities you move this is called stage 1. So, in stage 1 you actually move from A to either B or C or D. So, this is stage 1, in stage 2 you move setup cities B, C, D to set of cities E, F, G is it all right in stage 3, you move from E, F, G to H, I, any one of them and in the final stage you move from H, I to J if you are in H go to J, if you are in I, you go to J.

So, these are the 4 stages and what are the states you see at every possible you know stage after the conclusion of the stage you reach state. So, there is only one state to begin with and there is only one state to end with, but in after the conclusion of stage 1, you may be in one of the possible states that is B, C or D and after the conclusion of state 2, you can be in E or F or G and at the conclusion of stage 3, you can be in H or I, is it all right.

So, these are the stages and states, is it all right. So, I hope it is clear the stages are every lake of the stagecoach run states are you know at conclusion of the stage where what are the possible different states where one can be in, right.

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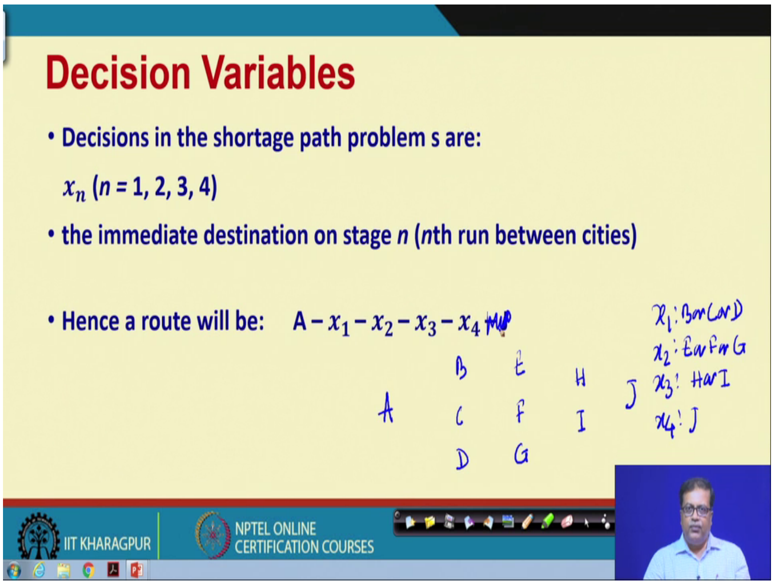

Now, the decision variables the decisions in the shortage path problem are x n which can be called as the immediate destination on stage n that is the nth run between cities; so, you know supposing hence a route will be A to x 1 to x 2 to x 3 to x 4 to J. So, to understand this let us draw these diagram here. So, this is the A, then B, C, D, and E, F, G, then H, I and then J, is it all right. So, x one could be either B or C or D x 2 could be E or F or G x 3 could be H or I, all right and; obviously, x 4 is J.

So, there is no x 4 separately because there is that is all, we do not have any other. So, we start with A and then in between there are B or C or D and then E or F or G and H or I and then finally, J right. So, there is no separate J here, then x 4 itself is J, is it all right. So, that is well that we have from A to B or C or D that is the destination city, then E or F of G, then H or I and then J, right. So, that after the completion or all the 4 stages, you know I find the x 1, x 2, x 3 and x 4 and then I find the final route which will give us the shortest cost hope it is clear that we decide the stages, we decide the states and we decide the decision variables and with the help of the decision variables we can find the optimal path, all right.

So, that is how it goes. So, now, that we know this.

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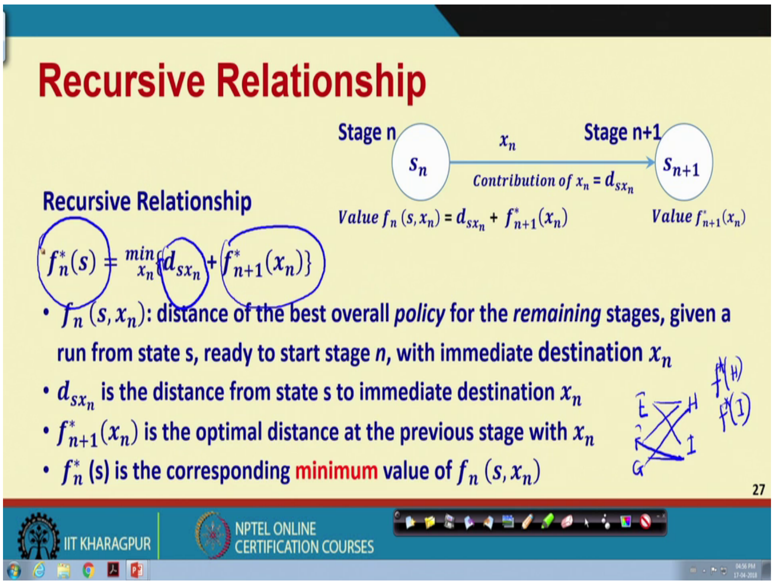

Let us see the recursive relationship. Now this is very important before even we discuss this, let us let us look at another very interesting thing you see in this particular diagram; what we find out if we move from one state to the other you know, we find out that as we go from in the stage 4, you know as we go in the stage 4, suppose, this is our consideration sorry this is our consideration in stage 4, you see this is our consideration in stage 4.

So, we find set and optimal decisions what are those optimal decisions that the if we start from H, then supposing.

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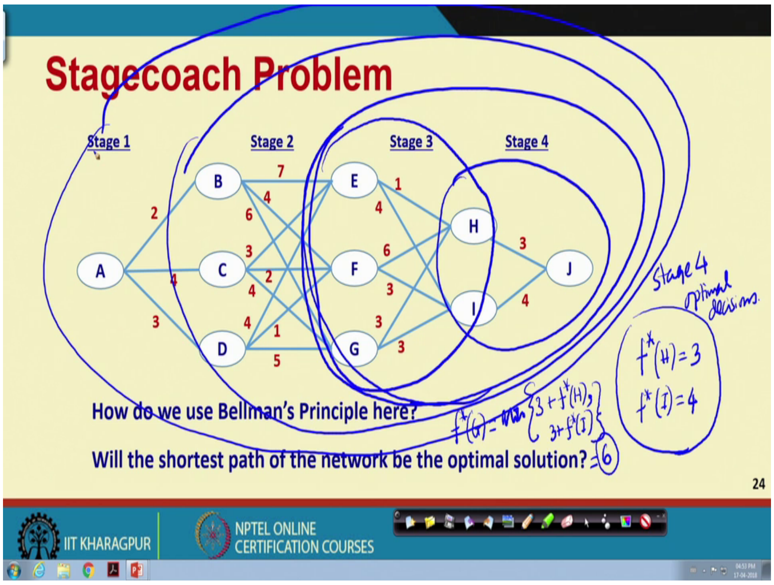

We can write F start H, then it will be 3 and the optimal decision from I will be F star 3. So, these sorry this will be 4. So, F star H is 3 and F star I equal to 4. So, what are they are my stage 4 optimal decisions.

So, you see; this is our consideration in stage 4, but then what is our consideration in stage 3; our stage 3 consideration are these please do not forget this that all though, we are really concerned about these parts. So, you see when at stage 3 our consideration also includes stage 4 decisions, but then out of that only this much we focus and these stage 4 optimal decisions, we actually make use of.

So, look here what is the optimal path suppose we are in stage 3 what is our optimal path from G 2 J, right, is it through I or is it through H, we only have to do; we do not have to really compare all the way from D, G to J through H through I not required I only look at G to H is 3 and H optimal distance is 3, is it all right. So, the optimal; so, what is F start G? F start G is minimum of 3 plus F star H comma 3 plus F star I.

Look here, we make use of these F star H and F star I which has stage 4 optimal decisions in stage 3. So, the optimal path from G to J can be obtained, you know by making use of D F star H and F star I values, is it all right and since you know F star H is 3. So, this value comes out to be 6. So, these F star G values you know, I can find out and I can keep it as an optimal decision for stage 3 which will make use in the stage 2 where considerations will be all this and when the entire thing is considered at stage 1 then we find the overall optimal decision from a to J is, all right. So, this is the kind of philosophy that we usually follow for dynamic programming right and how we go about it that is all we are going to see.

So, having said that let us see how we have we can forward or move further. So, exactly is same thing is written here in a slightly different language that you know if we have the optimal decision that we know which is F star n plus one suppose n equal to 3; that means, we want to find out the optimal decision of F n star S then it is making use of the you know F n star n plus one the next higher stage what is the optimal decision. So, optimal decision is here. So, this is our optimal decision at the next higher stage right and the distance from the current city suppose you know let us let us let us draw that once again. So, here is E sorry here is E, F, G and here is H and I, right.

So, there are paths from all these cities now we know F star H and we know F star I. So, you see F star H and F star I are actually this these values right. So, F star n plus 1 x n which is the next stage optimal value what we find out is the distance from S to x n; that means, G to J H x n is H or I. So, G to H or G to I then we add this portion and then find out of H and out of I from G which one comes out to be optimal. So, these will become our F star n s, is it all right?

So, exactly that thing is happening that is F n S x n distance of the best overall policy for the remaining stages given a run from stage state S ready to start stage n with immediate destination x n is it all right. So, D S x n is the distance from state S 2 immediate destination x n F star n plus 1 x n is the optimal distance at the previous stage with x n and F star ns is the corresponding minimum value of F n S x n all right.

So, like this by making use of the next stage optimal decision. So, this is our next stage optimal decision making use of next stage optimal decision and the distance from given point S state S to the destination x n; I can find out the value for the this F n S x n right and if I have many such values the because depending on the various combination the minimum amongst them of with regard to x n will give the new optimal value at the next stage.

So, that is going to be our recursive relationship and these recursive relationship we can make use of in our computations.

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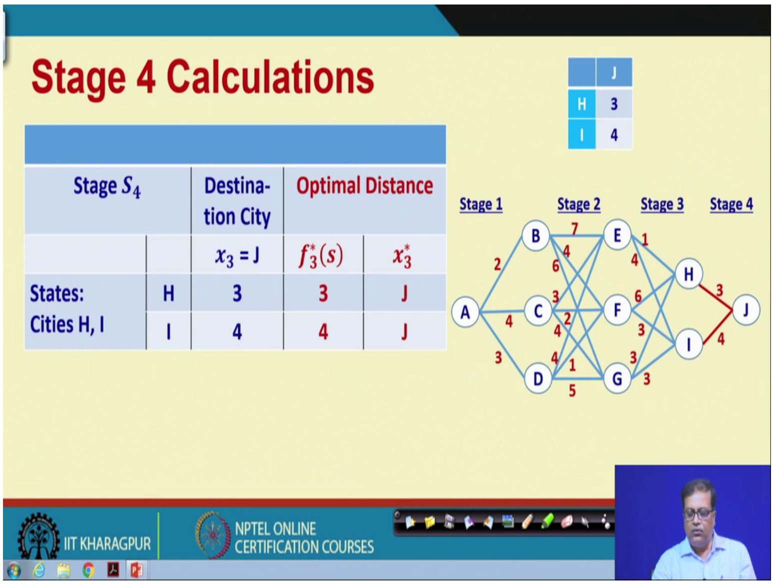

So, let us see how we compute right how we compute let us see that. So, that that is our stage 4 calculations the stage 4 is very straight forward the stage 4 calculations all we have to do is find out the optimal path from H and I to J and since this is the last stage, it is very simple. Obviously, from H to J there is only one path and therefore, the optimal is 3 and I to J, there is only one path. So, optimal is 4. So, you know the destination cities only 1. So, x 3 equal to J from H to I the distances are 3 and 4. So, F 3 star; S will be 3; that means, F 3 star H these are S actually, right.

So, if you take the pen this is S right. So, S is either H or I and F 3 star S is 3 or 4 and x 3 star is J, is it all right that is the destination city. So, this is our stage 4 calculation which is very straightforward and we can obtain it very easily. So, having obtained this; next see that how we go to our next stage that is stage 3.

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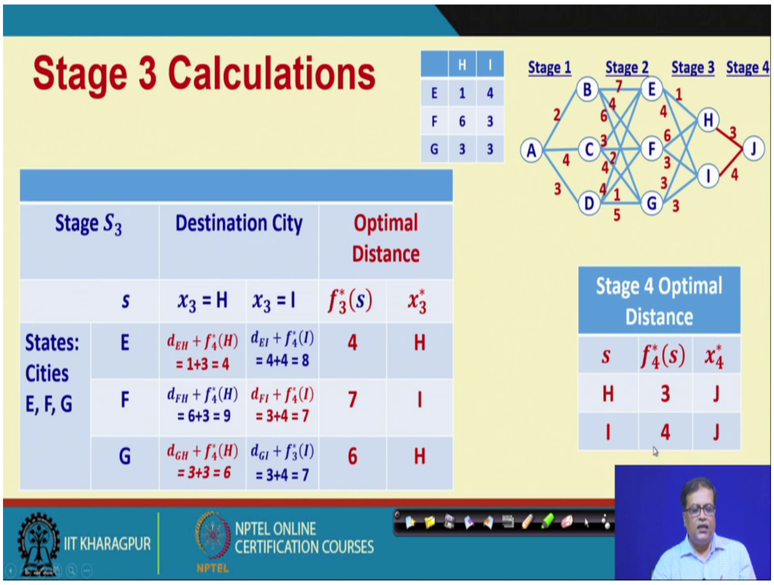

Now, while we make our stage 3 computations, we must remember that we have calculated our stage 4 optimal decisions where if S is H or I then F star 4 S is 3 F star 4 S for I is 4 and x 4 star was J. So, we shall make use of these decisions in our recursive relationship. So, what is those recursive relationship; now from E, we can go either to H or to I, is it all right.

Similarly, from F to H or G to H F to I G to I so, all this computations; we have to do, right, but please understand the good thing that is happening we are making calculations only between given stage of cities to another state of cities set of cities in the stage only right, you do not have to go beyond that is the advantage that we have.

So, we know F star H, we know F star I we are making use of them see F star H is 3 and F star I is 4 and D from D E to H is 1. So, it is 1 plus 3; 4 and here E to I is 4. So, E to I is 4; so, 4 plus 4; 8, right. Now, 4 and 8; what is the lower value? The lower value is 4 so; obviously, the F star F star E will become 4 and destination will be H why destination has become H because that H part has come out to be the minimum; all right.

So, similar calculation for F you know you will find the lower value comes with I so; obviously, x 3 star will be I; that means, we have to go through I and that value is 7; is all right and when it comes to G it should be D, G, H plus F star 4 and that comes to 6. So, this will become 6 and these are become H. So, we have now obtain the optimal decisions for the stage 3, right.

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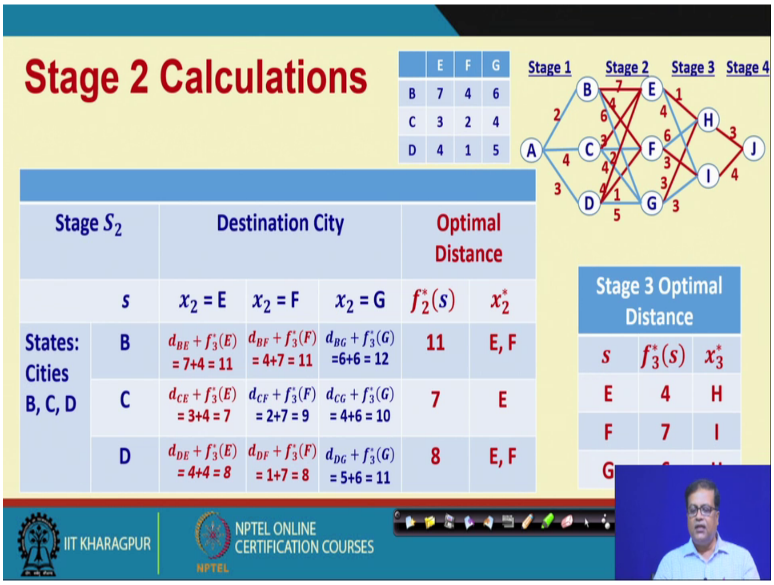

So, having found the optimal decisions for stage 3 again look here we noted down the stage 3 optimal decisions and which are also indicated in this plot now we have to see from the stage B, C, D to E, F, G when we see from B, C, D to E, F, G calculations again we make use of these optimal values which are 4, 7 and 6; So, 4, 7 and 6 4, 7 and 6, 4, 7 and 6 adding; the respective distances.

So, only distances are to be covered are between the set of cities B, C, D to E, F, G, you do not have to see any further. So, you see whatever really advantage we got by making use of dynamic programming we can only see the distances between a set of cities to the next set of cities which are finite, we really do not have to see combinations which are before or which are after because we already have the optimal value from the next set of stages and those optimal values, we are making of through the recursive relationship.

So, this is where the real advantage comes in and those advantages can be made use of. So, you know we see that from B the calculations that both B to E and B to F there minimum. So, the destination optimal is E and F both and the value is 11.

Similarly, for D the value is 8 and the destination cities are E and F to the stage 2 calculations.

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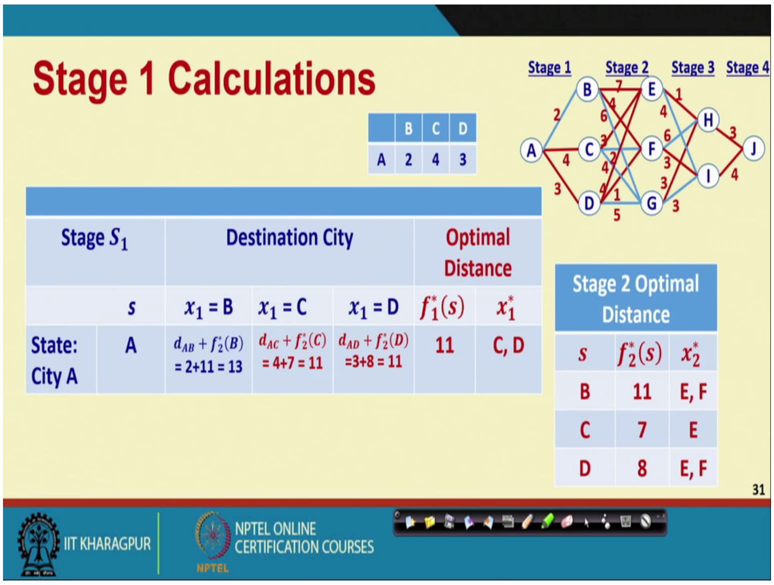

Coming to the stage 1 calculation, there is only one starting city, right and those optimals B, C, D has been made use of; right. So, F 2 star B is 11 F 2 star C is 4 7 and F 2 star D is 8. So, only 3 distances are there 2, 4 and 3. So, when you made use of them, you get 13, 11 and 11 and we know that 11 is the minimum.

So, 11 is the optimal distance and destination cities are both C and D is ok. So, this is our stage 1 calculations by making use of the recursive formula, now here we have listed all the optimal decision stage 4, stage 3, stage 2, stage 1.

So, from A you know we find that the there is a mistake here, let us correct it. So, this is not 29.

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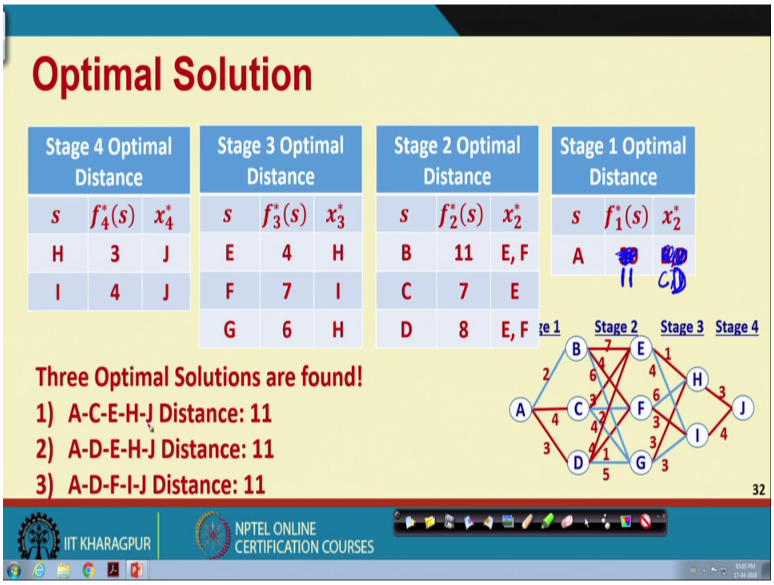

s	$f_4^*(s)$	x_4^*
H	3	J
I	4	J

s	$f_3^*(s)$	x_3^*
E	4	H
F	7	I
G	6	H

s	$f_2^*(s)$	x_2^*
B	11	E, F
C	7	E
D	8	E, F

s	$f_1^*(s)$	x_2^*
A	11	C,D

This is really 11, right. So, you know the previous distance. So, previous distance it was 11 C and D so, yeah. So, I do not know why it has any mistakes, any how this is corrected. So, you know it is 11 and this is C and D, right. So, the optimal is 11 and C and D, right. So, this is 11 and C and D, they are optimal.

So, now, you can combine that what is the if you go to see then from A, the distance is C and from C, it is E, from E it is H from H it is J. So, A, C, E, H, J, this is one, if you go to D, then A to D, D to E or F, if you go to E, then H, then J. Now if you go A to D, then from D to F and from F to I from I to J.

So, A to D to F to I to J and the distance are all 11, is it all right. So, those are our optimal distances that A to C to E to H to J 4 plus 3; 7 plus 8 plus 3; 11. So, this is one optimal then other one is A to D to F to H to J; this is also 11 and A to D to F to I to J; so, 3, 4, 7, 11. So, there are three optimal solutions which can be found out and all of them are really optimal solutions, right. So, this is how is solve the example stagecoach problem and we shall take up more problems in the subsequent lectures right so.

Thank you very much.