Management of Inventory Systems
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Lecture - 28
Dynamic Inventory Problems under Risk (Contd.)

So, during this the lecture station under Dynamic Inventory Problems under Risk, I will continue my discussion on cost based approaches for problem form formulation and solution.

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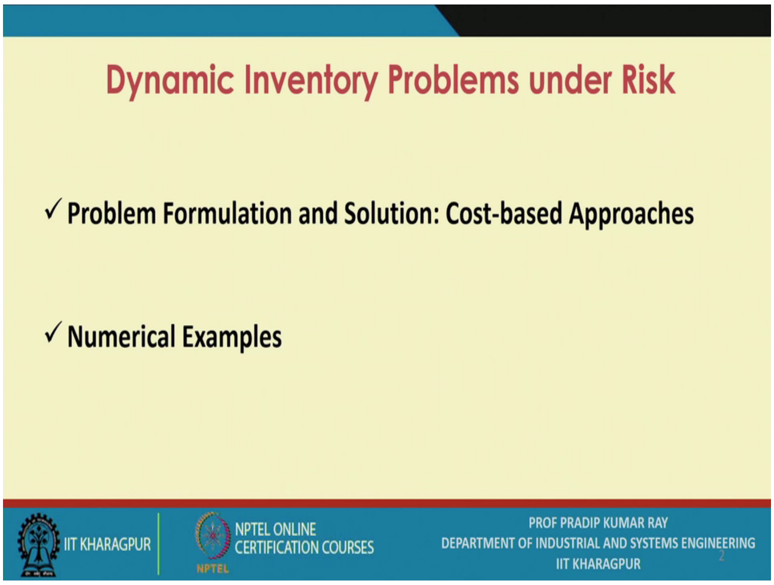

And we will also referred to a number of numerical problems so that your understanding is appropriate and you know you will come to know the intricacies of the problem.

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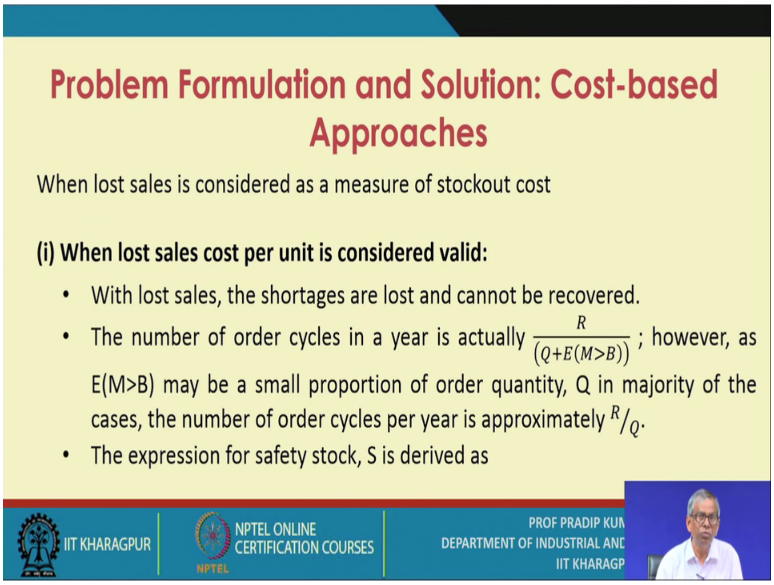

So, let us we have already the discussed two important cases as far as cost based approach is concerned that means, we have assumed that the stock out stock out situation will result in backordering. And against is backordering in one situation we have assume that the back ordering cost is to be computed per unit basis, and in the second situation we have assumed that that backordering cost may be computed as a fixed cost or per outage, ok.

So, both the cases we have considered and we have say the formulated the problem and we have also suggested the solution given a data set. And two specific numerical problems we have we have already referred to. So, now, during this lecture sessions we will be considered the lost sales case. That means, there is a stock out, suppose it say a Q systems we are always explaining the problem with the reference to a particular inventory control system that is so the Q systems.

So obviously the same approach you can use or the same conditions you may face for the P systems of inventory control systems, these P systems of say P systems of inventory control. So, right now, the third case is that when the lost sales cost per unit is considered valid that means, is the lost sales case and we have the ways and means or you have an information system support with which you can you know you can estimate with the reasonable accuracy the lost sales cost per unit. So, with lost sales the shortages are lost and cannot be recovered. So, this point is to be noted because, in not only in this case in general when you deal with the inventory control problem so the lost sales case for is this taken a very seriously, particularly for the critical items particularly for the important items.

The number of order cycles in a year is actually R by Q plus expected number of unit short, ok. So, this point is to be noted. How many cycles you have, order cycles in a in a particular year? Suppose the yearly demand is R and your order quantity meeting the demand for each order cycle is Q. So obviously, when there is the no shortages there is no the lost sales the total number of order cycles in a year will be R by Q, but here there is a lost sales. That means, if there is an negative inventory the negative inventory gets lost.

So, in order to avoid this, what you try to do? That means, in each order cycle you try to keep some extra stock and this extra stock is essentially expected number of unit stocks expected number of unit stocks. So, that is why when you try to in the lost sales case, when you try to compute the number of order cycle per year. So, the expression is R divided by Q plus the expected number of unit short.

So, this notation we have been using throughout. However, as this expected number of unit short the notation is E M greater than B maybe a small proportion of order quantity Q in majority of the cases, when majority of the not in all cases in the majority of the cases that means, maybe 90 percent of the cases the number of order cycles per year is approximately R by Q. So, these approximation if you if you have so it will hardly effect in many cases in majority of the cases the result of the results which you get.

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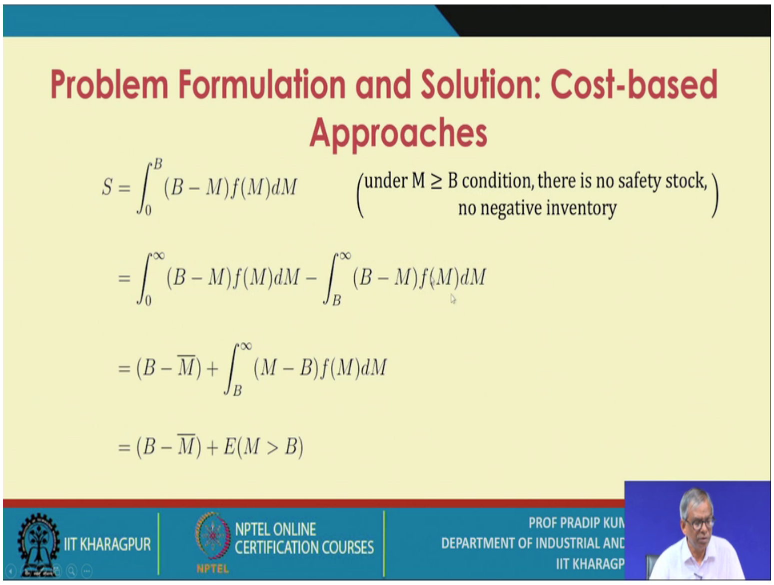

The expression for the safety stock S is derived as s equals to 0 integration 0 to B, that means, beyond B if you have a say the demand, so the demand will be lost; that means, if you refer to the inventory profile you will find for the lost sales case negative there is no negative inventory. So, it becomes instruments in 0 level. So, the expression is integration 0 to B, B minus M f M d M. So, under M greater than equals to B condition there is no safety stock no negative inventory, is it ok, this point we have been saying. Now, you follow the steps and ultimately you get an expression that means, the safety stock is B minus M bar plus the expected number of units shots. So, this is to be added.

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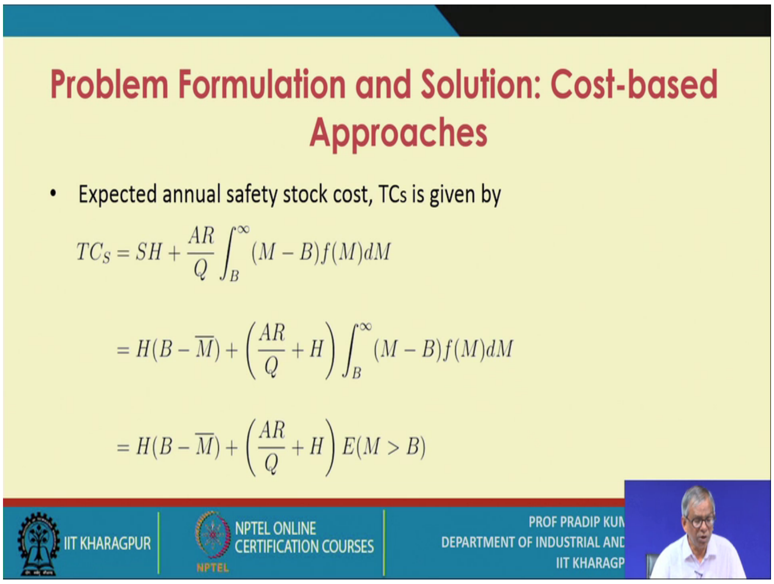

Now, what you do? You get an expression for the expected annual safety stock cost solid the notation is TC subscript s and this cost is given by SH plus A that means, then the you know we have been using the same notation that means, this is the say the lost sales cost per unit. R is the yearly demand, Q is order quantity and integration B to infinity M minus B f M d M. So, this is stand expressions. Now, you substitute s with its expression and ultimately. What you get? You get this expressions that means, H into B minus M bar plus AR by Q plus H into expected number of units short. So, this is you follow the steps you get this expression. Now, what do you do?

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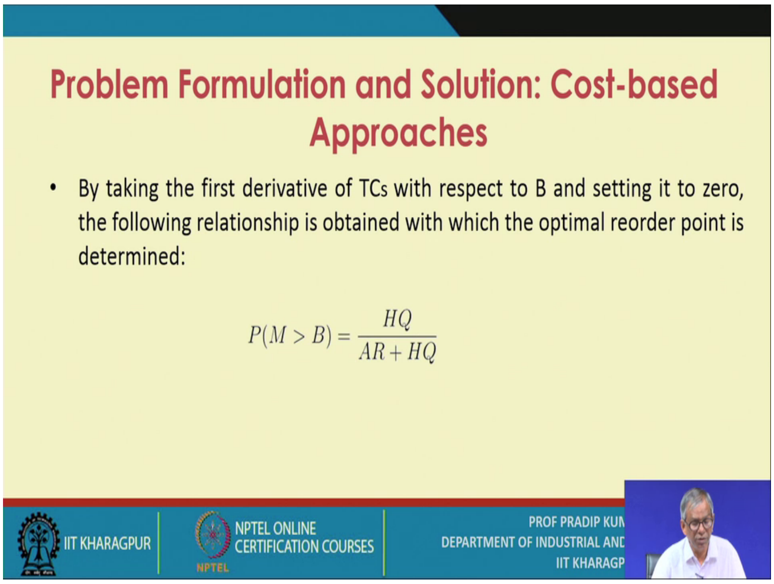

So, this you take the first derivative of the TC s with respect to B because that is the decision variable and setting it to 0 and you get this relationship that is the probability of going out of stock that means, the probability that M greater than B is equals to HQ divided by AR plus HQ. So, this is the expression you get. And with this if you use this expression you will get the optimal say the value of reorder point for the given inventory control systems.

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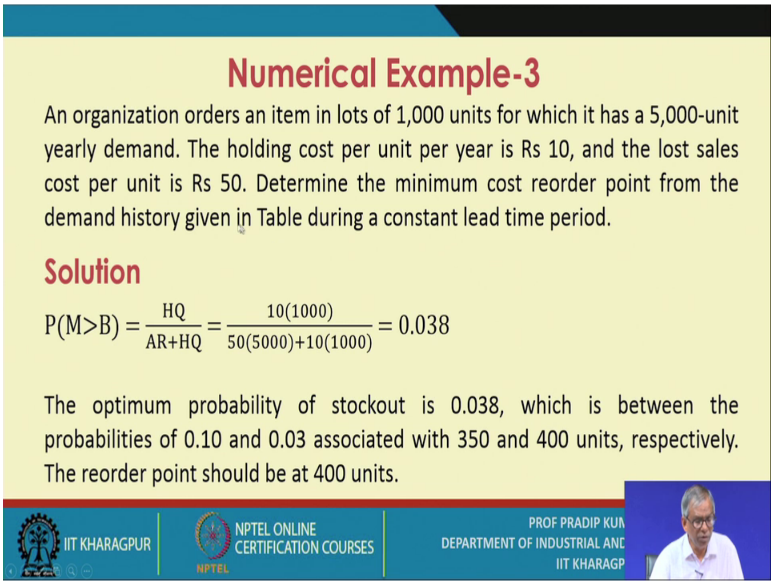

Now, the let us take one numerical example an organization orders an item in lots of 1000 units for which it has a 5000 unit yearly demand, ok. So, the go through this problem statement and you just get the different types of data. The holding cost per unit per year is rupees 10 this is given and the lost sales cost per unit is rupees 50, and what we are assuming that these estimates are more or less accurate there maybe some error, but this error is tolerable. Determine the minimum cost reorder point from the demand history given in table. We will show you the table data during a constant lead time period.

So, what we are assuming? So, we are in that domain that is the variable demand constant lead time, is it ok. So, what is the solution? That means, we have already derive the expressions for the optimal probability of going out of stock that is probability that M greater than B, there is HQ divided by AR plus HQ. So, these values are already known the values of H, values of Q values, of Q is essentially 1000 units that is a lot size and the R is yearly demand that is 5000 units a is 50 that is the lost sales cost per unit.

So, all these are known and you get a value of that of the that the stock out probability optimal stock out probability as 0.038. Now, what you do? You refer to the table given that optimal probability of stock out is 0.038 which is between the probabilities of 0.10 and 0.03 associated with 350 and 400 units respectively. Like in the previous case in the last lecture sessions the same approach we have followed the reorder point should be at 400 units.

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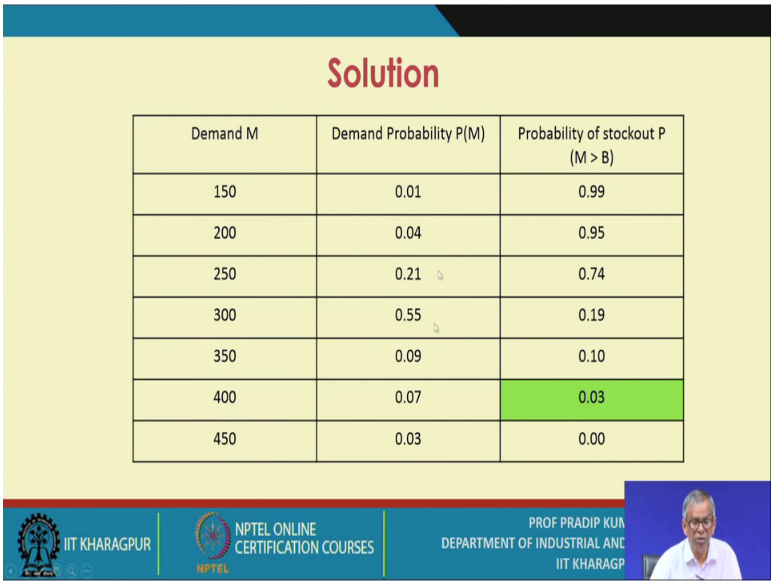

So, just you look at the table here that means, against the demand level. So, what are the demand levels? 150, 200, 250, 300, 350, 400 and 450, what we assume? This is the exertive exhaustive list that means, there cannot be any demand less than 150 as well as there cannot be any demand beyond 450. So, the corresponding probabilities are like this and then you add this column that is probability of stock out that the probability that M is greater than B that is here it will be 0.99, 0.95, 0.74, 0.19, 0.10, 0.03, 0.00 that means, when you have a the reorder point of 450 obviously, you cannot have any stock out situation.

But then again in order to say the operate at the minimum cost, is it sometimes you do not need to the keeps this maximum say stock has as a reorder point. So, you say that, that is my optimal probability of stock out assuming that the relevant cost is held at the minimum level. So, against 0.03 we have a stock of 400 and against the stock out probability of 0.10 you have a say the reorder point of 350, so obviously, you oft for say 400 units as your reorder point.

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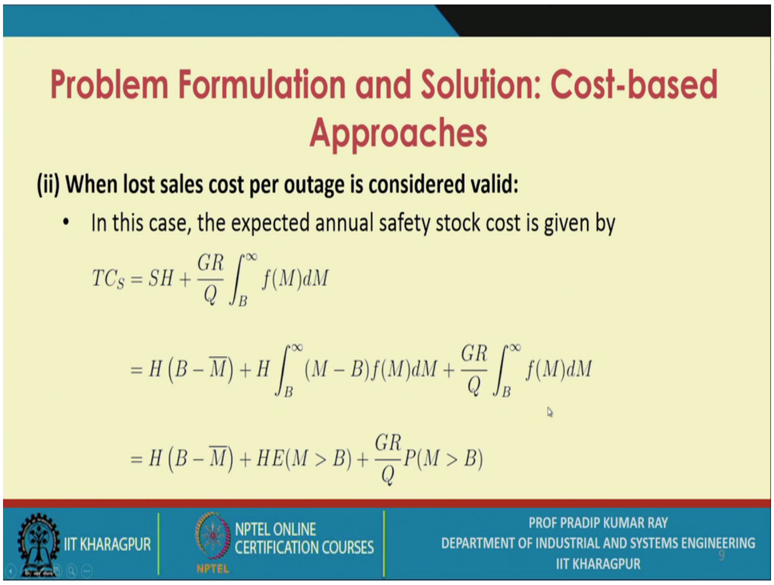

Now, the next important the case we will be dealing with right, now that is when lost sales cost per outage is considered valid. So, in this case the expected annual safety stock cost like in the previous case, so we have these expressions only here you will find that we have changed the notation of say the lost sales cost and it has become capital G, like in the previous case and this is the number of orders cycles per year and for each cycle what is the probability of going out of stock, is it ok.

So, this is the expressions. So, we have so ultimately we get this expression, right.

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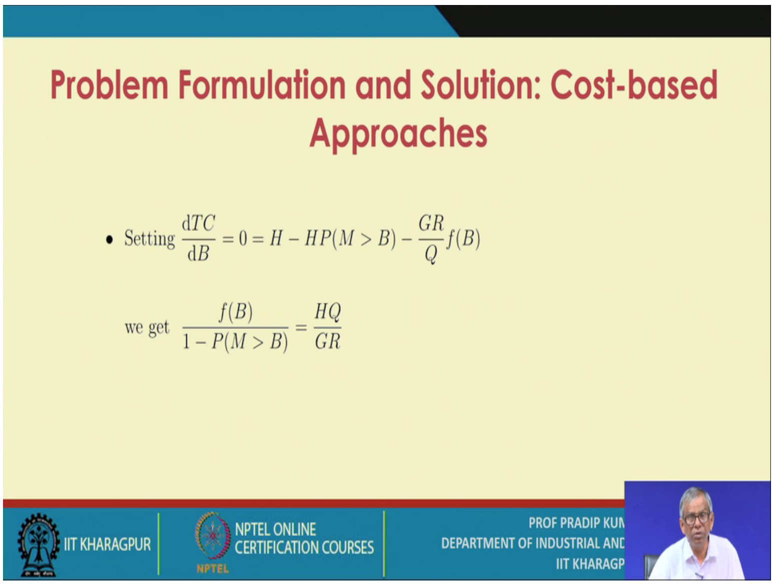

So, you just follow the steps and I am sure that you will understand all the steps involved. And then you take that say the first derivative with respect to the B of these equation, and set it equals to 0. So, you get these expressions and ultimately from these expressions you will get then if we use these expressions you will get the optimal the amount of say the reorder point. So, this is f B divided by 1 minus P M. So, probability that M is greater than B that is equals to HQ by GR.

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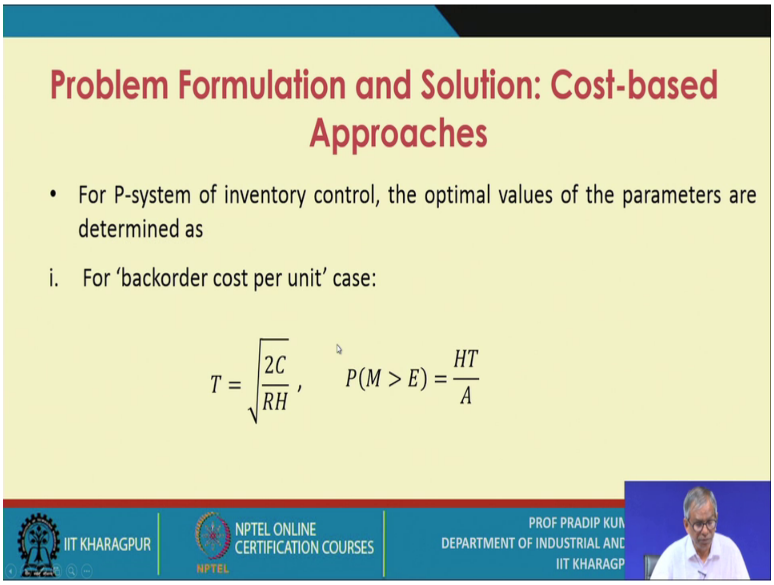

So, the for P systems now, what we have used this cost based approach which we have say, the discuss this is the entire formulation is for the Q systems of inventory control. Now, as an alternative to the Q systems as a pure systems you may also oft for the P systems of inventory control if that the P systems of the inventory control is valid, ok.

So, for the P systems of the inventory control the optimal values of the parameters also can be determined. So, for the back ordering cost per unit case, so this is the, so what we have assumed that that the lead time is constant the demand is variable. So, we can use this particular expressions for say the order period and this is the expression you get, that is the probability that that the demand during order period and lead time is greater than E capital E is the maximum inventory.

So, this is the probability of stock out. And where and where capital M here for this P systems of inventory control please make a note is refers to actually the demand during the order period plus lead time. So, the right hand side is HT by A, ok. So, this is the expression.

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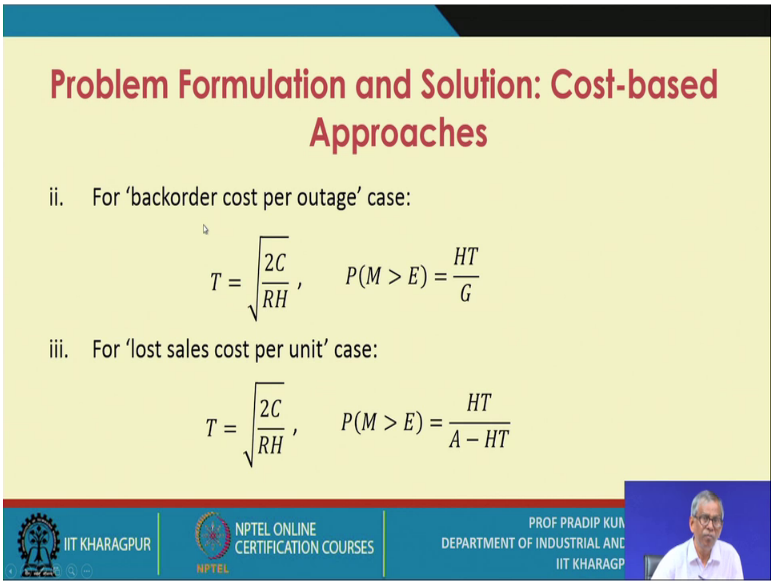

And similarly if for the back ordering cost per outage case, so this is the expression for T and essentially we have been say the using say the any EOQ formula if that is valid. So, in approximation and this is the probability of going out of stock and this is HT by G. For lost sales cost per unit case, the order periods optimal order period is root over twice C by R H and this is the probability of going out of stock, during the order period and the lead time. So, these expression is HT divided by A minus HT, ok.

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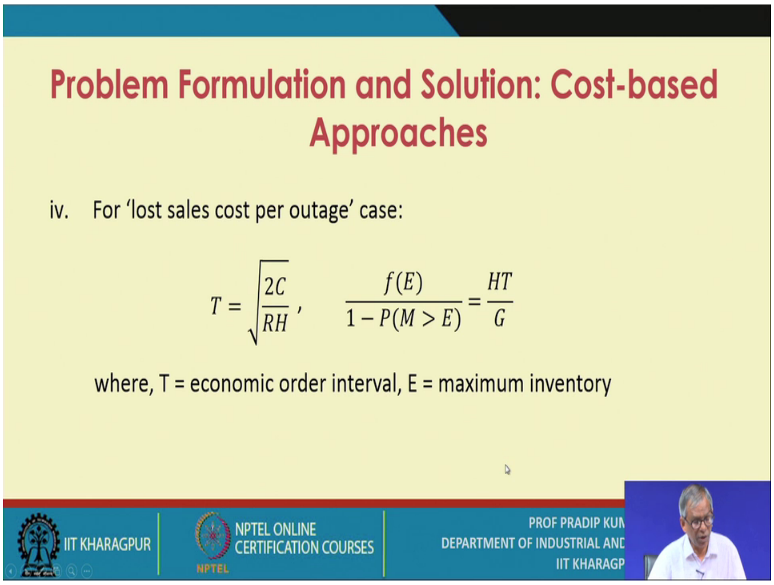

And for the last case that is the lost sales cost per outage case we have these expressions we have the same expressions for T that is economic order interval and capital E is the maximum inventory. So, this is one of the parameters of the P systems of inventory control. So, these expressions you use to determine the order quantity.

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Now, the next determine both the order quantity as well as the reorder point, and for the P system it will be say that that optimal order period as well as the maximum inventory.

Now, the next category problem is the category two problem that is the constant demand and the variable lead time. So, the inventory profile of the problem let us first say look at the inventory profile of this problem.

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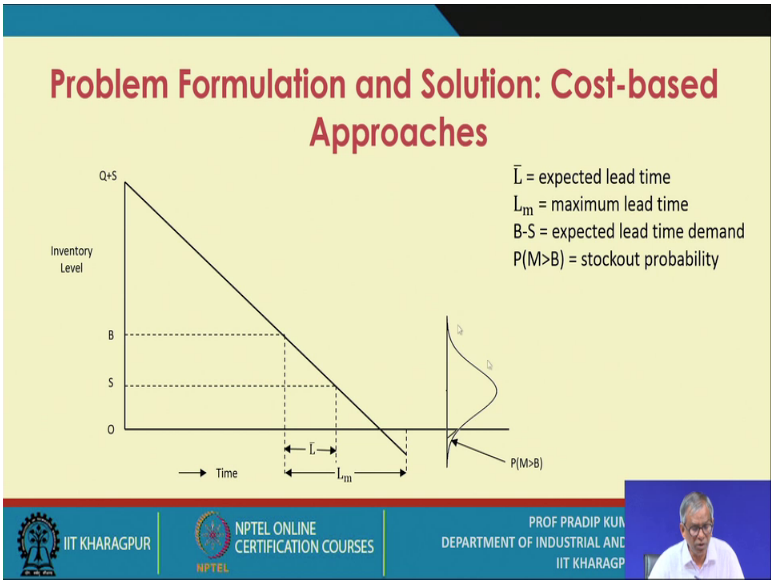

So, here on the y axis inventory level it refers to the inventory level. So, the maximum inventory level is Q plus S, Q is the order quantity and we are what we are assuming the this is Q systems of inventory control, S is the safety stock, capital B is your the reorder point and this is now, the lead time is wearing. So, L bar is the expected lead time, L bar is the expected lead time. L M is the maximum lead time, B minus S is the expected lead time demand, the B minus S that is the expected lead time demand and the, this is the stock out probability given as the probability that M is greater than B.

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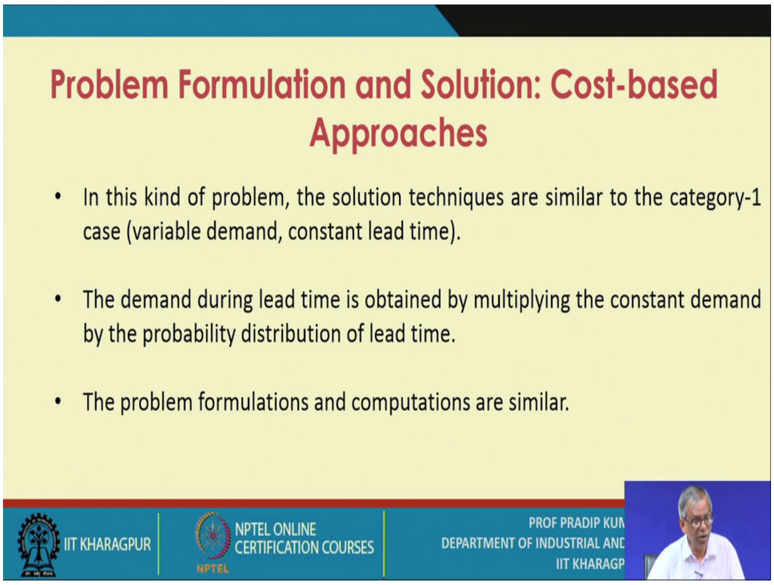

So, this is the inventory profile. Now, in this kind of problem the solution techniques are similar to the category one case. So, the variable demand and constant lead time the demand during lead time is obtained by multiplying the constant demand by the probability distributions of lead time and the problem formulations and computations are similar, ok. So, these points may please be noted.

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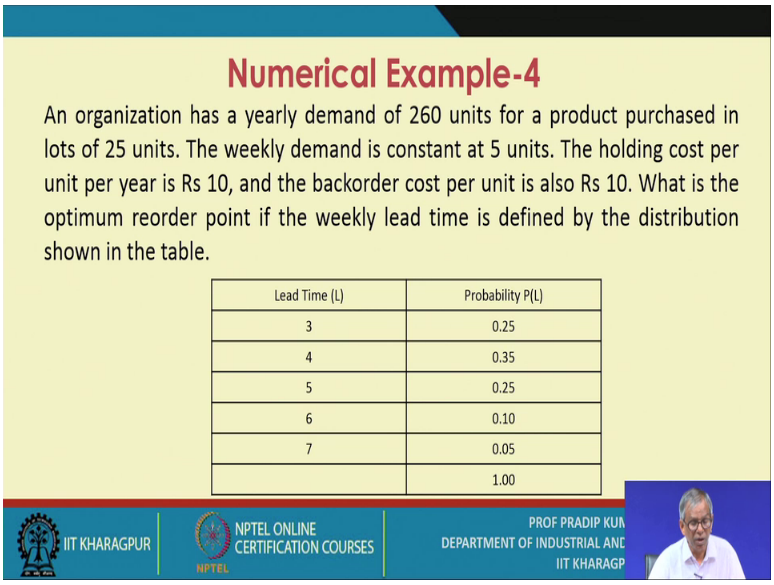

And here is one numerical example. So, you please go through this numerical examples all the data are given and this table that means, the lead time you know the lead time against the lead time probability say the distribution is given, that means, the possible lead times are 3 4 5 6 7 say the days and the corresponding probabilities also given.

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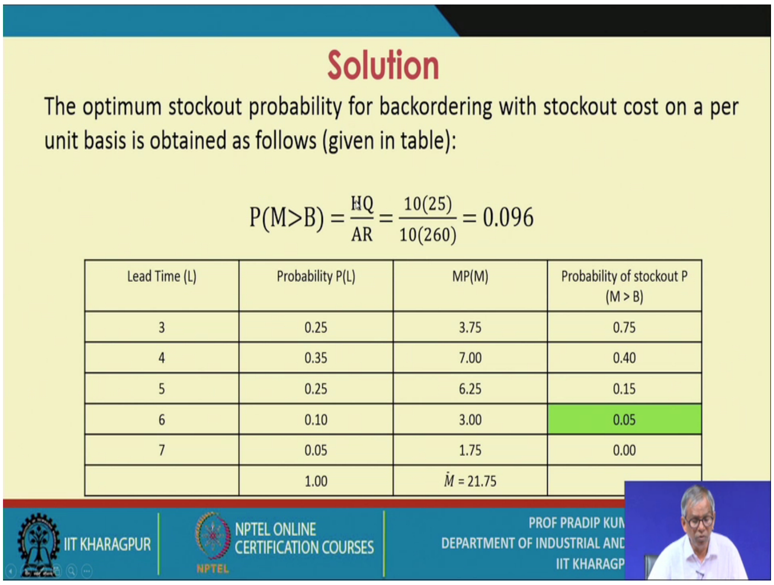

And what we try to do? That means, how to get the solution that means, optimal stock out probability you determine for the back ordering with stock out cost on a per unit basis is obtained as follows.

So, this is say the expressions already we have derived these expressions. So, this the optimal probability of going out of stock is point that is 0.096. And here in this table what you do that means, you add this the column that is M into P M and M is known. So, you multiply with the probability, ok, so 3.75 and all those. So, M bar calculation you do and the probability of stock out that is probability that M is greater than B so this value also you com compute. That is 0.096 that means, the lesser value is 0.05 as is obtained from the table and the corresponding value of lead time is 6, ok.

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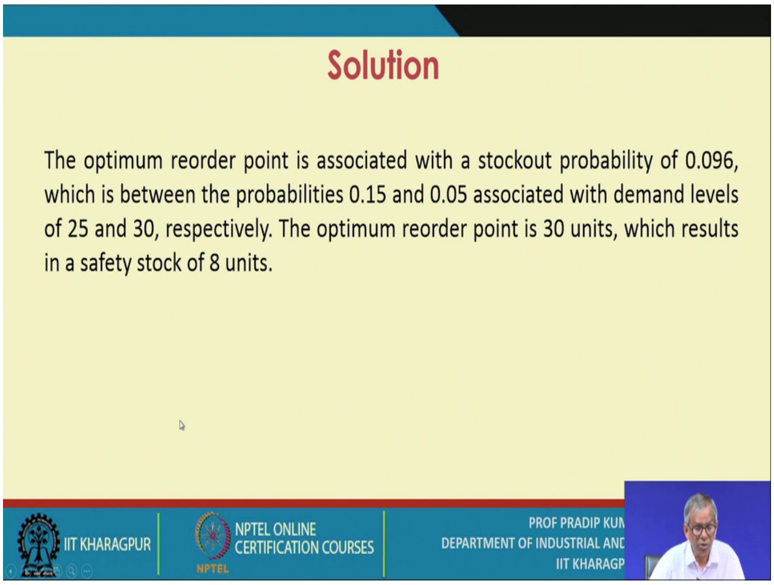

So, what do you calculate? That means, the optimal reorder point is associated this stock out probability of time 096 which is between the probabilities of 0.15 and 0.05 I have explained it very clearly, associated with demand levels of 25 and 30 respectively. So, the optimal reorder point is 30 units which results in a safety stock of 8 units. So, please go through this problem, ok. So, this numerical problem and I am sure that you will be able to understand all the details.

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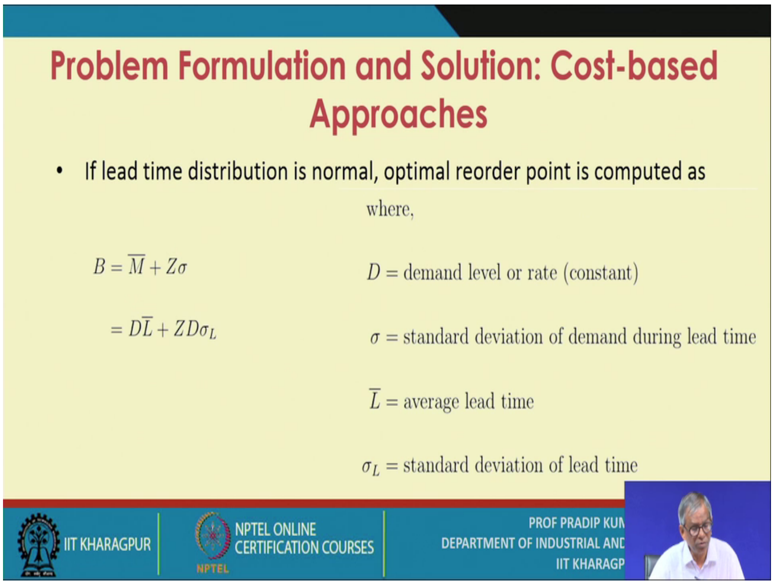

Now, as far as cost based approach is concerned. Now, if the lead time distribution is normal. So, the optimal reorder point is computed as, this is very important that means, reorder point is capital B and this is M bar plus Z into sigma, and M bar means essentially the demand into the average or the mean lead time, ok.

And here what we are assuming that the demand remains constant where is the lead time is a variable say if it is a variable then its standard deviation as well as its mean these two values are known and the mean lead time the notation is L bar and the standard deviation of lead time is sigma L. So, what do you try to do? That means, Z remain same and sigma means D into sigma L, the D remains constant.

So, D is the demand level or rate constant sigma is the standard deviation of demand during lead time and L bar is the average lead time and sigma L is a standard deviation of lead time. So, these are the notations are used. So, for the normal distribution, so you use these expressions for determining the reorder point.

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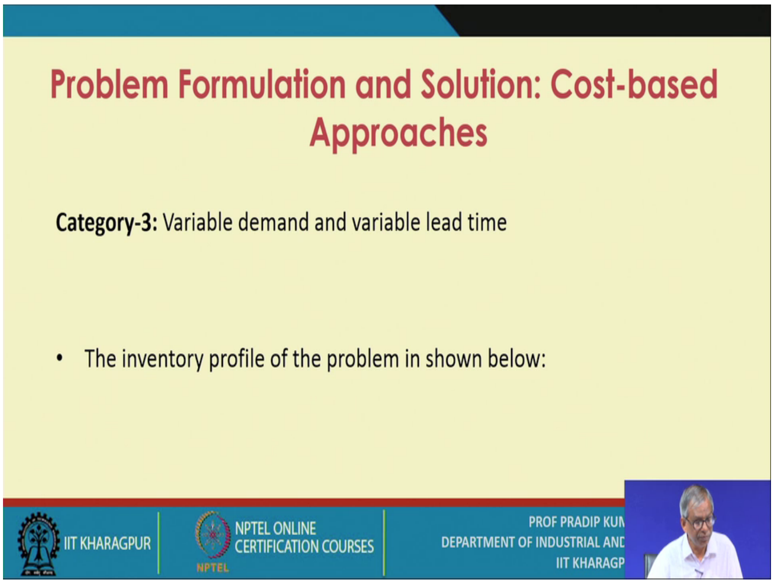

Now, in the category 3 problem what we assume that both the demand as well as the lead time both are a variable that means, they must be represented by the probability or say the probability distribution. So, the inventory profile let us first discuss or refer to the inventory profile of this particular problem, variable demand and variable lead time problem.

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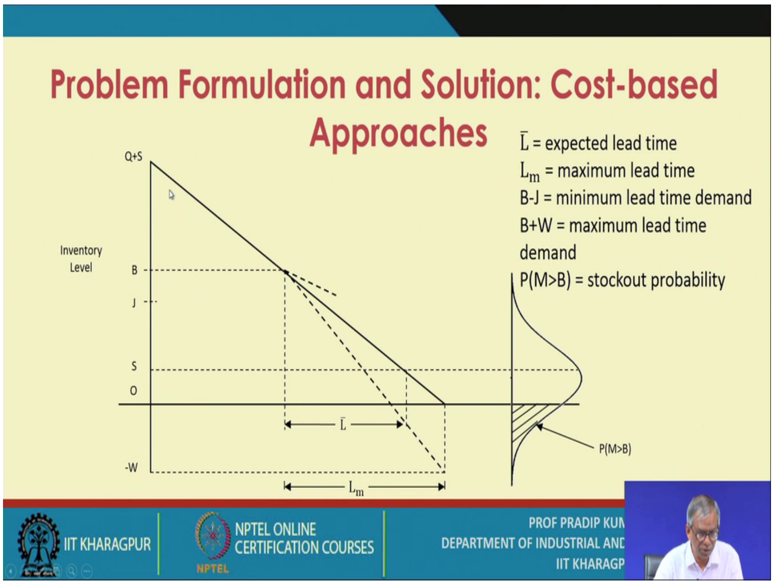

So, let us just focus on this particular inventory profile. So, here what you find as per as inventory level is concerned that means, the maximum the say the value is Q plus S, S is the standard is you know the safety stock and capital B is the reorder point. We have S that is safety stock and we have also the negative inventory that is W.

So, the both lead time as well as the demand varies. So, the demand could be during lead time could be say B minus J, that is B minus J is the minimum lead time demand. The demand during lead time could be B plus W and the W amount you cannot fulfill the demand of W amount, but it is a backordering case so that is why the negative inventory is allowed. So, B plus W is the maximum lead time and maximum lead time demand and this is the stock out probability that is the probability that M is greater than B L bar is the expected lead time at the maximum lead time is L m, ok. So, both lead time as well as the demand vary over time.

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So, the distribution of demand and lead time may be given either in empirical form or in standard form. For both cases the distribution of lead time demand that means, the joint probability distribution is to be computed from the individual demand and lead time distribution. So, this is to be say properly understood that means now, what happens that is the demand during lead time. So, this becomes a random variable and for which a joint probability distribution is to be known and you also must know that how to say or the determine this joint distribution joint probability distribution for a variable for a random variable called the demand during lead time.

Now, this the distribution of lead time demand can be computed from the individual demand and lead time distribution. So, there is already we have discussed in the previous lecture sessions. So, what is the procedure to be followed? So, please we have also given one numerical example related to this particular problem.

So, you please refer to that particular numerical problem given in one of the you know the past lecture sessions. There may be two conditions assumed, first one is the demand and lead time distributions are independent or the demand and lead time distributions are dependent. In other words what we are assuming that that the Q and say that means, the Q means order quantity and the reorder point they may be either consider to be independent or dependent or for the P systems that means, the order interval and the maximum inventory. They may be considered to be, so the independent or dependent. So, the two kinds of the modeling you may have.

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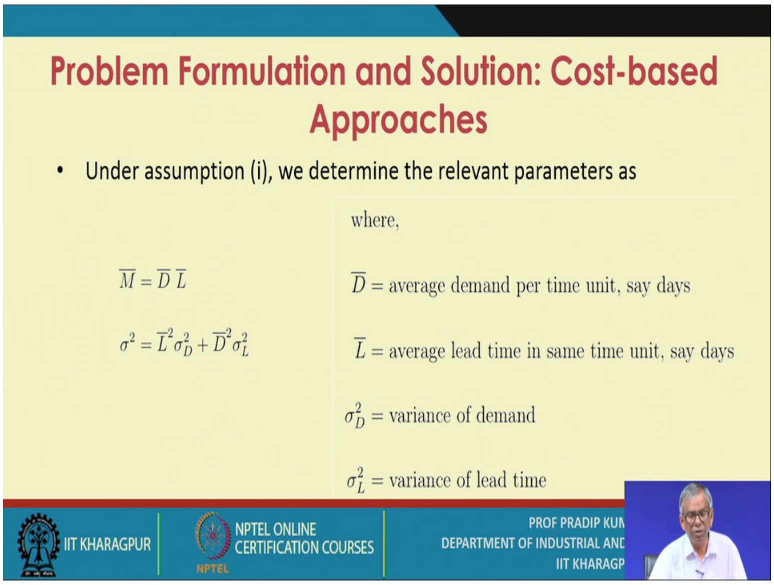

So, under assumption one that means, independence assumptions we determine the relevant parameters. So, what are the relevant parameters? That means, for the joint probability distributions, so you have distributions of the demand during lead time or the lead time demand that is a the random variable. So, you have the corresponding mean and the variance.

So, what is the expressions for M bar that means, say the mean demand during lead time and the variance of the lead time demand. So obviously, it is D, D bar into L bar. So, what is D bar? That is average demand per time unit say days and say one day, and L bar is the average lead time in same time units say days and sigma squared D is a variance of demand that means, the this is the variance of demand in lead time this means that L bar square into sigma D square

So, what is sigma D square? That is the variance of demand plus D bar square into sigma L square. So, what is sigma L square? That is the variance of lead time. So, from the given data set you have all these all these values and obviously, you can compute the value of M bar and sigma square.

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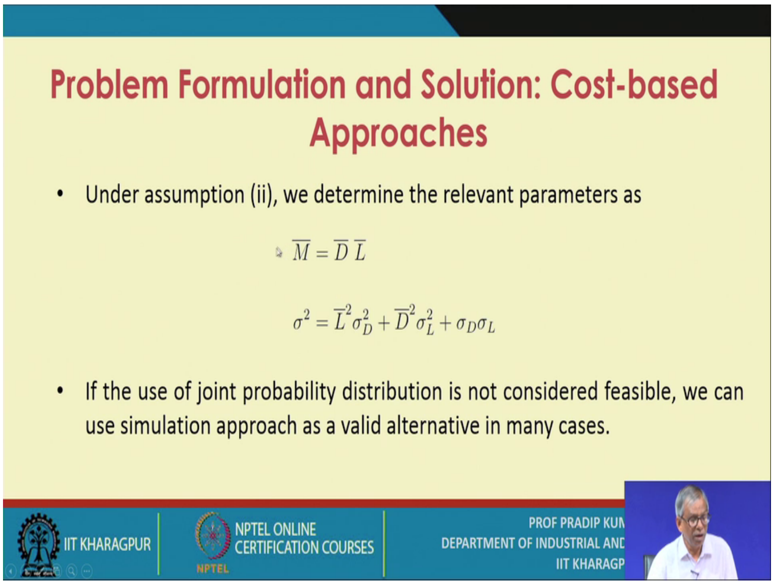

Under assumption 2, that means, the dependence assumptions. So, we determine the element parameters that means, the expression for M bar remains same that is D bar into L bar, whereas the expression for sigma square it is slightly different in the sense that you add the third term the first two terms they remains same like you have for the independence assumption that is L bar square into sigma D square plus D bar square into sigma L square.

Now, you have added one more term that is the third term that is sigma D into sigma L. So, what is sigma D? That is the demand the standard deviation. And this is what is sigma L? That is the lead time standard deviation. So, if the use of the joint probability distribution is not considered feasible in certain cases this may not be feasible. So, what you do? We can use the simulation approach as a valid alternative in many cases. So, later on you know in many cases you oft for say the simulation approach.

And so the first what you try to do? You try to formulate the problem and looking at the formulation with a given the set of assumptions, you can you can have an idea about the complexity of the problem whether you will be getting through mathematical modeling approach or through analytical approach whether you will be getting a closed form solution or not. So, if the close form solution is not obtainable, but the problem is very very critical.

So, as a next alternative, next best alternative many a time you go for the simulation modeling. So, in course of time we will also discuss these important aspect that in the field of the materials management what are the specific kinds of problems where you need to use the simulation approach.

Thank you. I conclude this session.