Management of Inventory Systems

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Lecture – 30
Dynamic Inventory Problems under Risk (Contd.)

During this lecture session, on dynamic inventory problems under risk as you will you are already aware of that, we have mentioned that there are the 2 types of approaches for problem formulation and solution.

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The first one is the cost based approach and the second one is a service level based approach. We have already discussed the cost level based approach; cost based approach. And now, we are discussing the service level based approaches.

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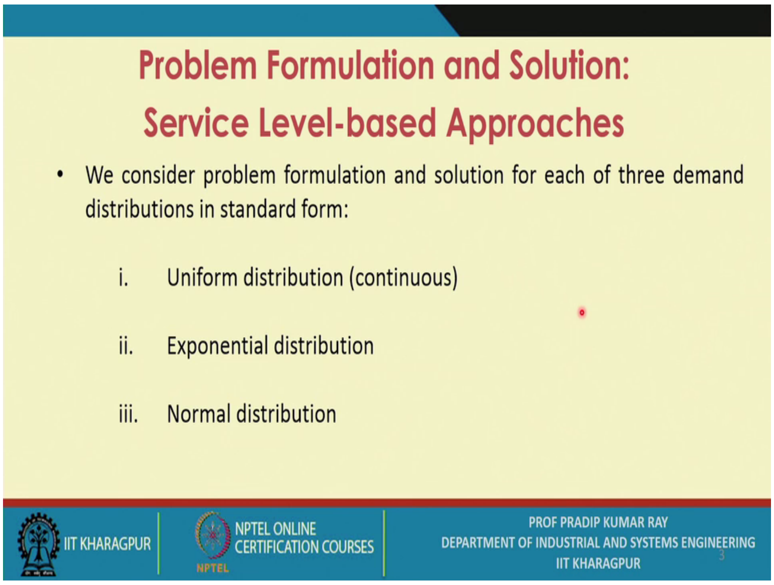

Now, as we have already pointed out that service level based approach is applicable only when the cost estimates or cost estimates do not have much reliability. In other words,, the cost based approach is not considered feasible. There could be many reasons for this one.

And that is why many a time we opt for service level based approach. We have already defined what is a service level; if you recollect there are the 2 possible the definitions and measures of service level. As the first one is the service for order cycle, and the second one is the service for units demanded. Now, when we try to incorporate the service level concept in the determination of the parameters of a given inventory control system usually, we use the second measure of service level that is service for units demanded.

In the previous lecture sessions, we have already defined the service the level the measure and you will and particularly this measure of the notations which you have used these are applicable for the q system of inventory control. And we have used a certain notations like SL bar sigma L N r f r mu.

So, my suggestion is you fly first get an idea about all these, you know say, the measures or the parameters. And the first thing you need to know that is, for which time period you are required to say that determine the safety stock. If it is a q system of inventory control; obviously, you know the during the lead time there is a possibility of stock out, and that is why, you know the service level based approach or the modeling is done exclusively for the lead time period.

And accordingly the problem is formulated. Whether, if you try to say you know the model the problem for say the p system of inventory control, you need to define all these terms for the time period which consists of 2 part S1 is the order period or the order interval and the second part is the lead time ok.

So, we will be referring to, we will be discussing the model formulation part with the reference to a q system in inventory control so that your understanding is appropriate. And I am sure that if you have appropriate understanding of the entire approach the same the approach with obvious the modifications you can you can use for say p systems of inventory control.

Now, we consider 3 types of say the demand distributions and for problem formulation and solution. The first one is a uniform distribution and we will be referring to the continuous type; that means, we will assume that you know the random variable is of continuous type.

Then, the next important the distribution we will consider that is exponential distribution or the continuous random variable. And the third one is the normal distribution again for continuous random variable.

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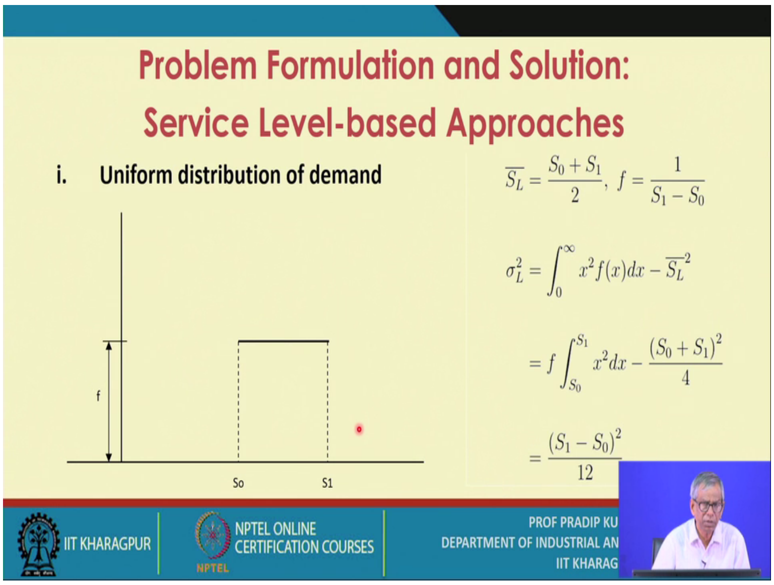

Now, let us first talk about the uniform distribution of demand. Now, the first thing you must know that is given a distribution what is the expression for mean, and what is the expression for the variance.

Now, here for this particular distribution; that means, continuous type uniform distributions, you have the interval between S0 and S1. And the for any value say the between S0 and S1, your the your probability the remains same. So, this is the notation that is the f; that means, probability or density function. So, obviously, you know we use this term SL bar; that means, the mean demand during lead time L stands for the lead time. So, the mean demand during that time that is SL bar is S0 plus S1 by 2 so, this is stations.

Now, what is f? F is 1 minus S1 1 by S1 minus S0, 1 upon S1 minus S0. So, similarly you can calculate the variance of say the lead time demand and it is integration 0 to infinity x square f x d x. This is the general formula, I am sure that you are aware of, minus the square of the mean so here, this is S L bar square, ok. So now, when you substitute the expression for SL bar; that is, S0 plus S1 by 2, and here what you find that f x is f; that means, 1 upon S1 by S0. And then the integration from S0 to S1 that is x square dx. So, if you integrate this expression, and you manipulate ultimately the expression for the variance during say the lead time demand is S1 minus S0 square by 12, ok.

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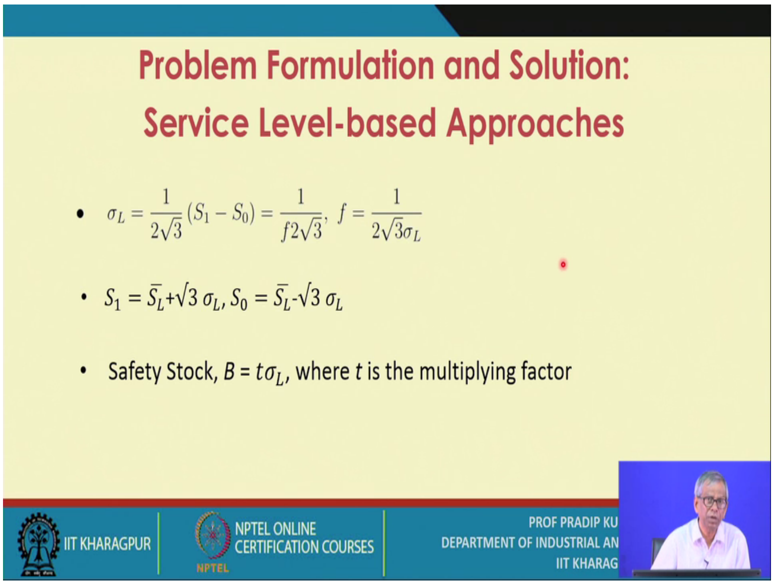

So, we have started from say we have use the fundamental principles in computing or in determining or computing the mean and the standard deviation or variance. So, so the standard deviation during lead time demand is 1 upon 2 root 3 S1 minus S0.

And S1 minus S0 is nothing but 1 upon f; so, it is 1 upon f 2 root 3, and hence f is equals to 1 upon 2 root 3 sigma L. So, what is the expression for S1 in terms of a SL bar and sigma L? That is, a SL bar plus root 3 sigma L. Further continuous uniform distribution, and S0 is a SL bar minus root 3 sigma L. Now, these are all well-known expressions for uniform or continuous uniform distribution.

Now, we have use these notation for the safety stock; that is, capital B, and the safety stock is in is expressed in terms of the sigma L; that means, the lead time the standard lead time demand standard deviation. And it is expressed as a t in to sigma L, where t is the multiplying factor. So, ultimately when you determine the safety stock, the safety stock is to be determine in terms of t into sigma L. If the value of sigma L is specified. You need to determine the value of t, ok. So, later on while we found the total cost equation so this point will be made very, very clear.

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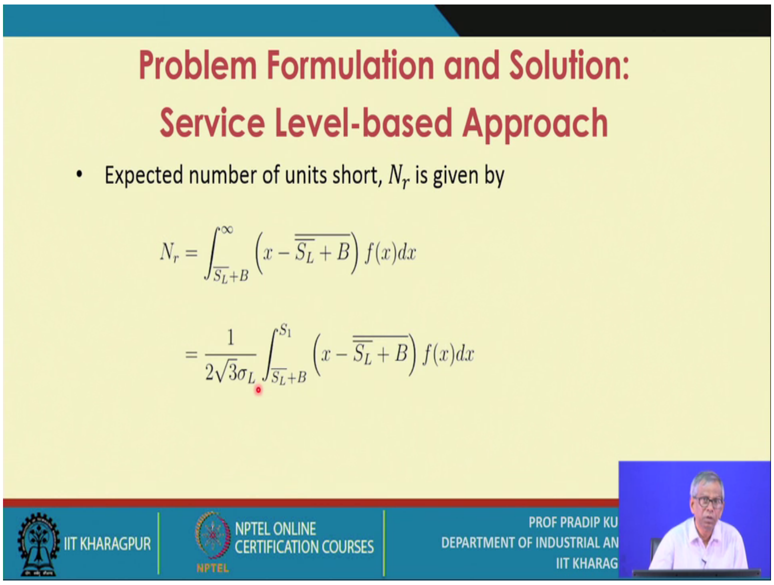

So, the safety stock the notation is capital B which is nothing but t into sigma L, ok; t is the multiplying factor. Now, you know in terms of N r we defined the service level.

So, with respect to so the in a continuous uniform distribution you must be able to say or the derive the expression for N r that is the expected number of unit short. Now, this expression is given by integration S L bar plus B to infinity; that means, during the lead time what do you consider the average demand during lead time? That is S L bar plus the safety stock, that is capital B.

Now, if the demand during the lead time is more than a S L bar plus B obviously, there is a stock out. And what we are trying to measure we are trying to measure the expected number of unit shot during lead time, because it is accused system of inventory control. So x is a random variable x minus SL bar plus B. And corresponding probability density function is a f x dx.

So now, these f x is 1 upon 2 root 3 sigma L already, we have derived these expression. So, integration SL bar plus B 2 S1, ok. So, that is the maximum value, x minus S L bar plus B 2 f x dx, so it will right.

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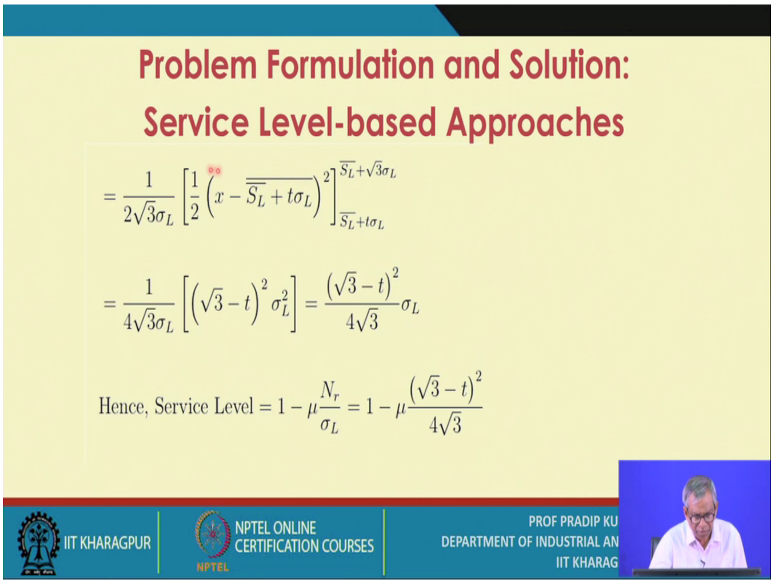

Next is, so obviously, you go for the integration, and this is the lower limit of integration, this is the upper limit of integration. So, this 1 is S L bar plus t into sigma L, t into sigma L is beta and the maximum value that is S1. So, S L bar plus root 3 into sigma L. Already, we have shown these you know the relationship between say S1 and S L bar and sigma L. So, the next step we get 1 upon 4 root 3 sigma L root 3 minus t square sigma L square, and this becomes root 3 minus t square by 4 root 3 into sigma L on simplification. So, hence service level is 1 minus mu into N r by sigma L.

So, the expression for N r is known, and when you substitute the this value of N r in this equation. So, what we get we get an expression for the service slab level so, when the demand is uniform. So, these expression is 1 minus mu into root 3 minus t square by 4 root 3.

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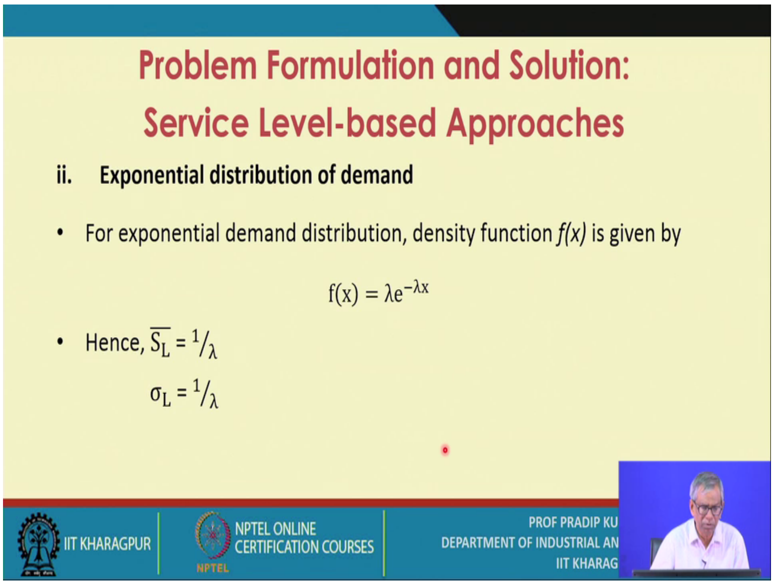

Now, the second so in the first case, when the demand distribution is uniform so we derive the expression for N r; that means, expected number of units or during lead time now when the demand distribution is considered exponential. So, what you need to do? That means, the probability density function f x or exponential distribution is given by f x equals to lambda e to the power minus lambda x.

Now, this I have written directly because if you follow the you know the steps to determine the mean or the determine sigma L. So, ultimately you will get the expression for the mean; that means, SL bar equals to 1 upon lambda and sigma L is also one up on lambda is it ok. So, it is a single parameter distribution and that the mean and the standard deviation are same for exponential density function.

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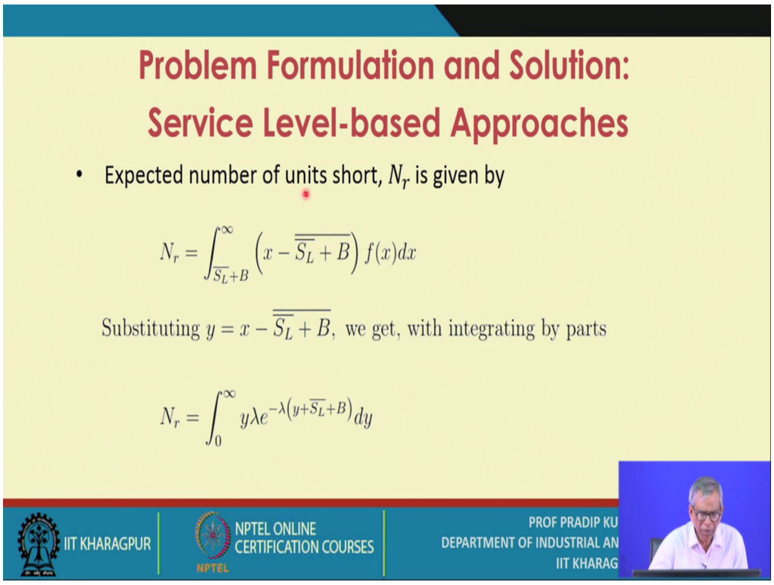

So, the expected number of unit short N r is given by we use the same the formulation that is N r equals to integration from SL bar plus B B is the safety stock and SL bar is the mean demand during the lead time of 2 infinity x minus SL bar plus B f x dx. Now, what you do actually you go for the integration by substitution followed by integration by parts ok. So, these are all well-known you know the techniques and with this the techniques by substitution and by parts are.

So, when you integrate this function ultimately these are the stapes that I have mentioned because the substitution is substituting y equals to x minus SL bar plus B. So, corresponding expression comes like this; that means, this become integration from 0 to infinity and this is the corresponding expression d y.

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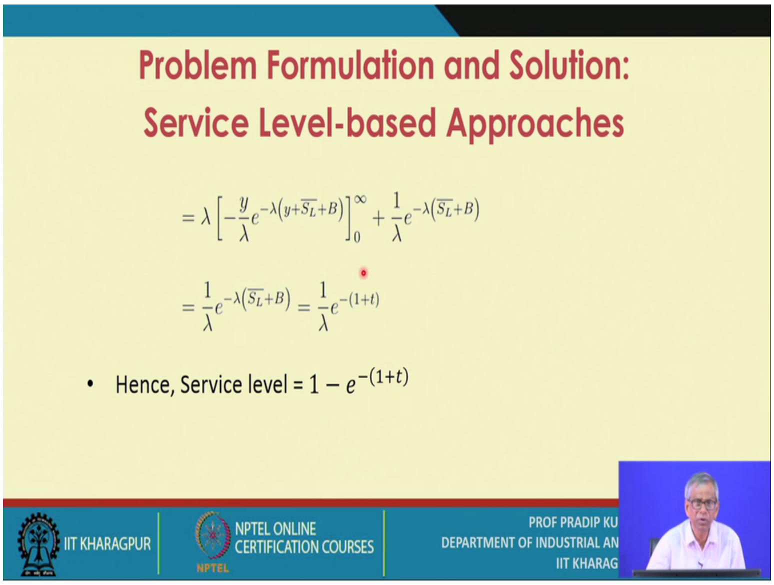

Then you go for integration by substitution so on ultimately you get an expression of N r As 1 by lambda into e to the power minus 1 plus t. So, what is t? T is the multiply factor and the if t is known; obviously, the sigma is value of sigma L is already specified. So, the t into sigma L will be known; that means, the safety stock will be known an average average the demand during lead time that is already specified that is a SL SL bar. So, SL bar plus B; obviously, will be your reorder point that is 1 of the aim one of the 2 parameters of q systems of inventory control.

So, here for exponential the density function demand density functions. So, what is the expression for the service level? That is 1 minus e to the power 1 plus t is it ok. It is essentially if you use that expression of SL equals to 1 minus mu into f r so; obviously, mu is equals to 1 mu is nothing but sigma L by SL bar. So, for exponential distribution both are same that is why mu is 1 and a f for is 1 say N r by f r is N r by sigma L. So, ultimately you get this expression of service level 1 minus e to the power minus 1 plus t e to the power minus 1 plus t.

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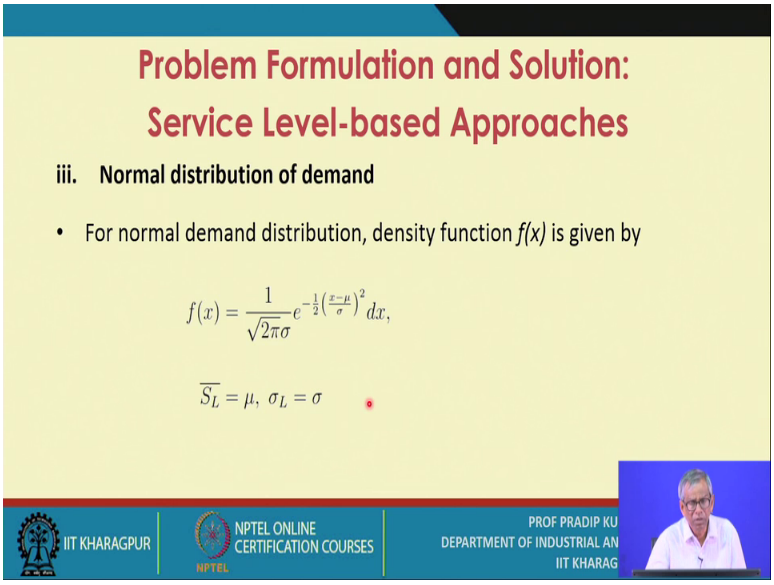

Now, the next important distribution you need to consider in many instances that is the normal distribution. So, if the demand is assumed to be normal.

So here, the density function is the probability distribution f x is 1 upon root over twice by sigma e to the power minus half into x minus mu see by sigma whole square into d x ok. So, this is f x not dx will come so this is just the f x and then SL bar equals to mu and sigma L is sigma; that means, this is the 2 parameter distribution and with you know the probability density function is this 1. Is it so f x is this now?

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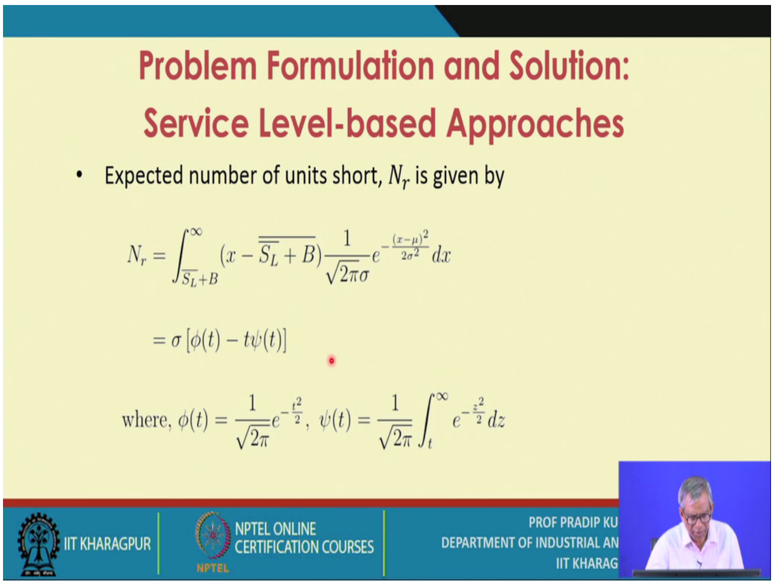

So in this case, the expected number of unit short N r is given by again the same way logic we use; that means, integration a SL bar plus B up to infinity x minus SL bar plus B ok. And this one is your f x is it for a normal density function into dx. So, if you simplify this expression ultimately you will find that N r is nothing but sigma into phi t minus t into phi t. Where, phi t is 1 upon root over twice pi e to the power minus t square by 2 and psi t is essentially the psi t is 1 minus capital f T. And so, the capital T is basically you know the distribution function and.

So, the psi t is 1 upon root over twice pi integration t to infinity e to the power minus z square by 2 into d z is it ok. So, if you follow the steps ultimately you will get this expression so, I am for the time being I am skipping the expression. So, so obviously, the N r if you want to get an. So, the N r for the normal distribution is expressed in terms of pi t and psi t; so, the expressions for pi t and psi t already given ok.

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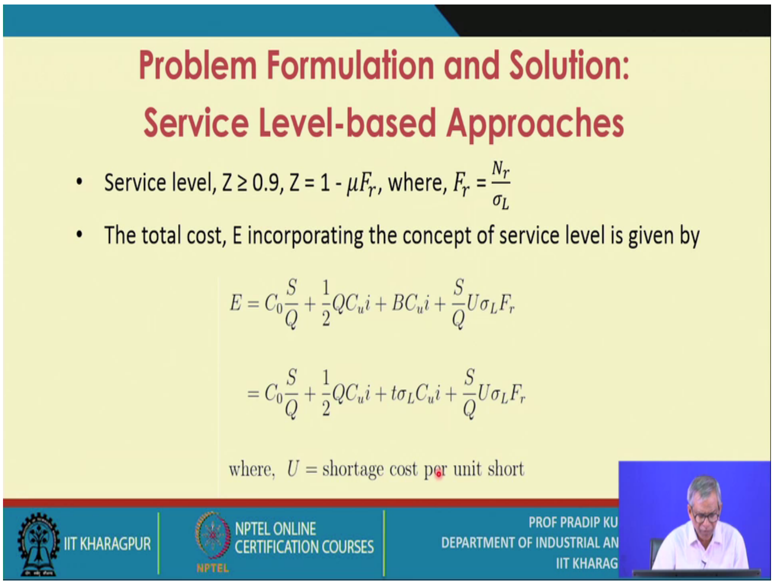

Now, once the distribution characteristics are known and for a given distribution, say the demand distribution when you know the expression for N r that is expected number of unit short. Now, our next step will be to develop the total relevant cost equation.

Now, here what we try to do actually that we opt for the exact method of analysis; that means, what we try to do; that means, the both the order quantity as well as the you know the safety stock must be the optimal values of their optimal values must be determined simultaneously. And so; obviously, you must have a total cost total cost equation in terms of say both the on the parameters.

The order quantity and the safety stock and then, you for follow the usual procedure to get the optimal the values of the inventory control or say inventory systems parameters. So, that the total relevant cost remains at the minimum level so, as I have already pointed out that that, when you try to design a system never you assume that the service level the value of service level is 1 the reason already I have explained.

So, when you design a system and that the service level based system inventory control systems say you may assume the service level as 0.9 or 0.95 or 0.99 and so. So, that is your target so based on this, what you can do? You can also determine the possible the values of the parameters of the given inventory control system. Now, if you so first thing you have to do at this stage that is the total cost expression. So, if you follow the exact analysis the total cost E incorporating the concept of service level is given by equals to this is the ordering cost. To meet the average demand; that means, C O into S by Q you are already familiar with this expression I do not need to explain it once again.

Then, the next one is the inventory carrying cost or inventory holding cost for the average demand that is Q by 2 into C u into I ok. So, Q by 2 is the average demand in an order cycle and the next one is B in to Cu into I; that means, this is the carrying cost for the safety stock of amount capital B and then what you have; that means, this safety stock carrying cost for the safety stock is opposed by the you know the out of stock cost. So, what is the total out of stock cost? That means, per year.

So, we have we are we are formulating the total cost equation per year so if S is the yearly demand and Q is order quantity. So, how many order cycles you have in a particular say year that is S by Q? What is capital U? Capital U is the shortage cost per unit short; that means, we are assuming per unit out of stock cost valid and this is basically the N r, what is F r? F r equals to N r by sigma L so;.

Obviously, sigma L into a F r is the expected number of unit short and for 3 different say demand distributions. Say, we have already derived 3 different expressions for N r ok. So, if you have these expressions for N r; obviously, you can you can write down the all the 4 components explicitly in a given of the of the total cost equation explicitly in a given situation.

So, after the simplification this is C O into S by Q plus half Q in to C u by 1 C u into i and i is inventory carrying cost as a profession of say, the average inventory which you hold. And now, the B is replaced with t in to sigma L C u into i and this is S by Q u sigma L into a F r so all the terms we have explained.

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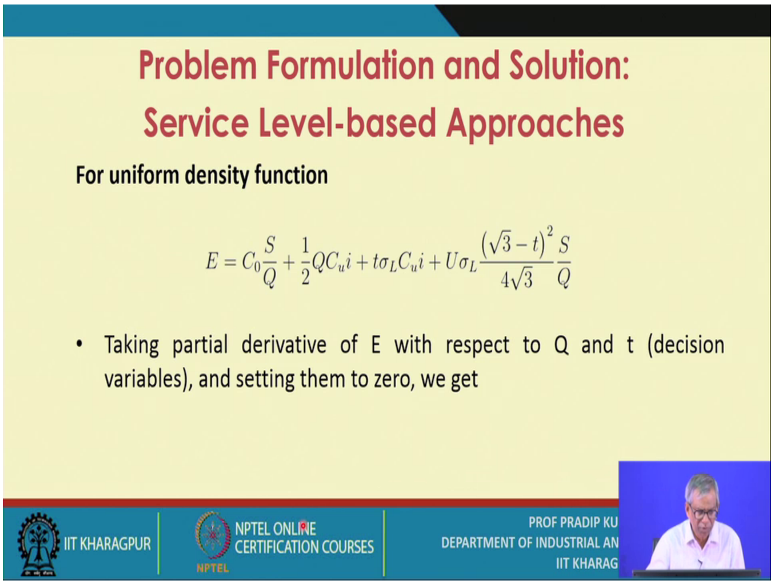

Now, for the uniform density function, what you try to do? That means, here you have the expression for f r so the uniform the distribution so; obviously, the you know what is sigma L? And this is expression for F r F r is essentially N r by sigma L N r by sigma L.

So, this is root 3 minus t whole square divided by 4 root 3 in to S by Q; that means, for each order cycle, what is the expected number of unit short multiplied by the total number of order cycles in a year? So now, once the total cost expression is known taking partial derivative of E with respect to Q and t decision variables. So, t is the multiplying factor which is to be used to determine the safety stock and setting them to 0.

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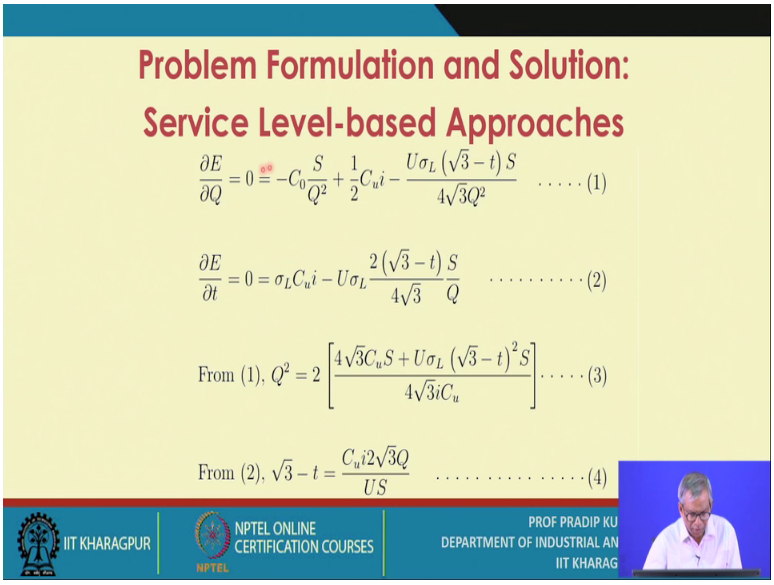

We get these 2 expressions so, this is the partial derivative of E with respect to Q and set it to 0. So, you have these expressions you just follow the steps and the second equation. You get by taking the partial derivative of e with respect to 3 and set it equal to 0.

And then you have the second expression and ultimately when these 2 equations are solved this simultaneous equations solved. So, the first you get the expression for the Q star Q square. That is question number 3 and from the equation 2 we get these expressions. In fact which is referred to as equation number 4.

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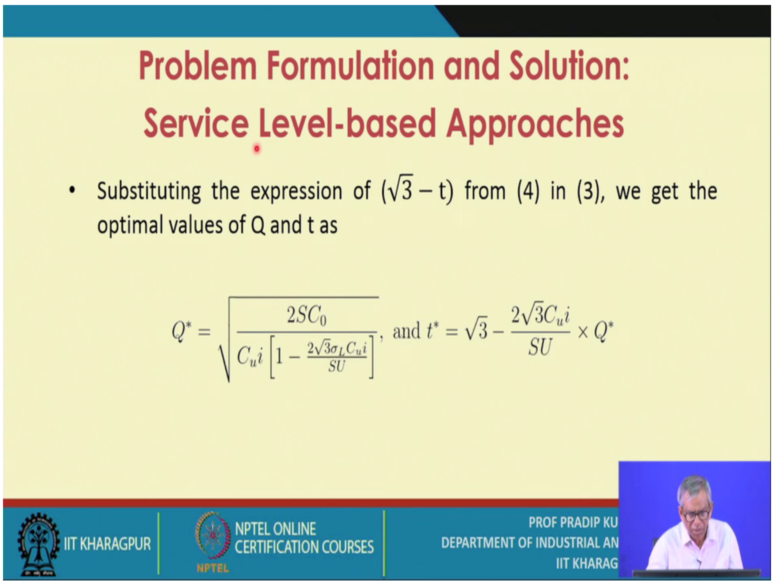

Now, substituting the expressions of root 3 minus t from equation 4 in equation 3 I have explain all the steps. We get the optimum values of Q and t as this one; that means, the Q star that is optimal value of the order quantity. We have use these notation root over twice S C 0 by i Cu into these expression. These expression is 1 minus 2 root 3 sigma L C u into i by S into U and what is the optimal? Say the value of t that is noted as t star equals to root 3 minus 2 root 3 C u i by S in to U into Q star is it ok; so, you can derive these expression.

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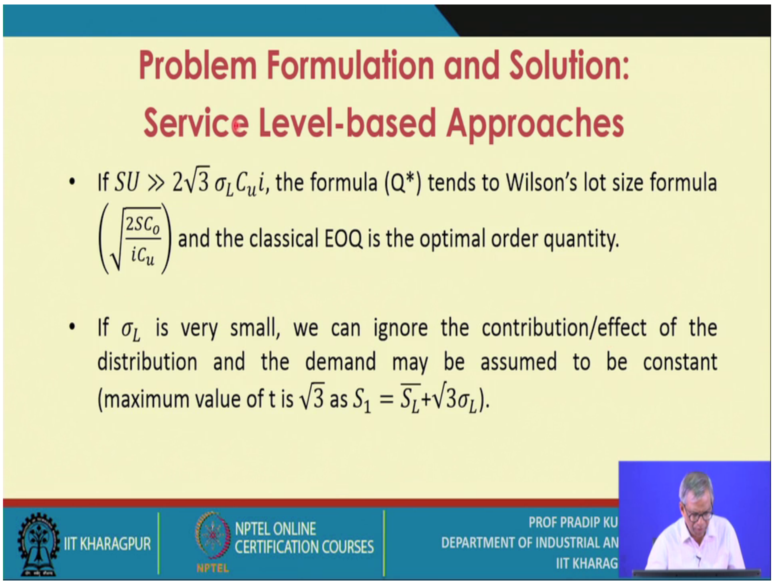

Now, there are 2 conditions you have the first condition is if S in to U S is the yearly demand in physical units.

And U is basically the out of stock cost per unit shortest cost per unit short is substantially greater than 2 root 3 sigma L C u into i. Then the formula Q star tends to Wilson's lot size formula ok. Have what is that formula we have already derived it in the previous lecture session? That is root over twice S C 0 i C u and under this condition, the classical EOQ formula you can use. And this classical EOQ is considered to be the optimal order quantity.

In the next case, if you find the sigma L is very small; that means, the variability is very less we can ignore the contribution or effect of the distribution and the demand maybe assumed to be constant. So, maximum value of t is root 3 as S1 equals to sigma SL bar plus root 3 into sigma L. Now, what you find? That SL bar is equal C is equals to in certain cases SL bar maybe equals to sigma L, but here what we do the maximum value is root 3; that means, the sigma L is very, very less ok.

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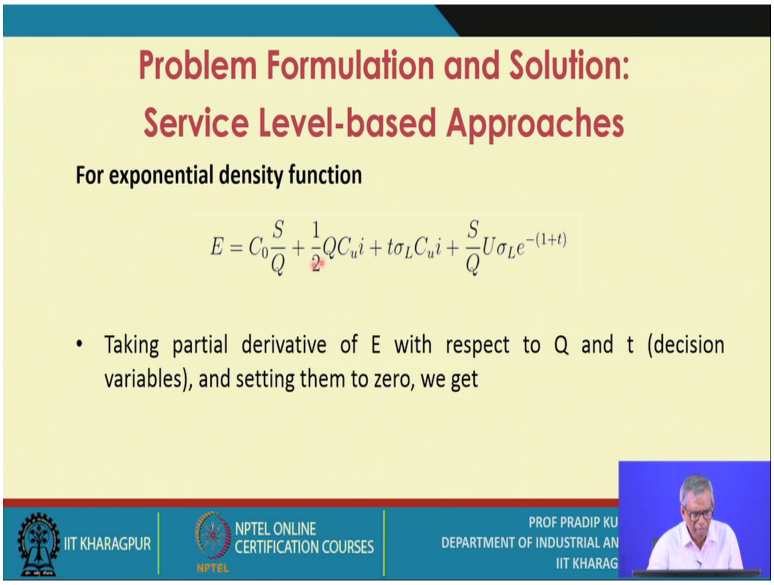

So, for exponential density function the total cost expression is almost similar accepting, you know what you do; that means, sigma L is already there S you get the expressions for F r. So, for this exponential density function so expression for a F r is e to the power minus 1 plus t. So, again you follow the same approach taking partial derivative of E with respect to Q and t and setting them to 0 we get the 2 equations like this.

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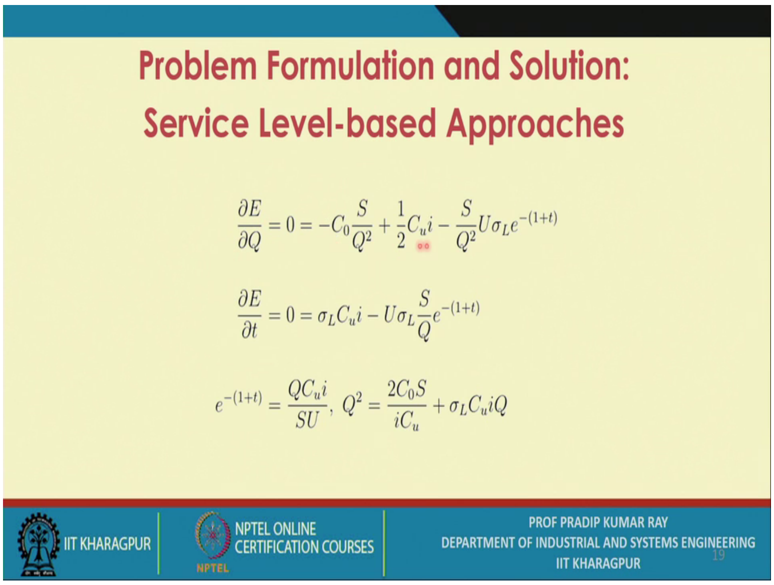

This is for after taking the partial derivative with respect to Q and the second one is after you take the partial derivative of E with respect to small t. So, ultimately you have these equations one for the Q square Q square in the second one is for e to the power minus 1 plus t.

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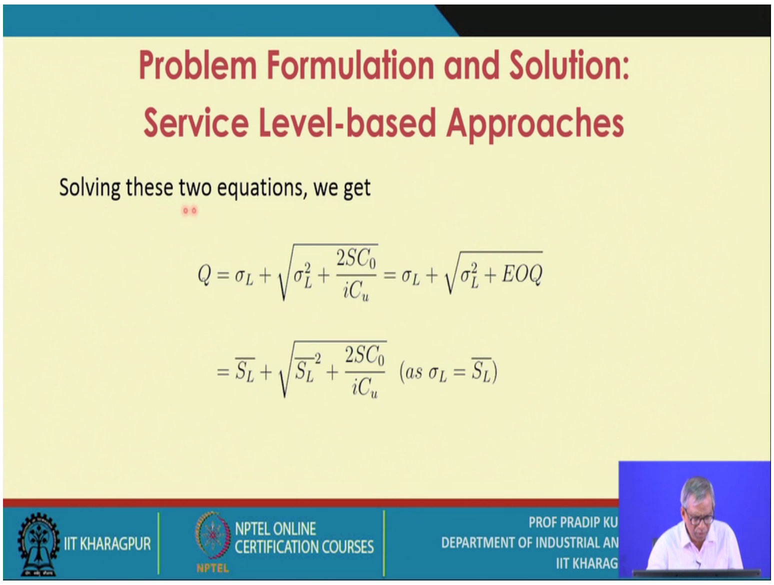

And once you solve this the 2 equations we get the expressions for Q you and that is sigma L plus root over sigma L square plus 2 S C 0 by i C u. In another words, it is sigma L plus root over sigma L square plus EOQ and as for exponential distribution sigma L is a SL bar. So, this expression is replaced with a SL bar plus root over SL bar square plus twice S C u by i C u.

Ah So, that means this is the classical EOQ formula, ok.

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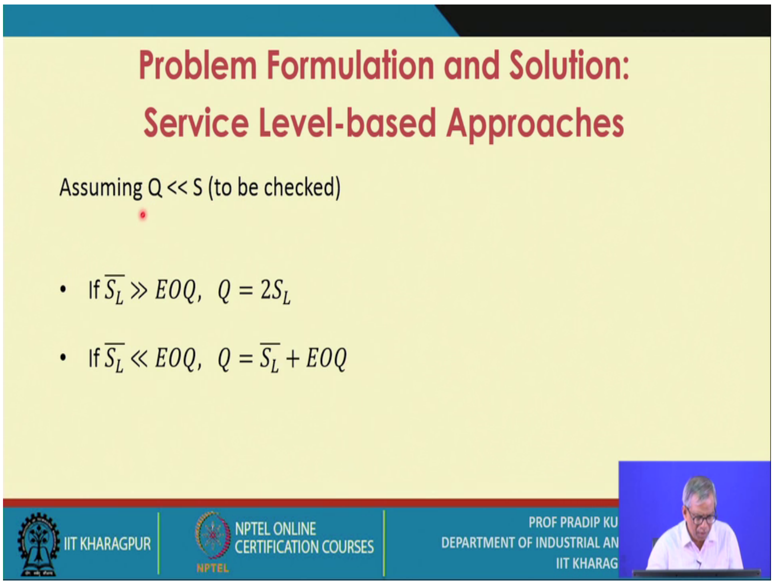

Now, assuming Q is a is significantly less than S. Now, this condition is to be checked if SL bar is greater than a significantly greater than EOQ then; obviously, the order quantity will be twice SL SL and if SL bar is significantly less than EOQ that is you know the classical EOQ so the order quantity becomes a SL bar plus EOQ ok.

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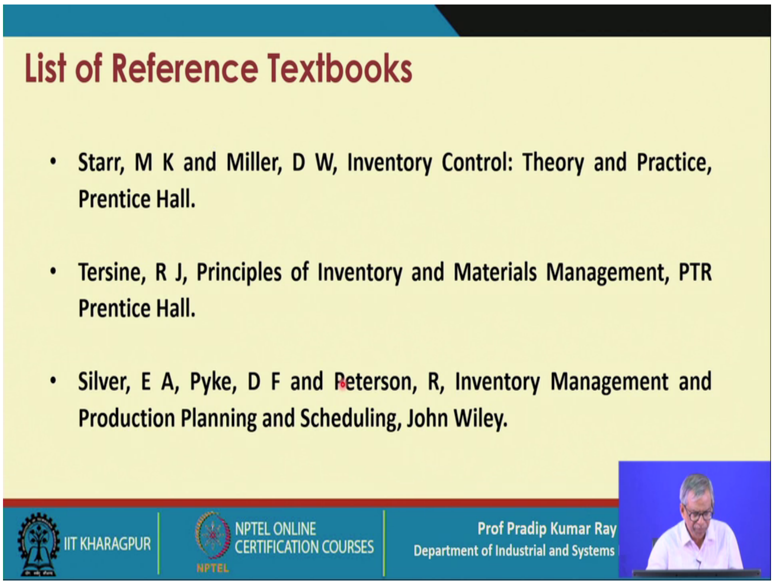

So, these are the 2 expressions we have.

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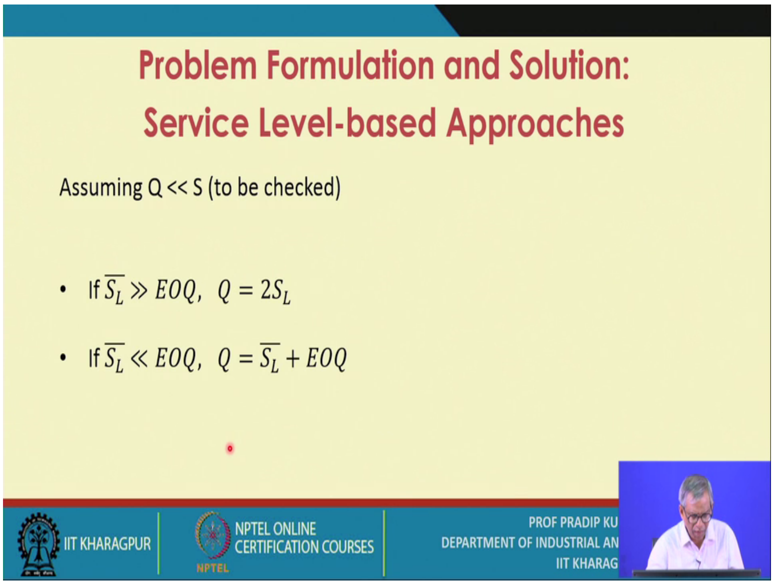

And similarly, what you can do? That means, we have already given the expressions of for N r. When the demand is true vision is considered to be normal. So, the same approach you follow when you assume that the demand to be say normal given demand as normal or the normal density function. You assume and the same approach you follow and to determine the optimal values of order quantity and the safety stock and.

My suggestion is that you try to extend this approach for the P systems of inventory control so; obviously, in a while you formulate the total cost expressions for the P systems of inventory control. So, the total cost expression must be you know the derived in terms of the order period and the safety stock is it ok. So, the later on we will referred to as such cost expressions so.

Thank you.