Management of Inventory Systems
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Lecture – 27
Dynamic Inventory Problems under Risk (Contd.)

During these lecture session on a Dynamic Inventory Problems under Risk, we will be referring to the 2 important issues one is the probability distributions of demand and lead time.

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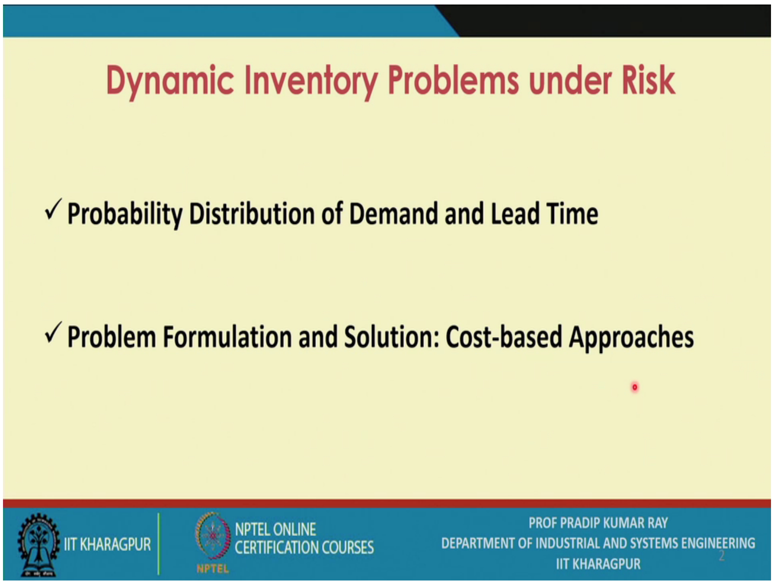

So, you must have good understanding of say the types of the distributions which say, but the demand probability distributions which are normally in use for modeling the problem or for formulating the problem.

So, this part we will discuss and the second one is we will be referring to the problem formulation and the solution part and first particularly the cost based approaches will be we will be discussing now.

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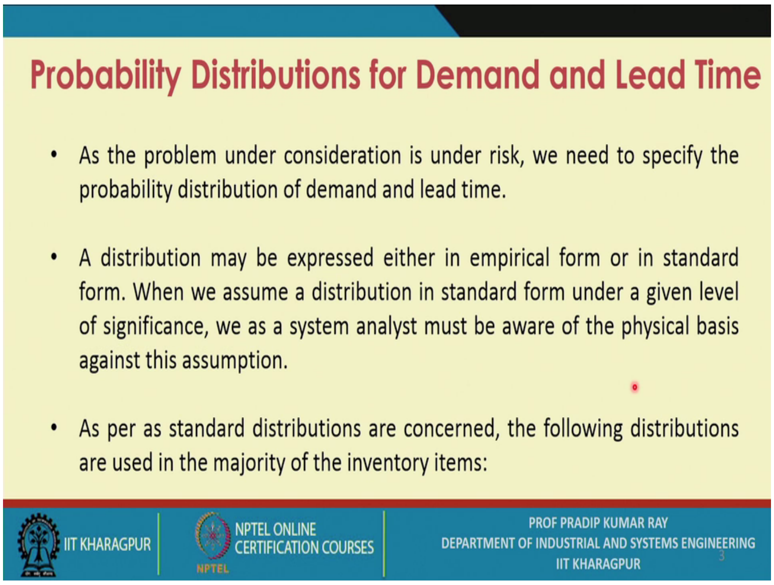

So, let us first talk about the probability distributions for demand and lead time. So, as we have observed that in any inventory control systems for modeling a purpose, so, you must have, say, thorough better understanding of the types of demand; that means, what are the characteristics of the demand and what are the characteristics of the lead time. And this, the 2 factors are essentially the determine the values of the inventory control parameters.

Now, as the problem is under risk; that means, in a given time period the, exact the demand is not known, but what is known as a probability distribution. Similarly, in a given situation. So, the exact value of the lead time is not known; what is known is that the distribution of lead time.

So, as the problem under consideration is under risk, we need to specify the probability distribution of demand as well as the lead time. A distribution may be expressed either in empirical form or in standard form. This point already we have highlighted. When you assume a distribution in standard form under a given level of significance; that is, say alpha, say 0.05 or 0.01.

So, whenever you say that this is the distribution in standard form, like say, exponential or normal or say uniform and so, obviously, you refer to, so, what is the level of significance under these assumption say alpha is 0.05, what is your confidence level; that means, 95 percent of the time, I conclude that the distribution is say the normal or the distribution is exponential.

So, whenever we conclude about the distribution in standard form, so, the level of significance must be specified. We, as a system analyst, must be aware of the physical basis against these assumption; this is very important. In fact, because any the distribution you assume, there must be a physical basis; that means, first you observe the physical basis and then the physical systems related to a particular random variable and then you can prove that this with all likelihood say a particular.

So, the random variable may be assumed to be normal or exponential, is it or etcetera. There are many kinds of say the distributions, you can you may be assumed as per the standard distributions as far as the standard distributions are concerned, the following distributions are used in the majority of the inventory items.

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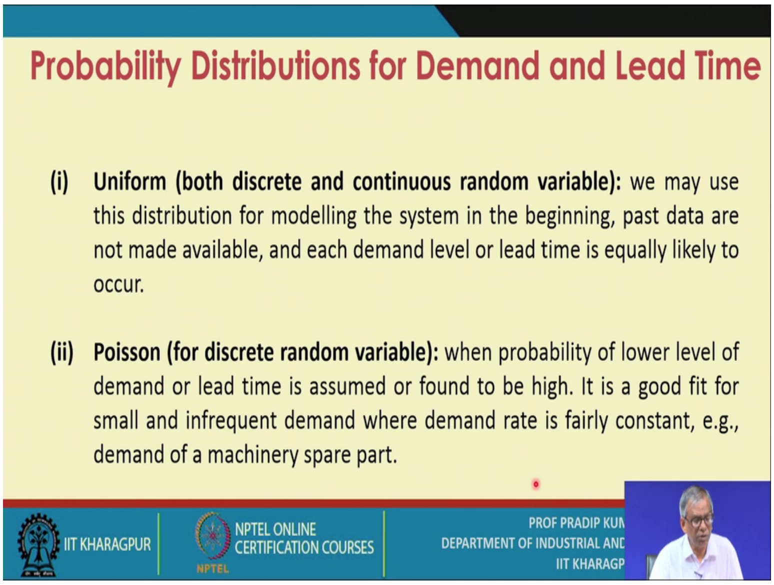

So, what are those? The first one actually 3 or 4 types are commonly used. The first one is the uniform distribution the distribution sometimes which is referred to as the that the distribution with the maximum ignorance; that means, this distribution you may assume at the starting point.

So, the both discrete and continuous random variable, you may use the uniform distribution. So, what we say that, we use these distribution for modeling the system. In the beginning, past data are not made available and each demand level or the lead time is equally likely to occur, is it ok? So, that is the basic assumptions we have while we assume it to be uniformly distributed.

The next important, say the distribution assumption is a Poisson and Poisson distribution is used for discrete random variable when probability of lower level of demand or lead time relatively speaking is assumed or found to be high. So, this is the condition you must know; that means, when you observe the system, say the physically. So, you have you must have these observations; that means, the lower level of demand or the lead time is there prove there are occurring say more frequently.

So, it is a good fit for small and infrequent demand where demand rate is fairly constant. For example, demand of a machinery spare part. So, later on, when you will we will take up, say the numerical problems. So, suppose, you want to have an inventory policy for the machineries for a machinery spare and so, many a time we assume that that the demand for the machinery spare is Poisson. Is it ok? So, there are many numerical problems.

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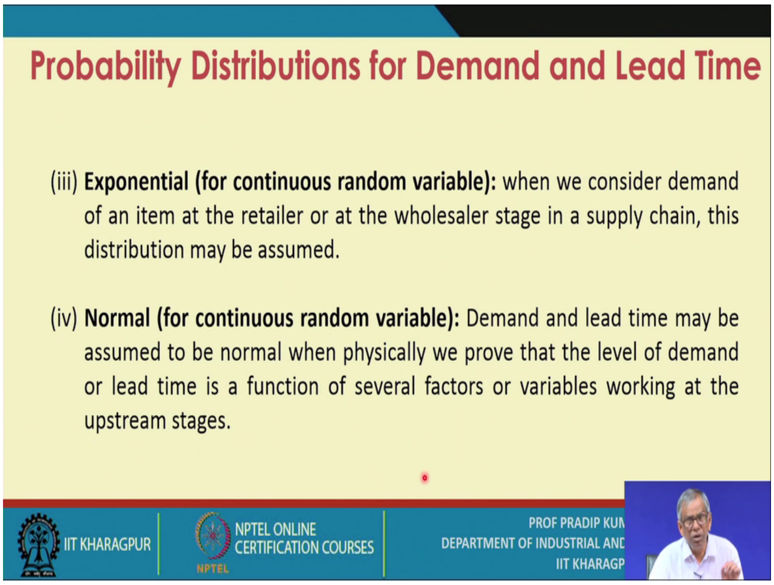

So, we will be referring to later on, the third important the distribution we assume in standard form that is referred to as a exponential distribution for continuous random variable, when we consider the demand of an item at the retailers or the wholesaler stage in a supply chain this distribution may be assumed ok.

So, there is must be some you know the physical the basis for assuming for in this particular case, that the distribution is exponential distribution of demand for the items being used at the retailer stage or the wholesaler stage. Then the fourth important, say the distribution that we assume in many cases, that is the normal distribution for continuous random variable demand and lead time maybe assume to be normal when physically will prove that the level of demand or lead time is a function of several factors or the variables working at the up steam stages it.

So, this is the scenario; that means, suppose Y is random variable, assume to be normal. It means, that the value of Y, Y is dependent on the several factors say n number of factors x 1 x 2 up to x n. So, and all these factors are occurring at the upstream stages not at the downstream stages. That is why, it these are called the effective or effecting variables.

So, if you find such a situation, physically you can assume it to be that particular random variable to be normal.

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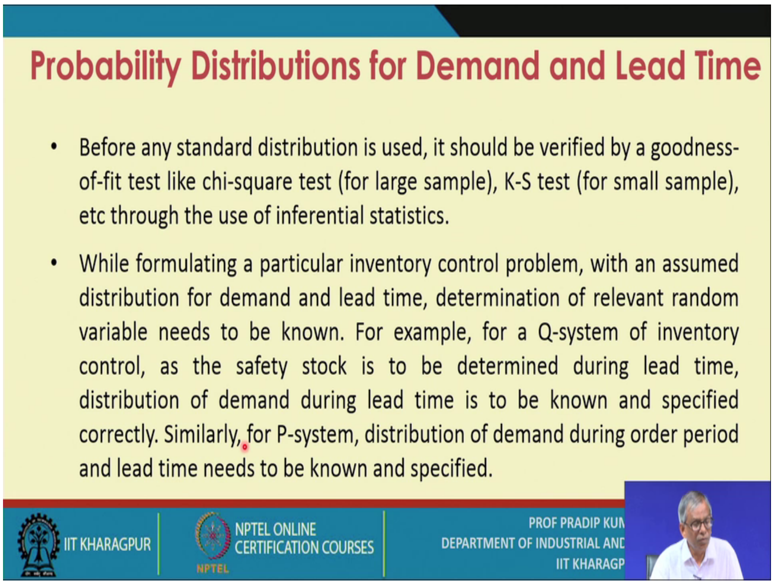

Before any standard distribution is used, it should be verified by goodness of fit test, you may be aware of a number of goodness of fit tests. Like the chi square test for the large sample and K S test Kolmogorov, similar test for the small sample this etcetera through the use of inferential statistics. So, I presume that you are aware of this one or more of these goodness of the fit test if you are not aware of.

So, please go through this tests, particularly the chi square test and the K S test which you find in any text books on probability and statistics you refer to the section on inferential statistics while formulating a particular inventory control problem with an assumed distribution for demand and lead time determination of relevant random variable needs to be known.

For example, for a Q systems of inventory control, as a safety stock is to be determined during lead time, ok. So, safety stock is one of the parameters distribution of demand during lead time is to be known and specified correctly; that means, for which time period the distribution of say the demand is to be known. So, that the time period you must be able to specify.

And this, this you know this you can specify only when you come to know the working of an inventory control systems and under what condition during which time period, there is a chance of say the over stocks or over stock or under stock situation. Mainly, the under stock situation and in order to avoid or the prevent the occurrence of the stock out situation so, you go for stocking some extra amount which is referred to as a safety stock.

Similarly, for the P systems, the distribution of demand during order period and lead time needs to be known and specified. Because you need to determine the safety stock for the entire period that for the P systems; that is, the order period and the lead time. Is it ok?

So, because the during this order period and the lead time, there could be fluctuations of demand and due to the fluctuations of demand, there could demand and as well as the fluctuations of lead time itself there could be that there could be chance of the stock out.

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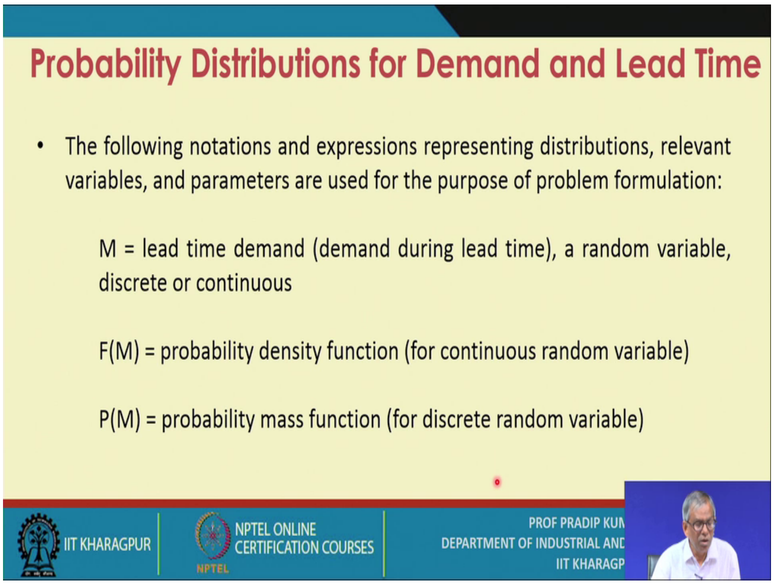

Now, we will be using this domain distributions while we model or while we formulate a problem so, we need to use the notations. So, what are these notations? So, let me explain. That means, this following notations and expressions; certain expressions like say, the mean or the standard deviation. So, the variance ok. So,. So, you will be using and obviously, you will be using their notations.

So, the following notations and expressions representing distributions relevant variables and the parameters are used for purpose of problem formulation, is it ok. So, so let us just discuss all these the notations and their the meanings in a systematic manner. The M, these notation is used to represent lead time demand; that means, the demand during lead time ok. So, this is demand in physical units say 100 units or 1000 units.

So, when we will refer to numerical problems is point will be made very very clear. So, the lead M is the lead time demand is considered a random variable discrete or continuous, ok. So, this is all about M. What is F M? That means, small f M, that is the probability density function for continuous random variable, ok.

What is P M? That means, the probability mass function for discrete random variable. So, this is the notations you used; that means, the M, the small f M and P M. So, probability density function as well as the probability mass function.

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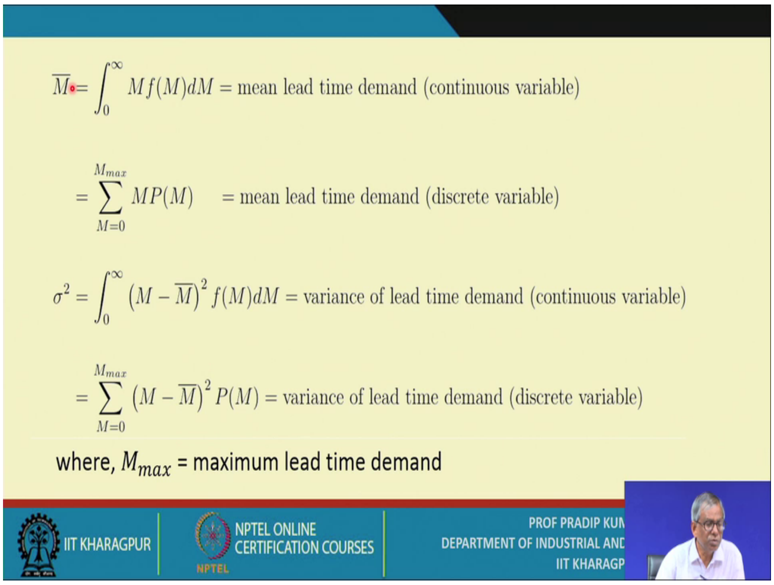

Now, what is the mean demand during lead time the notation is M bar. So, this is 0 to infinity M f M d M; that means, this is the mean lead time demand for the continuous variable for the discrete variable. What is the expression for M bar? That is sigma M equals to 0 to M max; that means, the possible demand levels M into p M. That means, for each demand level what is the corresponding the probability. So, that is p M what is the variance sigma square 0 to infinity M minus M bar square f M d M; that means, the variance of lead time demand.

When it is considered to be a continuous variable, when the lead time demand is considered to be a discrete variable, then the expression it is you know the variance is sigma M equals to 0 to M max M minus M bar square p M corresponding probabilities where, M max is the maximum lead time demand, is it ok. So, this is the expression. So, we have an expressions for M bar as well as the sigma square; that means, this is the mean, this is the variance.

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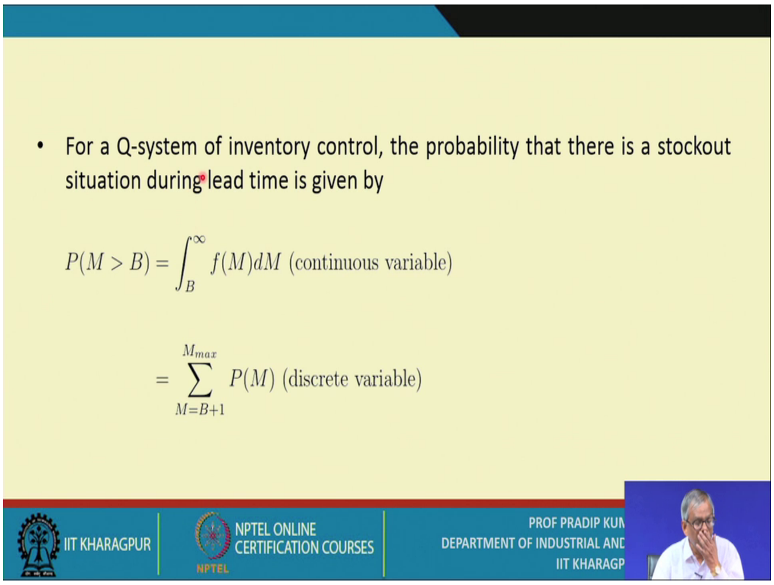

For Q systems of inventory control, the probability that there is a stock out situation during lead time this is very very important. Because, we must know the probability of stock out and we keep extra stock or the safety stock or the buffer stock to prevent the occurrence of this stock out.

So, the probability of stock out; that means, M is greater than equals to B. What is B? B is reorder point. That means, when during lead time, the demand is greater than the reorder point. Obviously, there will be stock out situation. So, this is integration B to infinity f M d M. So, it is a continuous variable and if it is a discrete variable, then obviously, this is sigma M equals to B plus on to M max P M.

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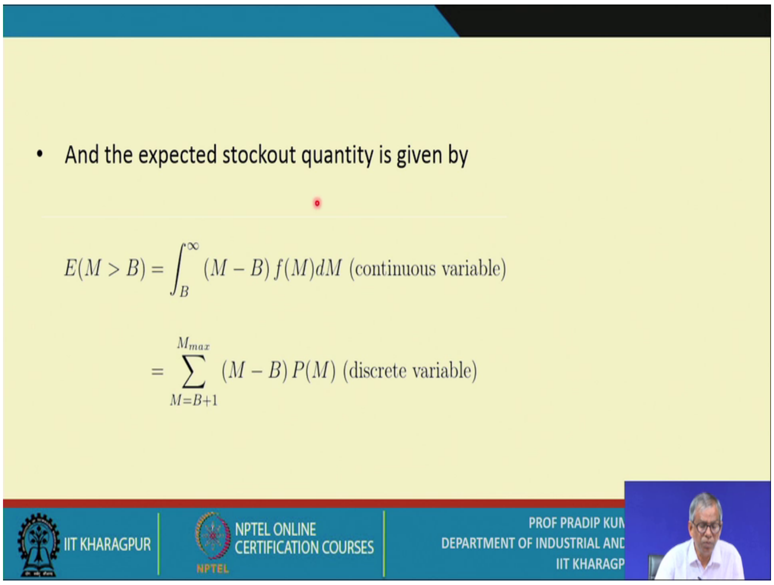

And the expected stock out quantity is given by this is the notations we have use; that means, expected stock out quantity the what is the condition; that means, M is greater than B. So, that is B to infinity M minus B f M d M that is for the continuous variable and for the discrete variable, this is sigma M equals to B plus 1 to M max M minus B P M.

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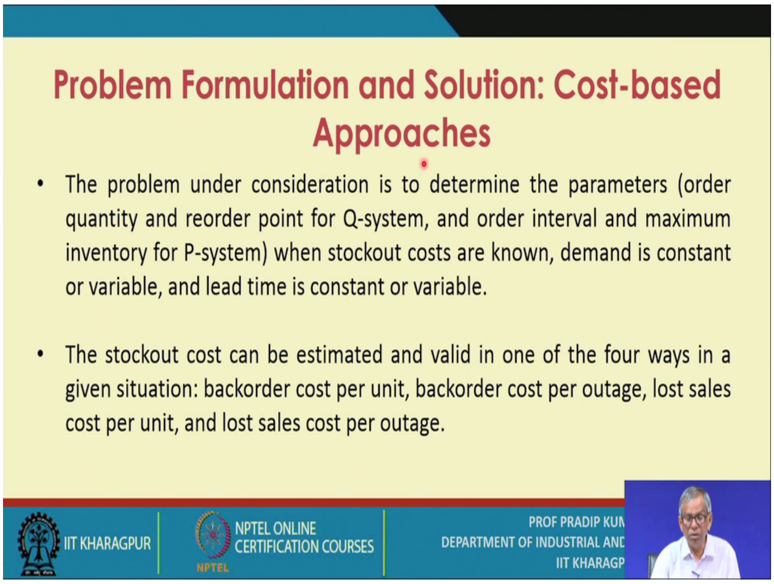

Now, let us now talk about with this the basic understanding of the distributions and their parameters with the notations the known. Now, let us talk about the problem formulation and the solution. So, that is the most important aspect and first, we will discuss the cost based approach. The problem under consideration is to determine the parameters order quantity and reorder point for Q system and order interval and maximum inventory for the P system.

When the stock out costs are known, demand is constant or variable and the lead time is constant or variable. The stock out cost can be estimated and valid in one of the four ways in a given situation; like, you know the stock out, how you know it is responded; that means, there could be back ordering case.

So, corresponding back ordering cost per unit that may be relevant or the back ordering cost may be assumed to be fixed one irrespective of the amount of shortage and so, this is referred to as the back ordering cost for outage. Now, the stock out may be responded by say, the loss cells. So, the corresponding loss cells cost maybe estimated per unit basis or in certain cases the lost cells cost can be estimated per outage basis.

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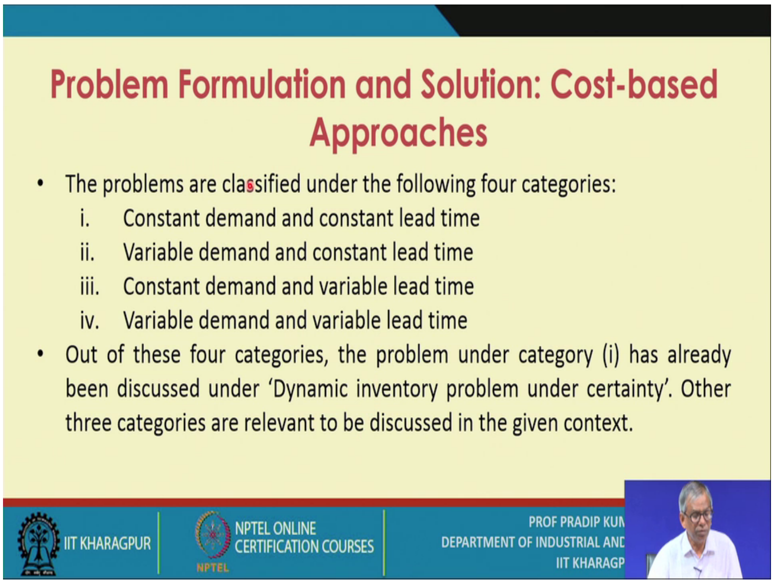

Now the so, the; hence, when we apply the cost based approach the problems can be classified under the following four categories. So, what are those? First one is the constant demand and constant lead time. Now, this particular problem already we have discussed; that means, when we refer to say the static inventory problem or now, dynamic inventory problem under certainty, so, essentially we refer to this kind of problem constant demand and constant lead time.

The second one, we need to discuss that is a variable demand, but constant lead time. The third alternative is constant demand and variable lead time and the fourth alternative is variable demand and variable lead time. So; obviously, is the number one type we have already discussed. The other 3 will be discussing during this week. Out of these four categories, the problem under category one has already been discussed on the dynamic inventory problem under certainty. So, the other 3 categories are relevant be discussed in the given context.

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So, let us the talk about say the category one problem; that means, variable demand and constant lead time, ok.

So, and what we are we assume that the demand and lead time are independent with each other; that means, the approximate method we will be using initially and later on, definitely the exact method also will discuss. So, the inventory profile of this problem; that means, category one problem is like this one.

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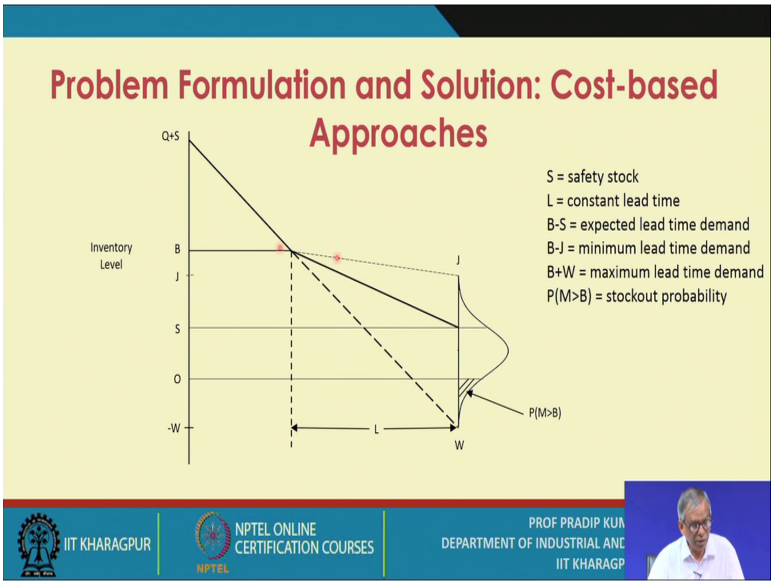

That means, here the Q is the order quantity, S is the safety stock, B is the reorder point and l is the constant. Say the least time lead time is constant. This is one B minus S is expected lead time demand; that means, this is S; that means, expected lead time demand is this one expected lead time demand is this one B minus J is a minimum lead time minimum lead time B minus J is the minimum lead time demand and B plus W is the maximum lead time demand; that means, W is negative; that means, is the backordering you are unable to fulfill this amount that is W, but what the customers are waiting; that means, in the next cycle, definitely you try to fulfill his or her demand and that demand is back ordered and that amount is W.

So, B plus W is the maximum lead time demand and probability that M greater than B. It means the stock out probability, is it ok. So, this is the stock out probability.

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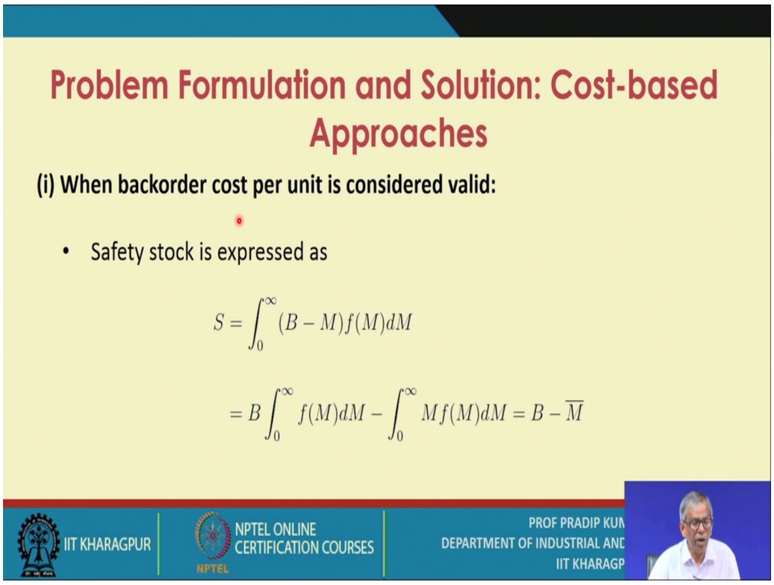

So, when back ordering cost per unit is considered valid now. So, that it is the first case. So, what is the safety stock? So, safety stock is expressed as S. S is the notation we use 0 to infinity B minus M f M d M. So, after the simplification, we have these expression B minus M bar. So, what is B? B is the reorder level and M bar is the average demand during lead time.

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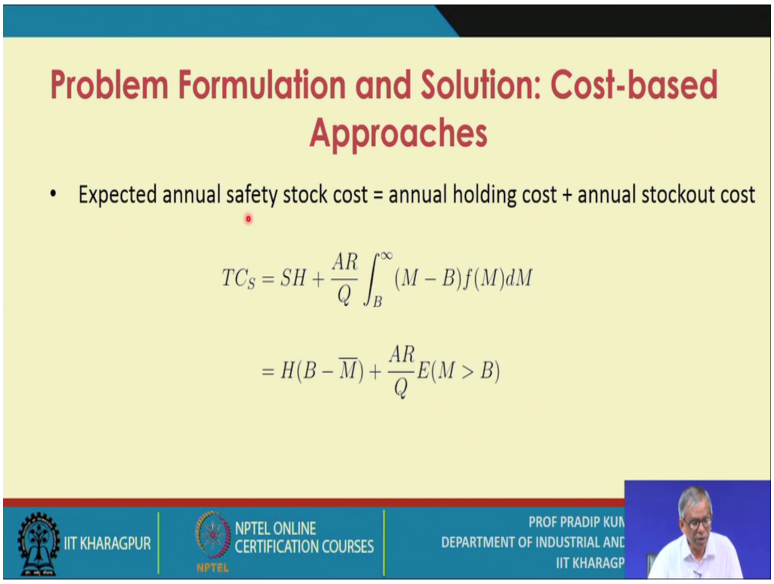

So, expected annual safety stock cost; so, that you first have say the expression for the expected annual safety stock cost and you try to minimize this cost. So, it consists of 2 parts; annual holding cost plus annual say the stock out cost. So, this T C s equals to S into H. H is the holding cost per unit per year. A is the safety say the say stock out cost say the per unit and R is the yearly demand and Q is order quantity which already you have determined and this is the expected number of unit short is it ok. So, A is basically the stock out cost per unit short. So, this is the expected number of unit short and you please go through these expressions and so, we have the expression for T C s.

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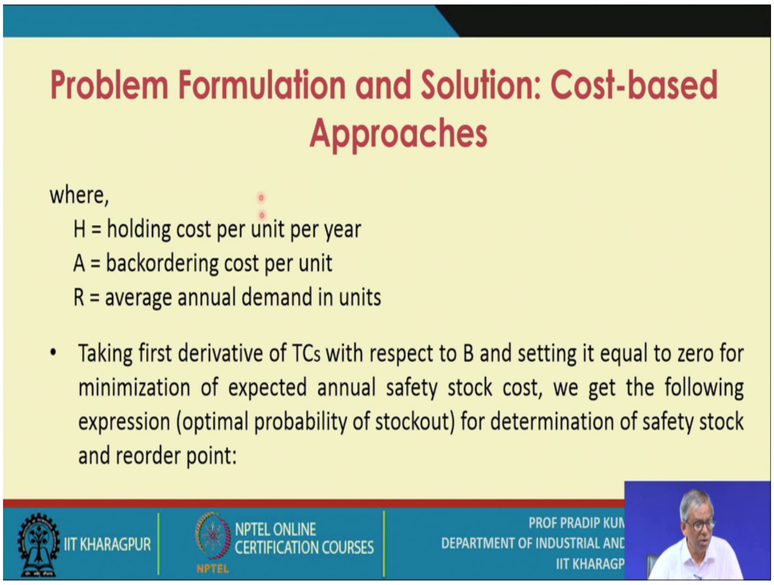

So, H is the holding cost per unit per year, A is the back ordering cost per unit, R is the average annual demand in units, already I have mentioned. So, taking the first derivative of T C s with respect to B. So, the B is basically your decision variable and setting it equal to 0 for minimization of expected annual safety stock cost, we get the following expression; that means, the optimal probability of stock out for determination of the safety stock and reorder point.

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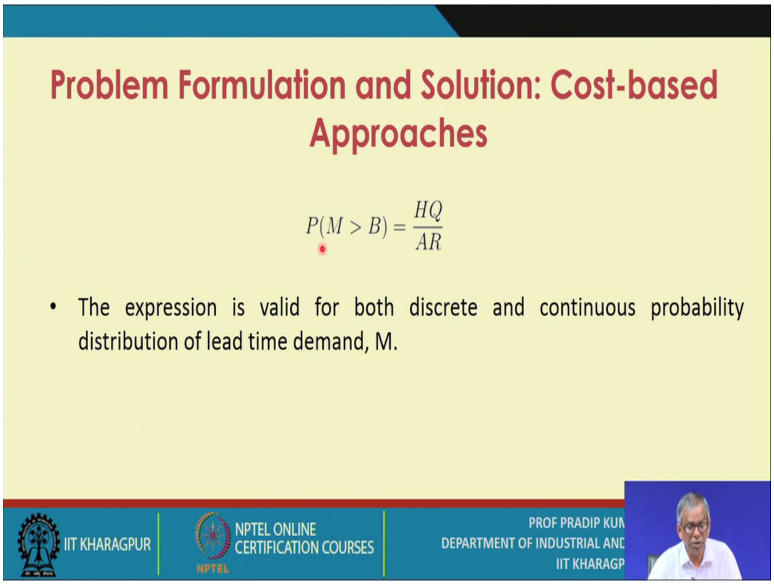

So, ultimately we get these expression; that means, probability that M is greater than B; that means, the probability of stock out is equals to H Q by A R. The expression is valid for both discrete and continuous probability distribution of lead time demand M. So, here is one numerical example.

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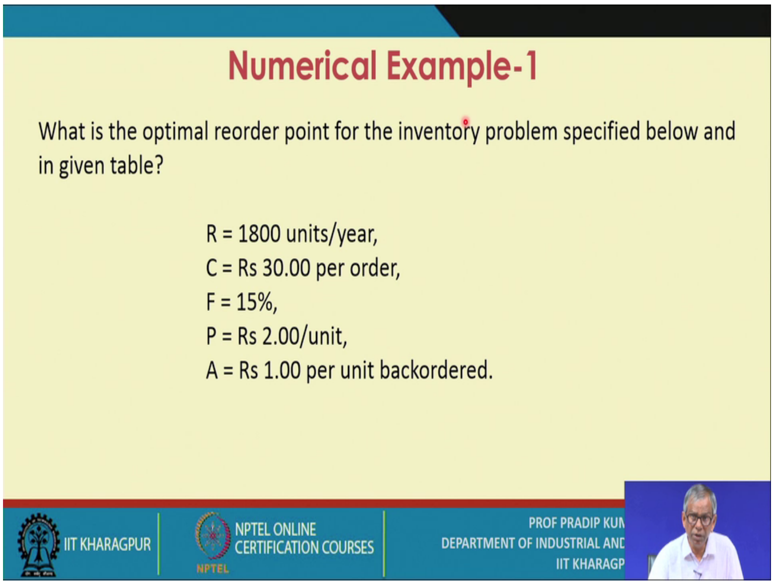

So, what is the optimal reorder point for the inventory problems specified below and in the given table? So, these are the values R 1800, C is 30, F is 15 percent; that means, the holding cost fraction P is 2 and A is just rupee 1 per unit backordered.

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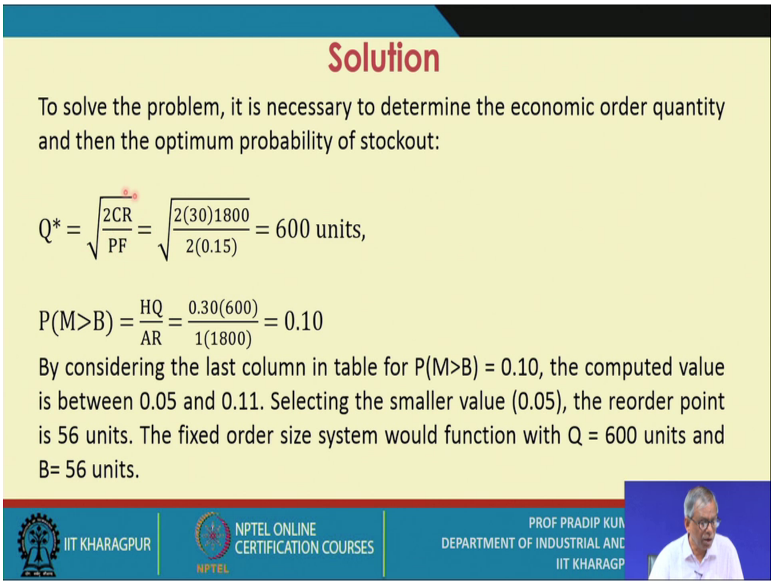

So, what we have; that means, first we determine the order quantity; that is, one of the parameters we use you know the basic the classical EOQ formula and we get a value of 600 units. So, all the data are known and you determine Q star; that means, optimal say order quantity for the given item and the probability of the stock out the notation is that probability that a M is greater than equals greater than B, there is H Q by A R. Already we have derived these expression. So, this value is 0.10 by considering the last column in table.

So, we will show you the table for probability M greater than B probability stock out; that means, this is the condition to be specified. And under this conditions, what is actually what will be your say the reorder point. So, the computed value is between 0.05 and 0.11. Selecting the smaller value 0.05 the reorder point is 56 units, the fixed order size system should function with Q equal to 600 units; that means, is a Q systems of inventory control for the given item and B is 56.

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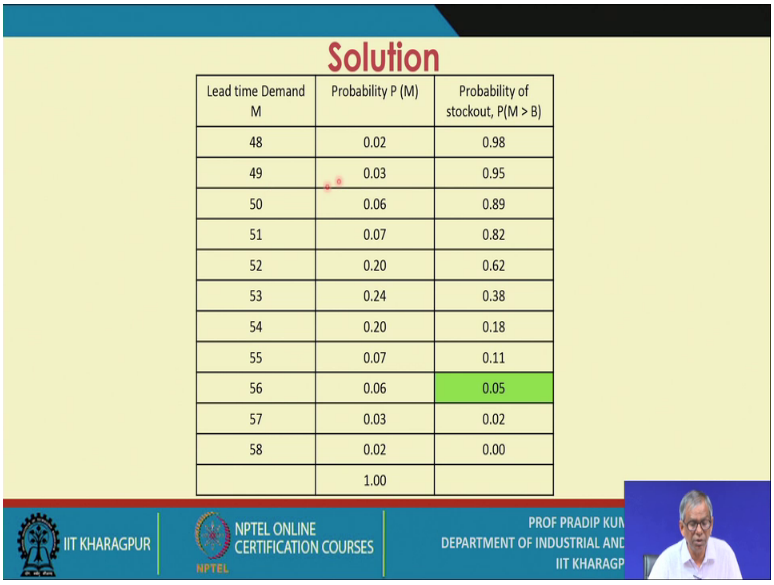

So, this is the table we have; that means, lead time demand M. So, these are the possible lead time demands we have corresponding probabilities are given. So, when you add all these probabilities, this is one you add one more column that is the probability of stock out probability that M is greater than B. So, these are the values.

So, if it is against 48; that means, what is the probability of stock out that is 0.98. If suppose, your the reorder point is 49, then what is the probability of going stop or going stock out that is 0.951 minus 0.02 minus 0.03, 0.95. So, this way you calculate and ultimately, you will find that against a 0.05 probability of stock out the corresponding value of M is 56 and against 0.11 probability of stock out the corresponding value is 0.55. Now, our benchmarked value is 0.10. So, that is why, our you say that again 0.05 probability of stock out you have the corresponding demand during lead time that is 56 and that is why the reorder point is 56.

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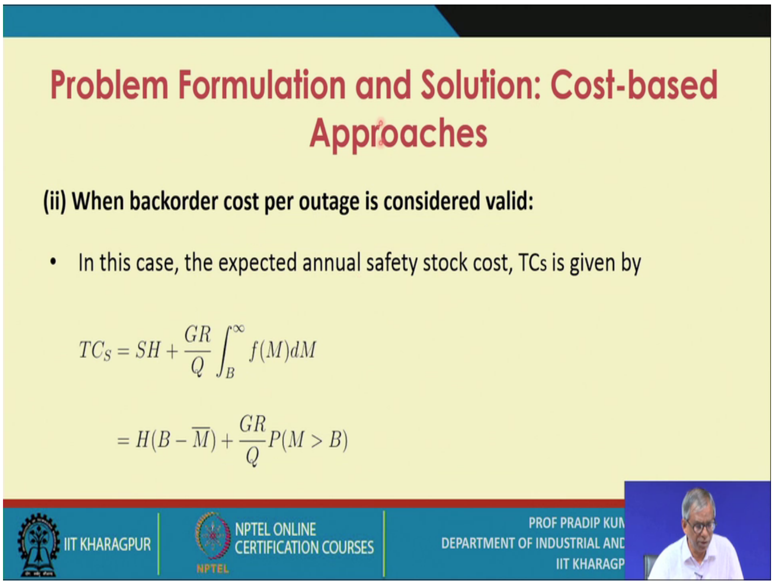

So, similarly when back ordering cost per outage is considered; that means, in the next case, we assume that the that the stock out cost remains the constant and the back ordering, it is a backordering situation remains constant irrespective of the amount shortages, ok. So, if this is the assumption then the expected annual safety. Safety stock cost T C s is given by S H plus G R by Q. Now, what is j G? Here, G is nothing, but the back ordering cost per outage per unit.

And this is basically the probability of going out of stock, is it ok? So, that means, integration B to infinity f M d M. So, this is the you know the simplified expressions.

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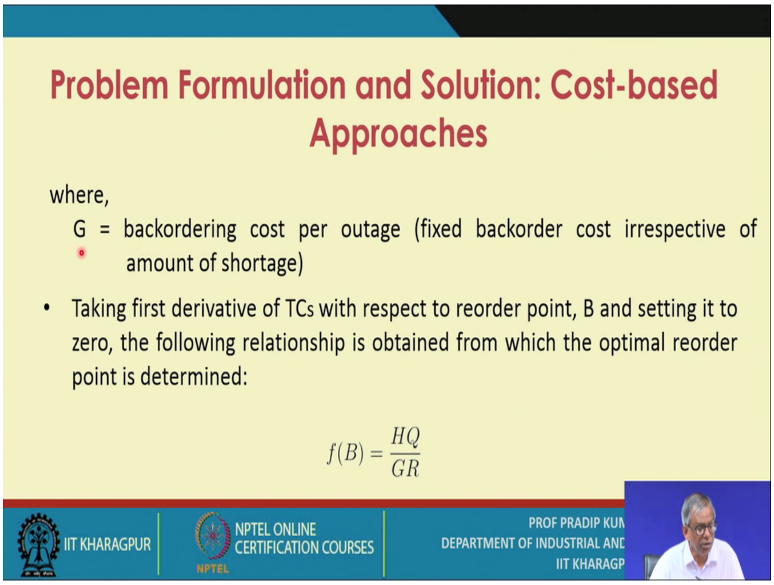

And then when so, when you take the first derivative of T C s with respect to the reorder point B and setting to 0 the following relationship is octane. So, what is that relationship? Relationship is f B is equals to H Q by G R, is it ok.

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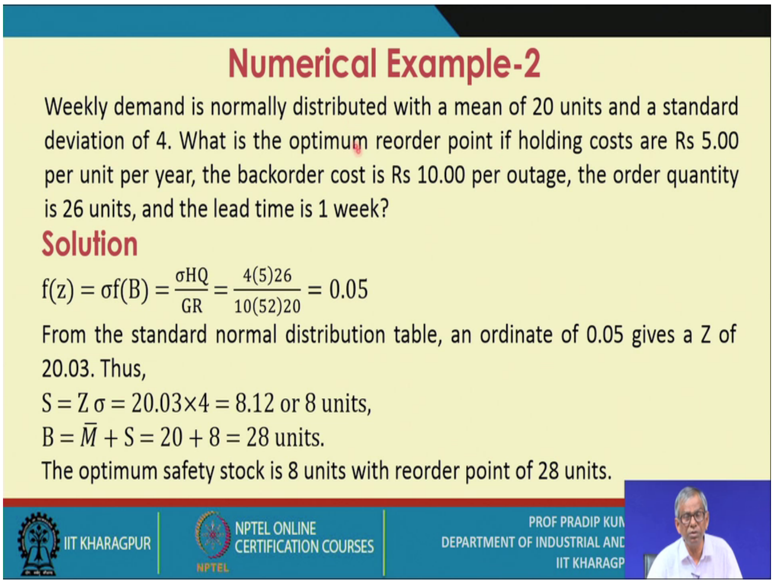

So, here is another a numerical example. So, the weekly demand is normally distributed with a mean of 20 units and the standard deviation of 4.

So, what is the optimum reorder point if holding cost are rupees 5 per unit per year; that means, the value of H is given the back ordering cost is also this is rupees 10 power outage the order quantity is 26 units; that means, you do not need to calculate separately. The order quantity is already given and the lead time is 1 week. So, here; your main purpose is to how to get the value of the reorder point. So, you have these expressions; that means, it is normally distributed.

So, what you try to do? You have the expression for f B. Now, this is to be expressed in standard deviation units as per as the normal institution is concerned. So, this is f z. So, we get the f z value. So, f B value is H Q by G R. So, when say the sigma is known already with a value of sigma is specified. So, f z value is 0.05. We refer to the norm normal distribution and this is the ordinate; that means, and ordinate of 0.05 the small f z are the gives a value of Z 20.03.

So, hence the safety stock is Z into sigma and this value is 8 units we have computed and the reorder point B is M bar plus S. So, M bar is 20 and S is 8. So, the reorder point is 28 units, is it ok.

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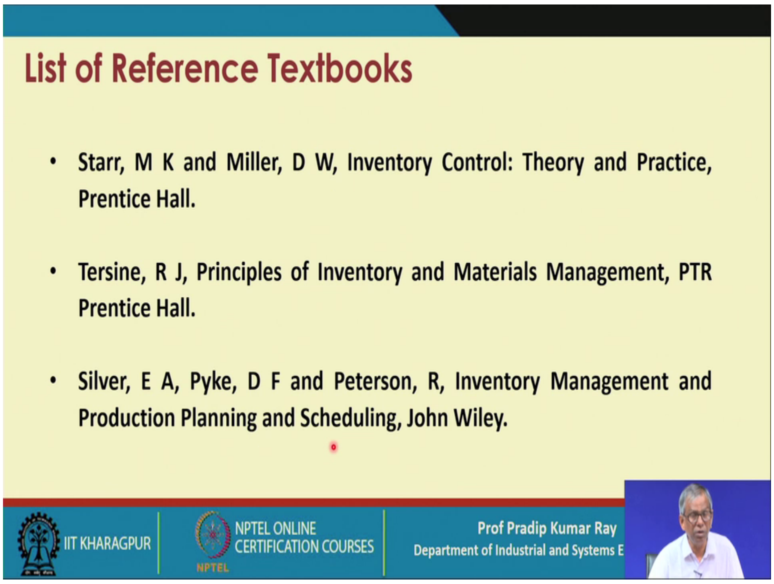

So, this is the solution and so, we have just introduced the concept of the cost based approach and under cost based approach, we have considered the 2 types of cost; one is the back ordering cost the per unit and the second one is the back ordering cost the per outage or we have assumed the there is the fixed out of stock cost or the back ordering cost. So, the other types of costs we will we will definitely consider in the next lecture sessions.

Thank you.