Selected Topics in Decision Modeling
Prof. Biswajit Mahanty
Department of Industrial and Systems Engineering
Indian Institute of Technology, Kharagpur

Lecture - 11
Integer Programming: Introduction

So, today as a part of our subjects, Selected Topics in Decision Modeling, we are going to start new series that is on Integer Programming. So, integer programming as a first lecture we are going to discuss the introduction of the topic.

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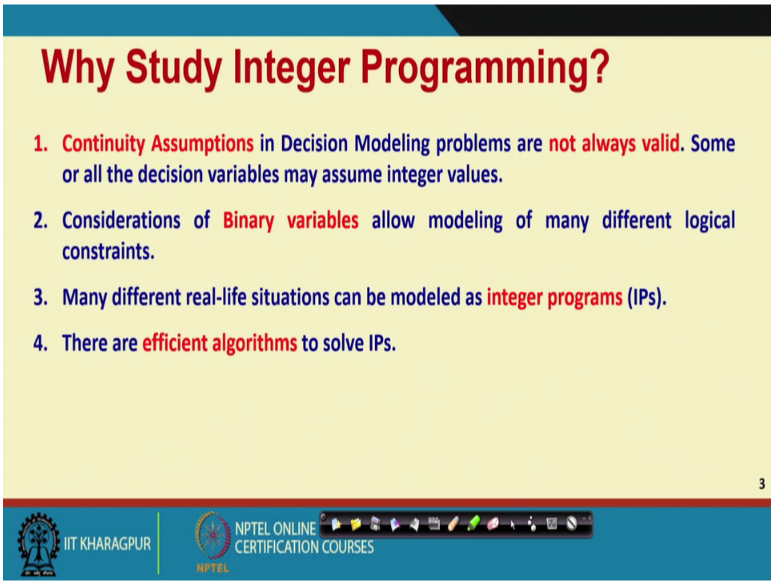

First of all in most of the decision modeling problems you know, particularly when we use linear programming we have a continuity assumption. What it exactly says that the decision variables you know they are divisible in nature, in the sense that you know they need not be integer; that means, they can also have what is known as fractional values.

So, when there are fractional values, then the continuity assumption holds. But when continuity assumption does not hold you know they kind of problems that we get where the solutions are integers, they are actually called integer programs. Now sometimes what happens that the problem the decision problem really does not require any integer values; that means, continuity assumption holds, but there are certain kind of constraints like either or constraints or some special type of constraints, where some of the variables are to be binary; that means, those variables will assume only a value of 0 or a value of 1.

So, when you have that type of situation; that means the value is either 0 or 1 that is binary variables we have what is known as mixed integer programming problems.

Then there are many different real life situations which you know you really require integer programs is it all right and we have efficient algorithms to solve integer programs. Now our lectures we shall divide like this, that in the initial part we discuss some integer programming problems. After discussing different types of integer programming problems, we shall see how to formulate integer programming problems.

Because formulation of such decision problem is very important exercise in you know solve solution is also very important, but even more than solution, formulation is I would say even more important. Because unless you formulate you know the solution is not even possible. And now there are lot of mechanization in the sense that we have software available, using which you can solve integer programming problems.

But you know real challenge lies in the practical world given situation, how do you model it as an integer program. So, given that we spent certain time on formulation of such problems and after that, we actually go into how to solve those problem. So, that is how we move that initially discuss certain integer programming problems, then we discuss how to formulate them and later on we discuss how to solve them.

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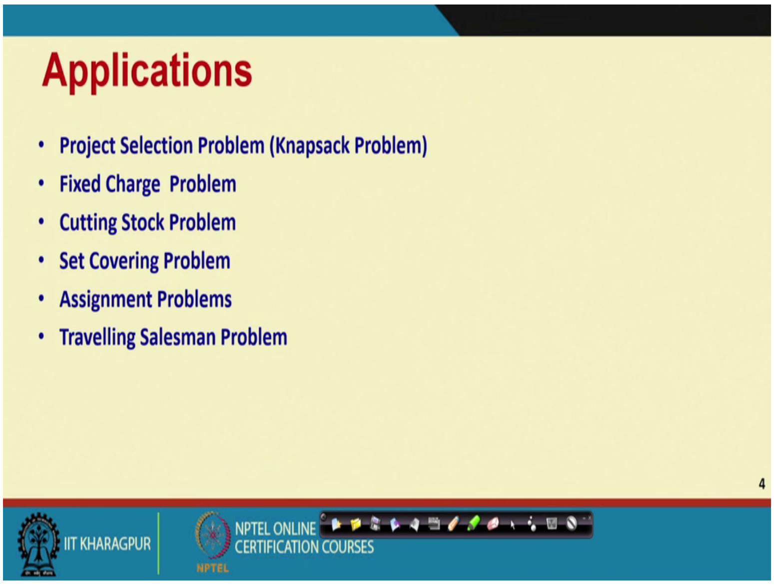

Now certain applications how be integer programming problems. So, here is a list that project selection problem, which is a type of knapsack problem you know where we have a number of projects have their certain input you know level costs or input level investments and there is certain amount of return, which projects to be selected. This is also a type of knapsack problem will discuss it further.

The second type of problem that we shall discuss is are the fixed charge problems. See what happens in fixed charge problem you know in many situations we see, there is a certain amount of charge which is fixed and certain amount of charge that is variable. But then if you do not go into that activity at all then there is no fixed charge is it all right.

So, therefore, fixed charge can come only when the variable value is above 0. If it is 0 then that activity itself does not start. And so, if the activity does not start, there is no question of a fixed charge so, how to model such situations and solve. Thirdly the cutting stock problem; see what happens in cutting stock usually even take the simplest one dimensional cutting stock problem, you know we have a certain steel or certain kind of plate of a certain length, but the customer demand is not in that length, the customer demand is a much smaller length.

So, question is how do you cut that stock so, that you know we have minimum trim loss. Trim loss is basically the amount that is remaining after cutting off. So, idea is that we decide certain cutting patterns, maybe number of cutting patterns and the integer programming problem is which patterns to choose is all right. The fourth type of problem can be called as set covering problem you know we have a certain set and we have to cover maybe all the elements of a that a given set, but then there is a criteria right the facility has a criteria. So, within that criteria, how do I cover all the elements you know will know what exactly it is as we see one problem.

Apart from that there are assignment problems, you know like assignment problems where we have a number of machines, number of jobs and with different capabilities to how to assign a given machine to a given job. So, this type of problems and then travelling salesman problem, where a salesman moves from a city to a number of cities and it has to go through all the cities exactly once, and come back to the starting city. So, what should be the route that the salesman should follow so, as to let us say minimize distance or minimize cost or things of that sort? So, these are some applications obviously, you know these are not the only applications of integer programs that would be many other applications, but you know we thought that these applications really give a good exposure to the integer programming problems is all right. So, having said that let us look some of you know these problems and what are the pros and cons of such problems.

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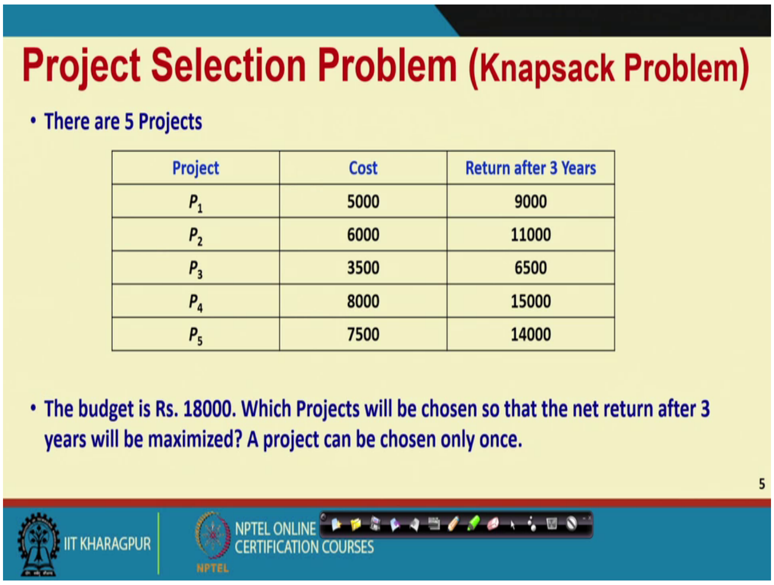

The first one can be called the project selection problem as I said it is a type of knapsack problem. Let us say there are 5 projects and you know each project has a cost and a return after let us say 3 years is not the cost could be 5000, 6000, 3000, 5000 8000 or 7500 and there are different returns. We have with us let us say 18,000 rupees. So, question is which projects will be chosen so that the net return after 3 years will be maximized.

Now I have put a condition here a project can be chosen only ones. Now in that sense its say a 0 1 type of integer program, but then you know you can also have you can choose a project more than ones right. So, that would be you know slightly more difficult project selection problem right. So, you understood that what is a project selection problem.

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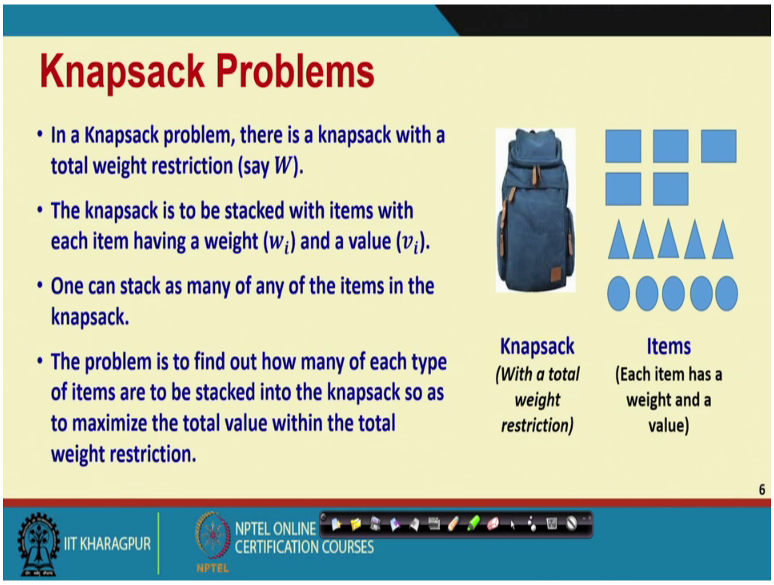

Now, a knapsack problem as we have discussed previously in our dynamic programming classes, you know is a knapsack with a total weight restriction say W and there are different types of items with individual weight and value. So, you can stack as many of any of these items in the knapsack. So, question is, how many of each type of items are to be stacked into the knapsack so, as to maximize the total value within the total weight restriction.

So, first of all why is it integer? Because you know if the number of items all let us say number of item 1 to be taken in the knapsack, number of item 2 number of item 3 they must be integers or if you can take only one of each item, then that is a 0 1, but then again an integer program; so how the project allocation program or the project selection problem, a knapsack problem? What is your knapsack? The knapsack is an imaginary parts, where you have your 18,000 rupees you know that is the budget.

And each project you know which is like individual item, they have a certain cost and a certain return. The idea is that within those cost, how do you obtain certain amount of return maximum amount of return using those 18,000 rupees is it all right. So, that is the precise thing that we need to do under this situation. So, in this you know way, you can say the project selection problem is also a type of knapsack problem right.

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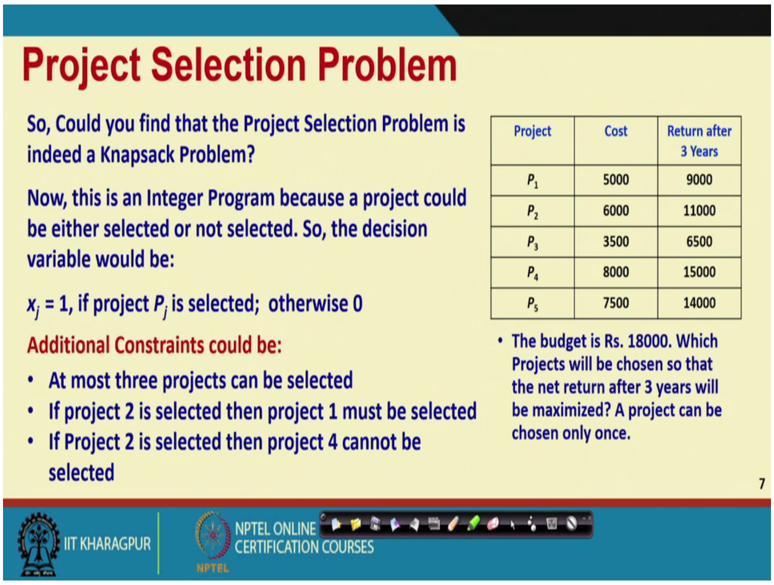

So, you have seen that project selection problem is indeed a knapsack problem right.

Now, this is an integer program because a project could be either selected or not selected right. So, what are some decision variables? The decision variables will be x j equal to one the project P j is selected otherwise 0 is all right. So, you know x j will have a value 0 to 1 which is like a decision variable if the project is selected. So, that could be our decision this thing.

So, you know what should be the exact formulation will not discuss it now, we are just discussing the type of problems that we have here. Additionally there could be some additional constraints, what are those additional constraints? You know something like let us say at most 3 projects can be selected is it all right how will you model this? Or a project 2 is selected, then project 1 also should be selected I mean must be selected or a project 2 is selected then project 4 cannot be selected.

So, you see there could be different types of conditions within those constraints and you know that would make an interesting integer programming formulation challenge, which we may have to solve in such kind of project selection problems right.

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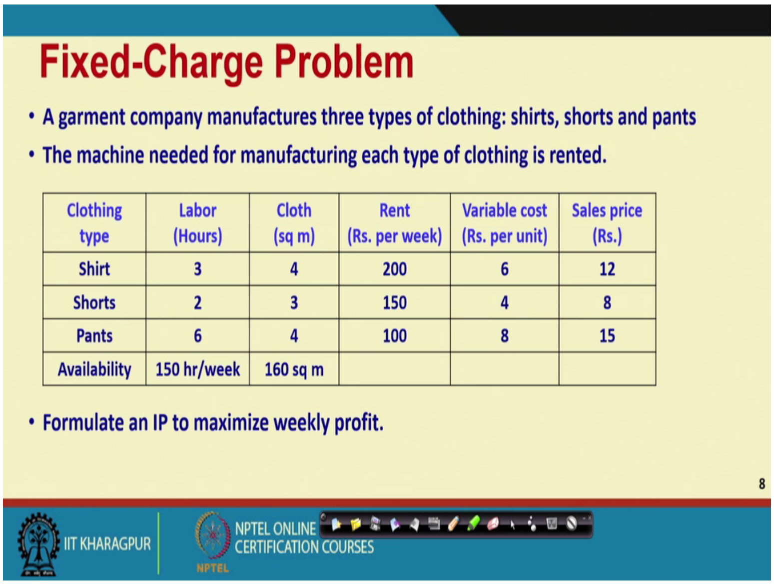

ah Now let us see another type of problem, which is known as the fix charge problem. So, let us take this problem that there is a garment company and it manufactured 3 types of clothing shirts, shorts and pants. The machine needed for manufacturing each type of clothing is rented right. So, you actually obtain the machines on rent and for each type of clothing, you requite certain hours of labor, certain square meter of cloth and there is sales price you know, and there is a variable cost and there is a fixed cost which can be called as the rent.

So, how will you formulate this as an weekly profit so, as to maximize weekly profit. So, how will you go about it? You see there are certain constraints, which may be called as a labor constant or cloth constraint there is a kind of variable cost and there is a kind of fixed cost and you know you can have a profit component also can be obtained. So, what will be our decision variables in this kind of problem? You know the decision variable could be you know the number of shirts, number of shorts and number of pants to be produced and as you understand that these are has to be integer quantities right. So, in that sense we have an integer program.

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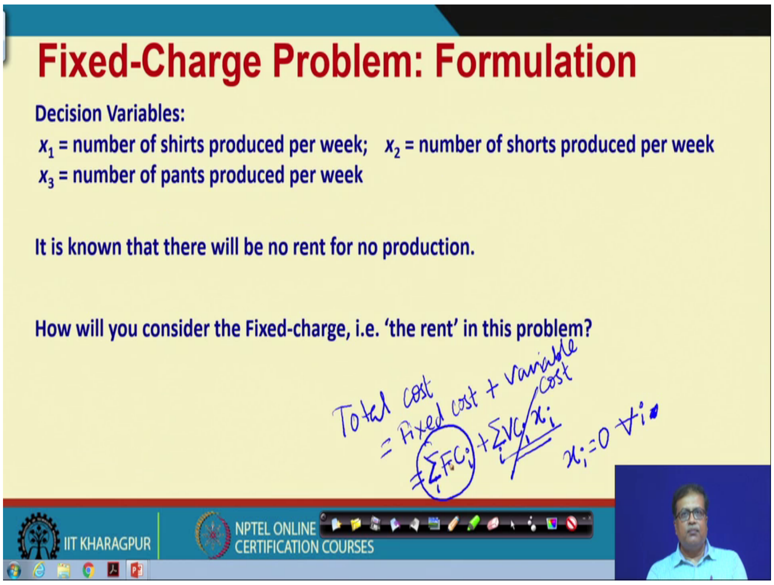

So, decision variables x 1, x 2, x 3 number of shirts, shorts or pants. Now it is known that there will be no rent for no production. So if you do not produce shirts, obviously; we need not have a machine to produce shirts, and if same thing holds for the other 2. So, question is how do you consider this fixed charge that is rent in the problem because you cannot hardcode it. If you if you simply add it to your program, there is a fixed cost, then it will it is not dependent on your decision variable is it all right. Since it is not dependent its fixed. So, when x is or equal to 0, you need not have that fixed cost equal to 0.

You know I hope you understand, let us see what at what exactly it means; supposing we have total cost equal to fixed cost plus variable cost. So, say fixed cost is FC, and variable cost is some VC i into xi sum over; obviously, we have to sum FC also for different i and sum of VC also. So, this is the total thing, but if you take it in this manner the problem is what happens when xi equal to 0 for let us say i equal to for all i let us say right.

Let us say for all i, x i equal to 0 take that situation where you do not produce anything then also these term because these term will becomes 0 because x i 0. So, this term will vanish, but this fixed cost will remain. So, you cannot model it in this manner is it not? So, this is the difficulty that you are going to face you know in a fixed charge problem, you know if you try to model it in this method is it all right. So, that is the difficulty we are going to have you know what is to model fixed charge problem in this manner. So, having known this, you know we have to then understand later on that how do I goead with such situations

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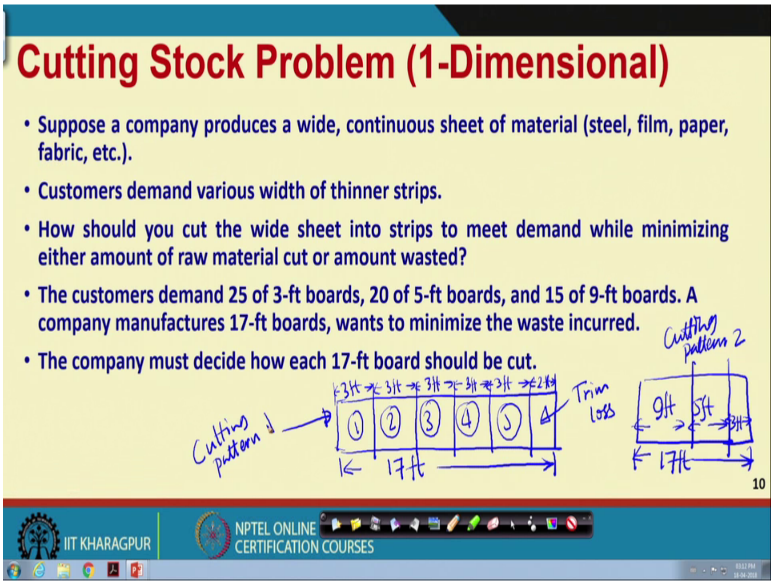

The next one is a cutting stock problem. So, suppose a company produces a white continuous sheet of material, it could be a steel or film or paper or fabric and customer demands various width of thinner strips right. How should you cut the white sheet into strips to meet demand while minimizing either the amount of raw material cut or the amount wasted right. So, let us take an example the customer demand 25 of 3 feet boards, 20 of 5 feet boards and 15 of 9 feet boards.

So, and the company manufactures 17 feet boards how can you minimize the waste? Let us look at this problem you know carefully, what it really means. You see assume this is your 17 feet. So, these entire length for u is 17 feet. So, you know you have particular what you call width of steel sheet and total length of that is 17 sheet. Now suppose you decide that I will utilize this entire sheet to cut what is known as 3 feet boards. So, what you can do? You can cut 1, 2, 3, 4, 5.

So, this is the first one, this is the second one, third one, fourth one and the fifth one each of them are 3 feet. So, like this is 3 feet this is 3 feet, this is 3 feet, this is 3 feet and this is 3 feet. So, you can cut 5 of them all right. And what will happen to this remaining 2 feet? This is you cannot utilize it will be your trim loss is it all right.

So, you understood this problem and this you can say is one cutting pattern. So, this is kind of cutting pattern, you know one of the cutting patterns that you can think of that you know you have a cutting stock problem, where you know you have generated one cutting pattern. Can you think of another cutting pattern you know involving 5 feet and 9 feet? You know you are correct you can you can have another cutting pattern, where you know you have this is your 17 feet.

So, you have 9 feet here and a 5 feet here. So, this is 9 feet and this is 5 feet and you can have a 3 feet also. So, good thing is this that you know with this 17 feet you can have another cutting pattern, where you can cut one 9 feet, one 5 feet and one 3 feet. You see you cannot really have these pattern this is the another cutting pattern.

So, these cutting patten 1 and cutting pattern 2. So, you know really speaking what you can do? You can take certain amount of cutting pattern 1 and certain matter of such an amount of cutting pattern 2 and you can decide may be you would like to have more of cutting pattern 1 and less of cutting pattern 2 or you can have other type of cutting patterns like 3 4 5 etcetera.

So, the real question is how do I really choose certain cutting patterns in how many numbers so, that we achieve our goal. What is the goal, how to minimize the waste; that means, that trim loss so, that we can fulfill the customer demand of let say 25 of 3 feet boards, 25 feet board and 15 9 feet boards. So, you can understand that a decision variables are how many of a given cutting pattern we need to choose is all right.

So, that is our cutting stock problem to really speak off.

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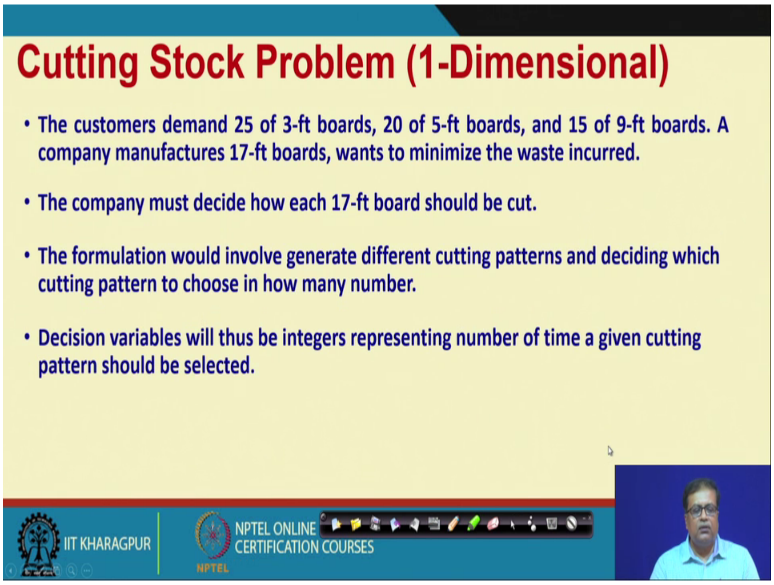

So, having said that you know let us move to you know this that, the formulation would involve generation of different cutting patterns and deciding, which cutting pattern to choose and in how many numbers is it all right. Now a decision variables will thus be integers representing number of time a given cutting pattern should be selected right. So, this could be another type of problem which we have to also take up.

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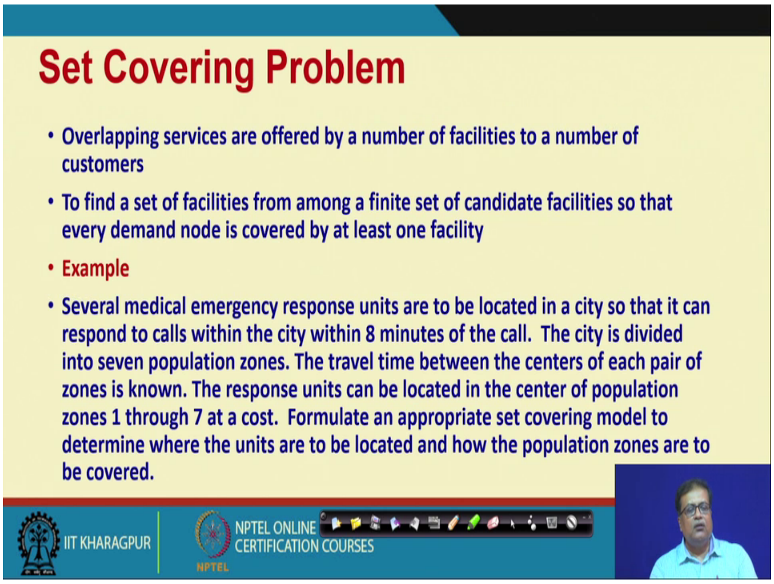

Then let us move over to the fourth type, which we make all a set covering problem. You know see the overlapping services are offered by a number of facilities to a number of customers. Now to find a set of facilities from among a finite set up candidate facilities, so, that every demand node is covered by at least one facility.

So, let us taken an example; suppose several medical emergency response units are to be located in a city, so, that it can response to calls within the city, within 8 minutes of the call. Now the city is divided into 7 population zones. The travel time between the centers of each pair of the zones is known. The response units can be located in the centre of population zones 1 through 7 at a cost is it all right. Now formulate an appropriate set covering model, to determine where the units are to be located and how the population zones are to be covered. So, that is the problem.

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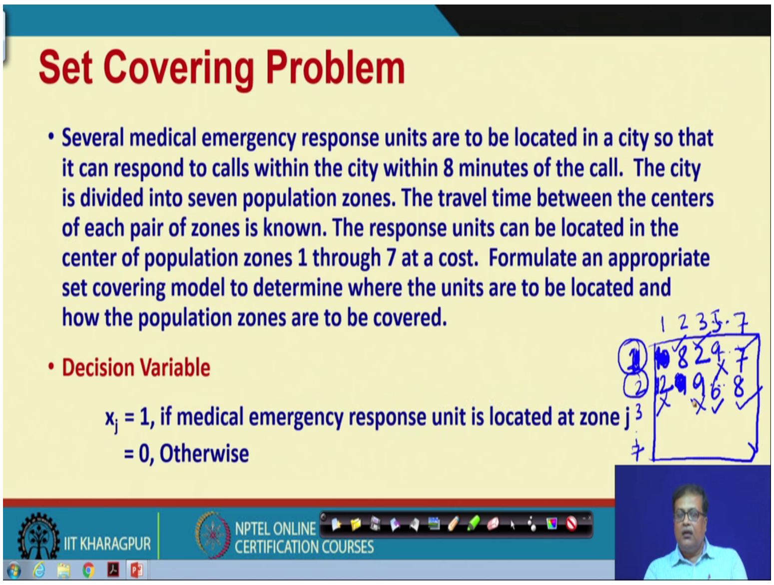

You see you know this problem will be better understood you know if you if you if you really look at the matrix. You see the matrix could be something like this, that you know you see I have you know before us say you know supposing these kind of distances.

Let us take only few distances, really speaking there will be 1 to 7 and the here also 1 2 3 7. So, there will be so, many distances. So, at least some distances let us let us put. So, suppose this is 10, this is 8, this is 2 and then there are others and this is 7, this one is 12, this one is 9 obviously one to one their cannot be anything. So, they can we into 0 actually then this is 3 to 2 yeah 2 to 3 is let us say 9, and 2 to 7 is 8 right. So, I think you can you can understand that these are zeros they do not exist. Now if these are some of my distances then the question is that, you know can I locate supposing I locate our emergency response unit at one you know this is the first one. So, I locate it here. So, when I locate here, you know can I put can I reach 2 within 8 minutes answer is yes.

So, yes can I go to 3? Yes. Can I go to 7? Yes right, but from 2 can I go to 1? No can I go to 3? No can I go to 7? Yes. So, you see I have given only some example that if I locate in 1, then I can reach 2 3 7 obviously, others I have not discussed. If I located 2 I can reach 7, but not one to you know not 1 and 3 because you know they are not within 8 minutes of call. So, essential idea is that to locate the medical emergency unit at a given location, it does not cover all the units is it ok. So, if I if I locate in 1 maybe it covers 2 3 7, but suppose there could be a 5 where it is cannot. Similarly a 2 it task cover maybe 5 and 7, but it does not cover 1 and 3. So, it is not really think that you know I have to just locate at 1 then I will not be able to cover all.

If I put in 2 again I would not be able to cover all. But if I put 1 at 1 and 1 at 2 may be I will cover all, but then you know the it its efficiency may not be the best. Maybe if I chose 1 and 7 or 1 6 and 7 by efficiency will be the best. So, where exactly I should locate my facilities so, that my efficiency is will be the best that is the essential idea of this set covering problem. I hope you understand which is also an integer programming problem.

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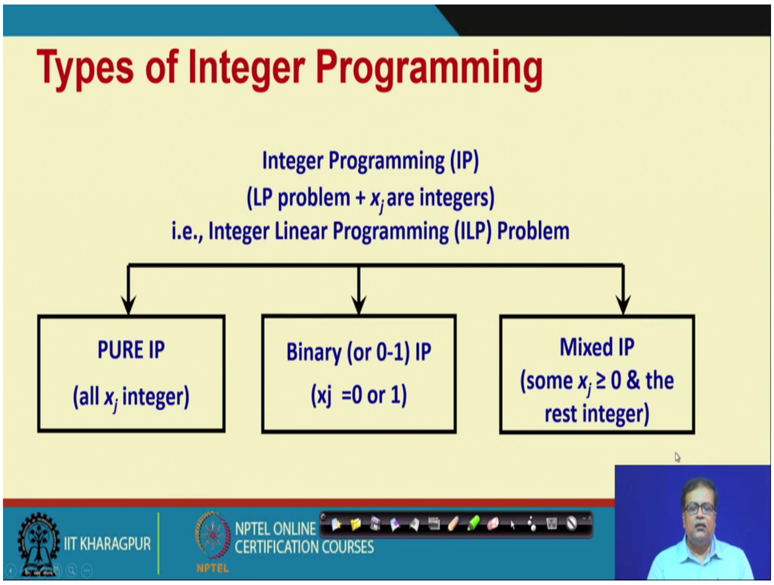

Now, let us look at the different types of integer programming problems that we can have you know, it could be pure IP where all x js are integers it could be binary that is 0 or one right that could be the second type or it could be mixed. Mixed there will be some x js that will be 0 I mean greater than equal to 0 and they can take continuous values or you know there, but the others the rest has to be integers is it all right. So, we have pure IP, binary IP and mixed IP. In mixed IP the risk would be integers or binary is it all right.

So, these are the different types of integer programming problem and we have discussed what is integer programming, how to formulate integer programming that will be our next set of lectures and later on how to you know solve them. So, with this I stop at this point.

Thank you very much.