Management of Inventory Systems
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Lecture - 23
Dynamic Inventory Problems under Certainty (Contd.)

Now, we are going to discuss about Dynamic Inventory Problems under Certainty we are going to discuss the EOQ modelling with planned shortages.

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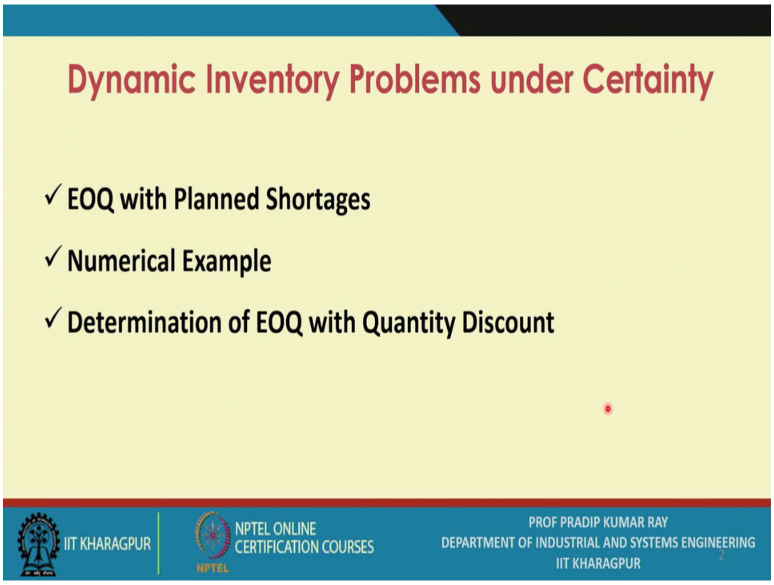

So, this is an important issue and because in many cases, you have no other alternative, but you go for planning for shortages, there are certain items and if this is a feasible alternative, that means the planned shortage is a feasible alternative for the given items; how to determine the order quantity. Then, we will refer to the numerical example. A typical numerical example we will go through and then, we will bring in the important issue called the quantity discount.

And quantity discount essentially says that if the order quantity is more, why do not you offer the unit price less or the lesser unit price you offer. So, this is a whole concept is referred to as a quantity discounts and it is a well adopted policy everywhere in fact.

So, how to determine knowing the quantity discount scheme and how to determine the order quantity for the given inventory item? So, essentially the two important issues we are going to discuss during this lecture session.

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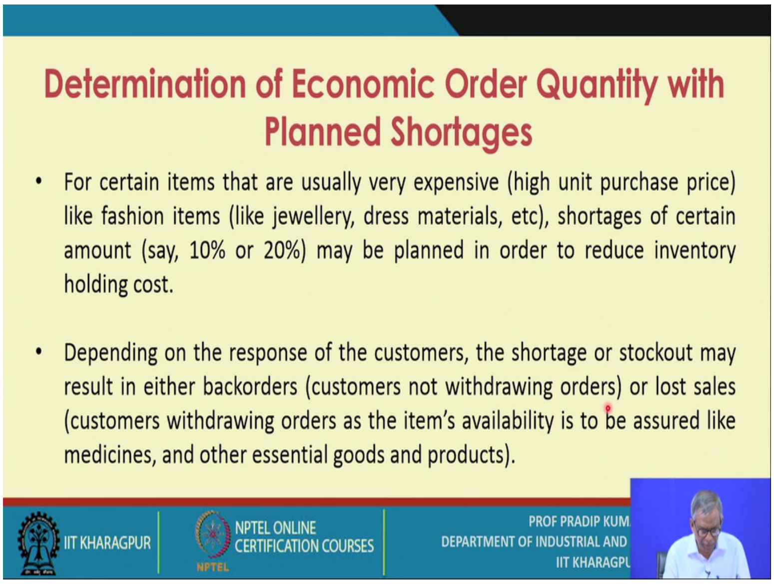

Now, let us first explain that why the planned shortages may be a feasible alternative. Usually you know the shortages should be avoided that sort or the policy you must have and what about the inventory control system you propose it and we make sure there is the no shortage, but in certain situations you can break this rule, but only in a specific case and these specific cases are not that uncommon. So, for certain items that are usually very expensive like and for which there is high unit purchase price, there are many such items like the fashion items like jewellery, dress materials, etcetera etcetera.

So, today if you visit any the shopping plaza or the shopping mall or for that matter any say any market in any city, what do you find? There are many kinds of items and some of these items are essential, but many such items may not be considered essential under the consumer goods category or the consumer product category. I am not referring to the industrial product category. So, mainly for the consumer product category, you may find such cases. And even for say industrial say the product category, the shortages may occur, but you never plan for shortage, for industrial say the product.

So, anyway so if the item is very expensive, the shortages of such items may be planned in order to reduce inventory holding cost. So, obviously the shortages cannot be 100 percent shortage. It may be 10 percent or 20 percent and this rules are followed and obviously, you make sure that even if there is shortage, there is only backordering and not lost sales.

So, depending on the response of the customers, now this point is to be understood very clearly. Because always when you go for the cost modelling for inventory item, there are two kinds of situations you come across; like particularly when you refer to probabilistic model. Later on we will take it up. We will find that this shortage cost can be of two types; one is the backordering cost and the second one is the lost sales cost, cost due to lost sales.

So, both are to be considered and these two say you know say types of shortages are defined with respect to the response of the customers, the shortages or the stock out may result in either backorders that means, here the customers not withdrawing orders because it is not that essentially he may wait; is it ok for another say even 2 months or 3 months, but the shortages or the stock out may result in lost sales. That means, customers withdrawing orders as the items availability, it is to be assured at that point in time like medicines you cannot wait if suppose it is not available at a particular store. You have to go elsewhere to get it supplied or the food items and other essential goods and products. So, these are the two situations you come across.

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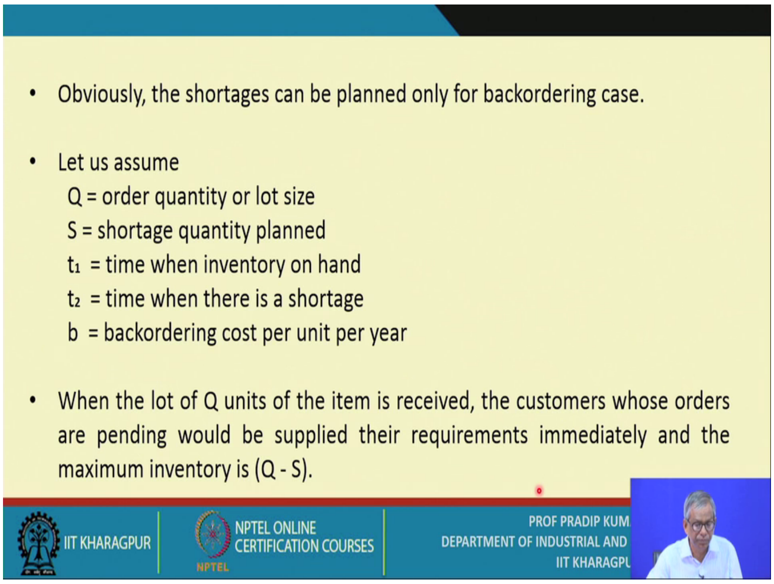

So, here what we are what we are assuming that the shortages can be planned for the backordering case and obviously, the shortages you may plan in certain cases, but these cases are backordering cases. Now, how to determine the order quantity? Let us first assume that Q is the order quantity or the lot size S is the shortage quantity planned. That means, another decision variable, you have that is how much the shortage is allowed to be determined and that notation is S.

So, here you have the two decision variables; one is the order quantity and the second one is the shortage quantity. t 1 is the time when the inventory is on hand that means it is available. There is no shortages whereas, you come across a time period called is noted as t 2. That means, during this time period, you face a situation called a shortage. Is it ok?

So, the shortage condition prevails only during t 2 time periods and what you are assuming that the demand is known with certainty that means, as a uniform demand. What is this small b? So, we have added say another cost related term called small b which is back ordering cost per unit per year, whereas we have already considered the small h. Small h is essentially the holding cost per unit per year, ok.

So, here for modelling this particular problem what you need to do? That means you must have an estimate for the back ordering cost measured per unit basis and per year basis when the lot of Q units of the item is received. That means, suppose at time t equals to t or equals to 0, so you receive Q units of item, and what you have that the order cycle starts.

The customers whose orders are pending at time t equals to t or equals to 0, now this customer should be supplied their requirements immediately and the maximum inventory is Q minus S. That means, as soon as you get Q number of units in one short, that means instantaneous replenishment what you try to do that, there is a back ordering of S units.

So, immediately the S units, so the demand for S units will be made out of Q; now you start the cycle with the Q minus S. So, this point we are trying to highlight.

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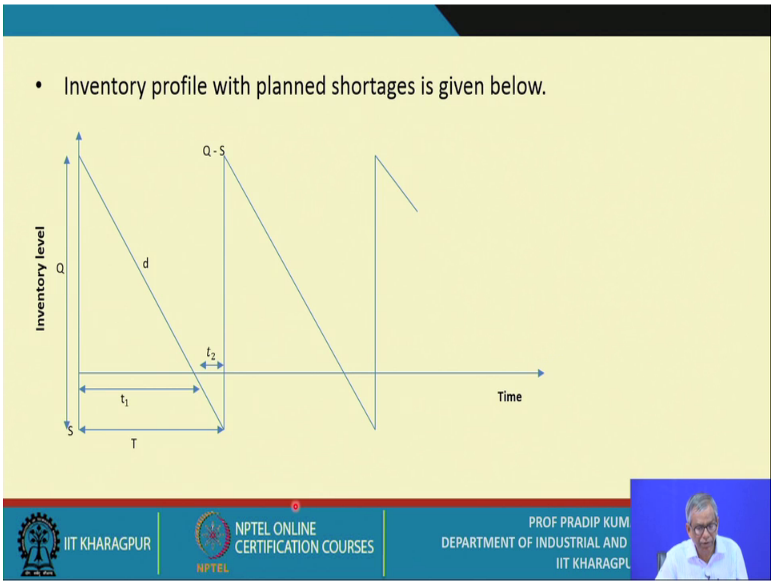

So, how does inventory profile look? So, with planned shortages, the inventory profile looks like this one. That means, at time t equals to 0, here what you get? You get an amount of item Q and this is the shortage amount. That means, this part, this is the positive one and this is negative one.

So, the shortage means negative inventory and as soon as you get this Q unit, so you subtract S number of units or shortage. So, you have Q minus S. So, this actual you know the inventory level on hand will be Q minus S.

Now, you start consuming this item with a demand rate of the consumption rate as d and you reach here. That means, during this period you have a positive stock. That means, this time period is t 1 when you know the units are available, there is no shortage, but then beyond t 1 what has happened? That you are getting these items; there is a demand; you are holding it as a backordering situation. That means, the demand that remains even if you are unable to fulfil the demand and during this time period t 2 time period, obviously this is the shortage. Is it ok?

So, what you are saying that t 1 is the time period during which there is no shortage whereas, t 2 is a time period where the order cycle is t, ok. There is a shortage and it repeats, this cycling repeats. Is it ok? So, this is the presentation of the Inventory Profile. So, just make a note that whenever you come across an inventory problems, so the first thing you need to do, that means the entire problem, the inventory problem must be explained or must be represented by an inventory profile.

So, by looking at the inventory profile and if it is correctly drawn and now, the person concerned, you will understand that what is the major problem, what are the decision variables and what are the assumptions under line. You know the modelling of the problem. Is it ok?

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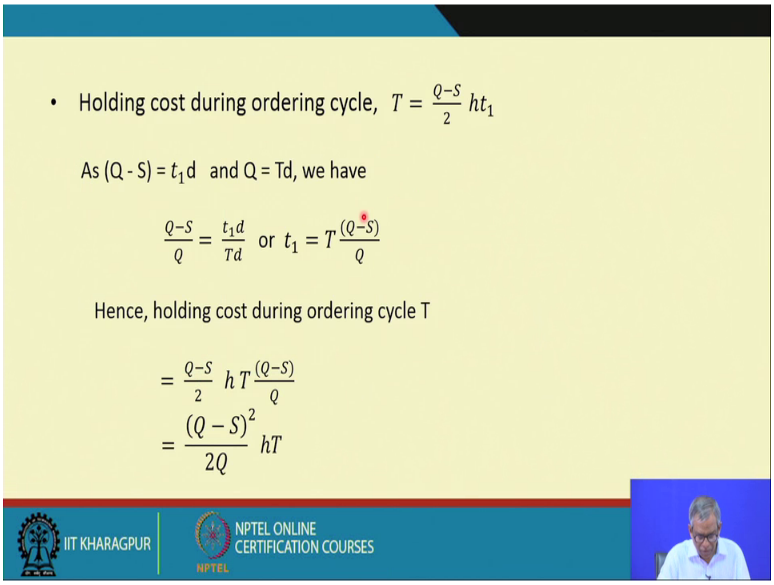

So, the construction or the knowledge of inventory profile is a must, right. So, now you have the holding cost, you have the ordering cost and you have also the backordering cost. So, while you get the expression for the total variable cost, you need to consider the back ordering cost also along with the ordering cost and the holding cost.

So, let us first determine the holding cost. So, the holding cost during the ordering cycle that is ordering cycle is capital T. It consists of two parts; t 1 and t 2. So, t 1 plus t 2 is equal to capital T. So, during this ordering cycle, so Q minus S by 2 is it ok? This is the uniform demand. Obviously, this is the average inventory and h is the holding cost per unit per year.

So, obviously here you need to consider the time component. That means, holding cost is possible, is valid only during this t 1 time period, ok. That is why h is multiplied with t 1. Now, Q minus S is the demand rate. So, how much time you require with this demanded rate to consume Q minus S? The timing is t 1 is a uniform demand and how long you take to consume capital Q number of units? Obviously, we consume or you require now within capital T time period with a demand rate of the d, you consume or you use the entire Q units.

So, Q is equals to capital T into d. Now, we have sub we have Q minus S by Q is equals to t 1 into d by t into d. That means, what you are trying to do that means we get the proportion Q minus S, that is t 1 into d divided by Q. Q is equals to capital T into d. So, we have an expression for t 1 in terms of capital T Q and S. So, this is the expression.

Now, what you try to do, that means now you have the holding cost expressions. So, here instead of writing t 1, you substitute t 1 with these expressions. That means h into T, capital T Q minus S by Q. So, ultimately you have this expression, right.

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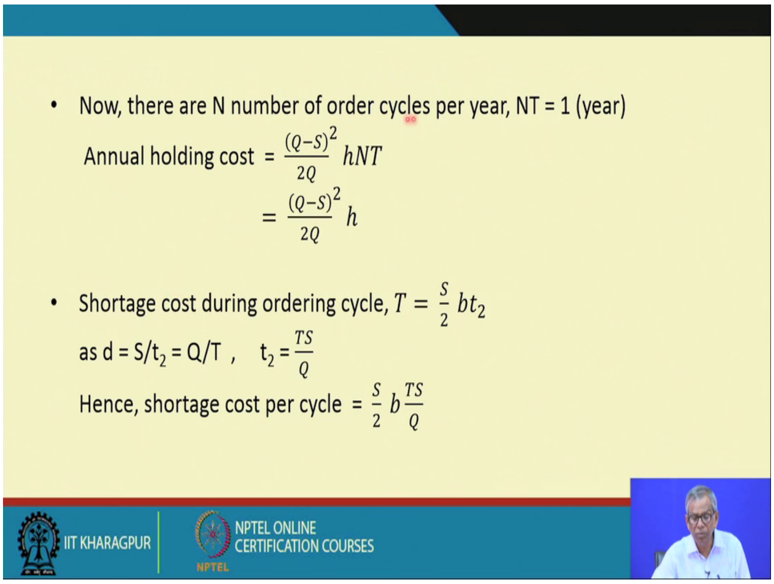

Next what you do? Now, there are n number of order cycles per years has obviously N into capital T equals to 1, ok.

So, 1 year consisting of say N number of order cycles and each order cycle is T time period, capital T time period. So, N into T equals to 1 year. Annual holding cost will be Q minus S square by 2Q h into NT. So, NT is equals to 1. So, ultimately you have these expressions for the annual holding cost. Similarly when you refer to the shortage cost, so that shortage cost is occurring or the shortage does occur during the ordering cycle T, but only during t 2 time period.

So, that is why the average shortages is S by 2. The b is the shortage cost or backordering cost per unit per year, and here it is to be time corrected and the corresponding time is small t 2. So, now you have small d is S by t 2 equals to Q by capital T directly you get these expressions and you have small t 2 equals to TS by Q.

So, this is the expression for shortage cost per cycle. Is it ok? So, instead of writing say t 2 over here, you write you substitute t 2 with T S by Q. So, you get this expression.

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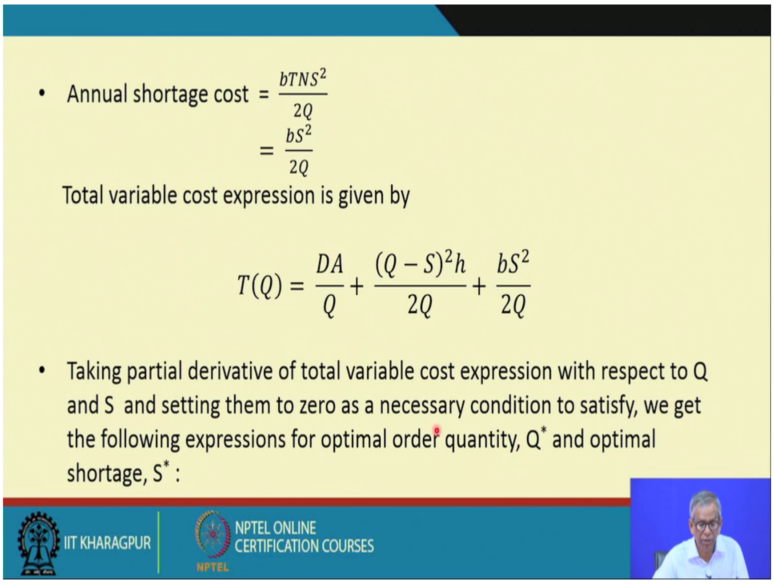

So, in the next day what do you calculate? The annual shortage cost and ultimately you multiply with capital N. So, N into T is again 1. So, ultimately it is b into capital S square by twice Q.

So, the total variable cost expression is given by this is the ordering cost. So, it is the annual demand that is Q D and Q is ordering quantity, is ordering cost for order like the same notations we have used in others expressions and this is basically the annual holding cost and this is the annual shortages cost. So, taking partial derivative of total variable cost expression with respect to Q and S and setting them to 0 as a necessary condition to satisfy, we get the following expressions for optimal order quantity Q star.

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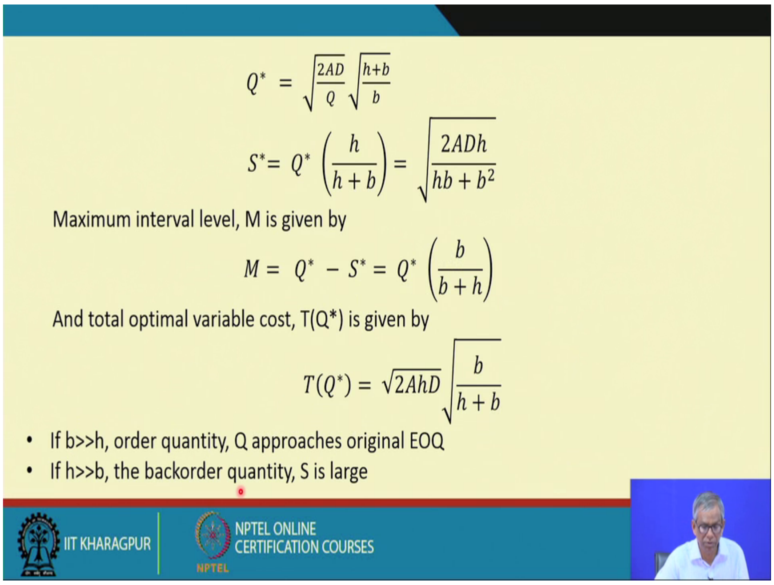

We use this notation Q star and an optimal shortages S star. That means it is a planned shortages. So, you have these two expressions. So, please you know to follow the steps. I have not written all the steps. I am sure that you will be able to get these expressions by taking partial derivative with respect to Q as well as with respect to S. So, you will have two equations and you solve these two say simultaneous equations to get an expression for Q star as well as S star. That means, optimal order quantity and this is the optimal say the planned shortage.

So, this is the expression you get. Obviously, S star depends on Q star with this particular factor as h divided by h plus b. So, this is the simplified expressions. So, you just try to get this simplified expression. So, maximum interval, interval say the level that is M is given by m equals to actually is actually it is the maximum.

The difference between Q star and S star, that is M and this is Q star minus S star equals to these expressions you have and the total optimal variable cost also you can calculate. This is the total optimal variable cost. Is it ok? So, please try to derive these expressions. It is very simple. You have the general expressions for total variable cost, you substitute the values of say Q and S, or you substitute Q and S in the original expression with the Q star and S star. Is it ok?

So, you get these expressions final. Now, these are two conclusions you draw. One is if the back ordering cost is substantially greater, then holding cost order quantity Q approaches original EOQ. So, this is one conclusion you draw, but if holding cost is substantially greater than b, the back ordering quantity S is large. That means, what you try to do if this condition holds you say almost the entire amount you keep as backordering, you do not keep any stock if the order arrives or comes or the customer approaches you.

So, we wait for a specific time period and then, you go for, you get the consolidated requirement and then, you activate your purchase action, and I get the consolidated requirements supplied by south side suppliers,.

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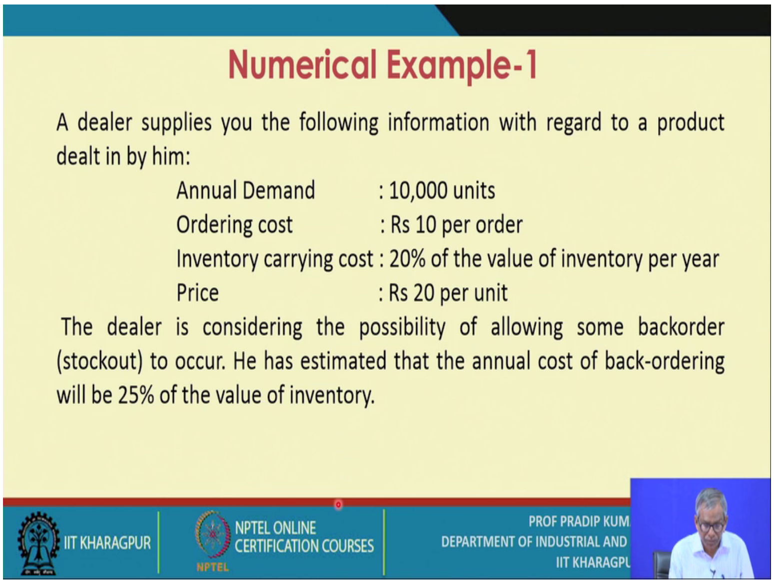

So, that is a rule. Now, here is an example. A dealer supplies you the following information with regard to a product dealt in by him. Annual Demand 10,000 units, Ordering Cost rupees 10 per order, Inventory Carrying Cost 20 percent of the value of the inventory per year it is considered to be high and the price is rupees 20 per unit. This is the unit price the dealer is considering the possibility of allowing some backorder. That means, the stock out to occur he has estimated that the annual cost of backordering will be 25 percent of the value of inventory. Is it ok? Whereas, the holding cost is 20 percent and now, the backordering cost is 25 percent.

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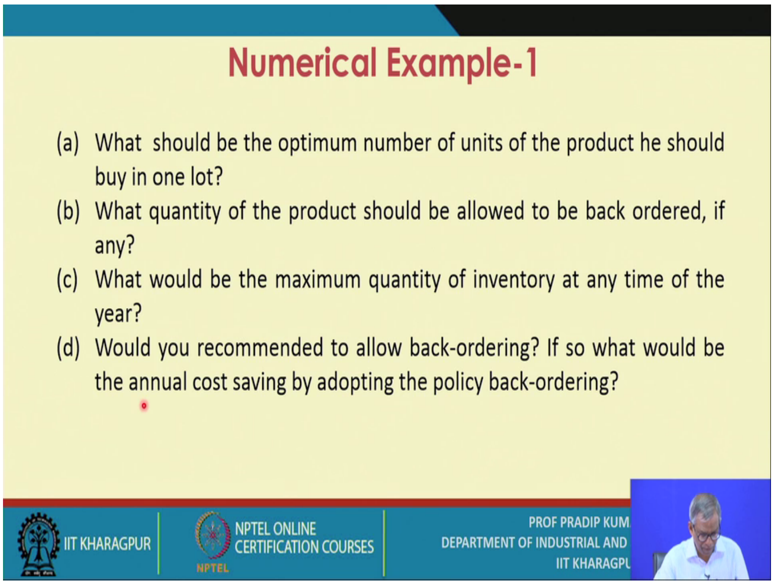

What should be the optimal number of units of the product you should buy in one lot? What quantity of the product should be allowed to be back ordered if any? What would be the maximum quantity of inventory at any time of the year? Would you recommend to allow backordering? What is your opinion? If so, what would be the annual cost saving by adopting the policy of backordering? So, these are your say the questions and you need to answer all these questions.

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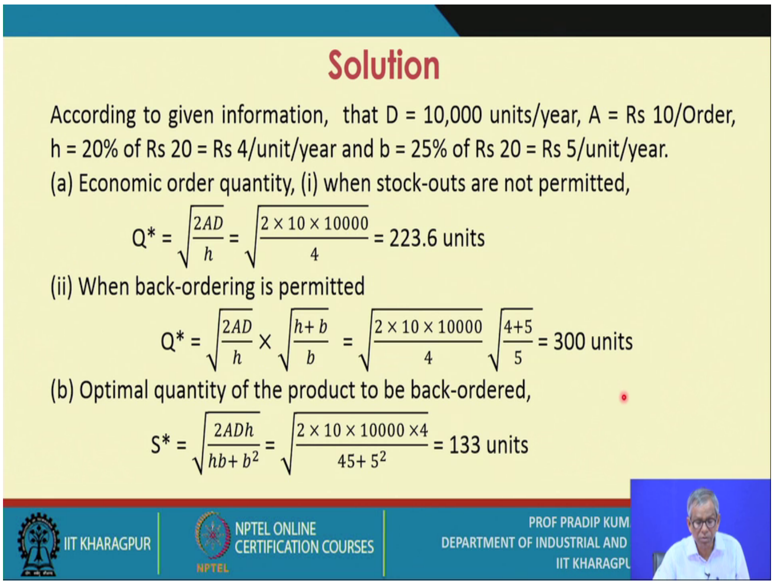

So, you have all these data DAh, then b, ok. So, this is in percent and it is 20. That means, a rupees 4 per unit per year, this is 25 percent of rupees 20. That means, rupees 5 per unit per year, is it in absolute term.

Then, you get the expression for Q star; you get the expressions of our say Q start when backordering is permitted. Is it ok? So, this is your Q star expression, right h plus b by b. An optimal quantity of the product we backorder, that is S star. That also we have this expressions. So, use this expression, so 133 units. Is it ok?

So, this is basically the economic order quantity, right when there is no backordering. So, that is the Q star whereas, when backordering is permitted what is the expression for Q star and what is the expression for S star?

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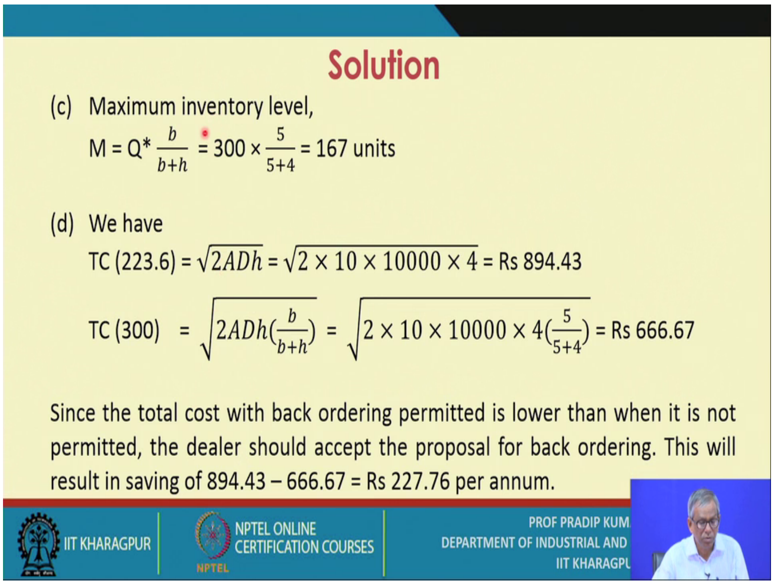

So, maximum inventory level you can calculate, the maximum inventory level that is Q star minus S star here the expression and we have the total cost expression when we use economic order, that is this one when we use say when you permit back ordering.

So, order quantity is 300. So, corresponding total variable cost is this and since the total cost with back ordering permitted is lower than when it is not permitted, the dealer should accept the proposal when back ordering. Is it ok? So, here the cost is this one whereas, if you allow the back ordering, that means the cost has come down and the difference is substantial.

So, this will result in saving of this one, this minus this one. That means 894.43 minus 666.67 and that means, approximately 227 rupees per annum.

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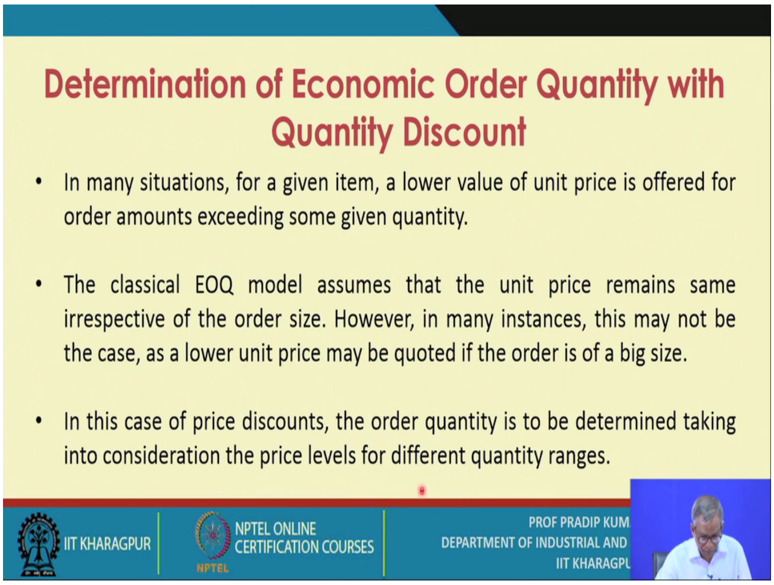

Now, the next one is you know the important concept is quantity discount in the original EOQ formula. What you have assumed that unit price remains same irrespective of the order quantity.

Now, in the real world what you find that there is a relationship with relationship between the order quantity and the unit price. That means, in majority of the cases if the order quantity is more, then unit price comes down whereas, if the order quantity is less, so say the unit price may be higher. Now, many a times what happens that if the order quantity is more, that means you are offered the quantity discount. That means, unit price will be substantially less. So, in many situations for a given item a lower value of unit prices is offered for order amounts exceeding some given quantity.

The classical EOQ model assumes that the unit price remains same irrespective of the order size, however in many instances this may not be the case. As a lower unit price may be quoted, if the order is of a big size, this point already we have elaborated. Now, in this case of price discounts, the order quantity is to be determined. So, that is our question. So, if the price discount is offered, how to determine the order quantity or whether you will offer discount.

So, if you offer discount, what will be the order quantity? If you do not offer the discount, price discount, what will be your order quantity? So, now what you need to do is, you need to take into consideration the price levels for different quantity ranges, ok.

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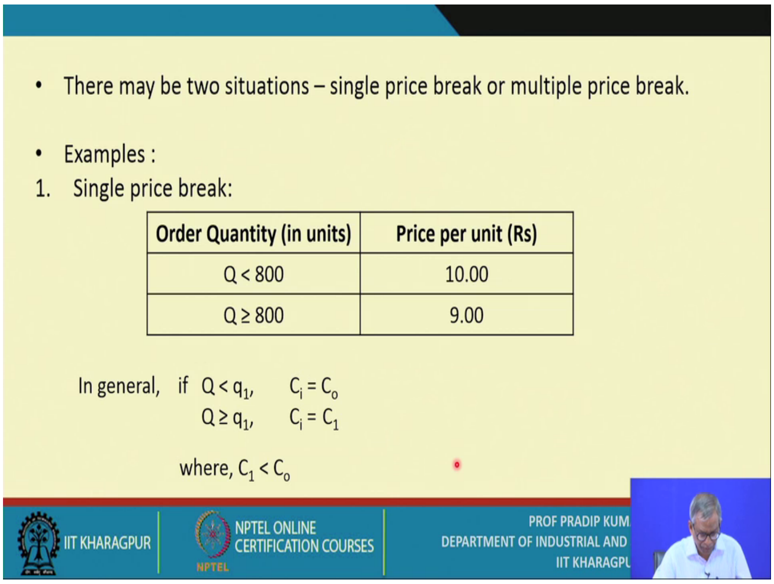

Now, there maybe two situations; single price break or multiple price breaks, ok. I will just write examples. So, that your understanding is good and the understanding will be appropriate. A single price break suppose order quantity is less than 800, you are offered a price per unit rupees 10, but as soon as the order quantity becomes greater than 800, any amount, from 800 to any amount, theoretically speaking up to infinity, so you are interested to get a unit price of 9 only. So, this is referred to as single price break. Is it right?

In general, we are generalizing if order quantity is less than q 1 C i that is the unit price for ith item is equal to C 0.

If order quantity is greater than or equal to q 1 and then, the unit price the notation is C i become C 1, where C 1 is less than C 0. Is it ok? So, this is the generalisation.

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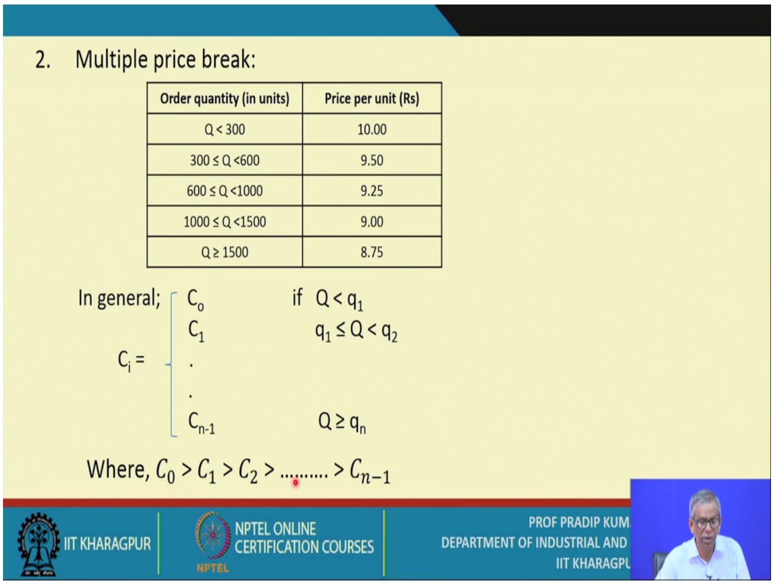

So, this is Single Price Break situation, where as in a Multiple Price Break just you refer to this particular table. What you find that the order quantity if it is less than 300, that means 299, 297 any value less than 300, the price is 10. If it is between 300 less than equals to, that means its Q maybe 300, but less than 600, then it is 9.5. So, this the prices are offered to you if the quantity is 600 and up to say less than 1000. That means, a 999 price is this. So, same rule you follow and ultimately, you find if the order quantity is greater than or equal to 1500 that means, 1500 or more price of per days 8.75 per unit.

So, in general what you find is that for the ith item, the unit price is C 0. If Q is less than equals to q 1, the unit price is C 1. If q 1, if capital Q is less than equals to q 1 between q 1 and q 2, q 2 is excluded and this process continues and ultimately, what you find that the unit prices C substitute n minus 1. When the order quantity is greater than or equals to say q n, where C 0, C 1 is C 0 is greater than C 1, C 1 is greater than C 2, C 2 is greater than C 3 continuing like this ultimately it is greater than C n minus 1. So, this is referred to as the Multiple Price Break.

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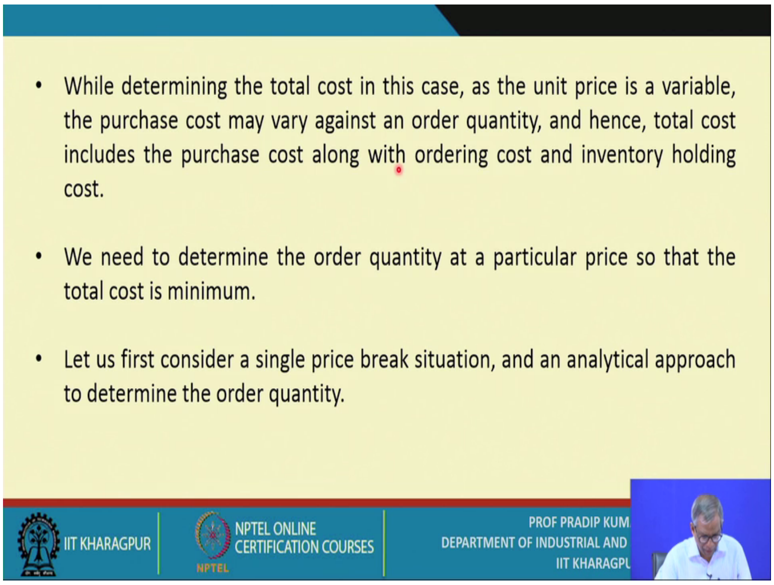

While determining the total cost in this case as a unit price is a variable, the purchase cost may vary against in order quantity. So, this point is to be noted and hence, the total cost includes the purchase cost along with the ordering cost and inventory holding cost. When we will formulate the problem, this point will be made very very clear. We need to determine the order quantity at a particular price, so that the total cost is minimum. This is your objective.

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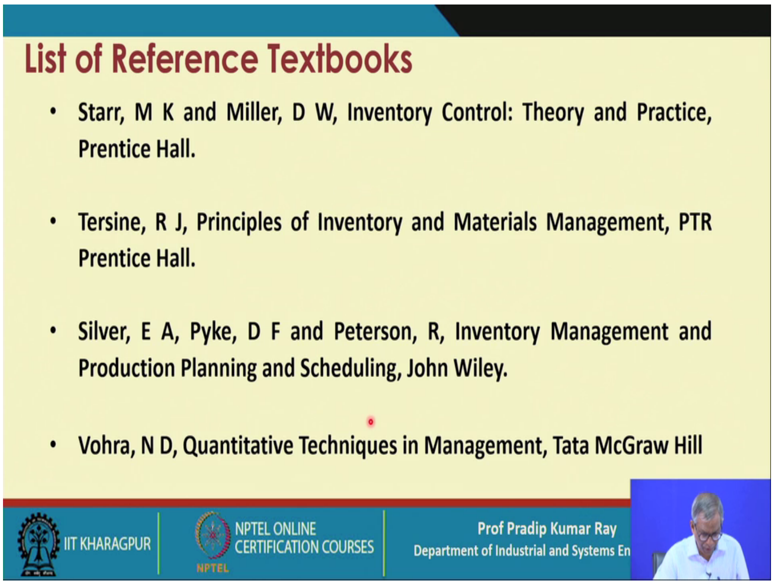

So, let us first consider a Single Price Break Situation analytical approach to determine the order quantity, ok. So, both for you know Single Price Break as well as for the Multiple Price Break, the situations there could be two approaches to determine the order quantity. The first approach is we will always prefer the analytical approach.

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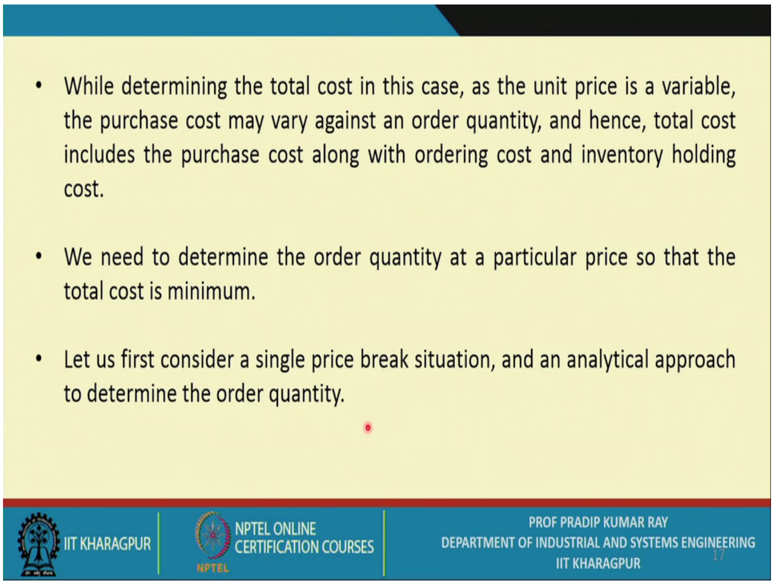

If you get the solution through analytical approach, find no problem, then that will be our preference, but if for some reasons the analytical even if you apply analytical approach, but getting a solution is a problem, then you will have to opt for alternatives and the alternative could be a graphical methods. So, both these approaches we will discuss in our the next lecture sessions.

Thank you.