Management of Inventory Systems
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Lecture - 29
Dynamic Inventory Problems under Risk
(Contd.)

During this session the fourth session of this week under Dynamic Inventory Problems under Risk, now we will be discussing in detail the exact method of analysis under cost based approach

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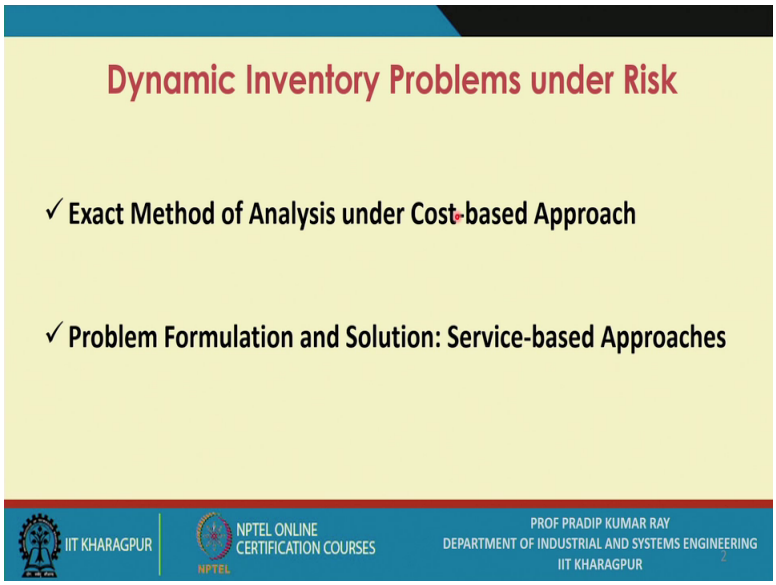

In fact, you know this is very important and when not only for the cost based approach, even for the service level based approach many time we prefer the use of exact method of analysis. We have already explained the, what is the exact method of analysis and what is the approximate method of analysis. So, this problem we will be dealing with during the session and then we will be just introducing the concept of the service level based approach for the problem formulation and in the solution.

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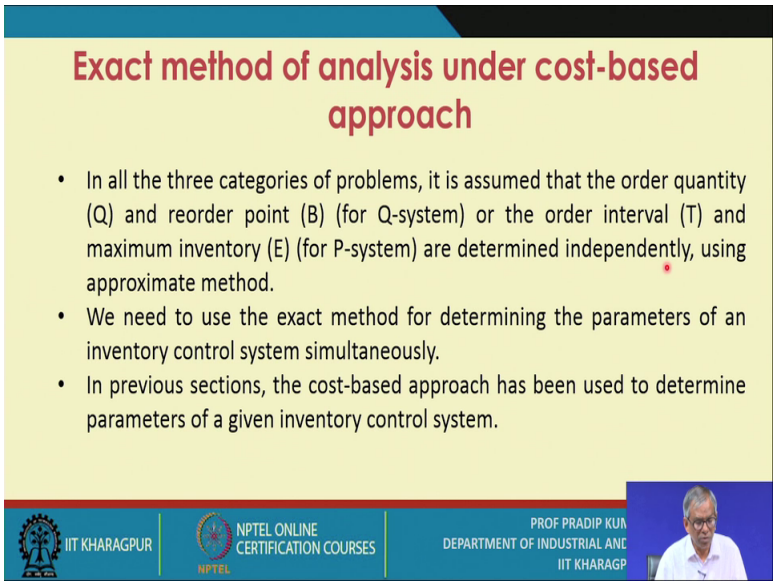

Now, let us first talk about the exact method of analysis and why it is needed in many cases. Now, let us go back to our say our discussions and our say you know say the description of the inventory control systems. So, and we have already in the past lecture session, in the previous lecture sessions we have referred two three categories of problem; category 1, category 2, category 3. Category 1 means the variable demand constant with time and then category 2 is the constant demand variable with time and category 3 means the variable demand and variable with time.

Now, in all this three categories of problems it is assumed that the order quantity and reorder period point for Q-systems obviously, or the order interval and the maximum inventory for the P-systems are determined independently. We have referred to a number of say numerical problems four specific numerical problems, we have referred to and if you say go through this numerical problems we will find that we have been using one the formula for determining Q and another expressions of formula for determining the reorder point as far as you know the Q system is concerned.

And the same approach we follow for the P systems of inventory control. So, we have you are familiar with all these the formulations. So, that is why you are saying that that this they took the parameters we are we are we determine independently using approximate method. So, as if there is no relation between order quantity and reorder

point or there is no relation between relationship between order interval and the maximum inventory.

Now we need to use the exact method for determining the parameters of an inventory control systems simultaneously. That means what we have? That means, one say we will just formulate the problem in such a way that we have one cost equation, the relevant cost equation and this cost equation we form in terms of both the decision we use. And, that is why we say that we are determining in this particular method called exact method the parameters of an inventory control systems simultaneously. In previous sections the cost based approach has been used to determine the parameters of a given inventory control system.

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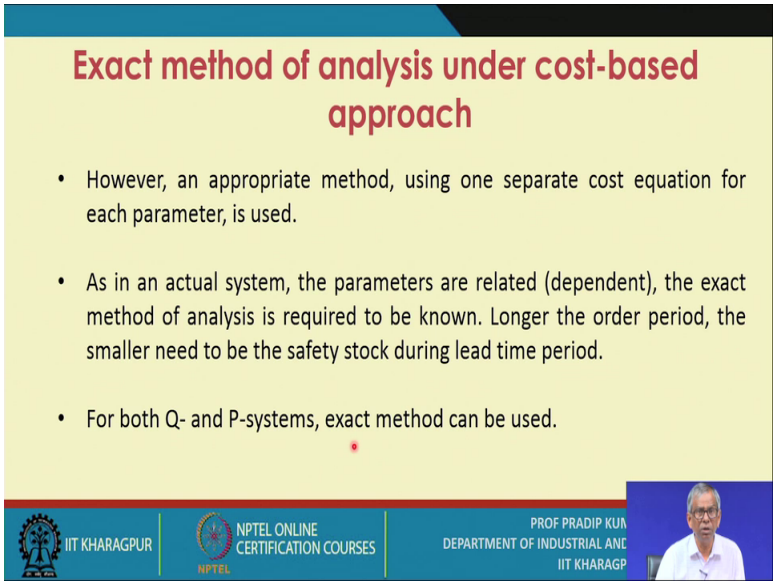

However, an appropriate method and approximate method using one separate cost equation for each parameter is used. Is it ok? So, that is why it is called the approximate method. As in an actual system, the parameters are related that is why they are dependent with each other; the exact method of analysis is required to be known.

That means, what we are saying like say you know if order quantity is more, obviously, you will find that the say T stock may be less or if the order quantity is less you can prove with the data say T stock will be will be more and similarly for the P system are related the parameters. Now, so, if they are related and suppose this relationship is very strong and we need to include this relationship the in the formulation of the problem;

obviously, we must use the exact method of analysis. Longer the order period, the smaller need to be the safety stock during lead time period. Is it ok?

So, there are many you know examples and these examples you always these examples are given in the text books and so, please refer to all these examples you will have a very clear idea of the dependency between the parameters in a given inventory control system. So, for both Q and P systems the exact method can be used.

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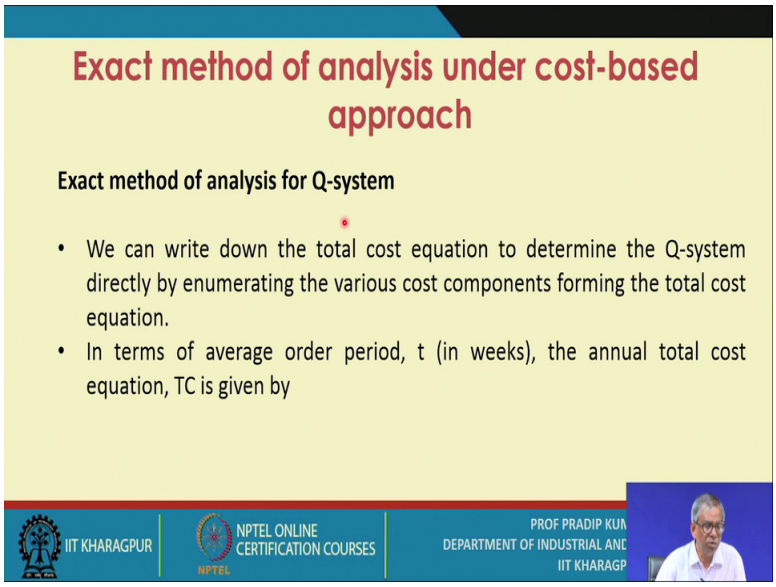

So, let us first talk about the exact method of analysis for the Q system. We can write down the total cost equation to determine the Q systems directly; that means, to determine the working of the Q system essentially because any you know any cost means, wherever there is some activity and there is always a related cost.

So, what we can do? We can write down the total cost equation by enumerating the various cost components forming the total cost equation; this is very clear. So, in terms of the average order period T in weeks, so, here we are assuming that the time unit is weak that is why you have written weeks, it could also in months or in years.

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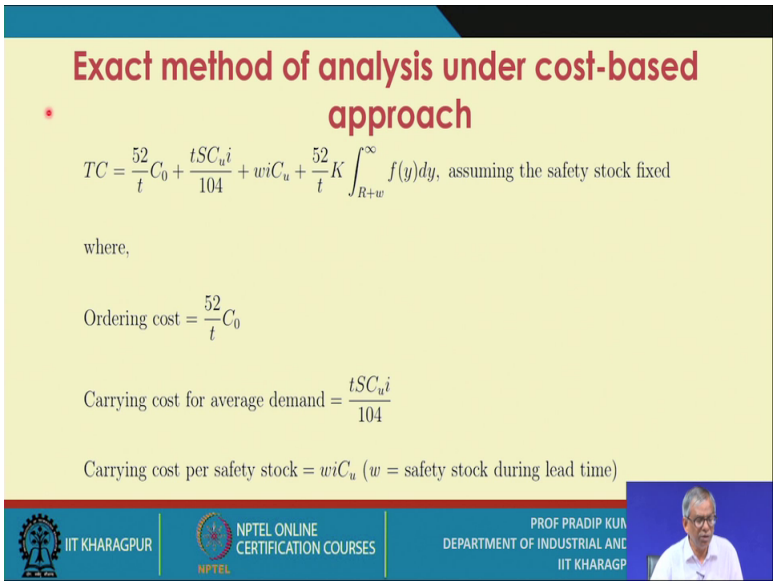

The annual total cost equation T C is given by this one this expression that means now, what we have? We have one total cost expression involving four specific components. So, the first component is that is 52 by T into C o ok; that means, C o is the ordering cost for order and 52 by T is basically the order you know the order cycle in or order period in weeks and what we are assuming that in a year there are 52 weeks that is why this term 52 comes.

Now, the next one is this is the inventory carrying cost for the average inventory. Now here the t S t by a say you know 52 is your average inventory. So and average that is inventory maximum inventory and average inventory half of that. So, this is essentially 2 into 52; that means 104. So, S into t into C u into i so, i is the inventory carrying cost as a proportion of the average inventory which you hold. What is S? S is basically the annual demand in physical units and C u is the unit price. Is it ok?

So you refer to our, the original expressions of EOQ, where we have use these notations. Then the next one is this is the safety stock. So, what is the safety stock amount? That is w and it is the carrying cost for the safety stock and why do you keep safety stock, to avoid or to prevent the occurrence of stock outs. So obviously, here what you are assuming? That in any order cycle there could be a stock out. So, how many order cycles you have per year? That is 52 by t and what is t; t is the order in weeks and what is K? K is essentially the fixed out of stock cost; this is a initially we have assumed that the fixed

cost the remains or the out of stock cost remains fixed irrespective of the amount of shortages.

So, what is the probability of shortages? That is R plus w up to infinity, f y dy; assuming that the safety stock remains fixed; that means, though even if you are keeping a safety stock, but you don't need to use it a in any order cycle. So, that is a one kind of a situation. In certain cases, you need to use certain portion of the safety stock in order to avoid or in order to say that it considered a situation where say the lead time demand is more than the average which you have assumed.

So, now what are these notion? That means, ordering cost is given by this, this is the first term. The second term that is this one represents the carrying cost for the average demand. The third term that is the, this one that is the carrying cost for safety stock, a w is the safety stock during lead time. So, i is the inventory carrying cost and the per unit the price is say C u.

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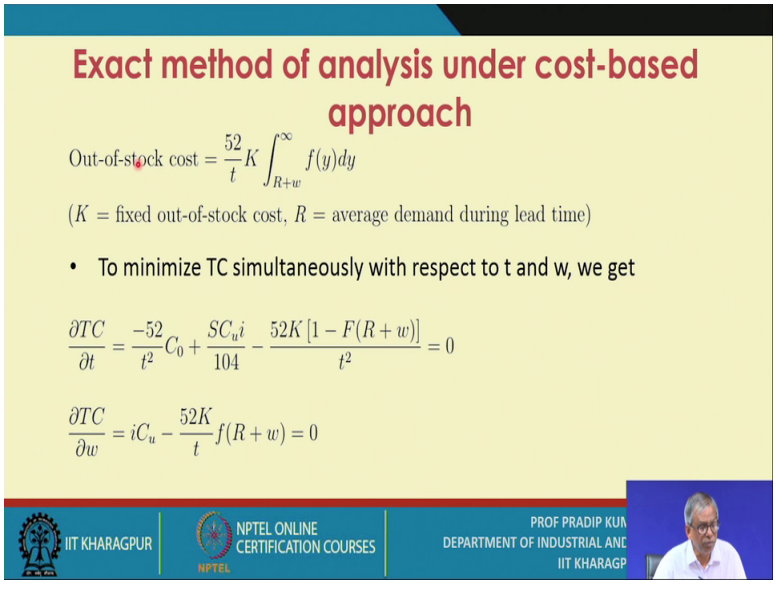

And what is the fourth term? Fourth term is the out of stock cost. So, 52 by t into K into say R plus w to infinity f y dy. So, what is capital K? Fixed out of stock cost that is our assumption, and R is the average demand during lead time. So, we have use the another notations ok, otherwise the same sort of expressions we have used in the previous case also.

So, to minimize the total cost simultaneously with respect to T and w, so what you need to do? You take the partial derivative of TC with respect to T as well as you take the partial derivative of T C with respect to w. So, these are the two unknowns or the two decision variables and you take the partial derivatives, set them equal to 0 and then you have one expressions like this one equation and the second equation is this one right.

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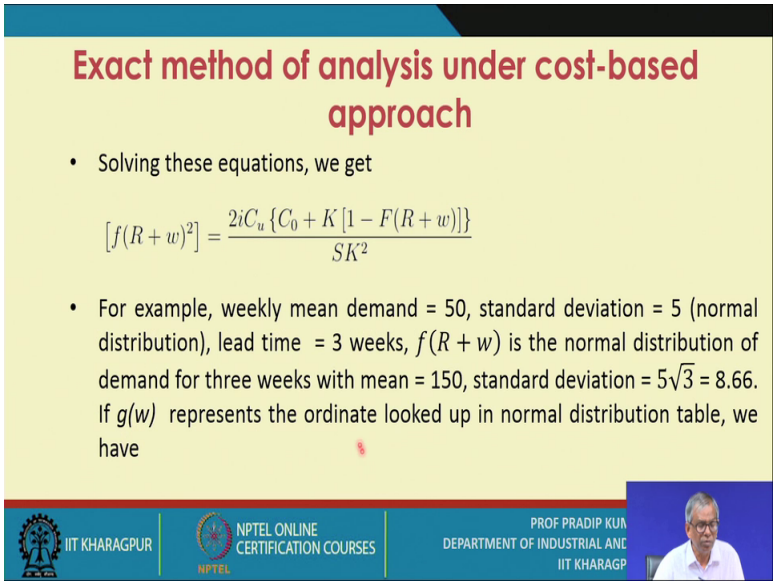

And then you follow the steps to get the solution. So, after you follow all these steps so, you get solving these week this two simultaneous equations, what you get? Ultimately you get these expressions, is it ok; that means, f of R plus w square equals to this one; this is the right hand expression in terms of the known parameters and some say parameters as well as some you know the factors. So for example, like you have these expressions and in order to get the values of say the order quantity as well as the safety stock say you need to use these expression.

So, for example, weekly mean demand is 50 in units physical units, the standard deviation is 5 that is also in physical units and we are assuming that the demand is normal distribution, lead time is 3 weeks assuming constant. And this one is actually that is the probability or the density function of R plus w f; R plus w is the normal distribution of demand for three weeks with mean equals to 150 and the standard deviation is obviously, you know the 5 into 3.

So, what we are assuming? That the normal distribution the mean demand; that means, the time period mentioned as 1 week. Whereas, the lead time is 3 weeks; so obviously, the normal distribution the values; that means, to be convoluted the 3 times and when you convolute the normal distribution 3 times, what happens? That its parameter values; that means, it's a mean becomes n into 50.

So, n is 3; 3 into 50 that is 150 and the standard deviation; obviously, it will be the 5 into root over n. So, here n equals to 3 that is why to 5 into root over 3 that is 8.66. So, if g w represents the ordinate looked up in normal distribution table, we have the solution; that means, g w square equals to 8.66 square into these expressions ok.

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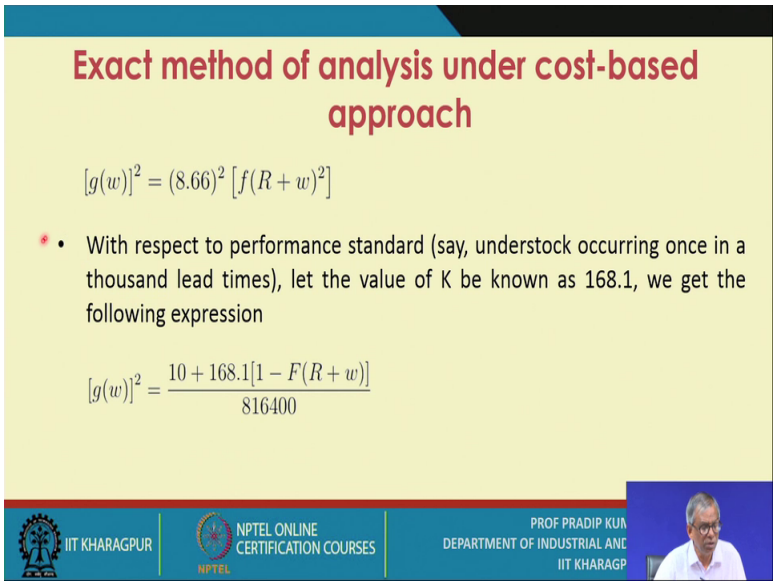

With respect to the performance standard say under-stock occurring once in a thousand lead times. So, what is the performance standard? That means, it is a kind of you know it is a service level that is 0.001; that means, in one order cycle there is a possibility out of 1000 order cycle, there is a possibility of going out of stock. So that is just the, we are referring to the event of stock out, but in the in that particular order cycle where the stock out the situation has occurred, we are not mentioning the amount of stock out.

So, we are essentially we are referring to a measure of service level which is which is the service per order cycles SLC. So, let the value of K be known as 168.1. Is it ok? We get the following expressions. Actually we get the value of g w square in terms of 1 minus f or plus w ok, the cumulative distribution function.

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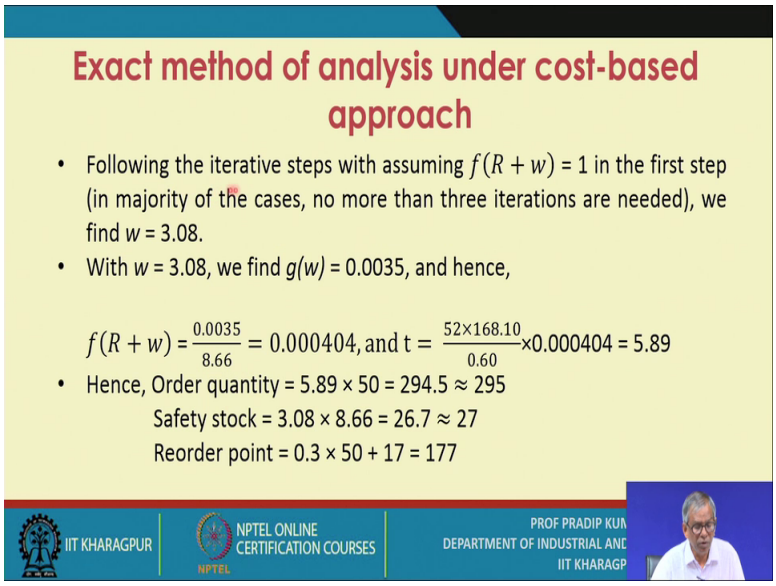

So, following the iterative steps with an assuming f R plus w equals to 1, because you have to use the iterative methods. In the first step, in the majority of the cases this point is to be noted, no more than three iterations are needed. Is it ok? So, you get you now you get a solution and we find following those, these iterative steps we find a value of w as 3.08. With w equal to 3.08, we find g w is 0.0035; that means you refer to the standard normal table and hence f R plus w is equals to this; that means this to be divided by the standard deviation that is 8.66. So, 5 root 3.

So, this value is this one 0.000404 and the value of T is this one that is for 5.89, 5.89 weeks. Hence, the order quantity 5.89 into 50 that is 295. Safety stock is 3.08 into 8.66. Is it ok? So, that is 26.7 approximated as 27 and the reorder point is 0.3 into 50 plus 17, is it is 3 into 50 plus 17 that is say 150 that is 27; 3 into 50 plus 27 equals to 177 ok. So, this is the 3 time periods that is the mean that is reorder point; that means, the demand the average demand during lead time. So, the lead time is 3 weeks and each week the demand is the 50 units; that is why it is 3 into 50 plus 27; 27 is the safety stock. So, 150 plus 27 that is 177.

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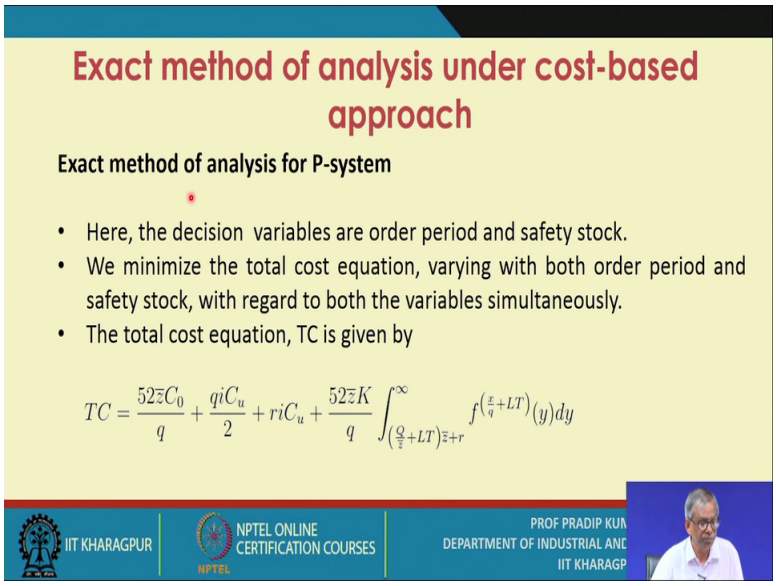

Now, for the P system, if you opt for the exact method of analysis, so what is the procedure? So, again what you need to do? That means, you need to say the form the total cost equation in terms of in terms of you know the decision variables. So, here the decision variables or order and the safety stock is it right. So, we minimize the total cost equation varying with both order and the safety stock with regard to both the variable simultaneously.

The total cost equation TC is given by TC equals to this one that this is for the average 1 that is the ordering cost, this is the inventory carrying cost for the average demand and this is R is basically the safety stock and the carrying cost for the safety stock. And, this is essentially you know say the out of stock cost and here what we have assumed that these expression is probability of going out of stock and this is capital K; that means, the fixed out of stock cost we have assumed.

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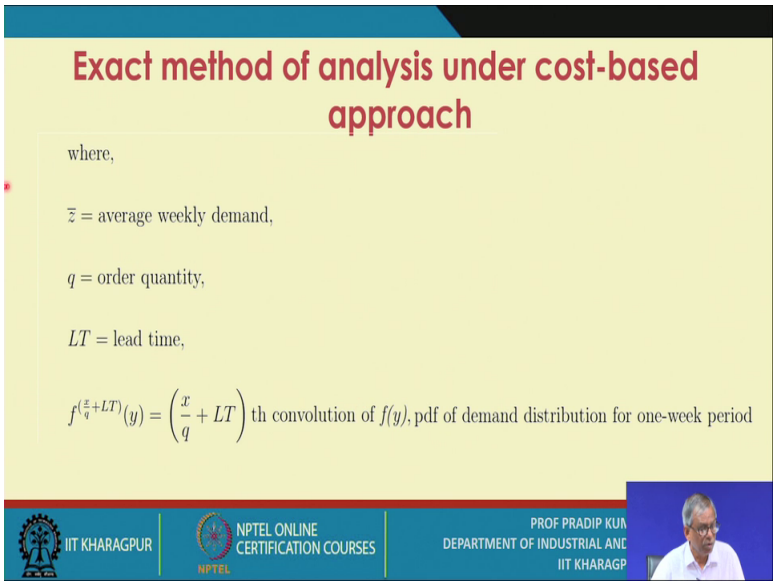

So, all these details are here; that means, z bar is the average weekly demand, q is order quantity, LT is the demand and this is essentially this these notations you have used. That means, it represents x by q plus LT, that is this is convolution of f y, f y; what is a the f y? F y is the pdf of demand distributions for one-week period; that means, this the time period you need to consider right.

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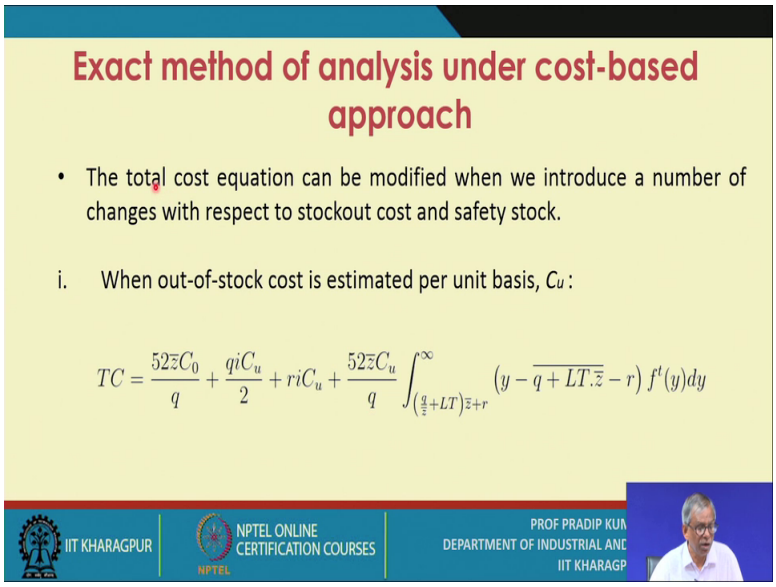

And what we have? That means, the total cost equation can be modified when we introduce a number of changes with respect to stock-out cost and the safety stock. This

point already we have referred to like the first case is when out of stock cost is estimated per unit basis, is it ok. So, what is the notation we have used? When it is a fixed cost so, we have an notation capital K, when the out of stock cost is estimated per unit basis the notation is C subscript u.

So, what is the total cost expression? So, the total first expression is this one; this is this expression is the ordering cost for the ordering cost, these expression is for say the inventory carrying cost for the average the demand. And, this is the inventory this is you know the safety stock carrying cost and this is say the this number of these expression is for the expected number of units short and this is your the per unit out of stock cost and this is basically the number of order cycles you have.

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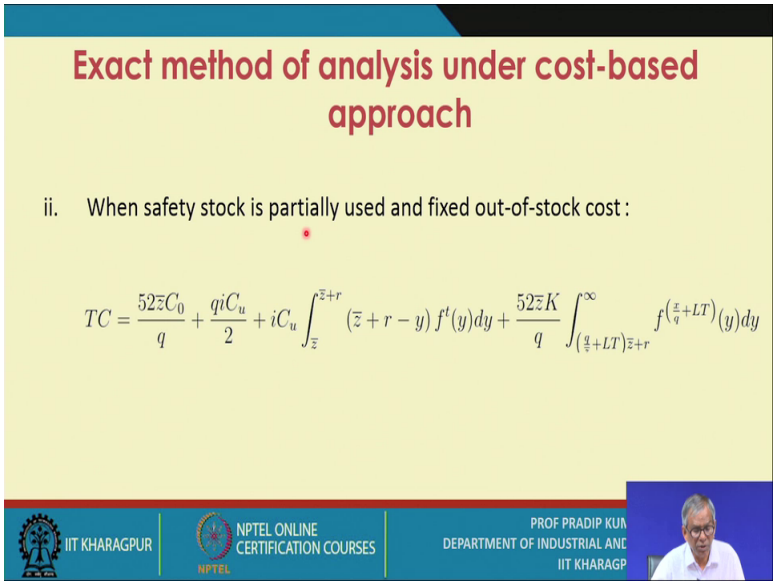

So, when safety stock is partially used and fixed out of stock; so, this is another combination. So, the first two terms in the total cost expression remain same. Whereas, the third term what do we use? That means, in this particular case you need to determine the safety stock for the entire time period entire; time period means the order interval plus the lead time. So, that timing is say small t and so, i into C u is the inventory carrying cost that is i is the proportion and C u is actually the unit price.

Now, the how many units are in the you have in the safety stock? So, sometimes you use the safety stock or sometimes you do not use a safety stock. So, what we are assuming that partially you are using the safety stock is ok. So, z bar plus z bar plus r. Is it alright?

So, any value we use; that means, it is greater than z bar, but it must be within z bar plus r, so that is the maximum amount you have during the order as well as the lead time. So, this is your actual you know the demand during the order and lead time and the corresponding you know the pdf is; obviously, it is to be convoluted the t times.

So, that is why it is f superscript t y d y and this is the part; that means, the fixed out of stock cost. So, the same expressions you have, this is the probability of going out of stock and that is the fixed out of stock per cycle and this is just one order cycle you have; with this one order cycle and there are say the total number of order cycles per year.

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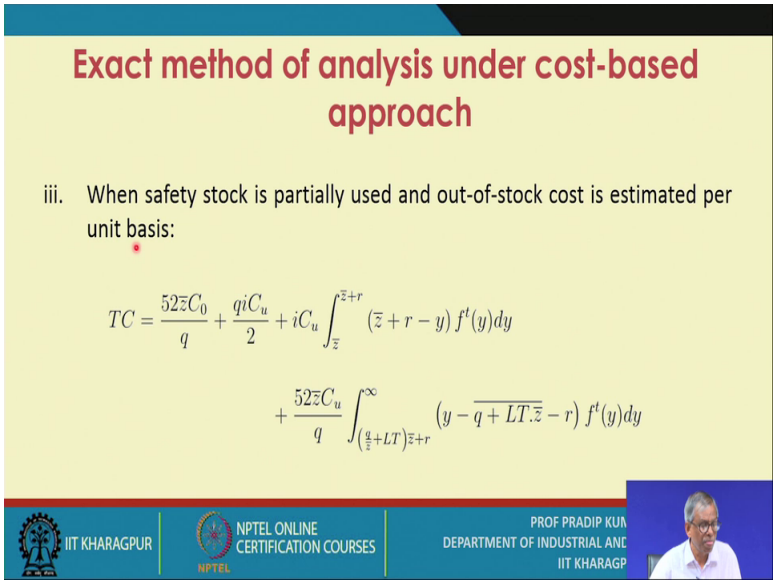

In the third case when the safety stock is partially used ok, we have already considered this case that is you have to change this expression to this one; that means we are assuming that there is a variation in the in the safety stock and out of stock cost is estimated per unit basis. So, per unit basis; that means, these expression; that means, this is the fourth term and per unit basis means this is the expected number of units short is it ok.

So, and this is the number of order cycles and this is the per unit out of stock cost. So, you please go through all the four the components in this total cost equation and if you have any doubts; obviously, you can you can write to write to me and definitely we will we will be able to say make you understand fully. So, as far as possible we have we have tried to make it as expletives explicit as possible and but the main point is that you must

be able to write down the total cost expression, total cost expression in terms of the decision variables.

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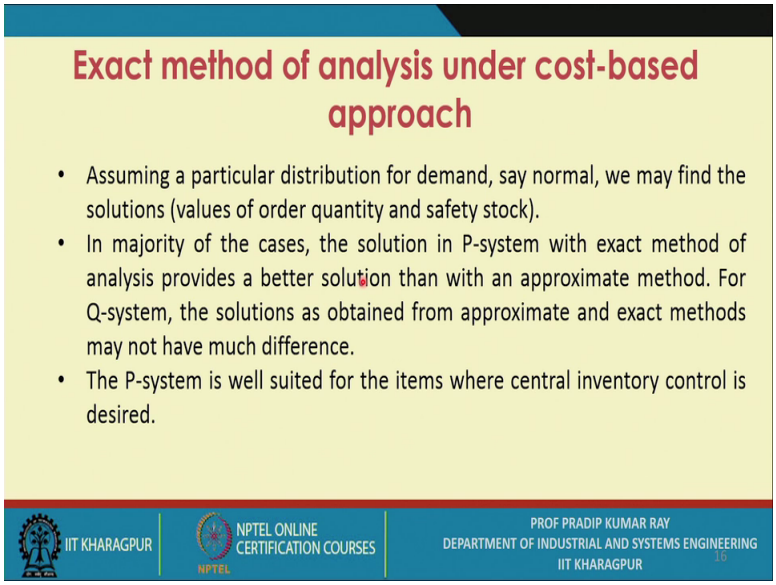

So, the first the condition is the given an inventory control system. You must be able to say identify the decision variables and in order to identify this decision variables; that means, you must be able to explain or you must be able to describe the working of a given inventory control systems. And for which we always suggest that you better you the present the inventory control systems the given inventory control systems with the help of the inventory profile. So, once you draw the inventory profile what the advantage you have; the advantages that immediately you will come to know that : what are the decision variables.

And then at the next step you try to write down the total cost expression involving of the four components and all these and involving component in terms of this decision variables. Assuming a particular distribution for demand say normal, we may find the solutions values of order quantity and the safety stock. In majority of the cases the solution in P system with exact method of analysis provides a better solution that with an approximate one.

So, this point is to be noted; that means, for many items you know that for what kinds of items or inventory items you prefer to use the P systems of inventory control and usually you know the for determining the parameter values of the P system. So, better you use

the you know exact method of analysis. Whereas, for the Q system the solutions as obtained from approximate and exact methods may not have much difference.

So, this we can prove with the data and when we take of numerical example, so, this point will be made more clear. So for the Q system, you prefer to use approximate method ok, whereas, for the P systems you prefer to use exact method. So, the P system is well suited for the items where central inventory control is desired. So, this point also is to be understood properly later on when we discuss you know the supply chain, the concepts later on. So, this point will be elaborated more you know the specifically or more explicitly.

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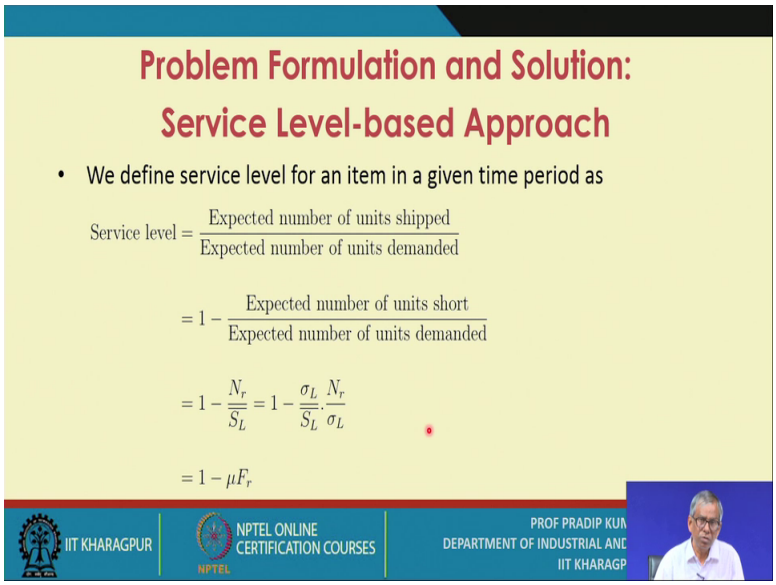

Now, you known by this; so, till now, what we have discussed that is for problem formulation and solution. We have we have suggested the cost based approach. You also know that under certain conditions that cost based approach may not be reliable and that is why it is to be avoided, particularly in a situation where the cost estimates are not reliable.

So, what is the alternative? Alternative we have mentioned that is the service level based approach. Now, let us before I conclude this session, so let us you know the, define the service level once again and let us get an expressions for the service level and we define service level for an item in a given time period. Now this is SLU basically, the service

per units demanded. So, the service level is nothing, but the expected number of units shift divided by the expected number of units demanded, already we have defined.

Now, this is 1 minus expected number of unit short divided by the expected number of units demanded. Now, as soon as you use the term expected, it means the demand is known with risk that means the, has having a distribution, the probability distribution. That means, exact demand is not known that is why you have used the term expected number of units shift. So, now we have used; that means, 1 minus N r subscript r divided by SL bar and this through manipulation what you do this means 1 minus sigma L by SL bar into N r by sigma L or it becomes 1 minus mu into F r.

So, what is mu? That means, it is we are referring to the Q system and this is basically the lead time demand; SL bar. So, average lead time demand is a SL bar and the standard deviation of the lead time demand is sigma L. Is it ok? So, this we have assumed. So, this is essentially sigma L by SL bar is nothing, but the coefficient of variation and that is why we have used this notation mu and N r is the expected number of unit short and if you divided by sigma L, so, it becomes F r. So, F r is nothing but N r by sigma L.

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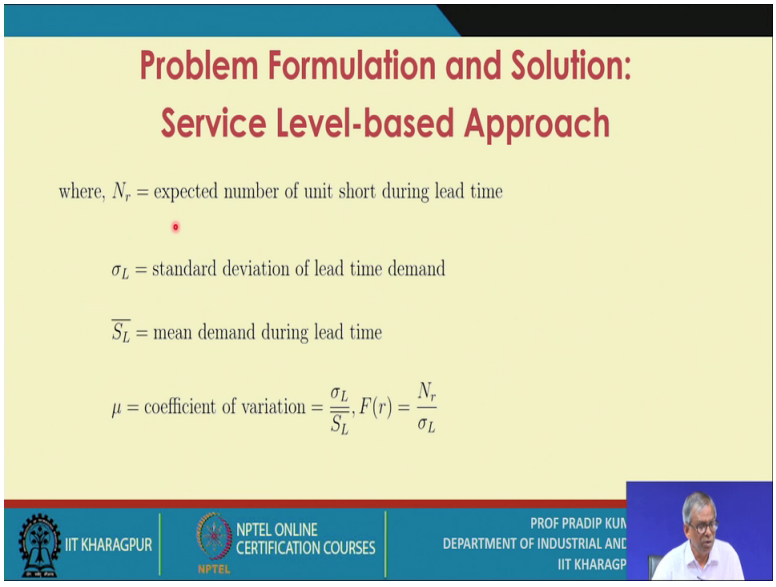

So, N r is expected number of unit shot during lead time; that means, we are bothering about only the lead time demand because we are referring to the Q system of inventory control. There is a possibility of stock out only during the lead time for Q systems of inventory control, sigma L is the standard deviation of lead time demand, S L bar is the

mean lead time mean demand during lead time and mu is coefficient of variation that is sigma L by S L bar ok.

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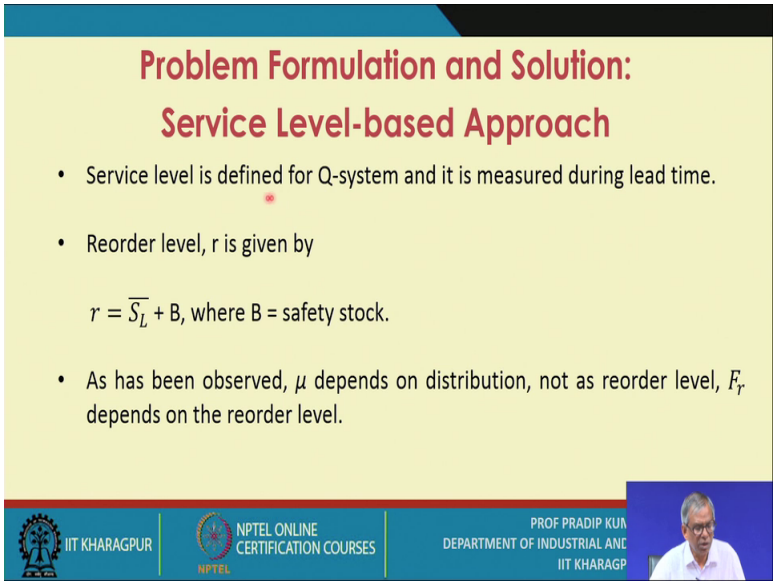

So, service level is defined for Q system here and it is measured during lead time and reorder level is we have use these notation small r, that is S L bar plus B, where B is equal to safety stock or the buffer stock. As has been observed, mu depends on distribution not on reorder level whereas; F r depends on the reorder level.

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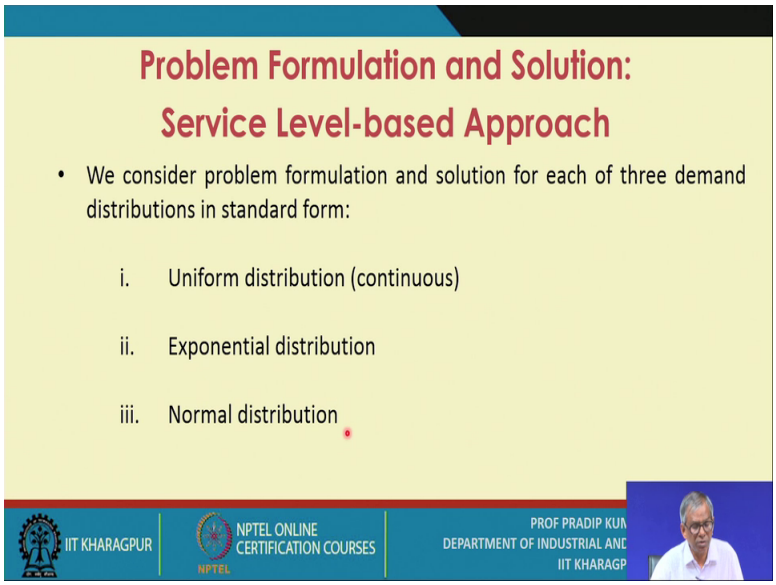

So, this is a conclusion. So, we have just started you know the discussing the procedures for say to be employed of for problem formulation and solution in service based approach. So, we considered problem formulation and solution for each of the three demand distribution in the standard form. Now, first one is uniform distribution we will consider, in the next lecture sessions and we also considered exponential distribution for demand and we also consider the normal distribution. So, the service level based approach and the exact method of analysis we will discuss in the next lecture sessions.

Thank you.