

Management of Inventory Systems
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Lecture - 38
JIT- based Approaches for Materials Management (Contd.)

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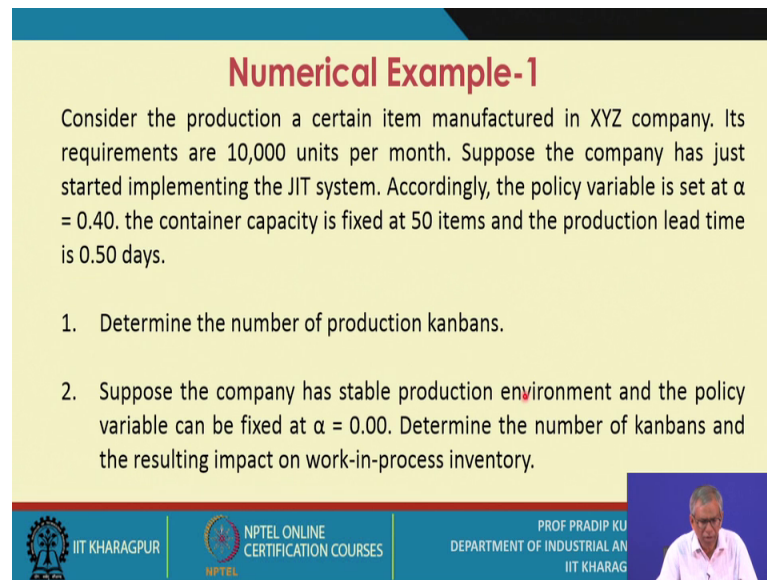
JIT-based Approaches for Materials Management

- ✓ Numerical Examples
- ✓ Determination of Number of Kanbans

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So, during this session I am going to discuss the few numerical problems, and how to determine the number of Kanbans when the probabilistic model is applicable. Now we have already explained that what is a Kanban system and the working of a typical Kanban system using the 2 Kanban cards. We have already explained in fact, so, the working is known to you.

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Numerical Example-1

Consider the production of a certain item manufactured in XYZ company. Its requirements are 10,000 units per month. Suppose the company has just started implementing the JIT system. Accordingly, the policy variable is set at $\alpha = 0.40$, the container capacity is fixed at 50 items and the production lead time is 0.50 days.

1. Determine the number of production kanbans.
2. Suppose the company has stable production environment and the policy variable can be fixed at $\alpha = 0.00$. Determine the number of kanbans and the resulting impact on work-in-process inventory.

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Now, when, but the understanding of the working of a Kanban system will be better if we take off a few numerical examples.

And these examples are very very common. And right at this point in time, we have also referred to the deterministic model for the number of kanbans, and this number of kanbans to be used in a particular say the production system or the pull system is a very important parameter. Now let us first talk about the new first a numerical example. Consider the production of a certain item manufactured in XYZ Company. Its requirements are 10,000 units per month.

If you refer to our discussions and the previous the lecture session's you might have noticed, that we are mentioned that that the average demand is to be known. And initially we do not consider the demand to be a variable so it? So, the 10,000 units per month is considered to be the average demand. We are not say the considering the standard deviation of the demand. Suppose the company has just started implementing the JIT systems; that means, essentially it was initially a pull systems and because of the erratic the demand pattern situations the company the coming across.

So, it tries to it changes it is systems from the push to pull. Accordingly, the policy variable is said at alpha equals 2.4; that means, still the company feels that that there is a significant at the strong effect of the external the factors on the performance of it is inventory and production control system.

So, that is why the value of alpha is considered as 0.4. The container capacity is fixed at 50 items or the 50 units it could be either 50 or 100. And the production lead time is half a day. That how another question is determine the number of production kabanass. Suppose the company has stable production environment; that means, it is initially it was unstable, that is why the value of alpha was 0.4.

So now, by taking some effective measures, the company has least in a least a stable condition. So, it is able to create a stable production environment, and the policy variable can be fixed at alpha equals 2.00. It will have a direct so, the impact on the safety factor, determine the number of kabanass and the resulting impact on work in process inventory.

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Numerical Example-1

3. What happens if the lead time is increased to 1 day because the labour shortages and failure of machines?
4. What happens if the lead time is reduced to 0.25 days because of process improvements? The value of α is 0.30 as a result of these process improvements.

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So, while do we determine the number of the kabanass. Obviously, till now we have learned the deterministic model so, we will apply that deterministic model only.

And this particular the kind of the formula which we will be using to determine the number of kabanass, this formula has been was the developed by Toyota many years back. Now the third question is what happens if the lead time is increased to one day, it was a half a day.

So now, it was become one day whatever may be the reasons, because the labour shortages and failure of machines. So, there could be many reasons that why so the lead time may increase. So, you have come to know that there are there are 2 specific reasons.

One is the shortage of labour or the workers and the second reason could be possible reasons could be the machine failure. Now you start thinking about other possible reasons. These are the 2 specific say the just examples, but you should also be aware off that in how many what are the specific reasons with the set of reasons you know for increasing or increased on the lead time. What happens if the lead time is reduced to 0.25 days because of process improvements?

So, these are the other extreme; the value of alpha is 0.30 as a result of this process improvement. So, there is a relationship between, say the safety factor and you know, say the safety factors and the lead time, is it? So, the 3 important factors or the 4 important factors, you need to consider to determine the number of say the kabanans.

And what do we are assuming that these 3 factors or these 4 factors are interrelated.

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Solution

1. We now know that number of kanbans is given by the following formula


$$\text{Number of kanbans} = \frac{(\text{Daily demand})(\text{lead time})(\text{safety factor})}{\text{container capacity}}$$

Assuming 20 works days in a month, the daily demand is $10,000/20 = 500$ parts. Accordingly,

$$\text{Number of kanbans} = \frac{(500)(0.50)(1.40)}{50} = 7$$




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So, what is the solution? So, the number of kabanans is given by the daily demand then the lead time and this is the safety factor. So, in the formula which we have used we are refer 2. So, the notation we have used that is capital D. The lead time has got the 2 parts t w and t p so, both are added. So, you get the lead time and the safety factor is specified as 1 plus alpha.

And the container capacity the notation is small a. Now assuming 20 work days in a month, the daily demand is 10,000 divided by 20. Now this is your assumptions there

could be 24 days or 25 days, but you have to specify what is your assumption. That means, what do you try to do; that means, already I have mentioned, that the JIT systems or for that matter the Kanban system is used for the short term planning and control purposes.

And so, the day wise you, you try to control you try to monitor the inventory as well as the production control situations. So, here so, you need to know the daily demand. So, it is 10,000 divided by 20 that is the 500 units. And so, what is the number of Kanban; that means, this is 50, 500, this is lead time is half a day and the safety factor is alpha. So, that is why it is one 0.4 0 and the continual capacity is 50 units so, this number is 7.

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Solution

2. If $\alpha = 0.00$, then the safety factor is 1.00. Accordingly,

$$\text{Number of kanbans} = \frac{(500)(0.50)(1.00)}{50} = 5$$

3. $\text{Number of kanbans} = \frac{(500)(1.00)(1.00)}{50} = 14$
 The implication if this change in the system operation is that the average inventory increases by 350 units $[(7)(50) = 350]$ compared with case 1. this is because extra containers are available as safety stock.

4. $\text{Number of kanbans} = \frac{(500)(0.25)(1.30)}{50} = 3.25$

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Next is if alpha equals to 0; that means, there is no impact of the external factor. And that is why it is an 100 percent the stable system. So, the safety factor is becomes one accordingly the number of kabanass is 500 into 0.5 into one divided by 50. So, it is reduced to 5, and in the third the case the number of kabanass is 500 that is the daily demand. And this the lead time becomes one day.

And your safety factor remains at one, because alpha value is 0. And the container capacity remains same that is 50 so, this is becomes 14. So, the number of kabanass become 14, the implication of this change in the system operation is that the average inventory increases by 350 units; that means, it was initially that means, initially it was 7; so now, it has become 14.

So, there is an increment of the 7 say the containers. And each container the contents 50 units so, this increases 350 units compared with case one. This is because extra containers are available as safety stock, ok. So, this is because what is important is initially it was 0.5 now it has become one.

So, these also effecting the number of kabanas and the average inventory. In the 4th case the number of kabanas is 500, the lead time is reduced to 0.25 and, but the alpha value is now is assumed to be 0.3. That means, some disturbances you are getting from the external sources container capacity remains same so it becomes 3 0.25.

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Numerical Example-2

Consider the manufacturing of product Z in a company. Product Z is assembled from two parts, X and Y, that are manufactured in the company. A schematic layout depicting preceding stage, staging area, subsequent stage, and the flow of production and withdrawal kanbans is given in the Figure. The data given in Table are available.

1. Determine the number of withdrawal and production kanbans.
2. Now suppose the assembly process is shifted to Mexico as a part of reorganization. The lead time to travel between the new location in Mexico and the present plant location is 4 days each for parts X and Y. determine the number of withdrawal and production kanbans and the impact of this policy on work-in-process inventory.

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So, this 3.25 change it to say 4. So now, you can manage the situation, you can control the situation with say the 4 containers, ok.

Or say was number of containers becomes 4 kabanas. Now so the next example that is the numerical example to let us discuss the next example consider or the manufacturing of product Z in a company. Product Z is assembled from 2 parts X and Y, ok. A very simple case, we are considering just to understand the concept clearly. Now this 2 parts are manufactured in the company.

That means there is an assembly operation, this is also a manufacturing operation. And what is important is, in any JIT based production systems of or inventory control systems, you must try to create a conditions where there is a perfect match between a

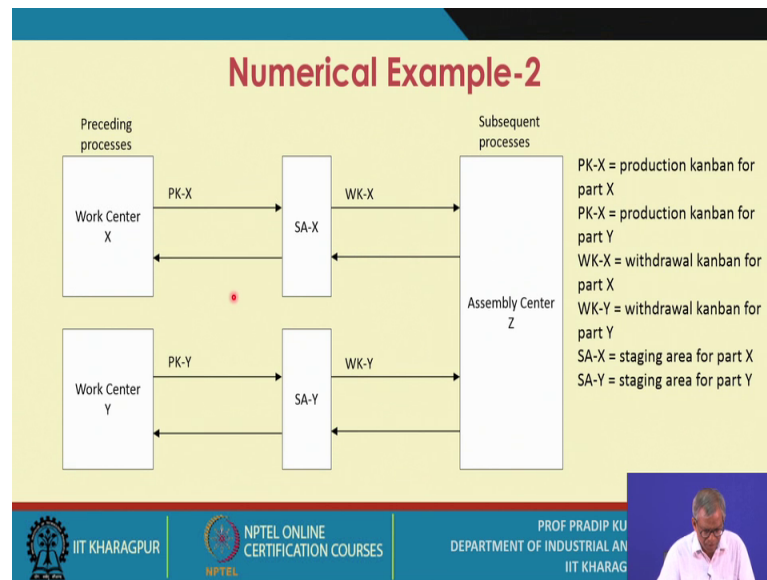
perfect balance between the part manufacturing and the assembly. Normally this is not the case in a pull systems, in the pull system what do you try to do? That means, you try to synchronize the operations of the part manufacturing with the operations of the assembly.

So, there must be a perfect matching there are many instances where these conditions the company could achieved by implementing JIT based production system. So, a schematic layout depicting preceding stage, the stacking area, subsequent stage and the flow of production and withdrawal kabanais is given in the figure. So, we will show you the figure the data given in the table are also all available; that means, those data also we will come to know. And with those the data you need to determine the number of withdrawal and production kabanais.

Now in the next condition which you face; that is, suppose the assembly process is shifted to another country say Mexico as a part of reorganization,; that means, you will be doing your manufacturing activities that you have plant, but the assembly operation now it will be done in another plant as a located in a another country in a foreign country, the lead time to travel between the new location in Mexico and the present plant location is 4 days each for parts X and Y, is it ok? Because the parts X and Y had they have sent you know you know in the same lot.

Determine the number of withdrawal and the production kabanais and the impact of this policy on work in process inventory.

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So, this is your the system; that means, you have the work center for manufacturing a part X, your work center Y for manufacturing part Y. And what is PK-X this is the production Kanban for the X item. And this is the production Kanban for the Y item. So now, this is the stacking area. So, the production Kanban moves to the stacking area with the container.

And then you get say this is the withdrawal Kanban for the X unit. And whenever you get the withdrawal Kanban; that means, from the stacking area this container moves to the subsequent process. So, and again this is the empty container and it comes to the stacking area, and subsequently with the say the production it goes to the work center X, is it? And what is Z? Z is an say the assembly center where both the units that is the both the parts X and Y are used.

And as far as the bill of material or the product structure, you are already aware of what is the product structure code, what is the product structure records. So, as per the bill of material so, you know what is the sequence of say the operations or the or say what are the process we will adapt for doing the assembly, which part comes first which part comes next. So, all these details are known, and then you carry out the assembly activity so a PK-X is production Kanban for part X.

PK say Y production Kanban for part Y, WK-X is the withdrawal Kanban for part X, and WK-Y is the withdrawal Kanban for part Y. SA-X equal to stacking area for part X, and SA-Y is a stacking area for say the part Y,?

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Numerical Example-2				
Part	Demand (Units/Day)	Lead time (Days)	α	Container capacity
Assembly Stage				
X	2000	1.00	0.00	100
Y	800	0.50	0.25	50
Manufacturing stage				
X	2000	0.50	0.20	100
Y	800	1.00	0.00	50

Now with this, now this is the data available; that means, for part X and Y demand in units per day, it is 2000 units this is 800 units, the lead time values are given that is for part X it is 1. And for part Y it is 0.5 days, this is at the assembly stage.

And the value of alpha the set factor related the value of alpha is 0.00. And this value is 0.25, and the container capacity for X it is 100 and for Y it is 50. So, this is at the assembly stage and for the manufacturing stage, this is 2000, this is 800 they remain same. But the lead time at the manufacturing stage it is 0.5 days and for Y it is 1 and 1 day. And value of alpha is 0.2 for part X; that means, the manufacturing area X and for manufacturing area Y the value of alpha is 0.

And the container capacity for manufacturing area X is 100 and for manufacturing area Y, it is just 50.

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Solution

- (a) $X = 20, Y = 10$
 - (b) $X = 12, Y = 16$
- Since the lead time is now 4 days, the number of withdrawal kanbans for parts X and Y are 80 and 80, respectively. The impact of this decision is that 60 containers for part X and 70 containers for part Y remain in the pipeline. There is no change in the number of production kanbans.

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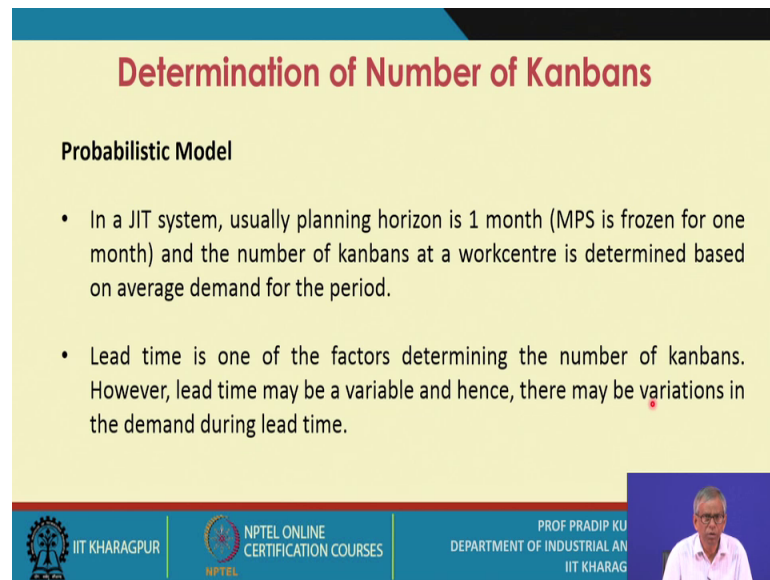
So, with this data you apply that formula, now this is the assignment given to you, and you just find out that X will be 20 and Y equals to 10. X will be 12 and Y equals to 60; that means, withdrawal Kanban and the production Kanban, ok. So, first you determine the number of withdrawal kaban, by applying the formula the deterministic model.

And then you use the same say the model, the same formula to determine the production Kanban. So, so the part 1 we will have 2 sections. The first section relates to so withdrawal Kanban determination, and the part b relates to the production Kanban so, the number of production Kanban determination. Since the lead time is now 4 days, the number of withdrawal kaban for part X and Y are 80 and 80 respectively.

So, you need to calculate again when you change the lead time. So, only here the same formula will be using, but you change the by the value of the lead time. The impact of this decision is that 60 containers for part X and 70 containers for part Y remain in the pipeline. Initially it was 20, now it has become 80 for part X. So, 60 more the containers you require, for part X. Now initially it was the 10 for part Y withdrawal kaban. And now it is become say 80 so, the difference is 70.

So, 70 containers for part Y remain in the pipeline. There is no change in the number of production kaban, it is obvious impact. So, this is just an illustrative example so, that you understand this particular formula.

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Determination of Number of Kanbans

Probabilistic Model

- In a JIT system, usually planning horizon is 1 month (MPS is frozen for one month) and the number of kanbans at a workcentre is determined based on average demand for the period.
- Lead time is one of the factors determining the number of kanbans. However, lead time may be a variable and hence, there may be variations in the demand during lead time.

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This is deterministic the model fully, and there is there is a perfect understanding that why how the factors are affecting say one another. Now the next important the model you may use for determination of number of kabanans is the probabilistic model. So, how to say get this probabilistic model in a JIT system usually planning horizon is one month. Please make a note, MPS is frozen for one month only. And the number of kabanans at a work center is determined based on average demand for the period.

The lead time is one of the factors determining the number of kabanans already, you know if you look at the formula you use for determination of number of kabanans. So, lead time is mentioned as one of the important factors. However, the lead time maybe a variable, and hence there may be variations in the demand during lead time.

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Determination of Number of Kanbans

- For determination of number of kanbans in this context, we assume that
 - i. Probability mass function (pmf) of the number of kanbans is known ($p(x)$).
 - ii. Holding cost per container per unit time is known (C_h).
 - iii. Shortage cost per container short per unit time is known (C_s).

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So, for determination of the number of kabanans in this context; that means the probabilistic model. We assume that the probability mass function pmf of the number of kabanans is known this is some standard method, you may use you refer to any so the good textbooks on probability and statistics. Also definitely you will know or you will get some standard methods to determine the probability mass function for the number of kabanans it is a discrete random variable.

Holding cost per container per unit time is known. So, this estimate you must have. Similarly, the shortage cost part container short per unit time is also known that is C_s so, these 2 notations we have used.

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Determination of Number of Kanbans

- At any point in time, let there be n number of kanbans in the system.
- If the actual requirement for the kanbans is x , there may be two possibilities:

Case-1: when $x < n$.
 Expected holding cost = $C_h \sum_{x=0}^n (n - x)p(x)$

Case-2: when $x > n$.
 Expected holding cost = $C_s \sum_{x=n+1}^{\infty} (x - n)p(x)$

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At any point in time, let there will be n number of kabanass in the system,. So, if the actual requirement for the Kanban is X , there is a n number of kabanass in the system, but the actual requirement may vary. So, it depends on what sort of you know that the demands imposed on you, and what is the lead time? So, a number of factors actually you know may determine the actual requirements during a particular say this time period. Now there may be 2 possibilities.

So, actual number the circulating within the system that is n , n number of kabanass, your system is having n number of kabanass. Where your actual requirement is X so, there could be 2 cases. When X could be less than n ; that means, your actual requirement is less than what is available. So, obviously there will be holding cost. And what is the expression for expected holding cost? That is the per unit it is C_h , already you have this estimate into $\sum (n - X) p_x$.

And X varies from 0 to n , is it ok? A simple logic so, X varies from 0 to n and in the case 2 when X is greater than n . So now, you have the expected the shortage cost, and the expected shortage cost is C_s into $\sum (X - n) p_x$, and X is varies from $n + 1$ to infinity.

So, you have the expected holding cost and you have also expected say the shortage cost.

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Determination of Number of Kanbans

- Total expected cost, $TC(n)$ is given by
$$TC(n) = C_h \sum_{x=0}^n (n-x)p(x) + C_s \sum_{x=n+1}^{\infty} (x-n)p(x)$$
- For determining optimal value of n , minimum $TC(n)$ satisfies the following conditions:
 - i. $\Delta TC(n) = TC(n+1) - TC(n) > 0$
 - ii. $\Delta TC(n-1) = TC(n) - TC(n-1) < 0$

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So, these 2 expressions you have, and then what is the total expected cost; that means, when you add these 2 cost; that means, expected holding cost, an expected shortage cost. So, you get this total the cost total expected for when there are n number of kabanans in the system that is if you add these 2 to 2 cost. This part is basically the holding cost expected holding cost. And this is the expected number of units actually you are for which the holding cost is applicable.

And this is the expected number of so, the unit short. And the part the unit short, you are incurring a cost of C_s . So, you have these expressions for determining optimal value of n , the minimum the total cost satisfies the following conditions, is it ok? This is the standard procedure you imply for determining says the optimal value of a say the n in a discrete case. So, the delta $TC(n)$ is $TC(n+1) - TC(n)$ so, this must be greater than 0.

And delta $TC(n-1)$ that means $TC(n) - TC(n-1)$ is less than 0, ok; that means, for n for $n+1$ you will incur a cost which is greater than the cost will incur with n . And similarly with $n-1$, the cost will incur should be greater than or the cost will incur with n number of kabanans. So, this is the logic you put forth.

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Determination of Number of Kanbans

- Taking first forward difference and simplifying, we get


$$p(n) > \frac{C_s}{C_h + C_s}, \quad p(n) = \sum_{x=0}^n p(x) = \text{cdf of } x$$
- Taking first backward difference and simplifying, we get

$$p(n-1) < \frac{C_s}{C_h + C_s}$$
- Optimal number of kanbans can be determined from the following expression:

$$p(n-1) < \frac{C_s}{C_h + C_s} < p(n)$$




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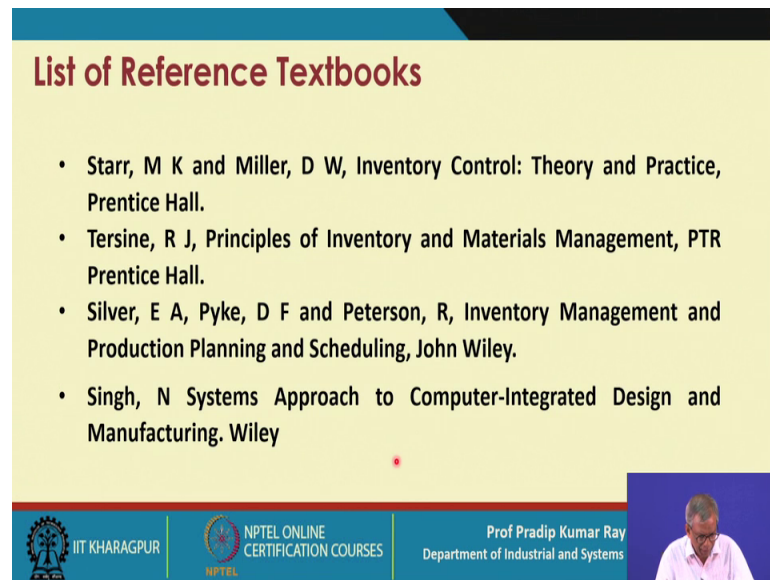


Taking first forward difference and simplifying we get these expressions,. So, this is a capital P n greater than C s by C h plus C s.

And this capital P n is equals to actually sigma P x, x varies from 0 to infinity; that means, it is basically the cumulative distribution function of x. Taking the first backward difference simply and simplifying we get capital P n minus 1 is less than C s by C h plus C s,. And obviously, this is the capital P n minus 1 is sigma P x equals to 0 to n minus 1. So, the optimal number of kabanans can be determine from the following expression. So, the capital P n minus 1 less than C s by C h plus C s less than capital P n, ok.


So, this is very very the simple expression. So, you have to use these inequality expressions to determine the number of n.


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So, what we have mentioned that we have considered the Kanban system and the Kanban systems lies at the heart of the JIT based production system. And the main purpose of the Kanban system is to is to control the inventory level as well as it also the it also controls the flow of inventory or the flow of materials in a production systems.

And in order to say they establish these Kanban systems, what you need to do? You must know how to determine the number of kabanans for which the 2 models we have we have referred to. The first model is a very very simple is called deterministic model. In mobility of the cases what the Toyota has observed that the deterministic model will be very very effective. Whereas, in certain other cases where you may you may not have sufficient control on your production systems.

And or the variability has to be allowed or the variations in the production system has to be allowed. And you try to fine tune say that the number of kabanans with or the variable demand, then so, you have no other options, but to go for using the probabilistic model.

So, both the cases we have discussed, and the other issues will be discussing in the subsequent lecture sessions.

Thank you.