Management of Inventory Systems
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Lecture - 26
Dynamic Inventory Problems under Risk

So, during the 6th week of our the lecture sessions. Now an important topic related to Management of Inventory Systems, we intend discuss in detail. And this topic is Dynamic Inventory Problems under Risk. So, there are various types of you know the inventory problems, we have already classified them and this is just one class of problem. So, during this week with 5 lecture sessions we will be discussing important aspects of this problem.

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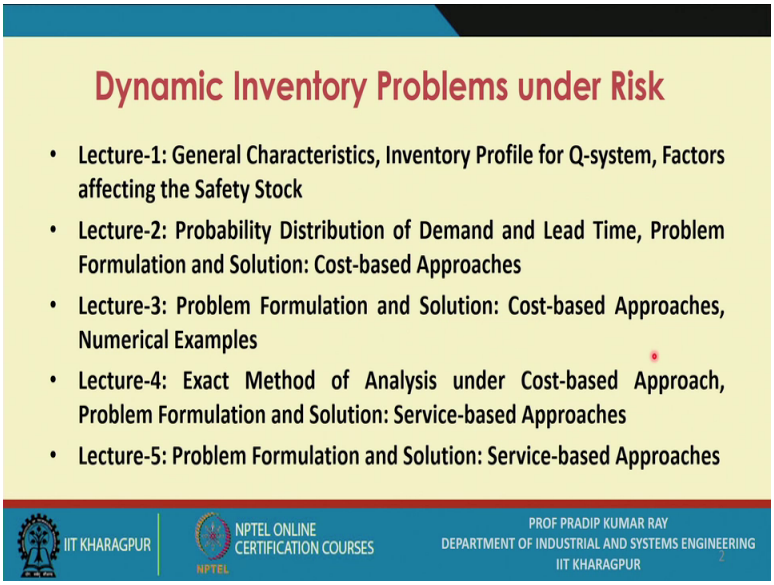

Now, before I start discussing all the important topics. So, let me just tell you what is the coverage; that means, the lecture 1, lecture wise the distribution of the topics. In lecture 1 we will specify the general characteristics of this kind of problem, dynamic inventory problems under risk. Then we will refer to inventory profile for the Q-systems and the factors affecting the safety stock.

As you may be aware that the safety stock is one of the important parameters to be determined for any kind of a inventory control systems, particularly under risk. During lecture 2 we will be referring to the probability distribution of demand and lead time,

there are various kinds of the probability distributions you need to use for the demand as well as the lead time so, this aspect we will discuss. Then we will discuss the problem formulation and solution approaches. And under this there are many kinds of approaches the first one we will refer to that is known as the cost based approaches.

So, in detail we will discuss all these approaches. In lecture 3, again we will continue our discussion on cost based approaches for problem formulation and solution, and with a number of numerical examples so, that will be the content of lecture 3. And during lecture 4 we will refer to the exact method of analysis and then cost based approaches. And the problem formulation and solution for the service based approaches.

And during the last session lecture 5, we will be continuing our discussion on problem formulation and solution service based approaches. And wherever required we will be referring to numerical problems, ok. So, let us first discuss the lecture 1 and the topics.

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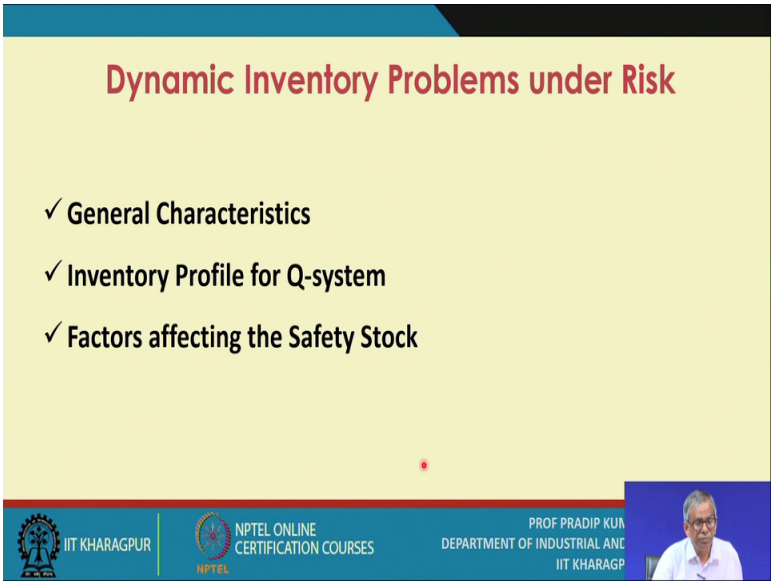

And under in lecture 1, we will be referring to 3 aspects one is we must know the characteristics of the problems from several perspectives. So, we will be highlighting all these characteristics one by one.

So, that we are saying the general characteristics of the problem. Then we will refer to the inventory profile for the Q-system. This is one of the one of the important or

sometimes this is referred to as the pure inventory control systems, Q-systems of inventory control.

So, we will be referring to the inventory profile. And if you look at the inventory profile, you will again come to know that, what are the characteristics of the Q-systems of inventory control and then as we have already the mentioned that the safety stock determination is one of the important exercise in any inventory control problem. So, that is why we must know at this stage that what are the factors that may affect the safety stock determination; so, this will be our coverage.

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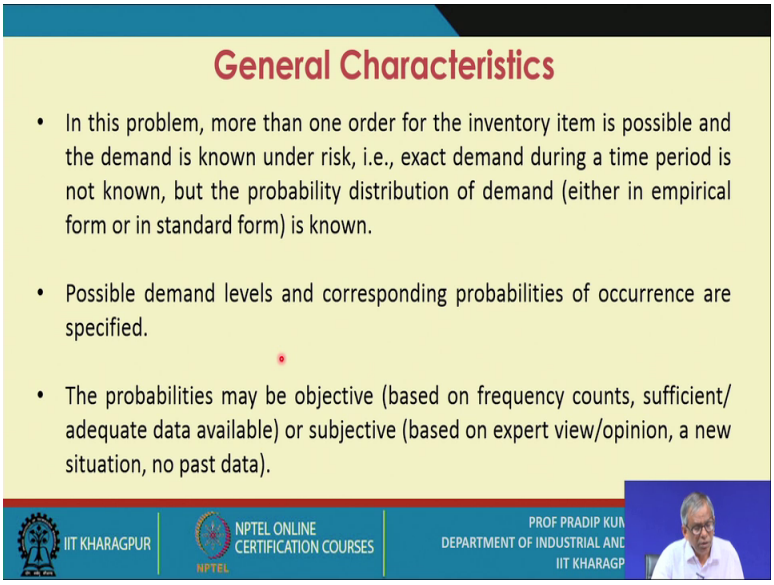

And then, so let us talk about the general characteristics. Now in this problem; that means the dynamic inventory problem under risk more than one order for the inventory item under consideration is possible. So, that is why it is referred to as the multi order systems or say dynamic inventory problem. And the demand is known under risk, as we have already pointed out that the problems can be classified under 3 categories.

The problem under certainty; that means, in the given context when the demand level or the demand load rate is known with certainty so, that is problem under certainty. Next you move to problem under risk that means the exact demand level or the exact demand rate is not known for the given inventory item. What is known is the probably key distribution of the demand for the given inventory item.

So, this problem is referred to as the problem under risk. So, the exact demand during a time period is not known, but the probability distribution of demand; that means, for the given you know the random variable, say the demand random variable the probability density function or the probability the mass the function is known.

And this when you referred to this probability distribution as you may be aware that either this can be expressed in empirical form or in standard form. So, while we discuss the problem so, both the forms we will be referring to and while you discuss the numerical problems so, sometimes the problem for the distribution, we refer to is of the standard form or many a time in the given problem the distribution the probability distribution of demand may be given in empirical form.

Now, the possible demand levels, and the corresponding probabilities of occurrence are specified. So, this is the main point and so, the exact demand is known not known, but against a particular demand level. The corresponding the probability of occurrence is known. The probabilities maybe objective now this is an important point if you should be aware of that is this probabilities may be objective; that means, when you have the frequency count for the given event. That means the based on the frequency counts and sufficient and adequate data are made available for the given event.

So, in this case, the occurrence of demand for the given say inventory item. Or in certain cases the probabilities against a particular or say the particular demand levels, they may be referred to as the subjective. Now the subjective means that based on expert view or the opinion, if you face a new kind of situation and the no pass data are made available. So, initially you may start with a subjective probabilities and as the system becomes the stable, and you are in a position to get the data. So, obviously you know as soon as you get the frequency counts so, you may change over from subjective probability to objective probability.

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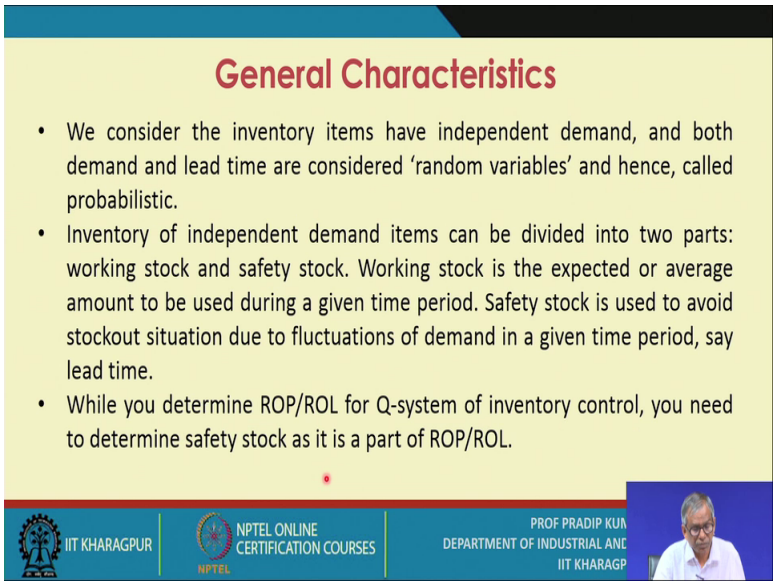

Now we consider the inventory items have independent demand. Now as we have say mentioned while we classify the inventory problems.

So, one of the important factor determining the class of inventory problem is the demand. And the demand maybe the constant may be assumed to be constant or demand may be assumed to be a variable. Or the demand may be considered as independent demand, or demand may be considered as dependent demand. So, right now what we are assuming that this is an independent demand case; that means, due to some external the factors mostly the market conditions the demand of say that particular so, the inventory item is influenced.

And obviously, you know the item which you considered we assume that for that item the demand is independently is occurring. And both demand and lead time and considered random variables, this point already we have mentioned and hence called probabilistic. So, please note this point because sometimes this problem is also referred to as independent or the demand case and the probabilistic model. So, the inventory of independent demand items can be divided into 2 parts. Working stock and the safety stock.

As you remember, that the inventory is have been classified from different perspectives. And so, one of the important say the way you define the inventory is its you know in which way you use or in which way or in which method you employee to keep the stock.

So, this is the working stock or it could be the safety stock. Now while you determine the parameter values of an inventory control systems, you will find that this inventory control system is defined with respect to the specific parameters. And these parameters have a is closely linked with the working stock like say when you determine the order quantity or the order size.

So, essentially you will be referring to the working stock and say when we determine the reorder point or reorder level. Now one so, the important part of the reorder point is the safety stock. So, you need to determine the safety stock also. So, that is why many a time the safety stock is considered to be one of the important parameters of any inventory control system particularly, when the problem you faced that is the problem under risk.

Now, working stock is the expected or the average amount to be used during a given time period say month. So, you have this stock and whether this and in a given time period say month, you are constantly using the inventory from the stock. And the so, at any point in time so, also during this one month, if you want to estimate if you want to calculate the average inventory, obviously we are referring to the working stock.

So, what is working stock? Working stock is the expected the average amount to be used during a given time period. And the safety stock is used to avoid stock out situations. Due to fluctuations of demand in a given time period say lead time; that means, as soon as you assume that the demand is probabilistic; that means, we are saying that the that the level of demand is fluctuating and if the, and in order to counter the impact of fluctuations of demand.

And these impact is nothing but essentially if it is you do not considered the fluctuations of demand while you determine the parameter values of an inventory control system. So, in all likelihood we will be facing the stock out situation, and how to avoid the stock out situation; obviously, you have to give some extra stock and these extra stock is referred to as the safety stock or sometimes this is referred to as the buffer stock. While you determine the reorder point or reorder level both are same for Q-system of inventory control.

So, one of the important parameters of Q-system of inventory control, you are already aware of. You need to determine safety stock as it is a part of reorder point or reorder

level. So, we will take out this case, we will you know the select has several kinds of numerical problems and this point will be made more clear.

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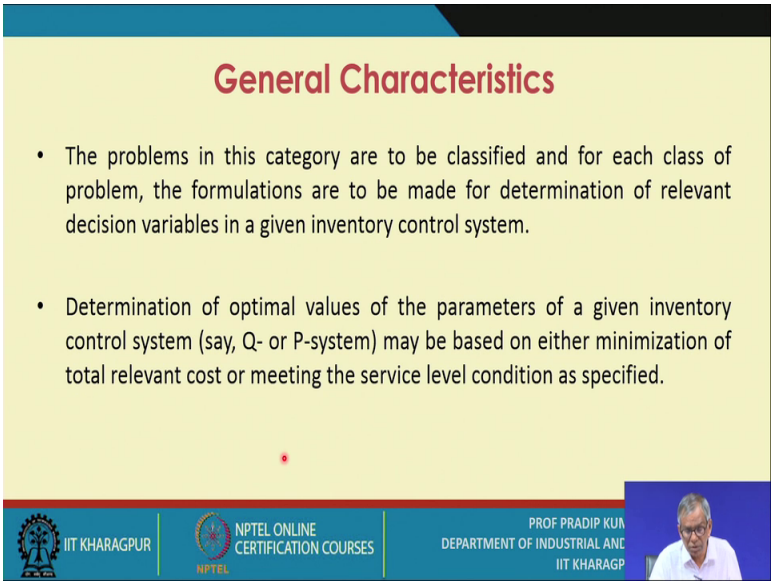

The problems in this category had to be classified and for each class of problem the formulations are to be made.

For determination of relevant decision variables in a given inventory control systems. Now we are referring to dynamic inventory problem under risk. Now again this particular problem can be classified under different categories. So what do will do? In during the lecture sessions, we will be referring to all sorts of possible the classes of problems or the categories of problems, categories of problems. And for each problem we will determine or we will specify the corresponding parameters.

And how to determine the optimal values of these the parameters assuming certain conditions to be met. So, all these details will be we will be discussing and you will and the whole thing is basically referred to as the problem formulation and solution. So, solution approach is also may vary from say one kind of formulation to another.

So, we will suggest the best possible solution approaches. Determination of the optimal values of the parameters of a given inventory control system. Say Q or P-systems may be based on either minimization of total relevant cost or meeting the service level condition as specified as. So, usually you know there are 2 the main approaches you employ. The

first one is referred to as the cost based approach. Means that with the working of say a particular inventory control systems known now, can you form the total relevant cost expression.

And you try to determine the say the parameter values in such a way that the total relevant cost remains at the minimum level. So, this is this could be one approach. In certain cases, even if you have the cost estimates, where these cost estimates may not be reliable. So, what you try to do? You try to get the optimal values of say the inventory control system parameters; in such a way that the service level condition is met.

So, as an alternative to say the cost based approach many a time, we refer to the service level say service level based approach, and in course of time we will define the service level, and we will also referred to the measurement systems for the service level.

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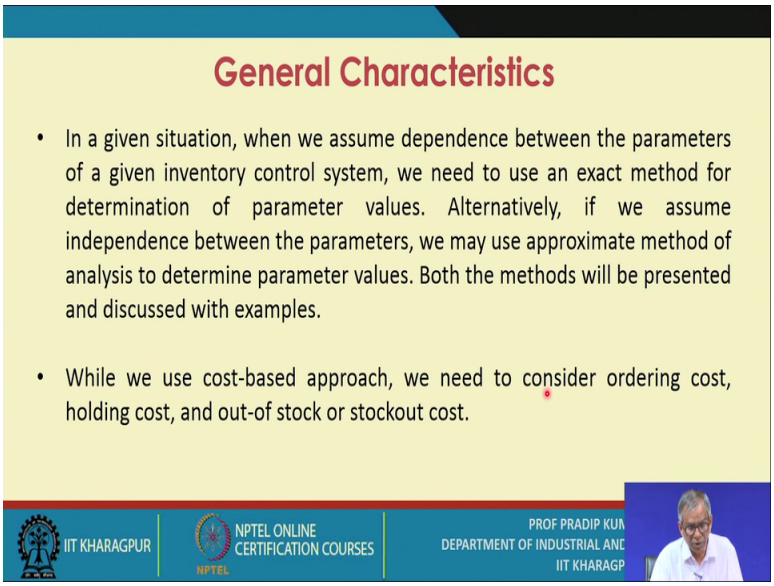

In a given situation, when you assume dependence between the parameters of a given inventory control system. So, this is an important point, you please note it down.

So, there could be say 2 important parameters. Say, the order quantity and say the reorder level for the Q-systems. Or for the P-systems it could be say the reorder or say the order interval and the maximum inventory.

Now, in the actual system, you can prove with the data and with logical the reasoning, that there is a dependence between these 2 parameters. So, we need to use an exact method for determination of parameter values.

So, when you assume the dependence between the parameters of a given inventory control systems. So, obviously, you can you need to use the exact method of analysis. So, the details of the exact method we will discuss in course of time. Alternatively, if we assume independence between the parameters that means what we assume that the order quantity say for Q-system can be determined independently of say the reorder point for the same system.

So, you have say one say the expression for Q, and another expression for reorder level. So, the 2 say the separate the mathematical expressions you develop. As assume these 2 mathematical expressions are not related to say each other. So, that is the independence assumption holds. So, if we have these assumptions, then we say that we are employing approximate method of analysis. Both the methods will be presented and discussed with examples.

So, next you will come to know that under what condition you may oft for approximate method. And under what condition you have no other alternatives, but to go for exact method. Now there are advantages as well as the disadvantages in both the methods. So, in course of time, we will come to know.

While you use the cost based approach, we need to consider ordering cost, holding cost and out of stock or the stock out cost. We have already in you know that the previous the lecture sessions which we have already covered.

So, we have referred to different kinds of inventory related costs. So in this particular case, while we form the relevant cost equation against a particular problem so, this relevant the total cost expression consists of usually the ordering cost, holding cost or inventory carrying cost and out of stock or the stock out cost. And your whole purpose is while you propose a particular inventory control system with its parameters. Now our main purpose is how to avoid stock out, ok.

So, in advance you must have an estimate of the stock out cost or out of stock cost.

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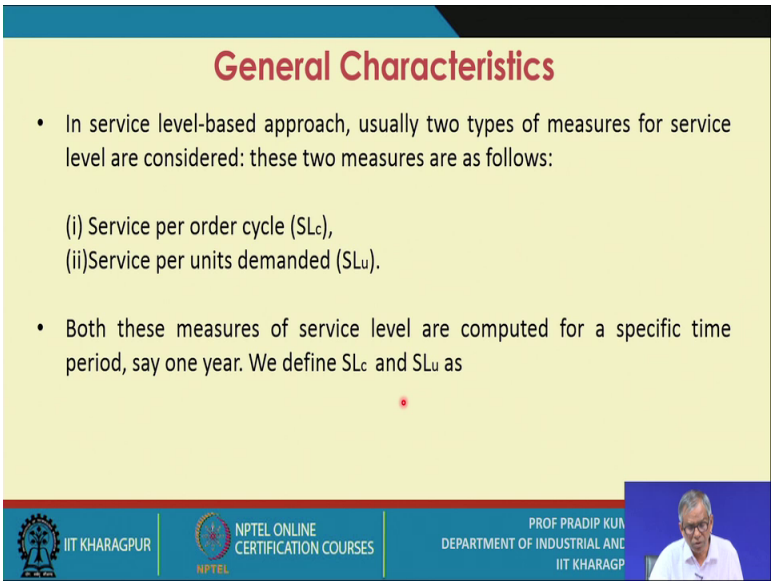

Now, let us also the defined at this stage the service level. As I have already mentioned that the cost based approach if you can; obviously, you can use, and alternatively when the cost based approach is not reliable. So, you go for say the service level approach.

So, what is this service level? So, usually when we use the service level based approach for determination of the parameters, parameter values of a given inventory control system, usually 2 types of measures for service level we refer to.

So, what are these 2 measures? The first one is referred to as the service per order cycle. I repeat it is referred to as the service per order cycles notation is SL subscript c. So, c stands for the cycle, and SL stands for service level. Alternatively, you go for another measure that is called service per units demanded and the notation is SL subscript u.

So, the u stands for the units demanded. So, both the; you know the service level measures we will define.

So, and usually what happens at the first level you go for the service for order cycle, and on the next level you use the SL u, is it ok? So, both these measures of service levels are computed for a specific time period. So, this time period has to be mentioned, otherwise the service level does not have any meaning. So, please make a note of these observations.

So, that is so the time; that means, of service level is measured for a specific time period. So, we defined SL c and SL u, service per order cycle and service per units demanded as; SL c total number of order cycles in a given time period, minus the number of order cycles with the stock out.

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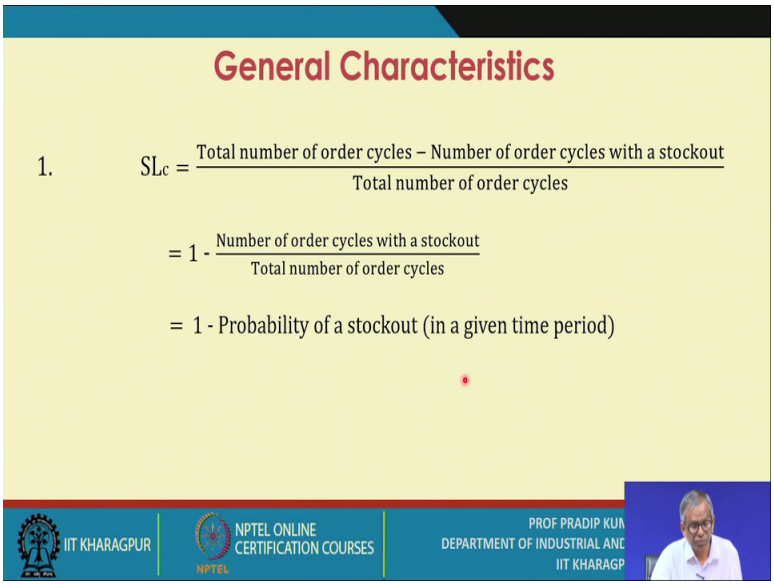

You know, while we refer to the inventory profile of EOQ model, the classical EOQ model.

So, we have already referred to the sawtooth curve. And if you refer to that particular inventory profile for that matter say any kind of so the inventory provide inventory profile against a particular inventory problem. If you refer to always you can identify a say the work cycle or say the order cycles, and how many order cycles you may have in a given time period say year that also you can determine.

So, the total number of order cycles in a given time period, minus the number of order cycles with a stock out, divided by the total number of order cycles in a given time period. So, it means 1 minus number of order cycles with a stock out divided by the total number of order cycles. Now what is this ratio? This ratio is nothing but the probability of stock out in a given time period. So, what is SL c service per say the order cycles; that is, 1 minus probability of stock out.

So, we will be using so the particular notation for the specifying probability of a stock out, ok.

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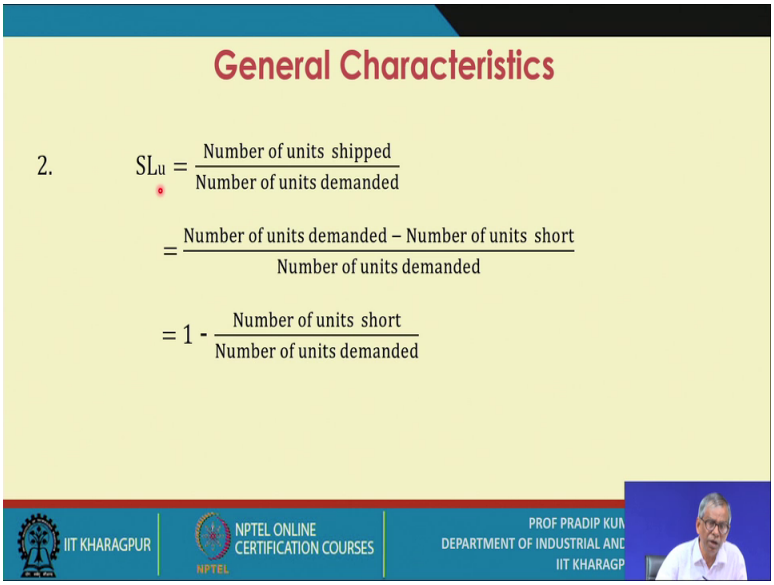

So now, our next the service level measure is SL u; that means, service per units demanded. This is referred to as the number of unit shift; the shift means the actually the supplied, right? And the number of units demanded it means the number of units demanded minus the number of unit short divided by the number of units demanded, ok. So, this is straight forward so, it means 1 minus number of unit short divided by the number of units demanded.

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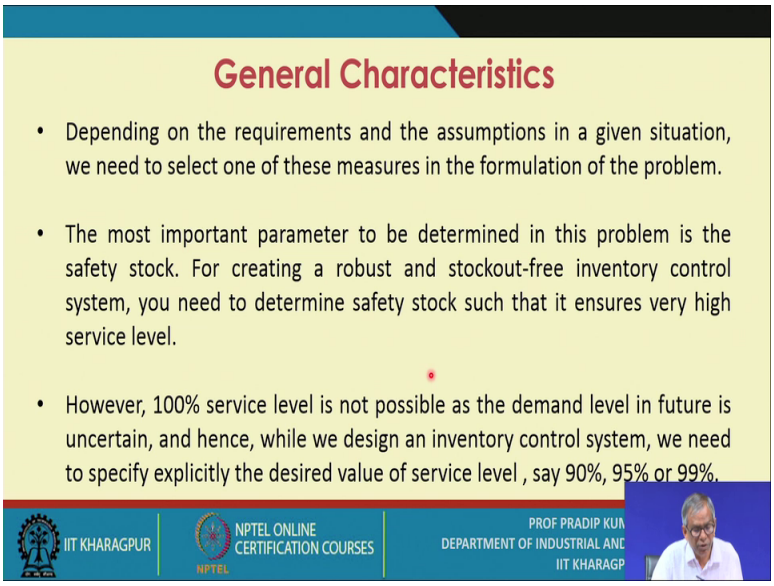

So, this is the expression for SL u and this is the general definition. Now the depending on the requirements and the assumptions in a given situation, we need to select one of these measures in the in the formulation of the problem. So, when we discuss the service level based approach so, we will be referring to these issue again. The most important parameter to be determined in this problem; that means, dynamic inventory problem under risk is the safety stock.

So, just make a note this one this important point for creating a robust and stock out free inventory control system. So, that is one of the major goals of any inventory control systems, within the overall materials management domain. You need to determine the safety stock such that it ensures very high service level. So, there is a relationship between the service level and the safety stock.

However, 100 percent service level is not possible. As that means, there cannot be any situation there, where you know this the shortage situation will never occur. Whatever may be the sophisticated of the systems so there is uncertainty, obviously in future. And that is why you it is implicitly known that 100 percent service level is not possible. So, and the main reason is the demand level in future is uncertain.

And hence while you design an inventory control systems, you need to specify explicitly the desired value of the service level say 90 percent service level is accepted or 95 percent or 99 percent.

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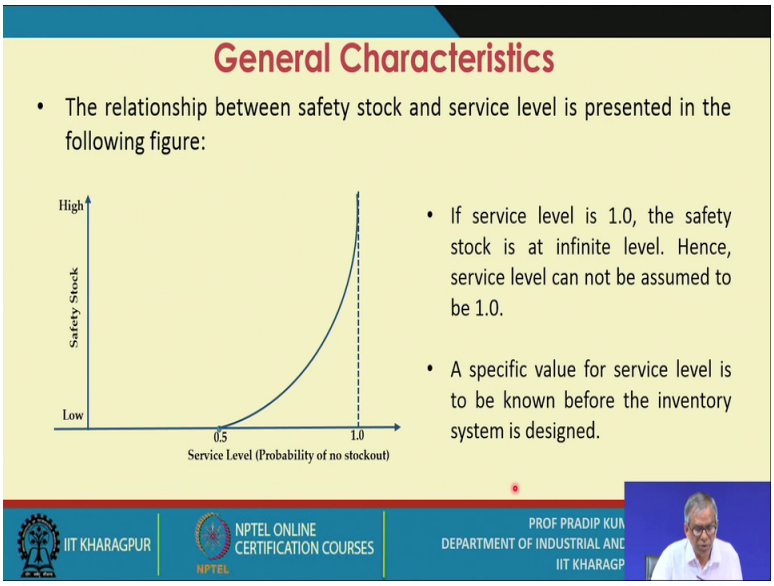

So, this is so, this graph or we will show or basically shows the relationship between the safety stock and the service level.

So, the relationship between the safety stock and the service level is presented in the following figure; that means, if the safety stock is low what you find that your the service level is a very poor. Whereas, if the service if the safety stock is more, the service level will be very, very high. Same with the service level is one the safety stock is at infinite level theoretically speaking.

So, that is why this value cannot be computed. Hence service level cannot be assumed to be 1 so, this the point to be noted. The second important point to be noted is a specific value for service level is to be known before the inventory system is design, so, this has to be pre specified.

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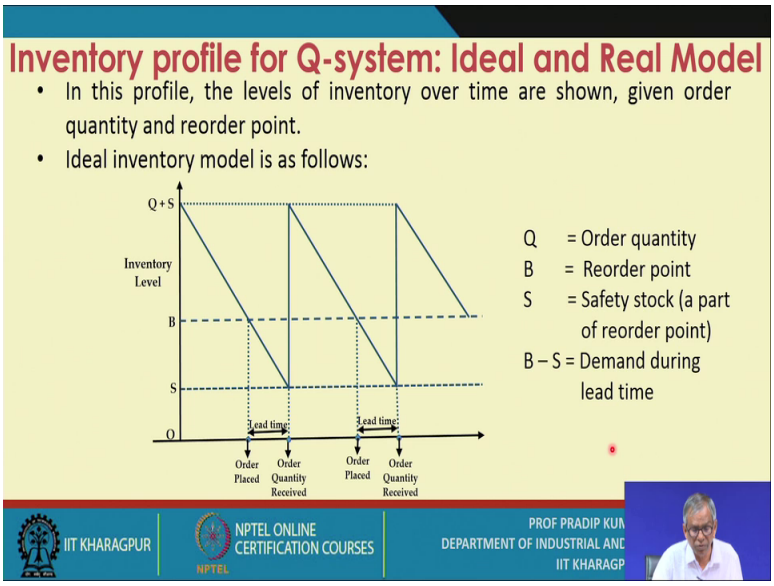

Now for the Q-system this is the typical so, the inventory profile for the Q-systems ideal and real model. So, here what you find? This is so, you please go through this particular inventory.

So, the inventory profile and the so, the ideal inventory model so, you have the inventory level, and you have the order quantity Q, S is the safety stock. So, the B is the reorder point, and B minus S is the demand during lead time. So, what do you find? That when the demand is known with certainty, so obviously, you know you keep a safety stock, and the lead time remains fixed. So, this is an ideal situation, and you have 2 parameter, one is the order quantity and the second one is the reorder point.

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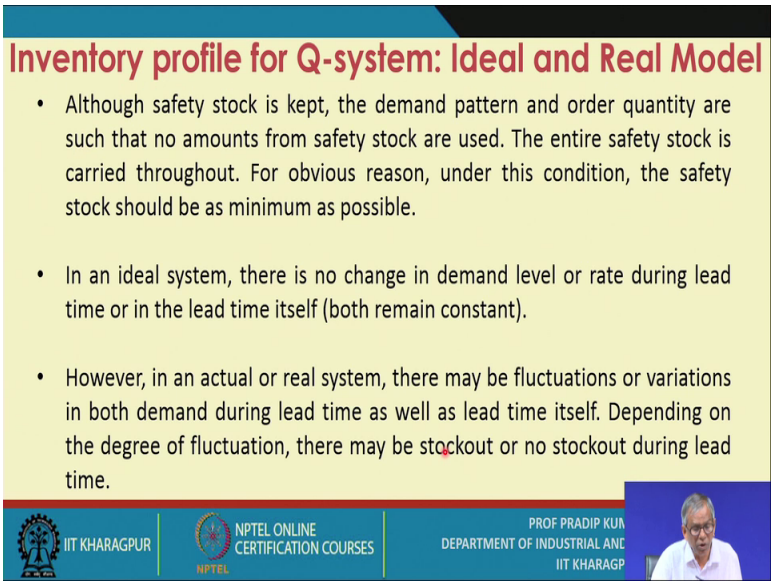

So, all though safety stock is kept, the demand pattern and the order quantity are such that the no amounts from the safety stock are used. So, this is the point, and the entire safety stock is carried though out. In an ideal system there is no change in demand level, or rate during lead time or in the lead time itself both remain constant.

However, in an actual or the real system, the system which you actually face there maybe fluctuations or variations in both demand during lead time as well as the lead time itself.

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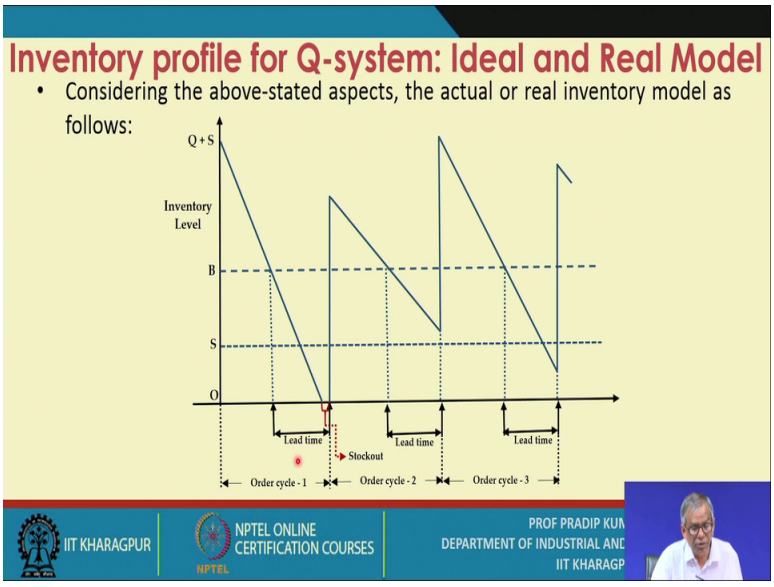

So, depending on the degree of fluctuations, there may be stock out or the no stock out during the lead time. So, this is the situation so, inventory profile for the Q-system ideal and the real model.

So, we are referring to an actual system, is it ok or the real inventory model. So, here what you find? During the order cycle 1 here we stockout situation, evenorder cycle 2, you do not need to use the safety stock. Depending on the demand rate, and during say the order cycle 3 so, you have been using partially a certain amount from the safety stock.

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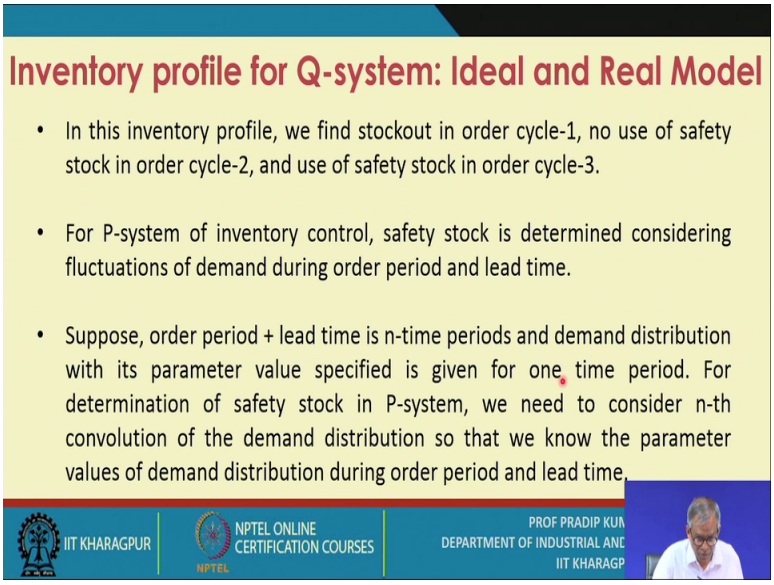

So, this is all about this. So, this all these detail points we have mentioned and for the P-systems of inventory control, the safety stock is determined considering fluctuations of demand during the order period and the lead time.

So, all these details are given and what we find that you must know that what are the time periods to be considered for determination of the safety stock for the Q-system and the P-system.

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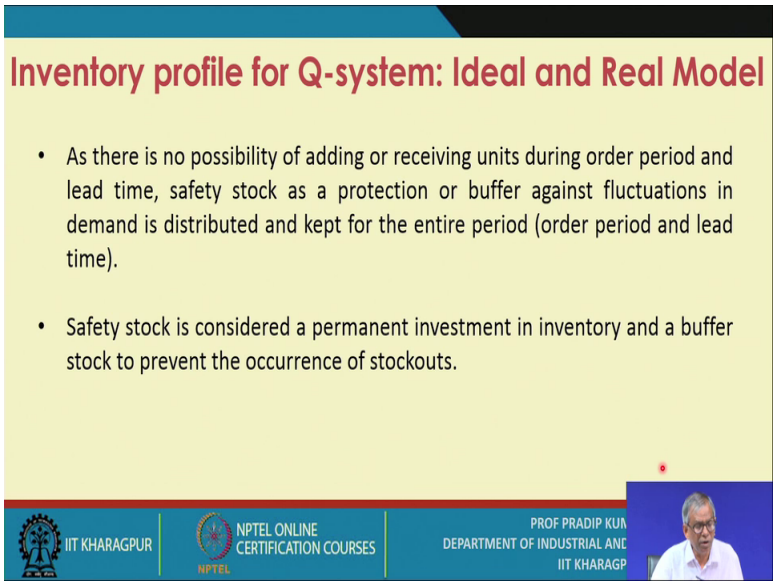

So, as there is no possibility of adding or receiving units during order period and lead time for the P-system safety stock has a protection or buffer against fluctuations in demand is distributed and kept for the entire period, is it? And the safety stock is considered a permanent investment in inventory and a buffer stock to prevent the occurrence of stock out.

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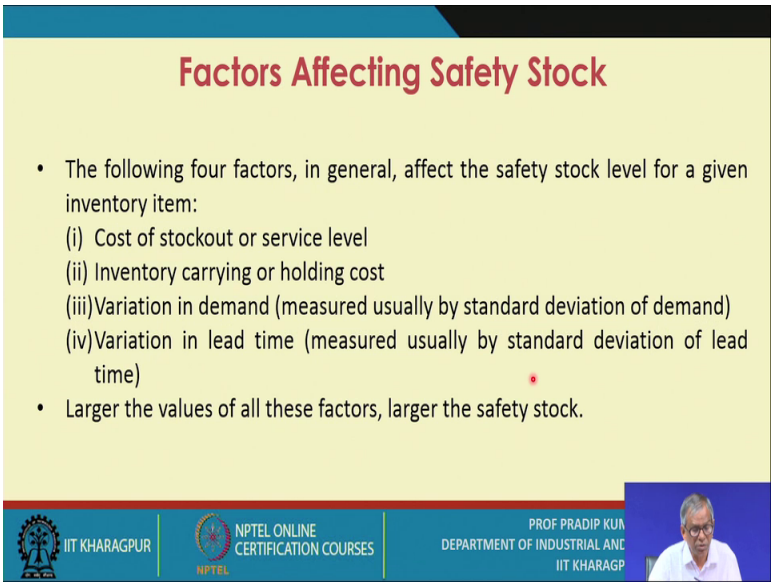

So now, as per as safety stock is concerned these are the factors which might affect the amount of safety stock. First one is the cost of stock out or the service level. Second one

is the inventory carrying or the holding cost. And the variation in demand if the measured usually by the standard deviation of demand. And then the last factory is the variation is lead time, measured usually by the standard deviation of lead time.

So, this is known to you and large the values of all these factors larger the safety stock.

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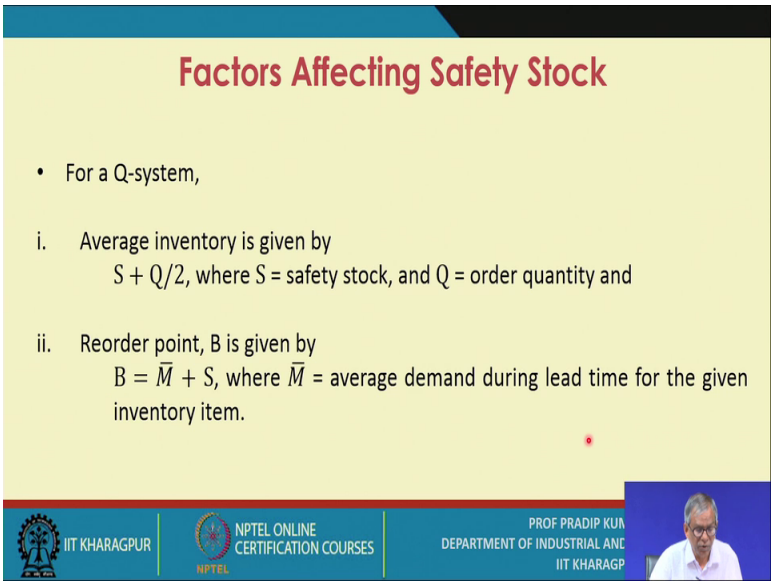

So, for the Q-system this is average inventory is given by safety stock plus Q by 2, and the reorder point is M bar plus S all these details we will discuss later on.

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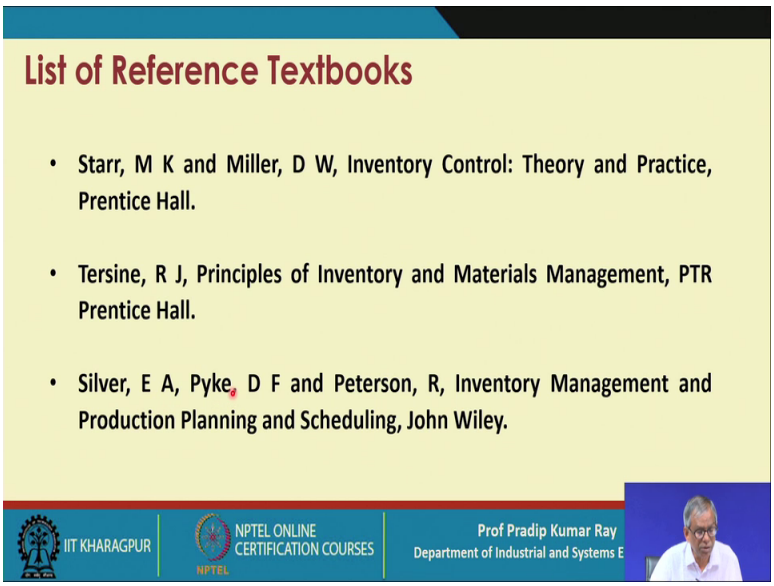

So, we have actually covered some important aspects for which these important aspects. You must be aware before you go for formulate problem under this category that is the dynamic inventory problem under risk.

Thank you.