Management of Inventory Systems

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Lecture – 22
Dynamic Inventory Problems under Certainty
(Contd.)

During this the lecture sessions, under Dynamic Inventory Problems under Certainty; I am going to discuss three issues the first one is EOQ.

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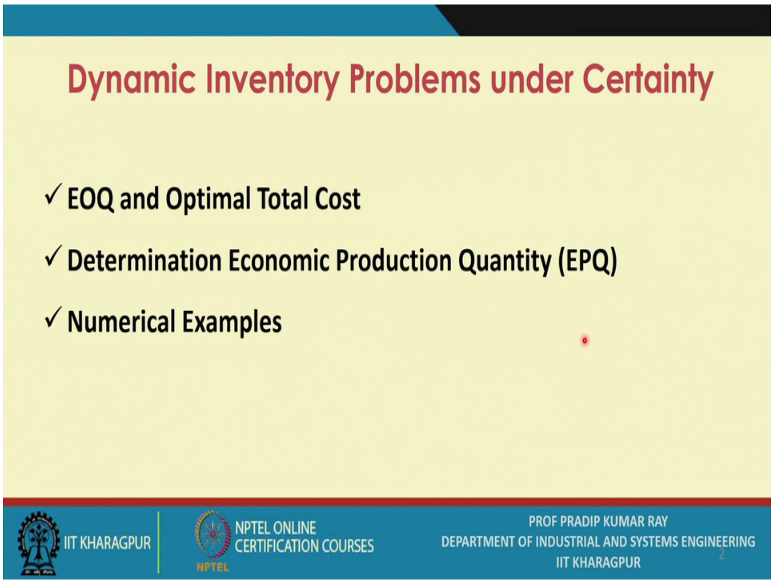

So, already we have developed the EOQ on the model. So, we will be discussing on some different you know the characteristics of EOQ model, and particularly we will be referring to the optimal total cost. So, how to determine the optimal total cost and what are the, you know the characteristic features of the optimal total cost, how EOQ as affecting the value of the total cost particularly the total variable cost.

Then the next important issue ~~you~~ we are going to discuss that how to determine the economic production quantity; that means, it is a self supply case, you need to produce certain amount with your own say the production unit and for the given inventory item. So, how to determine the economic production quantity is a self supply case and of course, we will be discussing or we will be referring to a number of numerical examples.

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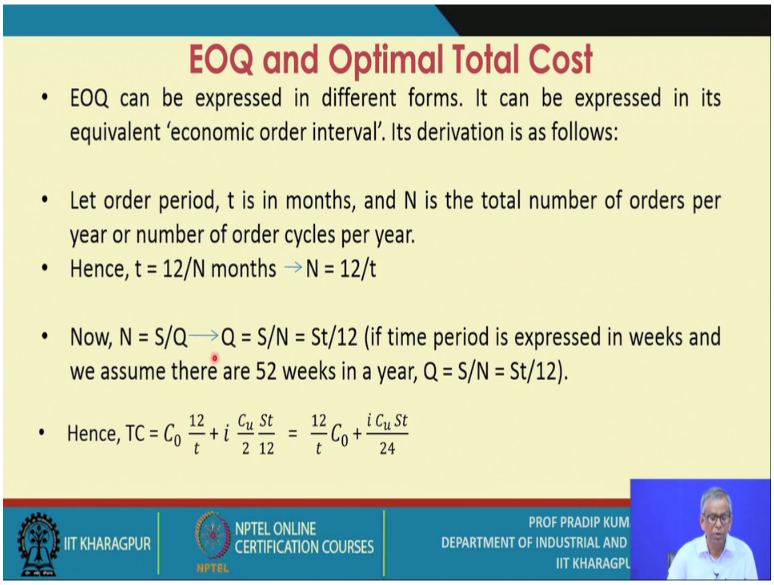

Now, continuing our discussions on EOQ already we have derived the EOQ formula that is Q ~~start~~ star equals to root over twice sc 0 by i Cu. Now, this particular the EOQ expression root over twice sc o by i Cu can be expressed in different forms. It can be expressed ~~any~~ in its equivalent economic order interval particularly you know if you referred to as the ~~p~~ P systems of inventory control

As already you have pointed out for the given inventory item, I can propose a Q systems of inventory control, I may also propose a ~~p~~ P systems of inventory control. ~~Say~~ So if it is a Q systems of inventory control, I say that the order quantity is your Q model. Assuming that all that in a ~~some~~ assumptions which we have already listed, all ~~this~~ these ~~in a~~things assumptions are valid.

Alternatively, suppose it is a problem with certainty; that means, the demand is known with certainty, you ~~save~~ say that the ~~persistence~~ P systems of inventory control needs to be followed for the same item, there is an alternative. So, what you need to determine? You need to determine the order interval, and in this case this is referred to as the economic order interval.

So; obviously, there is a relationship between the economic order quantity and economic order interval. So, how to derive the expressions for economic order interval? Let order period t is in months; like a if you refer to that inventory profile, what do you find that we have mentioned the order cycles. So, the each order cycle is nothing, but the order period.

So, suppose these order period is small t and capital N is the total number of orders per year or the number of order cycles per year. So; that means, you count how many order cycles accommodated in a particular say the year or years time is a 12 months times, so you find there are capital N number of say the order cycles per year.

So, hence the order period is nothing, but 12 by N months if say if the order period the measuring unit, if it is a if it is month. So, that is why capital N is equals to 12 by t. Now N is S by Q; S is the yearly demand and Q is the order quantity. So, hence Q equals to S by N now N is substituted with the say S t by 12; that means, S by N equals to S t by 12 if the 12 by t N is equals to 12 by t.

So, that is why Q is nothing, but S t by 12. If time period is expressed in weeks and we assume there are 52 weeks in a year. So, the Q will be S by N that is S t divided by 52 ok. So, S t it will be if you write S t by 12 this is the time period is in month, but if you write S t by 52; that means, the order is in weeks and we are assuming there are 52 weeks in the year.

So, what is the total variable cost expression? So, that it is TC equals to order the cost per order and how many orders you have? That is 12 by t per year is it ok. So, that is why it is C o into 12 by t plus the inventory carrying cost; that means, I into C u by 2 into Q. So, what is Q? Q is S t by 12 ok. So, on simplifications what you get? You get these expressions 12 by t into C o plus i into C u into S into t by 24.

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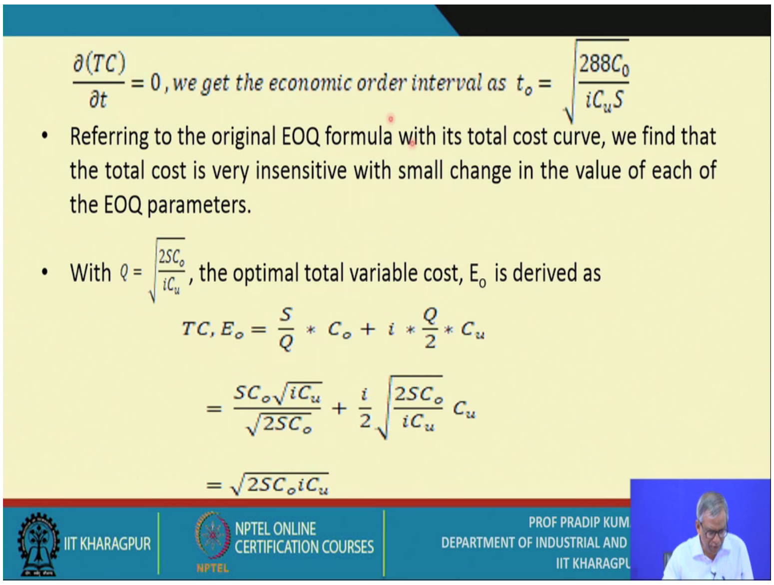

Once this expression is known what you try to do; that means, you try to minimise you try to the minimise the total variable cost, and for minimization condition what you have that the partial derivative with respect to t of say TC is said to be 0. And we get the economic order interval as to equals to root over 288 C o by iCuS.

So, this is another expression of so, the equivalent EOQ, that is referred to as economic order interval root over 288 C o by i Cu S. So, all these parameters we have explained what is i what is C u what is S and what is C o the referring to the original EOQ formula, with this total cost curve you have already shown to you the total cost curve.

We find that the total cost is very insensitive with small change in the value of each of the EOQ parameters around the economic order quantity that is the Q star. So, with Q equals to root over twice SC 0 by iCu this is the original EOQ formula the optimal total variable cost E o optimal total variable cost is derived as the total variable cost E o is equals to S by Q S by Q into C o plus i into Q by 2 into C o. Now, this is the replaced with; that means, the Q is replaced with Q star that is root over twice S C 0 By iCu.

So, when you substitute these expression of Q in this particular say the equation, what do you find that the total variable cost optimal total variable cost the notation is E substitute o is root over twice S C o into iCu ok; root over twice S C o iCu ok. So, this is the expressions we have.

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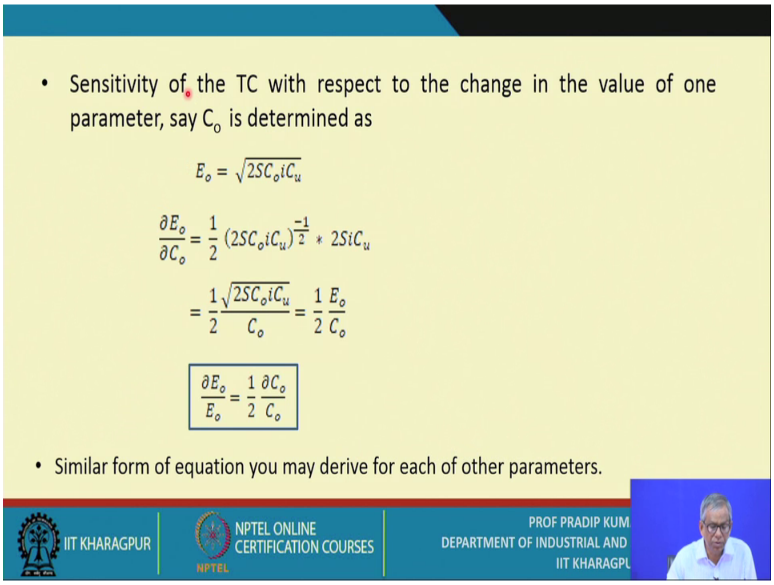

Now, the sensitivity of the total cost with respect to the change in the value of one parameters say Co. So, what we have mentioned that that if the total say optimal the total variable cost becomes highly insensitive to the change of say the parameter values. So, what we have say what we have assumed that one such parameter is C o, that is the ordering cost per order. So, this is the expression for E o, root over twice the C o iCu

now what you try to do; that means, you verify it sensitivity with respect to C o, you take the partial derivative of E o with respect to C o. So, this is the expressions you get when you get the partial derivative with respect to C o.

And so, what do you get; that means, the del del C o of E o is equals to half into E o by C o ok. So, in other words del E o by Eo; that means, this is a proportion equals to half of del C o C o; that means, supposing there is a change of 5 percent in C o. It will have say, but the E o will be; that means, the optimal total cost will change by change by half of 5 percent; that means, 2.5 percent and this is true this expression or these relationship is true not only with respect to Co.

But this is these expression is towards this relationship is true for all other say the parameters, all the other three parameters we have used in getting the EOQ say the formula. So, similar form of equation you may derive for each of the other parameters it is clear now.

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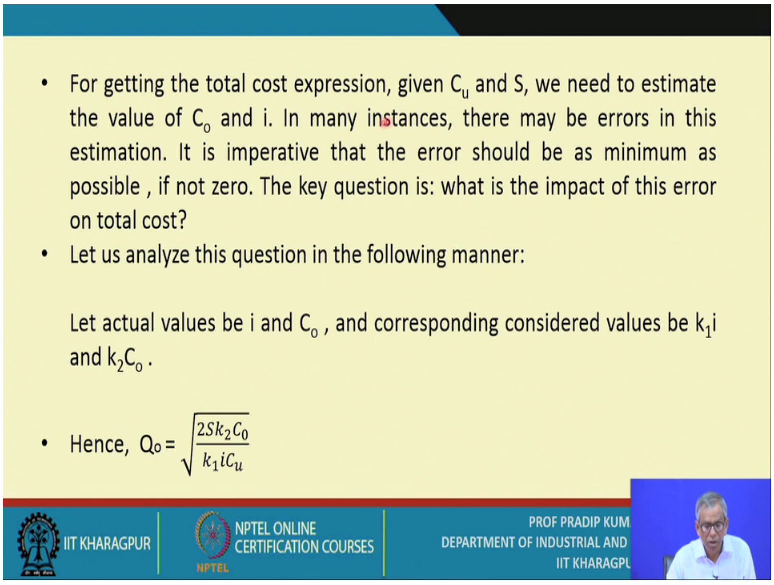

For getting the total cost expression given C u and S, we need to estimate the value of C o and i. In many instances; that means, C o is ordering cost per order and i is the inventory carrying cost as a percentage or as a proportion of the average inventory, which you hold. In many instances there maybe errors in this estimation this is quite likely. So, it is imperative that the should be as minimum as possible if not zero; that means, whenever you will deal with these are the parameters, C o or i; obviously, these are the estimates.

And make sure that these estimates are having the minimum error that is to be ensured. So, the key question is: what is the impact of this error on total cost? So, here is on derivation, let us analyze this question in the following manner. Let actual values be i and C o; that means, actual values of inventory carrying cost is i, and ordering cost per order is C o. And the corresponding considered values; that means, the estimated values is k 1 into i small k 1 into i and small k 2 into C o. Hence actually we will be using this two values k 1 into i and k 2 into C o; obviously, the order quantity which we will compute is given by twice 2 S k 2 C o by k 1 iCu is it.

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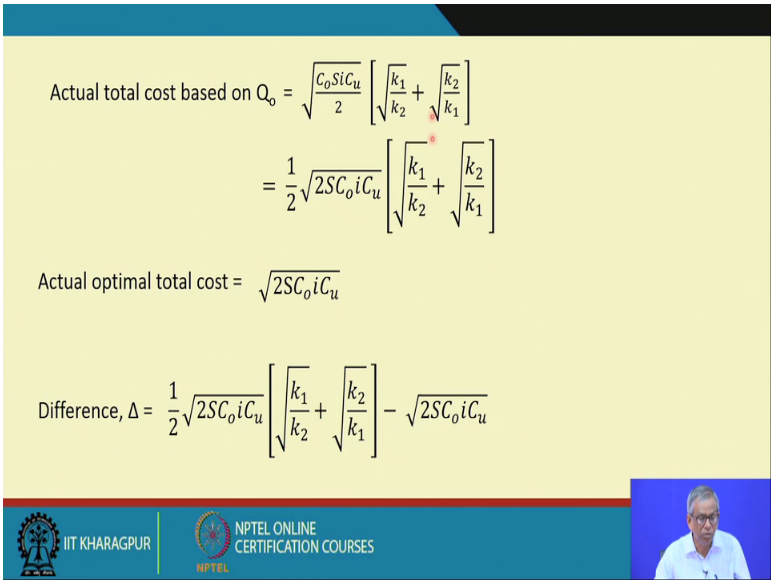

Now so, the actual total cost based on Q o so; obviously, you have a total cost expression general expressions and ultimately you get this expression ok. So, my suggestion is. So, that you substitute all these values, and you must check whether you are getting this expression or not is it so, you try on your own.

So, I am sure that if you follow the steps correctly, you will get these expressions. Now the actual optimal total cost so, these expressions already you have derived that is root over twice the C o iCu. So, what is the difference? The difference is the delta and capital delta is equals to this one is it alright.

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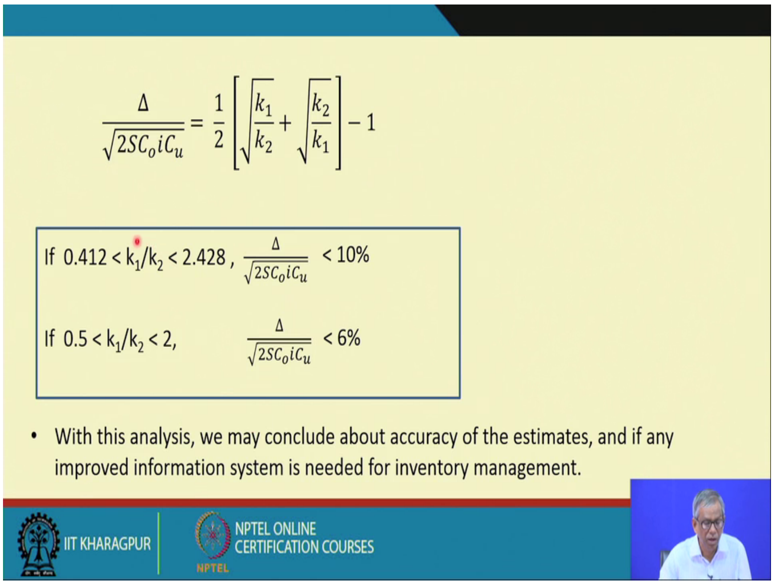

So, this is just you follow the steps and this is the difference as a proportion of the optimal the total variable cost is a proportion that is why you form this ratio and you get these expressions. So, half into root over k 1 by k 2 plus root over k 2 by k 1 minus 1 so, this is the expression. So, now you try with the several values of possible values of k 1 and k 2, and there could be certain the conclusions you may draw. Like if k 1 by k 2 that is the proportion k 1 is respect to i and k 2 is with respect to C o or the ordering cost per order.

So, if this k 1 by k 2 this ratio is between 0.412 to 2.428 consider to be more or less a wide margin what do you find that this proportion; that means, delta divided by the optimal total variable cost, will be always less than 10 percent. And if k 1 by k 2 lies between 0.5 and 2, then this proportion that means the difference with respect to as a proportion of that the total optimal total variable cost is will be just less than 6 percent.

So, with this analysis we may conclude about the accuracy of the estimates and if any improved information system is needed for inventory management, because when you go for such estimates you make sure that you are in that your information system is highly reliable. So, it reflects when you go for this sort of analysis, see I have an idea that how as far as inventory management system is concerned what extent it gets the support of the companies information system, and you may also conclude the level of reliability of such an information system.

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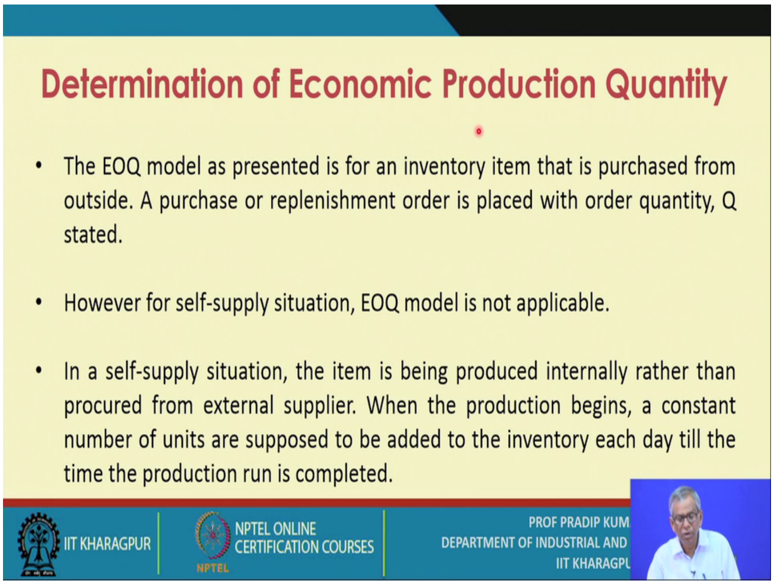

Now, let us the talk about the economic production quantity, certain remarks are mentioned. So, let me first you know discuss those points. The EOQ model as presented is for an inventory item that is purchased from outside ok. So, please follow these points. So, the original EOQ formula; that means economic order quantity.

So, as soon as you use the term order, it means that it is an outside supply case. As soon as you use the term called purchase order means that the item is to be procured from outside. So, a purchase or replacement order is placed with order quantity Q stated, you please refer to the flow chart or the flow diagram which we have drawn in respect of both Q system of inventory control and P systems of inventory control.

But there are cases where there is no outside supply, but there is a self-supply case; that means, suppose one work unit using one inventory item, and this work unit needs a certain quantity of the item. So, what it does, it places a work order to another work unit say the machine shop or the fabrication shop or the assembly shop and of course, based on this work order.

So, that particular shop starts the producing it with the required quantity, and when the production is over the required quantity is sent back to the work unit where from where the work order has been has been placed. So however, for self this is called the self-supply situation however, for the self-supply situation EOQ model is not applicable is it ok. So, it is very clearly understood. So, in a self-supply situation the item is been produced internally, rather than procured from external supplier. So, I have already explained to it.

And when the production begins a constant number of units are supposed to be added to the inventory each day till the time the production run is completed; that means, against the quantity to be produced that means, the. So, the production rate will be known and once the production rate will be known, immediately you can calculate you can estimate that how many what will be the time period or how many days or how many weeks you require to produce the entire quantity so, this points to be known.

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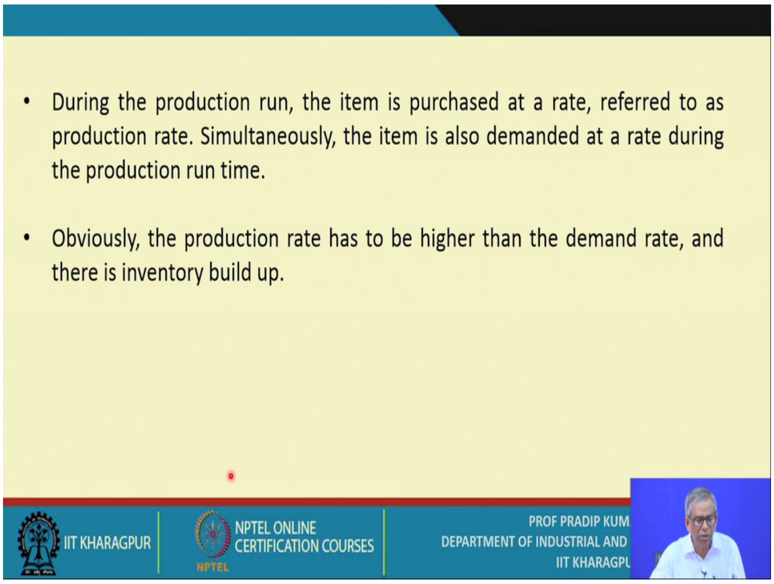

During the production run the item is purchased at a rate referred to as the production rate. Actually the item is produced at a rate referred to as a production rate; simultaneously the item is also demanded at a rate during the production runtime; that means, as soon as you start producing at a particular rate you start using it. So, what do you have? You have a production rate and simultaneously you also have a consumption rate or the demands rate ok.

So; obviously, the production rate has to be higher; now this point is to be noted; that means, the production rate has to be higher than the demand rate, and there is inventory build-up. Otherwise it is not a suppose you find that the production rate is less than the demand rate so; obviously, it is say not a problem of inventory. So, as soon as the production rate is becomes higher than the demand rate that means, there is inventory builder. So, and that is why it is referred to as one kind of inventory problem for which the order the production quantity, you need to determine optimally and this production quantity is referred to as economic production quantity.

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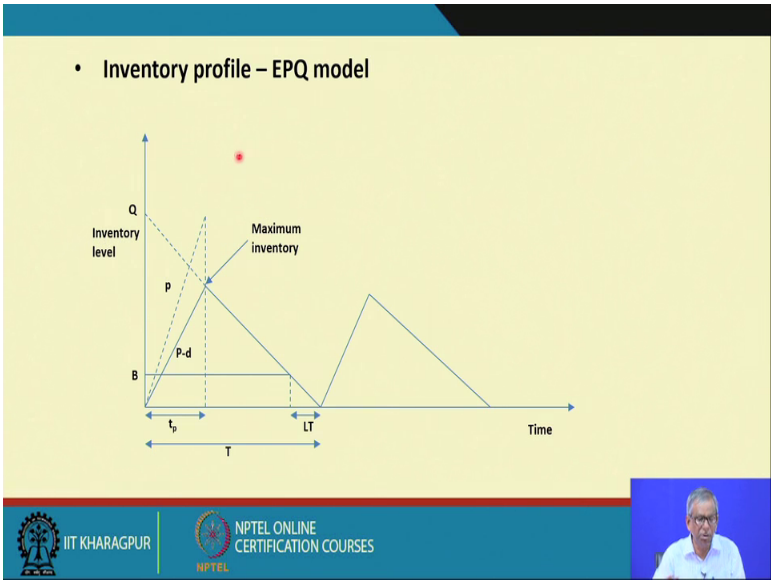

Now, this is the inventory profile and this is for the EPQ model economic production quantity model. So, here what you find over here. So, the for the so, your required quantity is Q, and for which you have a the production runtime and the production runtime is tp ok.

So, depending on which type of say the production unit you used. So, once the production rate is known, the run time you can calculate. So, at a production P so, are the inventory is suppose to find the path, but what is happening that you will never use this point, because as you start producing at a rate p you also start the consuming it; that means, there is a demand and this the demand rate is small d.

So, actually the inventory picks up in this manner at this rate the rate is P minus d and once the tp is over; that means, you have the entire the production quantity produced, and subsequently you start consuming it, but at a rate of at a demand rate of d. So, here you find it is d. So, this is the maximum inventory and so, this is one order cycle of the production cycle and similarly such production cycles repeat is it ok.

So, this is your inventory profile so, please the study this inventory profile and I am sure that you will be able to understand all the critical points over here. Basically you must know what is the production rate, you also must know what is the demand rate, you also you know you need to determine your decision variable is the production quantity that is referred to as the Q. And for the given a work unit of the production stage or the particular or machine so, the or machine tool you must know that what is the production run length here the notation is tp ok.

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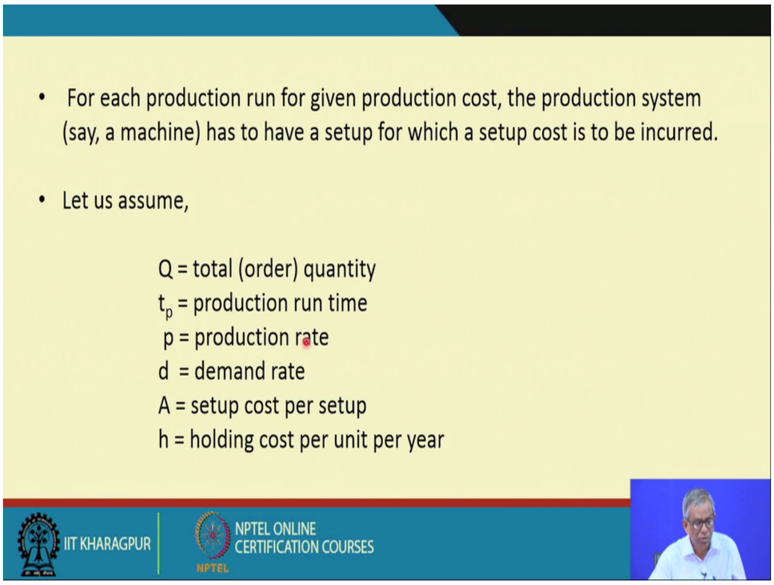

So, now for each production run for given production cost, the production system say a machine has to have a set up for which a setup cost is to be incorrect; that means, here instead of having the ordering cost you have a setup cost. So, you must have an estimate of this set of cost usually the setup cost is estimated per setup basis.

So, let us assume that the Q is the total order quantity; you must have an must get an expression for Q ultimately. The tp is the production term p is the production rate, small p is the production rate the small d is the demand rate or the consumption rate, capital A is the setup cost per setup. So, there must be a good estimate for this one and h small h is the holding cost per unit per year; that means, it is in absolute terms; that means, once the you know the purchase per unit basis also you can calculate, and so, once the price is the price is known or the production cost is known.

So, first which go for say percentage and then once the percentage is known and once the base price is known, you can calculate. So, the holding cost per unit basis and per year is it.

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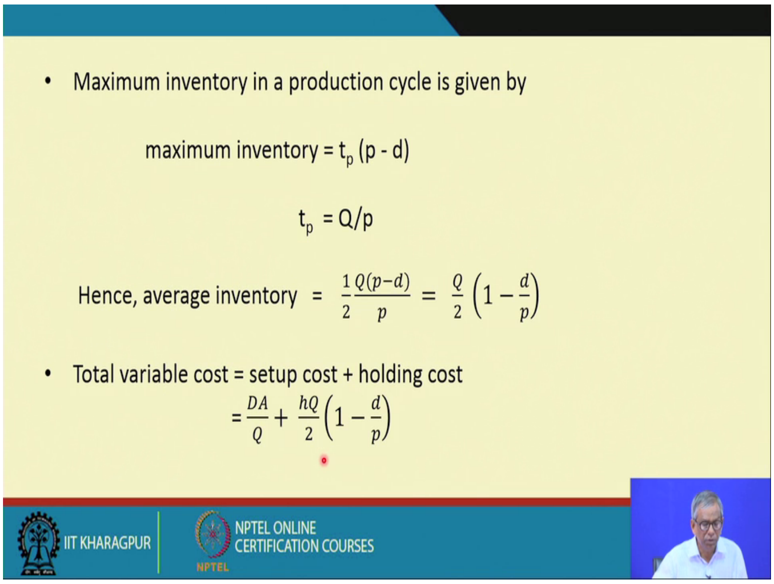

So, now the maximum inventory in the production cycle is given by; obviously, you just refer to the inventory profile. So, the maximum inventory is tp into p minus d, and the tp is Q by p is it you just refer to then inventory profile. Hence, the average inventory is half of the maximum inventory half of Q by p; that means, tp is replaced by Q by p over here into p minus d; this is p minus d and so, on further manipulation what you find Q by 2 into 1 minus d by p.

So, the total variable cost consists of the two types of cost the first one is the setup cost and the second one is the holding cost. So, the setup cost is given as; that means, the setup cost per setup is capital A and how many setups you have that is capital D by Q what is capital D? Capital D is the annual demand and Q is the production quantity.

So, D by Q into A plus Q by 2 into 1 minus d by p is the average inventory average inventory and you is a incur and holding cost of each per unit per year. So, that is why it is multiplied by small h.

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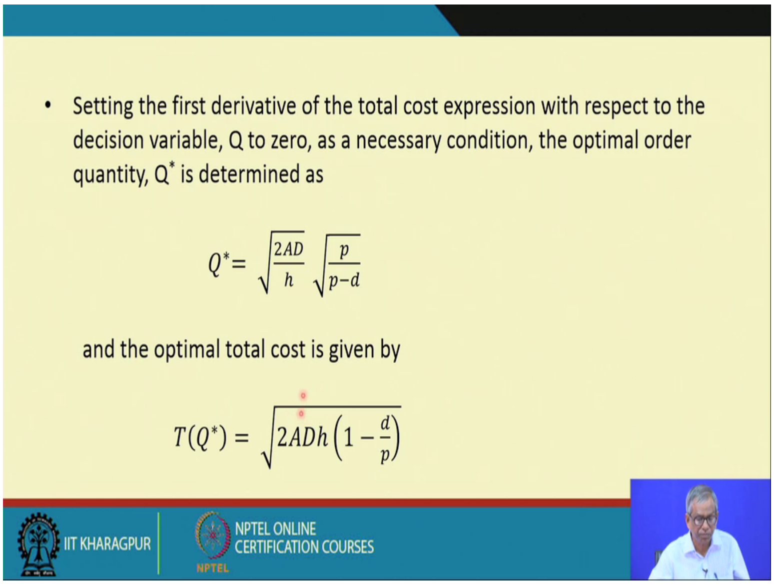

So, once you have these expressions then what you do? The remaining steps are very simple setting the first derivative of the total cost expression is it assuming that the, this is a they continues the function and that is why differentiable. So, once you have the total cost expression, now what you do? You differentiate with respect to the decision variable, as a necessary condition the optimal order quantity Q star is determined as this expressions you have the. So, you have an expression for Q star and when we use this expression for the Q star, you get the expression for the optimal total cost.

So, this is the optimal total cost so, please you substitute you have an expression for the total variable cost in terms of Q, and you substitute the expression of Q with this expressions. So, ultimately you will get the optimal total cost expression ok.

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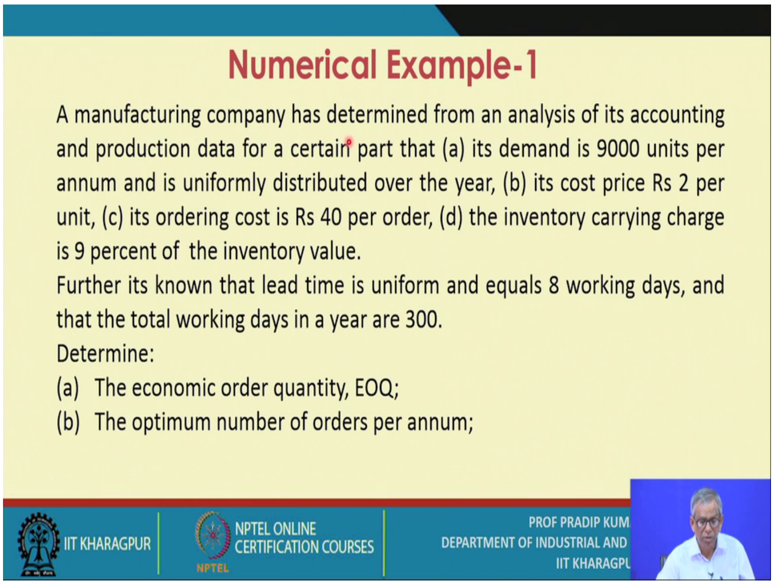

So, this is a numerical example. So, the manufacturing company has determined from an analysis of its accounting and production data for a certain part that is demand is 9000 units per annum and is uniformly distributed over the year; that means, the demand rate is uniform. Its cost price is rupees 2 per unit this is just an example, you go through this example all the steps you follow its ordering cost is rupees 40 per order.

The inventory carrying charge is 9 percent of the inventory value or say average inventory, which you hold; further its known that the lead time is uniform and equals 8 working days, and then the total working days in a year are 300 determine the economic order quantity EOQ, the optimal number of orders are annum.

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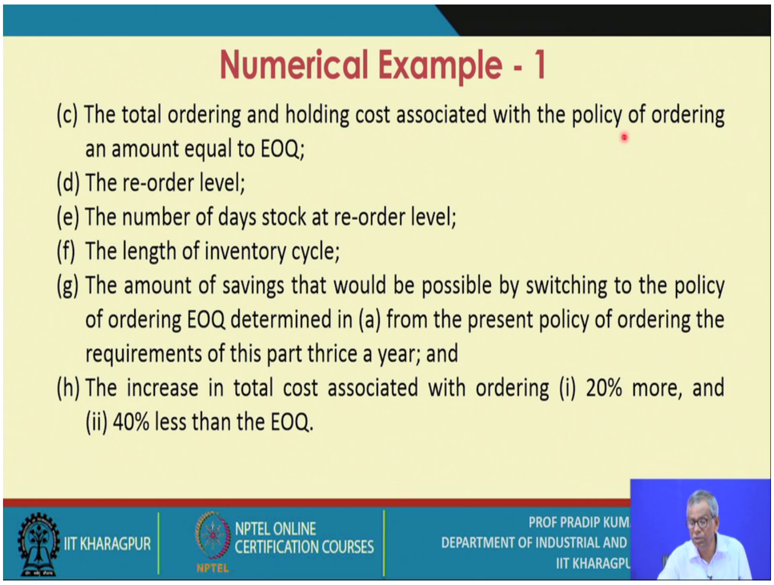

The total ordering an holding cost associated with the policy of ordering an amount equals to EOQ. The reorder level you need to determine, the number of days stock at reorder level the length of inventory cycle, the amount of savings that would be possible by switching to the policy of ordering EOQ determined in a from the present policy of ordering the requirements of this part thrice a year, and the increase in total cost associated with the ordering 20 percent more or 40 percent less than the EOQ.

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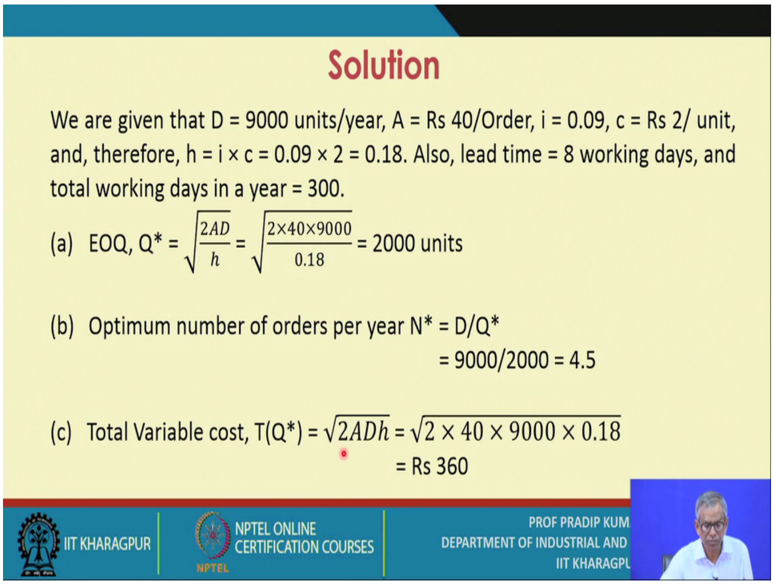

So, these are the so, these all the solution steps are given. So, please go through all the steps all the details we have written.

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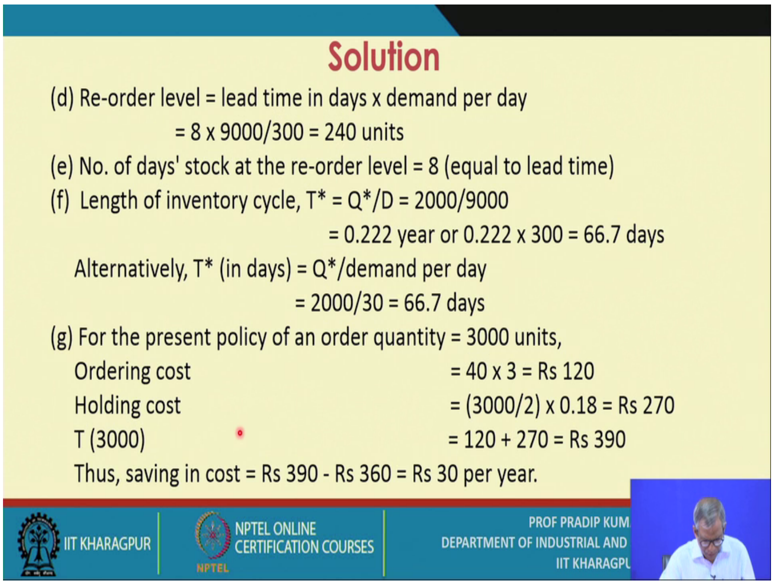

So, these are the solutions I am sure that you will be able to follow the steps, all the detail calculations are given and the kinds of the formulations or the expressions you will given already expressions you are supposed to supposed to use to solve this problems and; obviously, these expressions are already been derived and explained.

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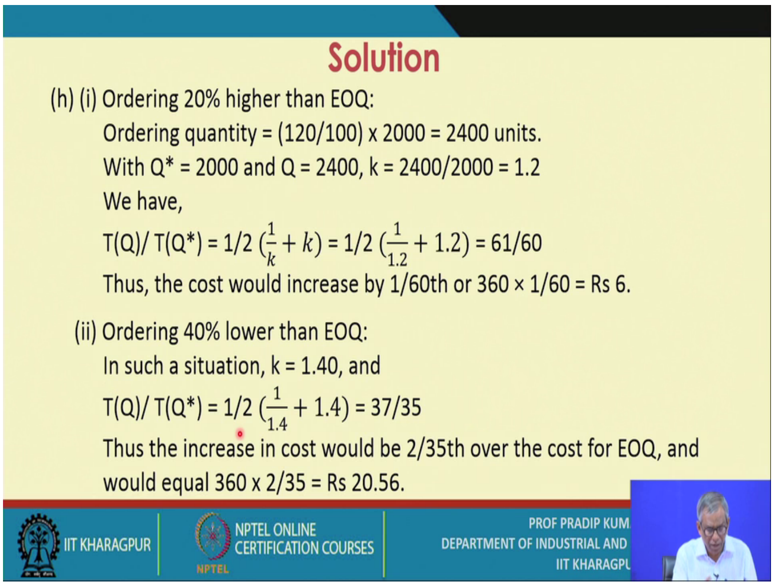

So, these are the solutions.

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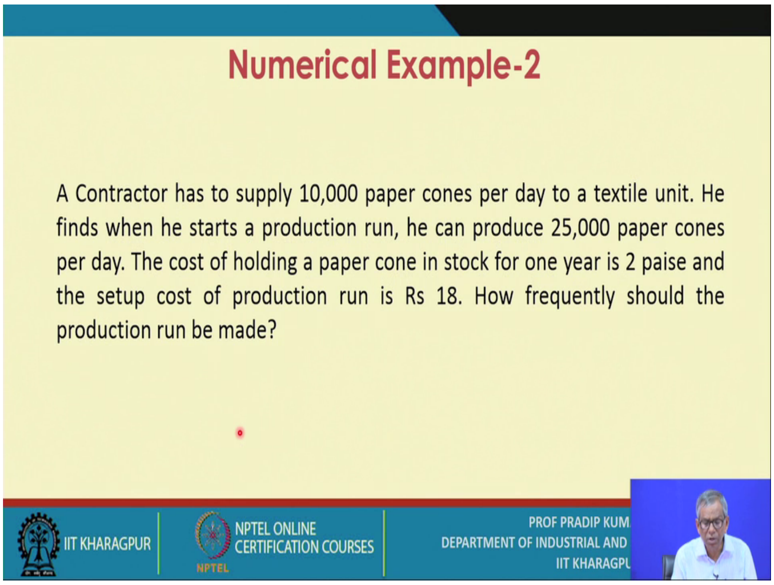

Now, this is the second numerical example, a contractor has to supply 10000 paper cones per day to textile unit, he finds when he starts a production run he can produce 25000 paper cones per day this is another typical problem. The cost of holding a paper cone in stock for every one year is two price 2 paise and the setup cost of production run is rupees 18. How frequently should the production run be made?

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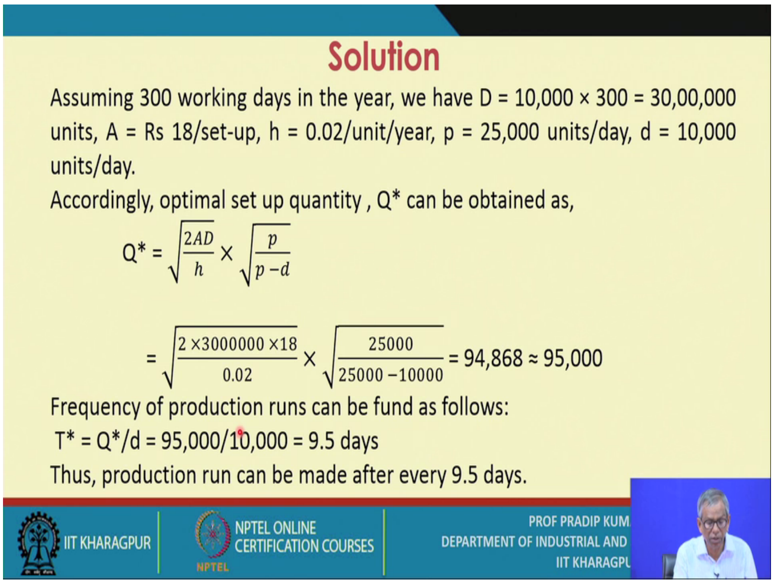

So, this is the typical problem. So, here it is what you try to do? You try to determine the, that economic production quantity. So, this is a problem related to economic production quantity and that is why it is related to this problem is related to self-supply case. So, you have the expression for Q star and you determine that the frequency of production run that is that is production run length that is 9.5 days. So, this way you solve the problem ok.

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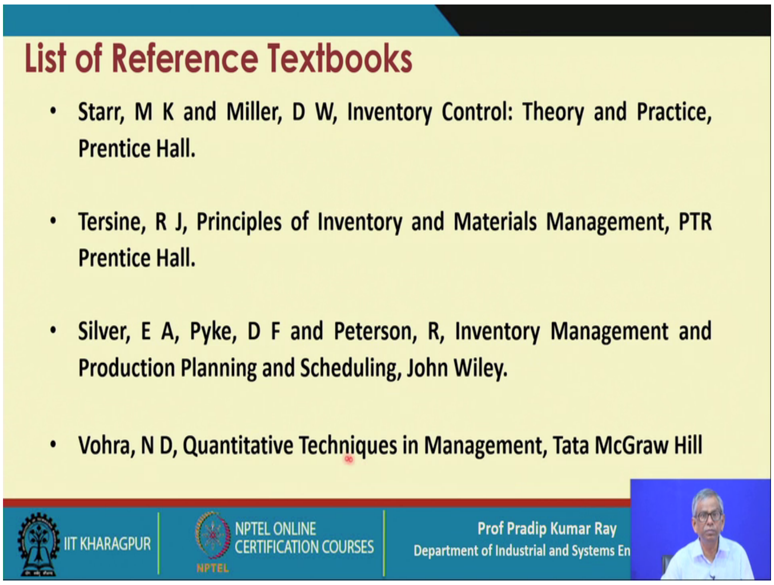

So, what is important is that you follow. So, we have discussed what is EOQ its application, what could be a typical the numerical the problem are related to EOQ and the second one that we have discussed that is self-supply case, then how to determine the economic production quantity. So, the all the steps we have explained and we have also taken up one the typical numerical problem.

So, I hope that when you go through this the typical numerical problems. So, here understanding will be better and always you know I have explained the steps meticulously point by point. So, that you do not have any doubts all the steps if it follow so.

Thank you. So, these are the text books.