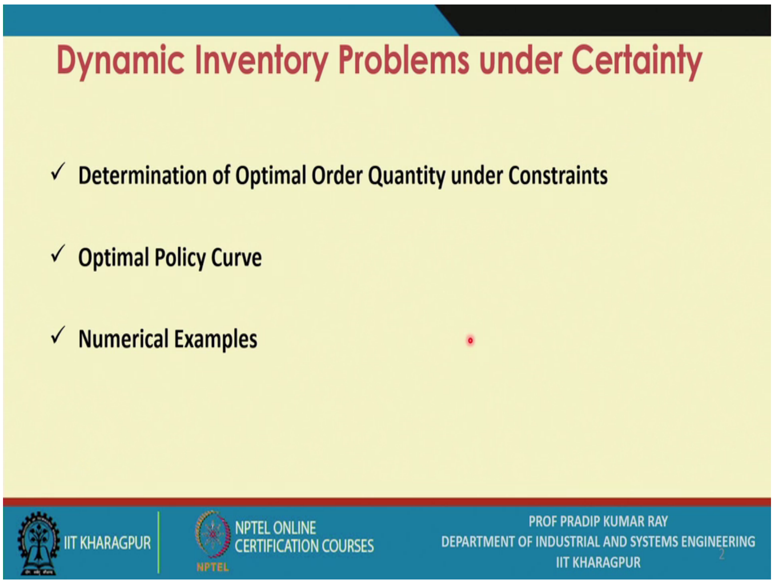
Management of Inventory Systems
Prof. Pradip Kumar Ray
Department of Industrial and Systems Engineering
Indian Institute of Technology, Kharagpur

Lecture – 25
Dynamic Inventory Problems under Certainty
(Contd.)

During the, this week we discussed a number of issues related to Dynamic Inventory Problems under Certainty.

(Refer Slide Time: 00:31)



Now, during this session that is the fifth the lectures sessions are specifically I will be dealing with the determination of optimal order quantity under constraints. The till now while we determine the order quantity or optimal order quantity for a given inventory control system, we have assumed that there is no constraints. But in a real world situation the many a time we have many types of constraints and when these constraints are acting this is quite natural that that you determine the optimal order quantity in the presence of these constraints. So, what sort of the approach we should follow? So, this is the first the topic we are going to discuss and then we will bring in the concept of optimal policy curve.

And if you can use if you can develop the optimal policy curve for a given inventory for an inventory item, it will help you in accessing the represent the level of performance and for improving the performance what approach you should follow. So, anybody who is considered to be an expert in inventory management; he or she must be aware of the concept of optimal policy curve. And then obviously, we will be taking up a number of numerical examples.

(Refer Slide Time: 02:18)

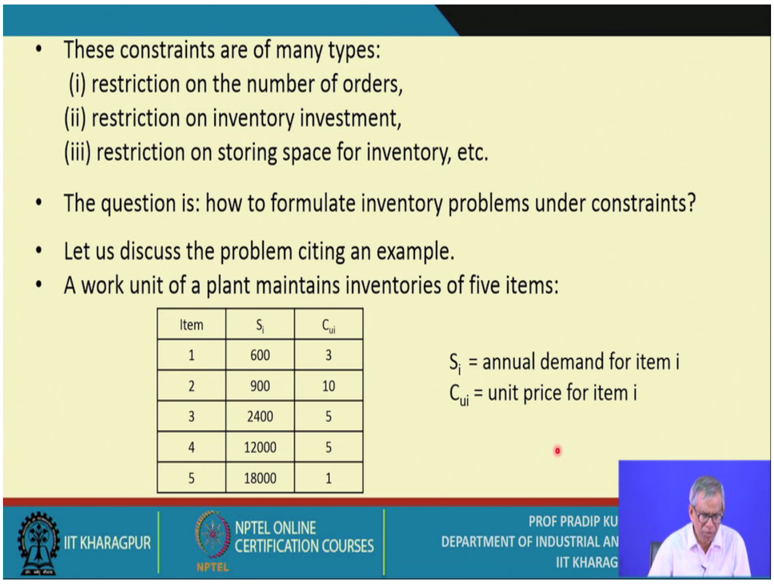
Now, let us the talk about the optimal ordering policy for more than one item ok. So, we will definitely explain the approach which you need to opt for determining optimal order quantity under constraints, but what we will try to do? That means, while explain the procedure, so we will bring in certain examples. So, that your understanding is proper and you know: what are the critical issues to be considered, while you deal with such case.

So, in the problem formulations as mentioned in the previous lectures sessions; that means, in the last the previous four lecture sessions, what we have assumed? We assumed to a two specific aspects or two specific assumptions. The first we have assumed there is a single item case; that means, whatever the formulae we have used they are applicable for one item. And what you are assuming that the same are the set of formulae you also can use for other items, but the item wise that being single item wise you have to use those say that formulae.

So, that is one assumption and the second assumptions already you have mentioned that is no constraints; that means, you refer to the assumptions related to the classical EOQ formula you just you will you might have noticed that there are some 10 assumptions against the classical EOQ formula. And if you look into all these assumptions you will find that essentially there is no constraints we have assumed. Now, we will have to relax these assumptions one by one and the first you now we will assume that that constraints are present of different types and if the constants are present what will be the modified say the EOQ expression?

Now, in the majority of the cases we come across a situation where more than one item, the multiple items may be considered jointly while determining the optimal order quantity item wise. So, this is the everywhere when we study an existing inventory control systems for an organisation. So, you come across this situation and there may be one or more of the constraints of varieties; as I have already pointed out that these constraints may be of the different types. So, the varieties of constraints you come across under which the ordering policy is to be developed ok.

(Refer Slide Time: 05:18)



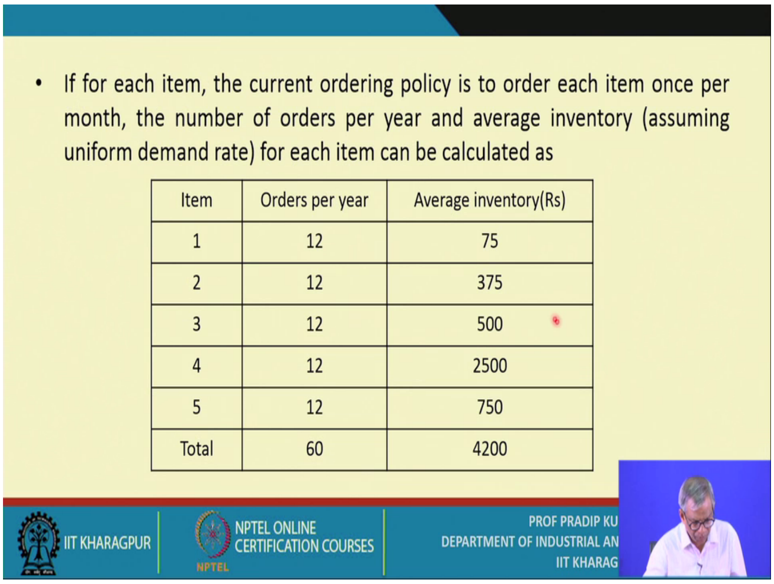
Item	S_i	C_{ui}
1	600	3
2	900	10
3	2400	5
4	12000	5
5	18000	1

Now, these constraints are of many types. So, there are three specific types of constraints we need to consider many a time, in the majority of the cases. The first one is the restriction; there could be restriction on the number of orders. There could be restriction on the inventory investment like at any point in time, so, you need to compute the average inventory in monitory terms. So, sometimes this is referred to as the inventory investment.

So, one of the objectives of the inventory management is to control the inventory investment maintaining say an acceptable condition or acceptable performance level. So, I need to consider restriction on inventory investment and the there could be restriction on the storing space for inventory. So, this is just I have given three examples, but there could be many such examples on the restrictions. The question is how to formulate inventory problems under constraints?

Now let us discuss the problems writing an example. I have already explained that let this problem be explained are the referring to one typical say the problem. So, what is this example? A work unit of a plant maintains inventories of 5 items. What are these 5 items? They are specified as 1 2 3 4 and 5. What is the size? That means, for the ith item, what is the annual demand or say the per say per or per each item and what is C ui that is what is the unit price of the purchase price for ith item? So, against each item we have the values of S i and C ui as given in the table.

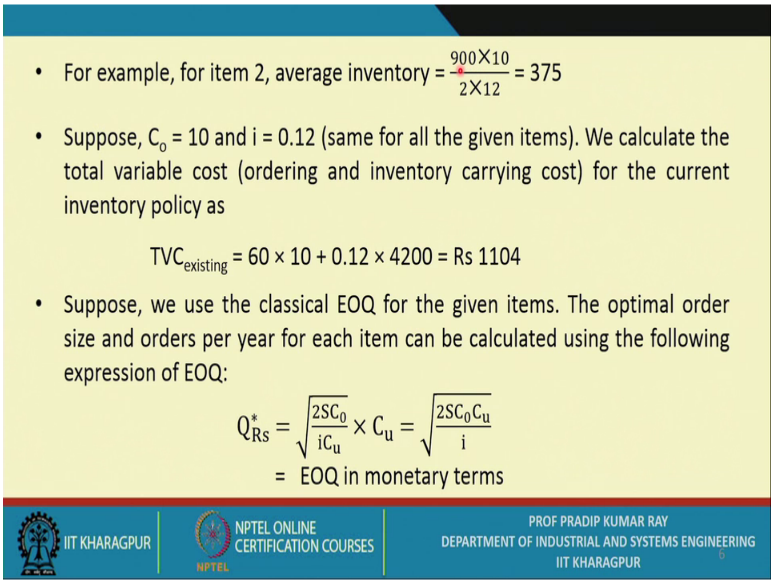
(Refer Slide Time: 07:46)



So, precise the annual demand for item i and C ui is the unit price for item i ok, it is very simple. Now for each item the current ordering policy. So, the first anywhere you go with respect to a particular item, if you find there is a stock of item; obviously, there is an inventory policy inventory control system. Whether the inventory control system is good or bad? That is a different issue, but there is an inventory systems. So, this is referred to as the existing ordering policy.

So, suppose that under existing or the current ordering policy, you need to place an order for each item once for month ok. So, that is the existing policy. So, the how many orders you place for each item per year obviously, there 12 orders per year. And so, the number of orders per year that is 12, for each item under current ordering policy and average inventory you can also calculate, assuming that the demand is uniform and so, you can calculate the average inventory. So, these values are 75, 375, 500 in monetary terms rupees 2500, 750 and the total average inventory investment is 4200. So, this is just an example.

(Refer Slide Time: 09:24)



Now, how do you calculate this average inventory? That means, just for example, for item 2, average inventory is computed as 900 into 10 that is the price by 2 into 12; that means, the month wise there is an order. So, you have the total demand as 900 divided by 12; that means, it is the each mans demand in physical units. And the price is actually 10 and half of the order quantity is the average demand that is that is why it is divided by 2. So, the total value is 375, so, same will you apply for other items also.

Suppose C 0 is 10; that means, ordering this is just an illustrative example. So, if the order ordering cost per order is just 10 and i that is the inventory carrying cost is 0.12, same for all the given items. Now, this i and the other i is slightly different that i stands for the ith you know the item; whereas, this i actually the stands for the inventory carrying cost as a proportion of the average inventory which you hold. We calculate the total variable cost. So, it consists of ordering cost and the inventory carrying cost. So, under current ordering policy, so, what is the total you know the variable cost? That is 60; how many orders you need to meet the demand of all the 5 items? That is 60 order and for each order separately replacing you are in carrying a cost of placing an order. So, that is the 10 per order; that means, 60 into 10 and this is the 12 percent is of the 12 percent is your inventory carrying cost.

So, the 0.12 into 4200 ok, this is the average inventory. So, this is rupees 1104. Suppose you use the classical EOQ, classical economic order quantity; Wilsons slot size formula for the given items. The optimal order size on orders per years for each item can be calculated using the following expressions of EOQ. So, that you can do; that means, this is in monetary terms order quantity; that means, what you do? This is your original EOQ expressions in monetary terms. If you multiply it with the unit price, you get the expressions of EOQ but in monetary terms. So, you use this formula to calculate the EOQ for each item in monetary terms.

(Refer Slide Time: 12:33)

Item	Optimal order size (in Rs)	Orders per year
1	548	3.28
2	1225	7.35
3	1414	8.49
4	3162	18.98
5	1732	10.39
Total	8081	48.49

- Here, the orders per year for an item is calculated as

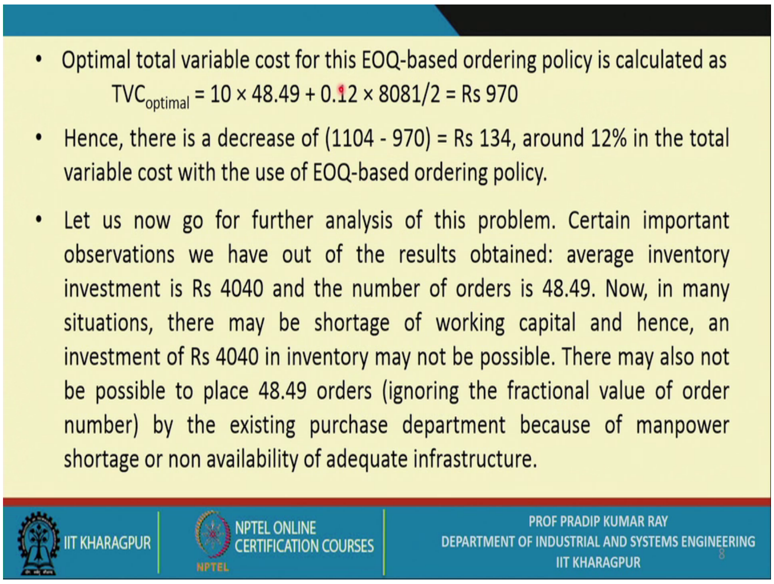
$$\frac{S_i C_{u_i}}{Q^*_{i,Rs}}$$

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | PROF PRADIP KUMAR RAY DEPARTMENT OF INDUSTRIAL AND SYSTEMS ENGINEERING IIT KHARAGPUR

So, when you do this so, what you what you get? That means, item wise optimal order size that you get and orders per year when the total say that the demand is known yearly demand annual demand is known. So, this way you compute for each item and what you get that the optimal order size is 8081 and orders per year is 48.49, when the classical EOQ of formula is used for determination of the order quantity for each item.

So, here the orders per year for i an item is calculated as S i C ui; that means, annual demand in monetary units and divided by the order quantity; that means, that EOQ expression in monetary terms. So, this formula we use.

(Refer Slide Time: 13:33)

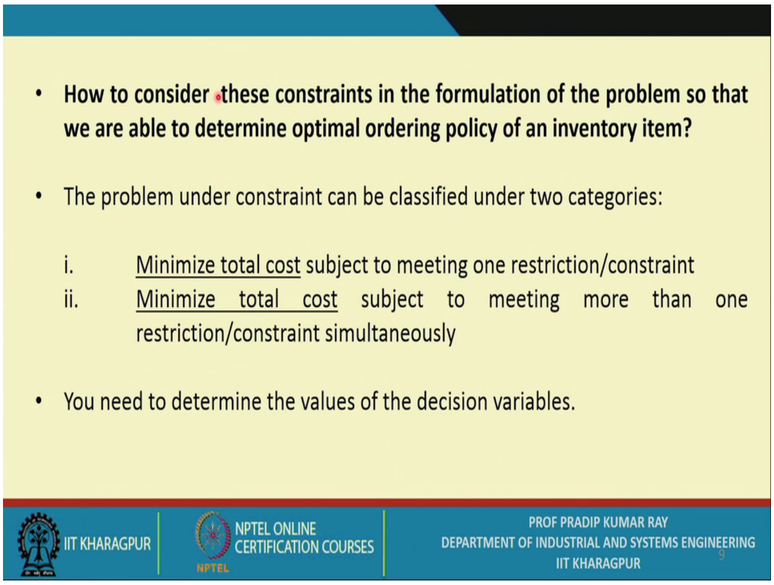


And then again you calculate the optimal total variable cost for this EOQ based ordering policy. So, what is this the total variable cost to the optimal? That is the 10 per order into the number of orders that is the 48.49 per year and this is the inventory carrying cost. Is it ok? So, this is the total inventory and divided by 2 is the average inventory. So, this is total value is 970; hence there is a decrease of say 134, the previously it was 1104. So, now, it has become 970. So, there is a decrease of 134 around 12 percent in the total variable cost with the use of EOQ best ordering quality situation.

So, this is an improve situation there is this improvement, if you if you use the EOQ based ordering policy. Now let us now go for the further analysis of this problem. Certain important observations we have out of the results obtained. So, these important observations I am explaining one by one. First one is the average inventory investment is rupees 4040 and the number of orders is 48.49.

Now, in many situations there may be shortage of working capital and hence an investment of rupees 4040 in inventory may not be possible. So, there may also we may not be possible to place 48.49 orders because of many reasons, so by the existing purchase order department. So, mainly because of say the manpower shortage or non availability of adequate infrastructure.

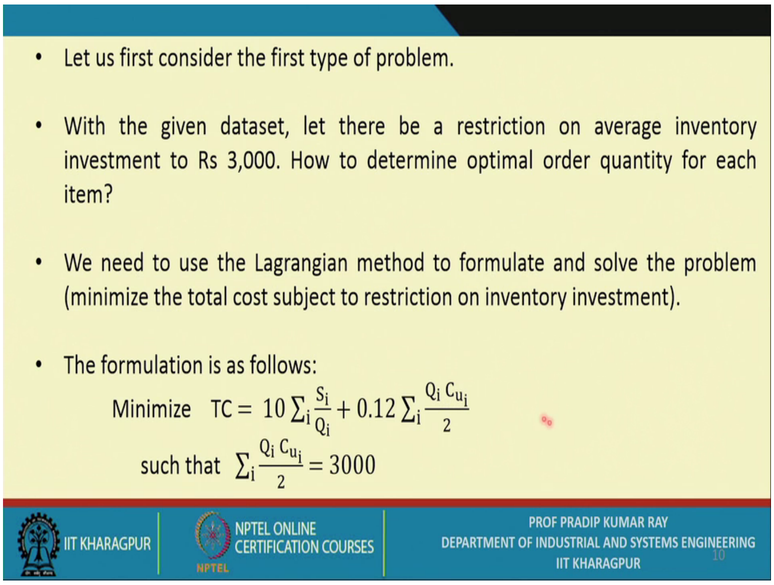
(Refer Slide Time: 15:36)



Now, if you have this sort of restrictions or the constants, the question remains how to consider these constants in the formulation of the problem so that we are able to determine optimal ordering policy of an inventory item? Now, the problem on the constraints can be classified under two categories.

So, you should be aware of that means, what are these two categories of problems? First category is minimize total cost subject to meeting one restriction or the constraints. So, this is the first category and the second category is you need to minimize the total cost or the total relevant cost subject to meeting more than one restriction or the constraints simultaneously ok. Now, you need to determine the values of the decision variables so, mainly the ordering quantity.

(Refer Slide Time: 16:34)

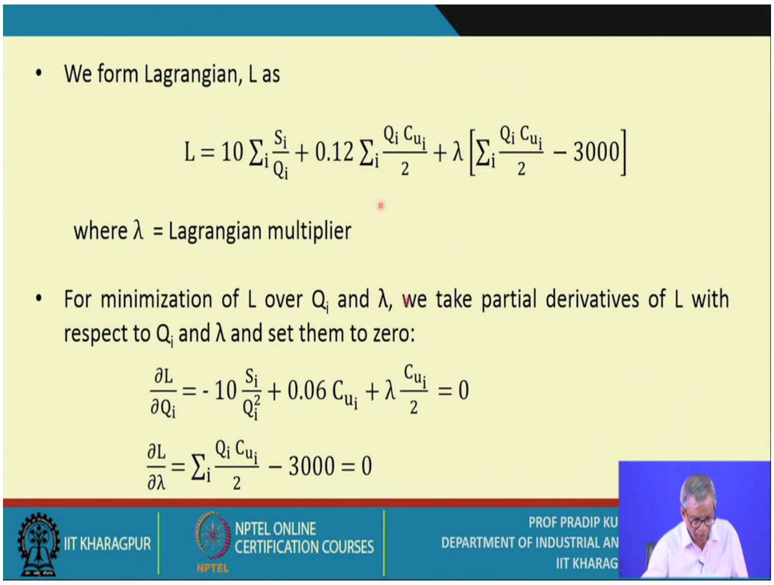


Now, let us first consider the first type of problems; that means, just one set of constraints you have. With the given data set already the data sets data set is given, let there be a restriction on an average inventory investment to rupees 3000. So, in existing one it is 4040, but just you restrict it to 3000. How to determine the optimal order quantity for each item? Now what you need to do? That we need to use the Lagrangian method to formulate such a problem; I have to solve the problem. So, what is that problem? That is minimize the total cost subject to restriction on inventory investment.

So, what is that restriction? Restriction is there inventory investment is limited to just 3000. So, how do you formulate the problem? Minimize the total cost, total cost expression is this one. This part is for the, i further for you know or the total number of items what is the total ordering cost and this is for the set of items that is total number of items what is the total you know so, the inventory carrying cost or inventory holding cost? Is it ok?

So, subject to or such that that this is essentially this part represents the average inventory investment. So, this average inventory investment; that means, the total for all the items all the 5 items plus we have taken up is restricted to 3000. So, this is the formulation of the problem.

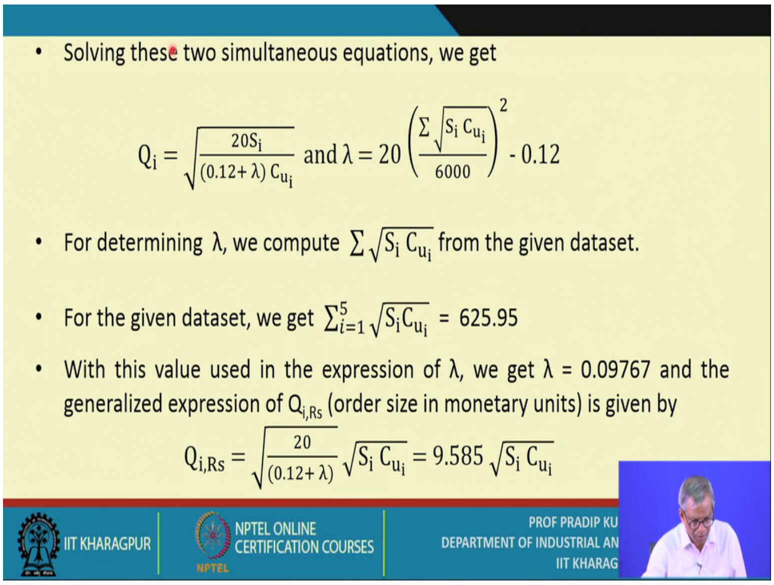
(Refer Slide Time: 18:31)



So, what you try to do? We form Lagrangian, there is a there is a well known the technique well known say the method we used. So, we form the Lagrangian denoted as capital L; that is this part that is your condition; that means, you try to minimize this total cost that is ordering cost and the inventory carrying cost plus so, the lambda time this is the constraint; that means, this is the average inventory investment restricted to 3000, so where lambda is the Lagrangian multiplier.

So, what you try to do for you need to minimise capital L over Q i and lambda? So, we take partial derivatives of L with respect to Q i; that means, for individual say the inventory item i and lambda and set them to 0. So, when you take the partial derivative of the Lagrangian with respect to Q i, you have this expression; set it equals to 0 and when you take the partial derivative of the Lagrangian with respect to the lambda that is the multiplayer. So, we have just one constraint that is why one multiplayer and this is so, the expression; that means, this is the restriction on the average inventory and you have these expression the partial derivative and you set it equals to 0.

(Refer Slide Time: 20:10)

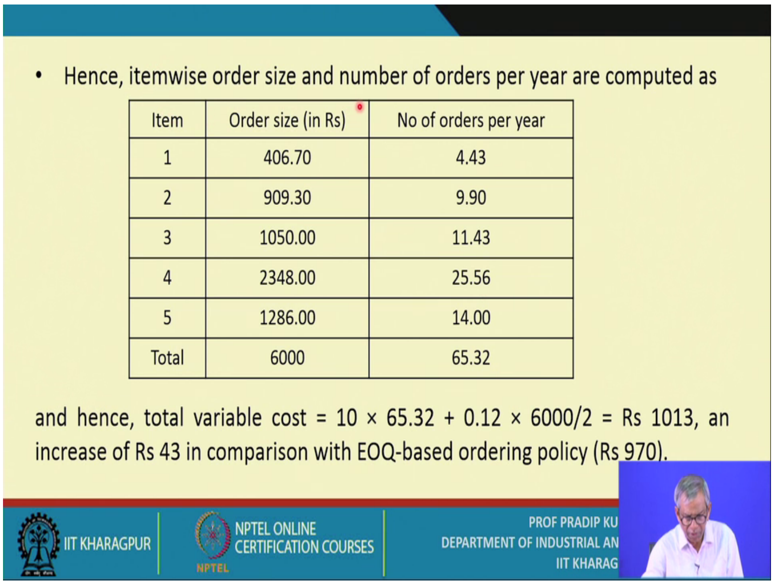


So, you have these 2 equations and ultimately what you get after solving these 2 simultaneous equations? We get Q i; that means, the order quantity for the ith, ith items Q i this is the expressions you have; S i is what is S i? S i is the for the ith items, what is the annual demand and ah; obviously, in this expression you must be able to find out the value of lambda. So, first you get the value of lambda and for lambda you have this expression; that means, that this is the expressions. Now on the numerator what you have? The sigma root S i C u I square this items you have. So, what you try to do? That means, for each item you get the value of root over S i C u I and you sum them sum them up and then these summation u square.

So, you get the expressions or the values of the numerator; minus 0.12, that is for a particular case you have this factor these values. So, for determining lambda we compute this one as I have already told you from the given data set ok. So, for the given data set, we get this value the sigma i equals to 1 to 5; there are 5 items we have consider. So, this value is 625.95 and when you use this value in this expression that is ah; that means, the lambda you calculate.

So, you get the value of lambda as 0.09767; that means, once this is known you substitute these value over here, you get the value of lambda as 0.09767 and the generalized expression of Q i; that means, this is the expression of Q i in physical units and so, the order size; that means, in monetary unit. So, the you have these expressions and this is S i C u I root over S i C u i so, you this general expressions for each item and expressions for the order quantity.

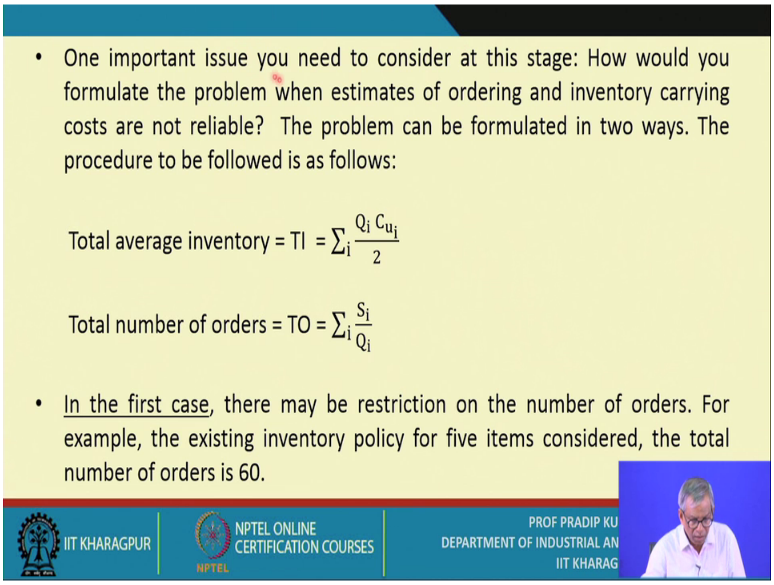
(Refer Slide Time: 22:38)



So, hence item wise order size and the number of orders per year are computed as you form you create this table. So, against each item you have the order size you determine you have the formulation and you can easily calculate the number of orders per year when the yearly demand is known. So, you add all the order sizes. So, you get a value of 6000. Is it ok? 6000 and the number of orders total number of orders is 65.32.

So, hence the total variable cost is 10 per order into 65 into 32 per order and plus you know this is your the order size; order size by 2 is the average the inventory. And so obviously, this 6000 by 2 and the inventory carrying cost is just 12 percent of that; that means, ultimately you get where the total variable cost as 1013 an increase of rupees 43 in comparison with EOQ by best ordering policy that. So, in that so, EOQ based ordering policy you get there is no restriction and that is why you get the minimum cost that is 970.

(Refer Slide Time: 23:54)

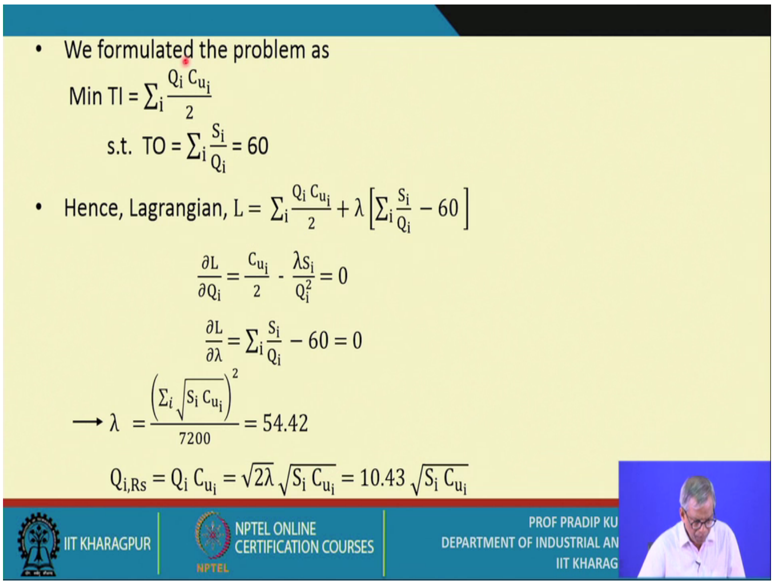


Now, one important issue you need to consider at this stage. So, this is the point you must make a note how would you formulate the problem when estimates of ordering and inventory carrying cost are not reliable? So, this is the on the real problem we face because you know this ordering cost for inventory carrying cost these estimates are not available in the accounts department accounts department, you cannot locate them. So, you have to make an extra effort to identify the activities the relating to the placement of an order as well as for holding the inventory in good conditions.

So, once you know these activities, so, separately you have to estimate the values of the ordering cost and inventory carrying cost. So, whether these estimates are reliable or not depends only on, surely on that how effective your existing information system information system support. So, this is a real problem. Now so, what you try to do? You try to the formulate problem as far as possible without you know without considering the estimates of such costs if these estimates are considered unreliable. The problem can be formulated in 2 ways; the procedure to be followed is as follows.

So, the total average inventory this notations we have used for say for a group of items and so, that is basically the total average inventory this is the expression and the total number of orders we have used this notation TO and that is S i by Q i, for the ith item. So, in the first case there maybe restrictions on the number of orders, for example, the existing inventory policy for five items considered, the total number of orders is 60.

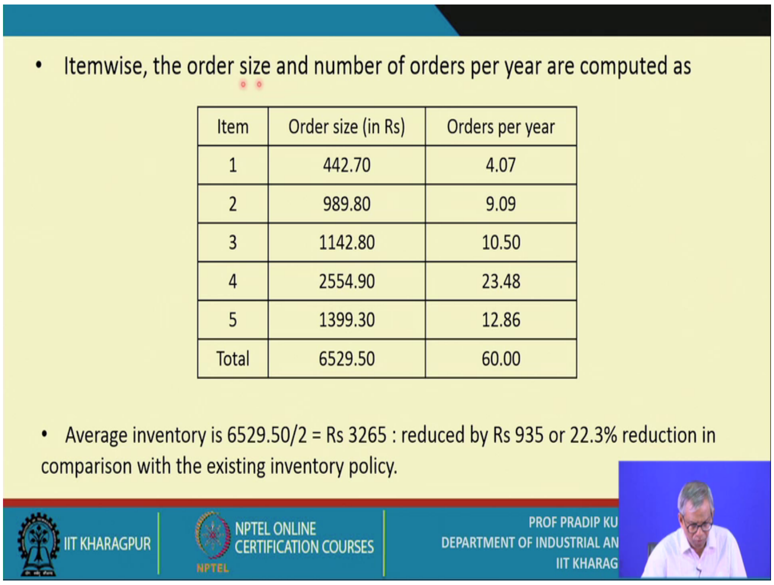
(Refer Slide Time: 26:03)



So, we formulate the problem has minimum TI is equals to this one, subject to total number of orders S i by Q i sigma i over i equal to 60. So, hence again the same procedure you have applied that is you from the Lagrangian and this is your condition that is this one, you have minimise this and plus this is the restrictions on the total number of orders. So, that is why we have used one say you know Lagrangian multipliers lambda.

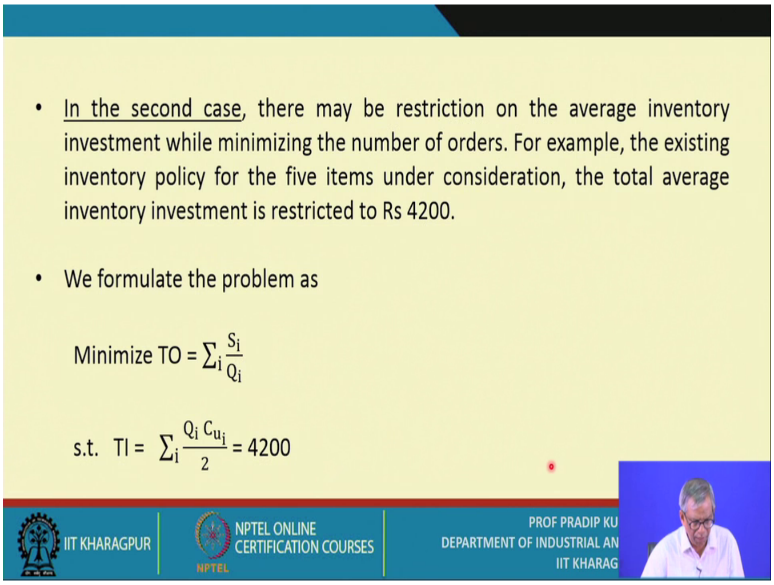
So, following these steps when was the taking partial derivative with respect to the decisions the variables, in this case Q i and lambda. So, ultimately you get an expression of lambda like this and when you have these values already you have completed these values. So, you get a value of 54.42. So, please follow the steps we are we have been using you know this similar we have been using all the steps has already explained in the previous problem, the same approach you follow and ultimately at the order quantity for the ith item in monetary terms you have these expressions.

(Refer Slide Time: 27:23)



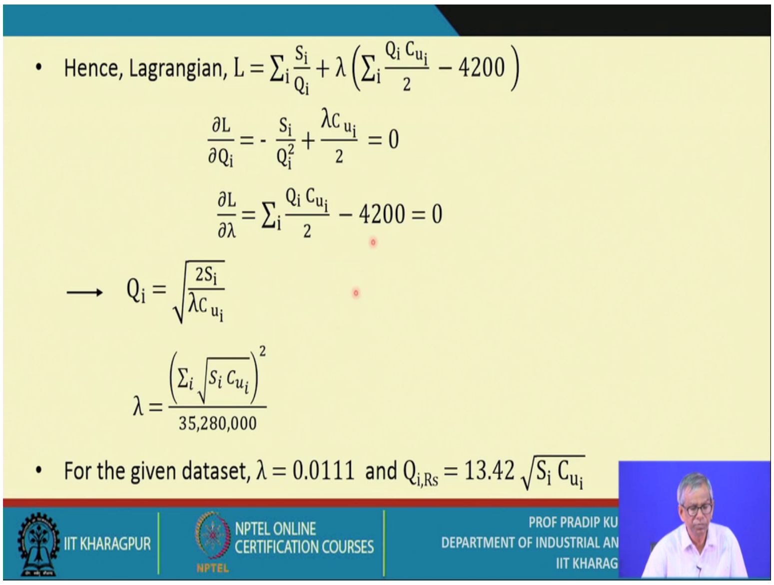
Next what you do? Item wise, you calculate the order size and the orders per year as you have done the before and so, you have all these values and what you find? That order the number of orders is restricted to 60. So, you will be getting a value of 60, whereas the order size is 6529. So, the average inventory 6529.50 by 2 that is rupees 3265; that means, the average inventory is reduced by 935 or 22.3 percent reduction. So, this is your analysis.

(Refer Slide Time: 27:58)



In the second case, what you do? Here there is restriction on the average inventory investment. So, what was the restriction? Restriction was 4200, whereas under these restrictions what you try to do? You try to minimize the total number of orders. So, this is the expression for the TO, subject to the total inventory for which you have these expressions.

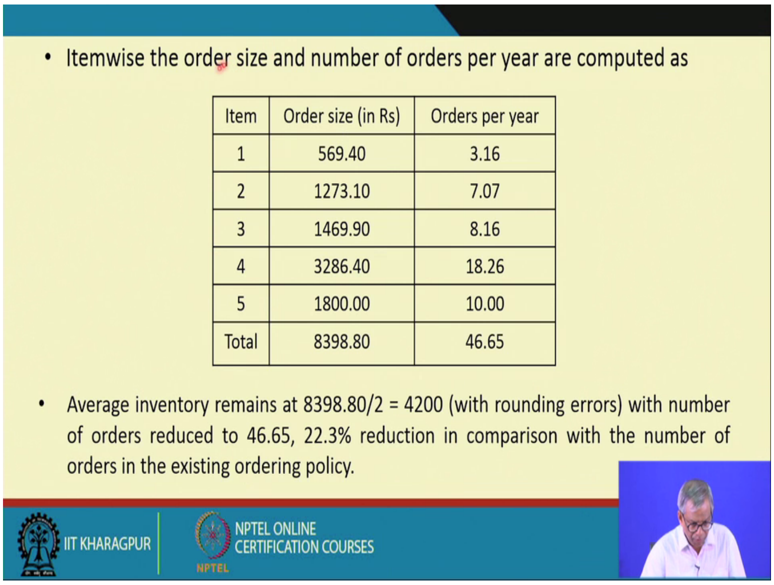
(Refer Slide Time: 28:34)



And this so, the total inventory is restricted to say 4200. So, again you apply the same procedure, you found the Lagrangian with one lambda and take the partial derivative with respect to Q i, as well as the partial derivative with respect to lambda and you get an expressions for Q i as well as in lambda.

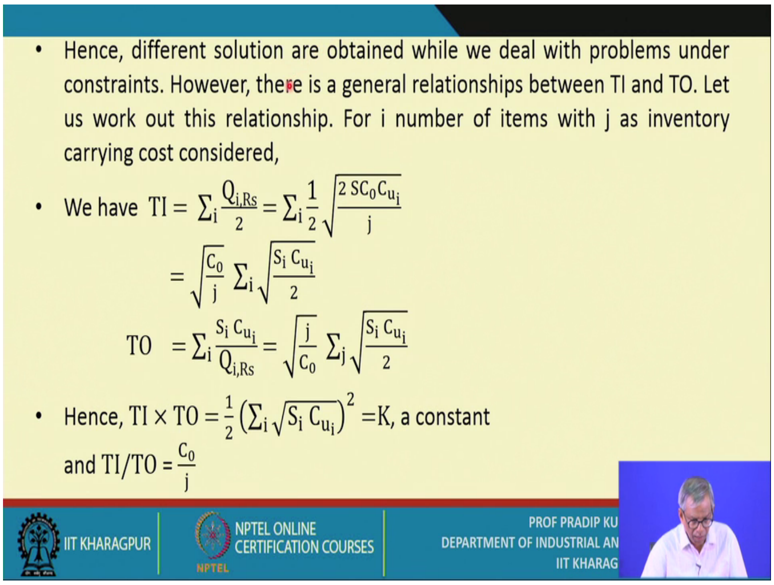
So, once the lambda is known from the given data set, you can easily calculate the order quantity in monetary terms. So, this is the expression for the of the order quantity for the ith item.

(Refer Slide Time: 29:07)



And then for all other items you follow these steps and you get the item wise order size and the number of orders per year. So, the average again what you find that the average inventory investments is restricted to 4200; obviously, and but the orders per year number of orders is reduced to 46.65 that means, 22.3 percent reduction.

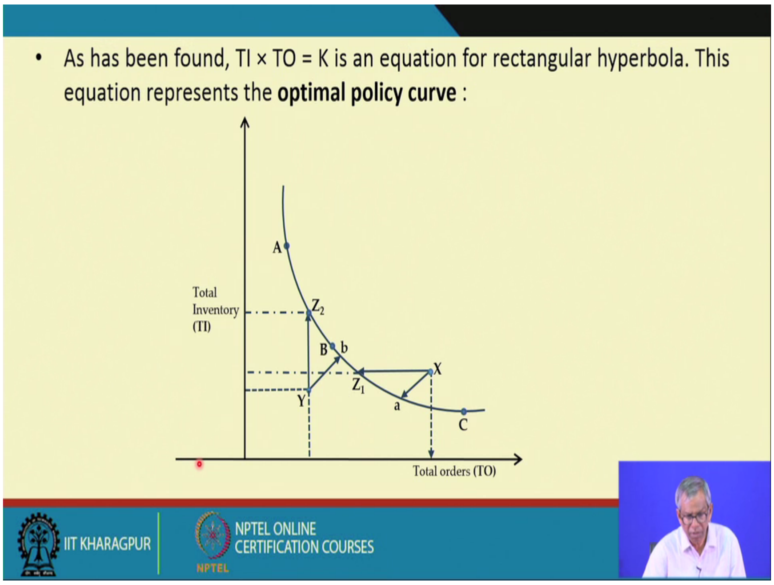
(Refer Slide Time: 29:38)



Hence, the different solutions are obtained while you deal with problems under constraints. However, there is a relationship between TI and TO.

So, what you can do? We have a expressions for TI, assuming that EOQ formula holds. So, you have these expressions for TI and another expression for TO; please go through this 2 expressions and hence if you multiply TI with TO, you have these expressions and what about maybe you are ordering policies. So, this remains the same; that is why it is treated as constants, the notation is capital K and if you divide TI by TO, you get an expression of C o by j.

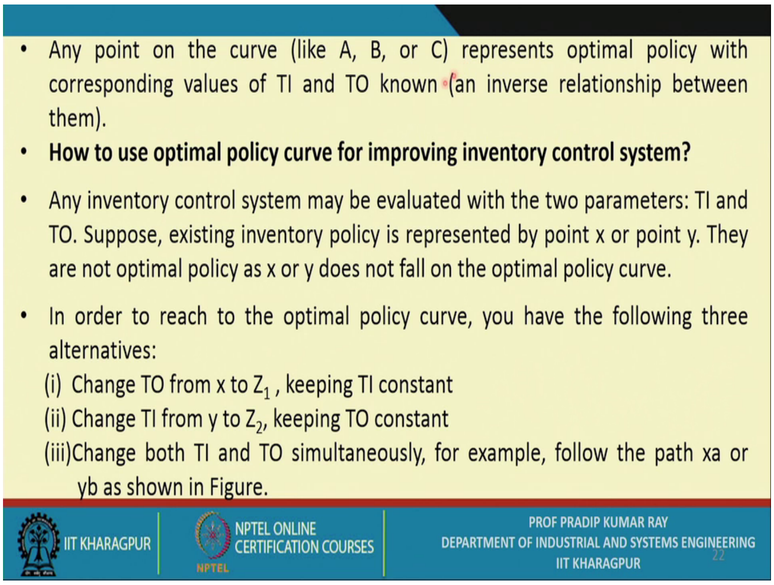
(Refer Slide Time: 30:21)



So, here the j is nothing, but actually the inventory carrying cost as a proportion of the average inventory which you hold. So, what you have? You have the expression call TI into TO is a constant. So, it is an equation for rectangular hyperbola. So, this when you have this curve; that means, TI into TO equals to K curve, this is actually referred to as actually optimal policy curve.

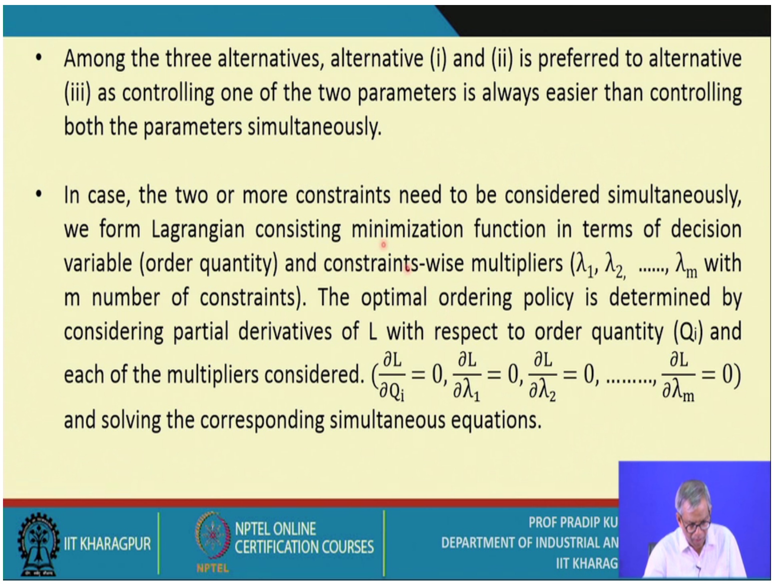
So, what you try to do? You try to develop these optimal policy curve and if you can now are you in a given situation you can calculate the total inventory as well as the total orders and so, this value could be at X; obviously, the value of X this X or the Y; these are not you know say they do not fall on this particular curve. So that means, any point falling on this curve represents an optimal policy ok. So, you are deviating. So, how to say get the optimal policy? So, there could be several policies.

(Refer Slide Time: 31:36)



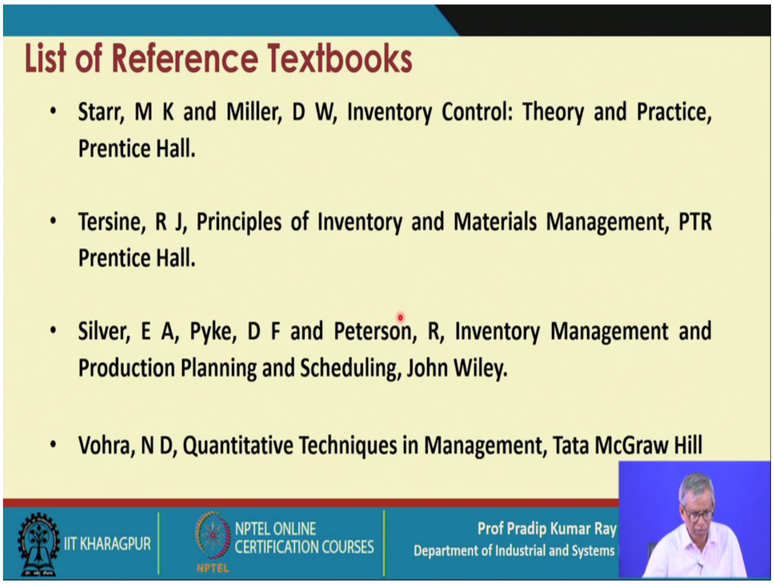
So, you please study this optimal policy curve and you have several examples like you know if you go through this; that means, we have considered 3 points A, B or C; optimal policy. So, how to use optimal policy curve for improving inventory control system, it is very simple. So, either you change the say order number of orders or you change the average inventory and or you can change both.

(Refer Slide Time: 32:10)



So, what we what we suggest that if you there could be you know it would be easier for you to change only just to one particular factor and so, if you if the same approach you follow; that means, when you have more than one restriction. So, I have written down all this particular the steps and if you have n number of restrictions; obviously, the total number of say the equations you need to consider that is n plus 1 and so, an ultimately the same approach you follow; that means, Lagrangian multiplier technique you follow.

(Refer Slide Time: 32:54)



So, later on we will refer to such cases with certain other examples.

Thank you.