Management of Inventory Systems
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Lecture – 21
Dynamic Inventory Problems under Certainty

So, during that the fifth week of our the lecture sessions on the management of inventory systems, we are going to discuss the important topic called dynamic inventory problems under certainty.

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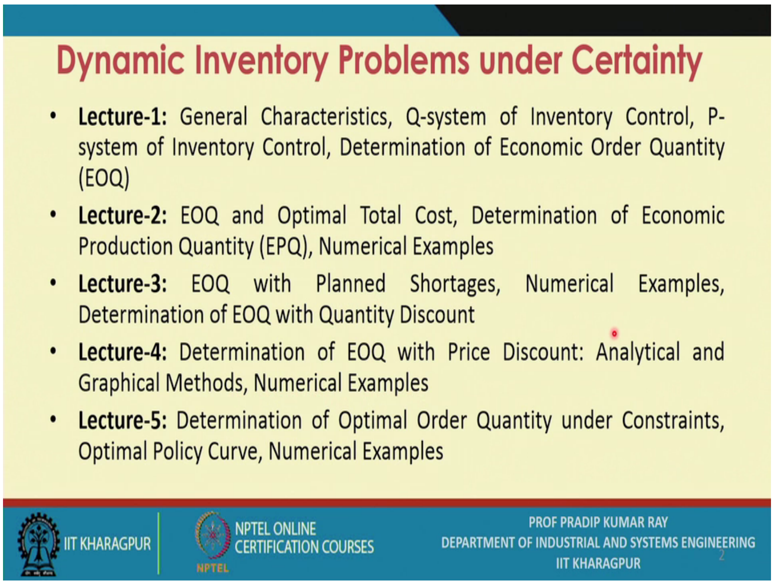

As we have already mentioned that the inventory problems are essentially classified under four categories. And one important category is dynamic inventory problem under certainty. So, during this week we are going to discuss the several important aspects are related to this problem; particularly or the specific topics lecture wise, I want to tell you what are the specific topics we are going to discuss in detail.

In the first lecture, we will mention the general characteristics of the problem. Then two types of inventory control systems they are referred to as a pure inventory control system and they are this systems are called Q-systems of inventory control and P-systems of inventory control. Q stands for the quantity; and P stands for period. So, we will explain it in detail theses two types of inventory control systems.

Then the next topic we are going to discuss that is determination of economic order quantity or EOQ. During the second lecture session, we will discuss the EOQ and optimal total cost, what is the relationship between EOQ and optimal total cost; Then determination of economic production quantity particularly when it is a self supply case and we will cite several numerical examples.

In the third lecture session, we will discuss a specific model called EOQ or economic order quantity with planned shortages. There are situations where you know the shortages can be planned for certain kinds of items. So, this particular order quantity model with planned shortages, we are going to discuss. And there will be numerical examples and how to determine EOQ with quantity discount; this is also an important topic.

During forth lecture session, we will continue our discussion on the determination of EOQ with price discount. There are several issues one after another we will take up all these issues; and both analytical and graphical methods we will discuss. And there will be obviously numerical examples. During the last lecture session - during this week, we referred to determination of optimal order quantity under constraints. And optimal policy curve this particular concept we should be aware of; and we will take up several other numerical example. So, this will be our coverage during the fifth week.

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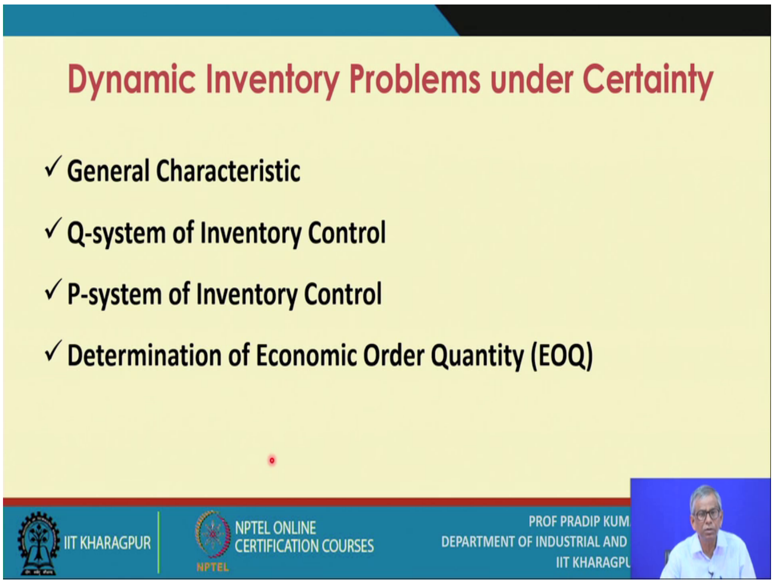

Now, let us first talk about certain general characteristics of this kind of problem called dynamic inventory problem under certainty. And then once characteristics are known, then we will explain in detail the working of two kinds of inventory control system. The first one is Q-system of inventory control; and the second one is the P-system of inventory control. Then we will explain in detail how to determine the economic order quantity or EOQ. So, during the first lecture session, this will be our coverage.

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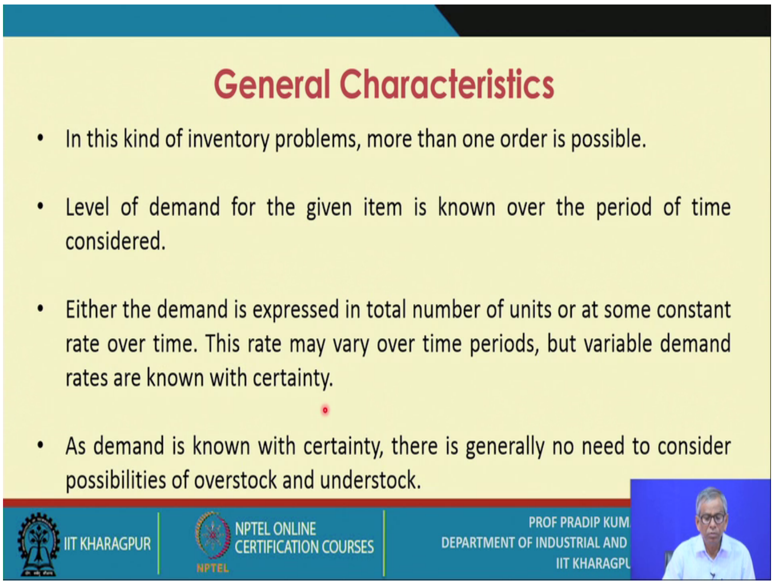

Now, let us first talk about the general characteristics of this type of problem called (Refer Time: 04:54) inventory problem under certainty. So, several points we have listed. So, let us first discuss all this points one after another. Now, in this kind of inventory problems more than one order is possible; we already we have defined that is why it is referred to as a dynamic inventory problem. The level of demand for the given item or the given inventory item is known over the period of time considered say it is 1 month suppose that period.

So, what is you know the demand during 1 month period that is to be known. Similarly, suppose the demand level of an item of an item for the next 12 months or 1 year this is this demand level is known with some certainty, so that is our assumption at this point in time. Now, either the demand is expressed in total number of units say 1000 units or 10,000 units physical units or at some constant rate over time like someone might say that the daily demand the rate is for a given inventory item may be say just 1000 units or

2000 units. So, the depending on the production rate, so you specify this the rate like it could be on the hourly basis or daily basis or weekly basis, whatever it is so the total demand is not known total demand level is known what is known is actually the demand rate.

And what we will assume that when we say that whether it is a dynamic inventory problem under certainty, we will assume that this demand rate does not vary over time. So, this rate may vary over time periods, but variable demand rates are known with certainty. Like in certain cases like so you are given the next 12 months the data on demand level. And you find that month-to-month, there is there is a change in demand.

But this is the demand is known with certainty, but it is a variable demand in the sense that this month wise there will be month wise variation, but year wise there will be the no change in a month’s demand so that sort of situation or that kind of so the demand level is also referred to as the demand with say certainty ok. So, as demand is known with certainty there is generally no need to consider possibilities at overstock or under stock.

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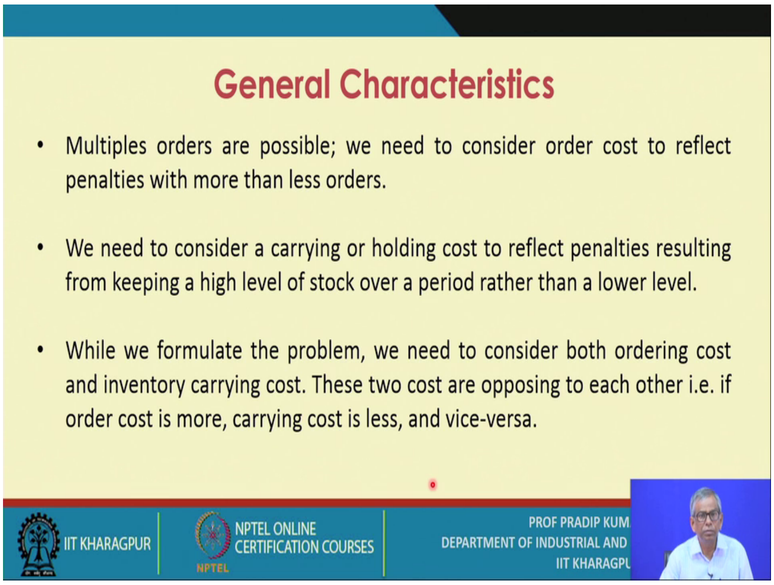

In the previous lecture sessions, we have explained that why there is a an over stock or there is an under stock situation. And what we have mentioned that you know the effectiveness of any inventory control system for that matter, and there is a quality of any inventory control system is just by the amount of say the overstock as well as the amount of under stock that you generate with respect to a given segmentary policy for an inventory item, is it. So, here in this case as the demand is known with certainty, there is no question of having under stock or overstock.

Multiple orders are possible; we need to consider order cost to reflect penalties with more than less orders. Now, now you are referring to the, what are the kinds of relevant costs you need to consider while you formulate this problem. So, as multiple orders are possible; obviously, you need to replace an order. And while you place the order for a given item; say outside supply case, obviously, you need to carry out certain activities. And any activity you carry out there is a cost associated with it. So, I need to be aware of all these activities related to placement of an order. So, when you consider all the individual cost elements, you can an estimate of the ordering cost so that is the first type of the cost you must consider.

We need to consider a carrying or holding cost or inventory carrying or holding cost to reflect penalties resulting from keeping a high level of stock over a period rather than a lower level. That means, if for keeping the stock in good condition, so I need to take certain actions and any actions, you take there is a cost associated with it. So, I need to be aware of all these actions all these activities. And when you consider all these activities and corresponding cost elements, you can have an estimate of the so called inventory carrying or the holding cost ok. So, this point also we have we have discussed in the previous lecture sessions.

While you formulate the problem, we need to consider both ordering cost and inventory carrying cost. And these two costs are opposing to each other; that means, if the demand remains constant and if you have more number of orders, obviously you know the ordering amount per order will be less. And if the per order the ordering quantity remains less, obviously, the average inventory which you hold will come down. And if the average inventory the decreases, the inventory holding cost also will decrease.

And but similarly if you place a less number of orders, obviously the average inventory will be more at any point in time or how much the inventory you keep on an average and if the average inventory which you hold is more, obviously, the corresponding inventory carrying cost will be more. So, that is why we say that these two costs are opposing to each other that is if the order cost is more carrying cost is less and vice-versa.

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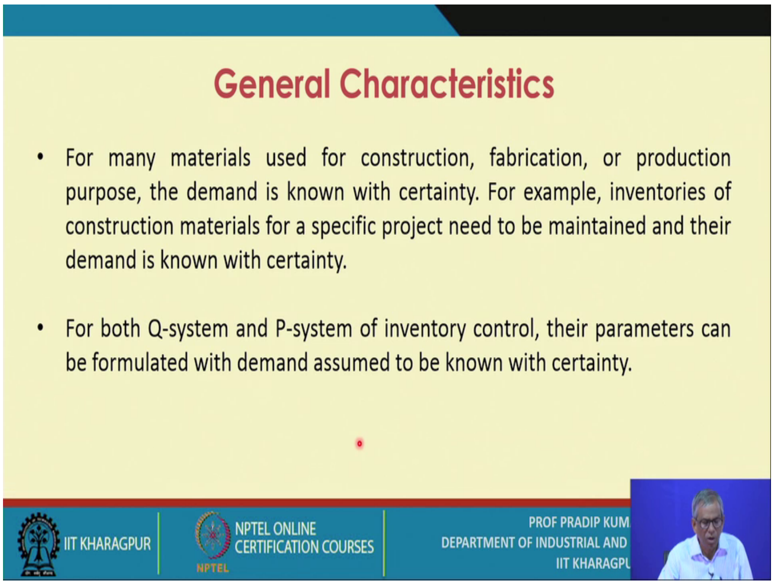

For many materials used for construction, fabrication, or production purpose, now we are citing examples that where you may assume that the demand remains constant or it is a problem under certainty. So, for many materials used for construction, fabrication, or production purpose the demand is known with certainty. For example, inventories of construction materials say steel girder.

So, the amount of steel girder that you that you need for constructing a house so that is known and with certainty; And this the amount you need to be maintain that means, the inventory of this particular material you need to it needs to be maintained and their demand is known with certainty. So, there could be several examples of the demand the remaining so the constant at a particular during a particular time period. For both Q-system and P-systems of inventory control, their parameters can be formulated with demand assumed to be known with certainty.

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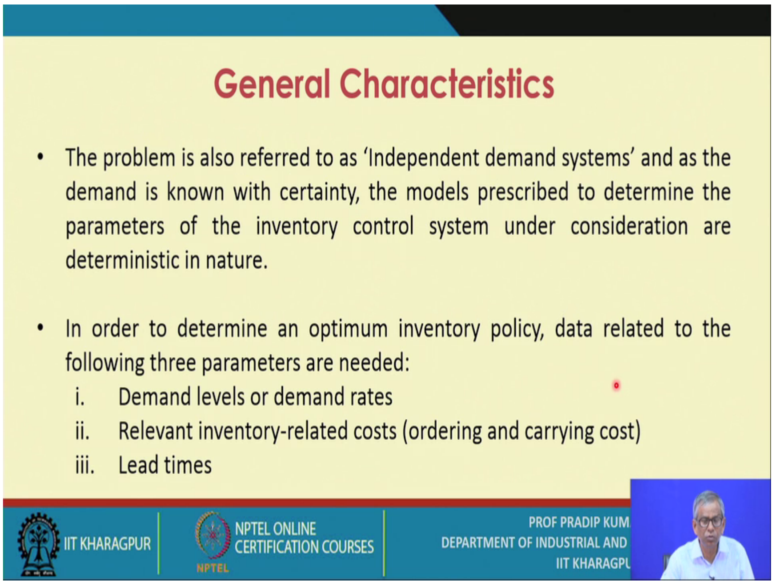

As we have already mentioned that for any kind of inventory control systems so the two questions are raised. The first one is when to going to other; and the second one is how much to order. And as well as Q-system is concerned the when to order that is to be decided that means, you really do not know in advance, when to place the order. But if you want you want to decide of you decide to place an order, obviously at a particular point in time the order quantity is known and remains constants, so that is a Q-system.

Whereas, for the P-system what we have that means at what point in time you need to place an order that that timing is known, but the order quantity may vary, is it ok. So, the problem is also referred to as that means this kind of problem dynamic inventory problem under certainty this is also referred to as an independent demand system. So, please note this particular say the point that is it also referred to as in many text books in many the case studies, these are referred to this problems are referred to is the independent demand systems and as the demand is known with certainty, the models prescribed to determine the parameters of the inventory control systems under consideration are deterministic in nature.

So, in this particular say during this week, in all these during the lecture sessions, we will refer to several kinds of numerical problems. And you will find that that the demand the level of a particular item considered does not vary over on the time periods. So, in order to determine an optimal inventory policy, data related to the following three parameters are needed. So, please note this point. First one is the demand levels or the demand rates has already we have mentioned. Relevant inventory-related cost like in this case the ordering costs as well as the inventory carrying cost of inventory holding cost you need to consider. And the third important say the aspect you need to consider that is the lead time.

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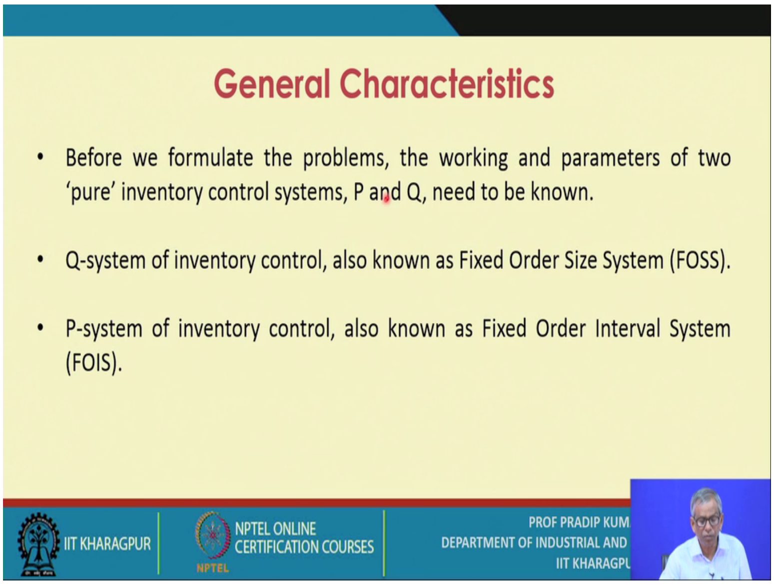

Before you formulate the problems, the working and the parameters of the two pure inventory control systems has already I have pointed out, this P systems as well as the Q-system of inventory control, they referred to as the pure inventory control system. And either in any inventory in any production also the department or any work units related to a particular so the inventory items. Either you come across the Q-systems of inventory control or P systems of inventory control, and in many instances, you may also find a combination of P and Q is it ok. It is not exactly Q or it is it is not exactly P, but it is a combination P and Q. In course of time we will be referring to such inventory control systems.

Q-systems of inventory control also known as the fixed order size systems - FOSS. Please note this particular term that means, this is referred to as a fixed order size systems. Sometimes this is also referred to as a continues review system. So, whenever you place an order, the order quantity or the order size does not change that is why it is a refer to as the fixed ordered size systems. Whereas, when you use the P-systems of inventory control, we know that what is the order interval, at what point in time you are supposed to place an order, so that is why this is also referred to as the fixed ordered interval system FOIS. So, the both the systems we will discuss.

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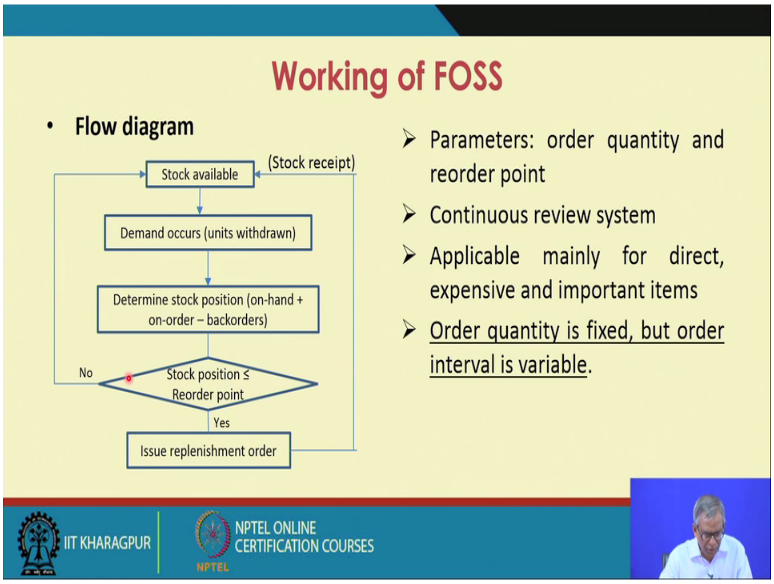

So, the first we will refer to the working of fixed order size systems or FOSS continuous review system. Now, this is the working. So, we have explaining the working of FOSS with the flow diagram. So, let me first clearly you know the explain this flow diagram. So, what are the activities involved. Like the first you start with the stock available for the given inventory item.

So, now, if the demand occurs, obviously, from the stock the units will be withdrawn. And this demand may occur at any point in time. So, you are ready with the stock for the given inventory item. So, the demand occurs means units withdrawn. Then what you do that means, you just check what is the current the stock position; that means, you need to determine the stock position. So, what is this stock position? The stock position is the on hand ; that means, what is physically available at that point in time plus the on order that means, for the given item already you have placed an order and the order has not yet arrived ok. So, obviously, it is pending what do you need to consider that amount so that is referred to as the on order minus the back orders ok.

That means, what is happened that due to some point in time does the customer has arrived, he or she is asking for that particular item, but there is no stock. So, obviously, you cannot meet his or her demand and that is why you ask them to wait, and they are waiting. So, this case is referred to as the backordering case. So, as soon as you the stock position that means the back order quantity should be subtracted.

Now, you check whether the stock position is less than or equals to reorder point or not; if it is not, then obviously the stock is available. But if you find that the stock position is less than the reorder point that means, it touches the reorder point has already have pointed out, they are two parameters of Q-systems of inventory control. First one is order quantity and the second one is the reorder point. So, the stock position at any point in time when the units are withdrawn, you check with the reorder point. So, if the stock position is less than or equals to reorder point, obviously, what do you do, you issue replenishment order is it ok. And obviously, with the laps of this leads time the stocks are received, is it ok. So, this is the flow diagram.

Now, the certain important points you please note down. First one is the what are the parameters of this particular system. First one is order quantity, and the second one is reorder point already I have mentioned. It is also referred to as the continuous review system applicable mainly for now this is the important point. So, applicable mainly for direct material that means, these are directly used for producing the product at the production stage and that is and this material is the part of the product. So, this those materials are referred to as the direct the inventory item or the direct material.

Or if it is an expensive item, that means, units purchased price is very high, and if it is considered to be an important item. Already we have referred to the classification of the inventory items the classification schemes. And we have extensively discussed the selective inventory management. Now, order quantity here in this case the order quantity remains fixed, but the order interval is a variable ok. So, this is the working.

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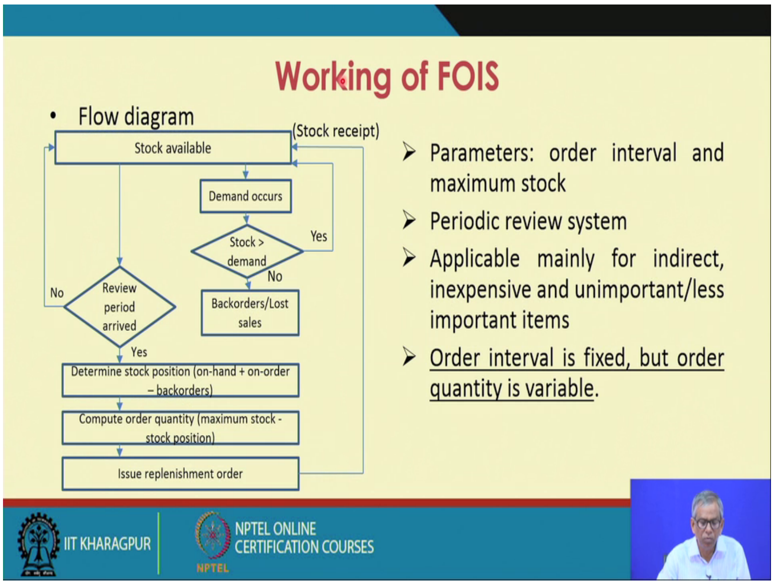

Now, what is the working of fixed order interval system? So, here also we have drawn this flow diagram. So, what you have first the stock available that is the starting point. The review period arrived the first question you ask that means suppose yearly just once replace the order, there are many such items mostly indirect materials you use the periodic review systems or FOIS. So, there what you do that means, a every year just once you place an order and meeting the requirements of the entire year. So, the review period arrived.

So, you need to place an order on first April of any year. So, the reorder the financial year of the fiscal year starts on first of April. So, that to the review period you just take whether it has arrived or not. If it is no, that means, you go back to the stock available you do not need to take any action. Now, during this time what might happen that the demand may occur. And if the stock is greater than the demand, there is absolutely no problem that means the stock is assured; but supposing the stock is less than the demand, then obviously, you are you are unable to fulfill the demand. And depending on the responses of the customers there could be two situation; first one is the back ordering and alternatively we can have also the lost sales.

So, now, if the review period arrived, suppose it is yes, then determine the stock position. So, again stock position is on hand plus on order minus the back orders then compute order quantity. Now, how do you compute the order quantity now in this particular system, there is one parameter called the maximum stock or the maximum inventory. So, later on we will take up this case and we will tell you how to determine the value of this particular parameter called the maximum stock.

So, the maximum stock is one of the parameters; this value this value will be known. So, how do you determine the order quantity maximum stock minus the stock position is it ok. So, obviously, the stock position varies from one period to another and so also the order quantity. And then once order quantity is known we issued replacement order; and then when the lead time is over the stock is received so as per the order quantity.

Now, what are the important points to be noted? First one is here the parameters are the order interval and the maximum stock as already I have pointed out. It is also referred to as the periodic review system applicable mainly for indirect inexpensive and unimportant or less important items ok. We have already explained or do you mean by the importance of an item from which perspective you define are the importance of an inventory item. And here the order interval is fixed that is why we are saying that you just check whether the review period arrived or not that means, it is known in advance, but the order quantity is a variable.

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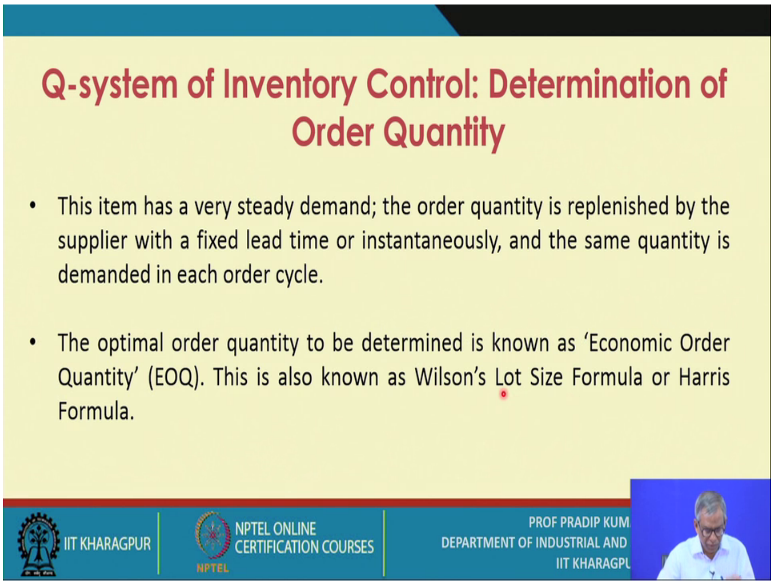

Now, so let us I determine the order quantity the Q-systems of inventory control. Now, this item is a very steady demand the order quantity is replenished by the supplier with a fixed lead time or instantaneously. And the same quantity is demanded in each order cycle. The optimal order quantity to be determine is known as the economic order quantity or EOQ. So, please note this point that many a time we use the term economic order quantity. And this term has become very, very popular. So, this is also known as the Wilson's Lot Size Formula way back in 1913, it was the first time it was introduced by Wilson and that is why it is refer to as the Wilsons lot size formula. It is also refer to the Harris formula, because the he actually the person who extensively used this particular say EOQ the model.

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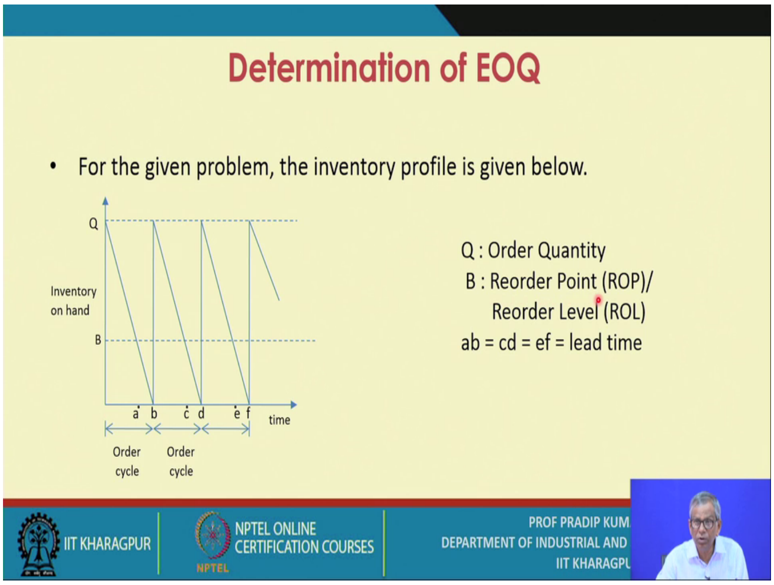

So, for the given problem, the inventory profile is a given below. Like here Q is the order quantity B is the reorder point ROP or reorder level. So, these are the two points. Now, ab, cd, ef these are called lead time that means, you have you starting point at time t equals to 0, you start with the order quantity Q and then you start consuming it and it is a demand with certainty that means, the consumption rate is uniform.

And then as soon as it reaches the reorder point, you place an order at time a. And the lead time is known obviously so at time b, you get the replenishment and so immediately and one shot yeah this from it from 0, it becomes Q again. So, this is one order cycle, his is the first second order cycle. So, this way you repeat ok. So, this is referred to as the order cycle. So, in a year there could be several such order cycles depending on the production rate.

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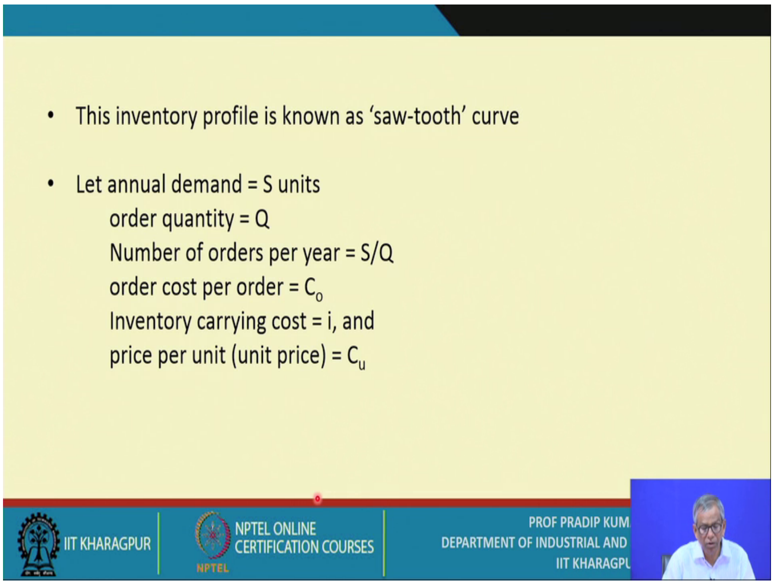

Now, this inventory profile is known as the saw tooth curve; obviously, it looks like the saw tooth. So, let us use certain notations. Let annual demand is S units, order quantity is Q, number of orders per year is S by Q, order cost per order is C o, inventory carrying cost is i; that means, as a percentage as a fraction of the total so average inventory which you hold this point already we have mentioned in the previous lecture sessions, and unit price for the given item is C u.

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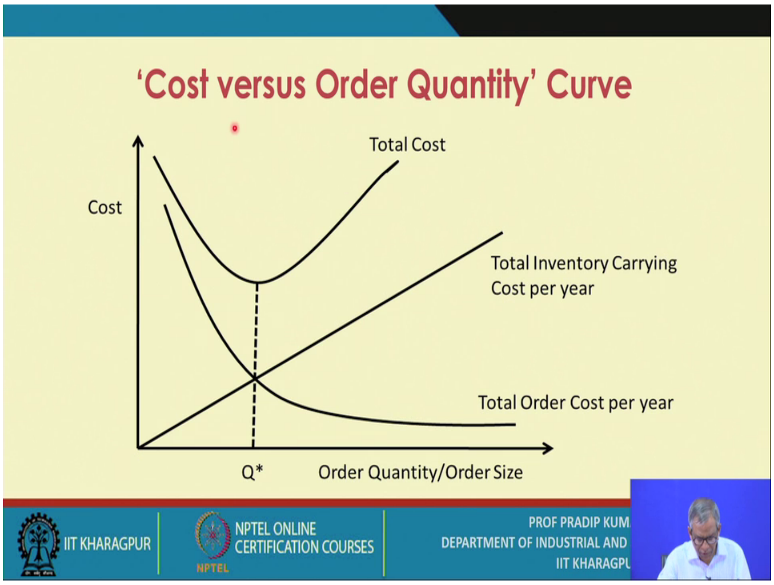

So, what you have so this is the cost versus order quantity curve. So, you have the total inventory carrying cost per years. So, as the order quantity increases, so the total inventory carrying cost increases. And as the order quantity increases or the order size increases, obviously the total ordering cost per year the decreases, is it ok. So, this is an when you add both these cost, so this is the your the cost function or the total cost curve ok.

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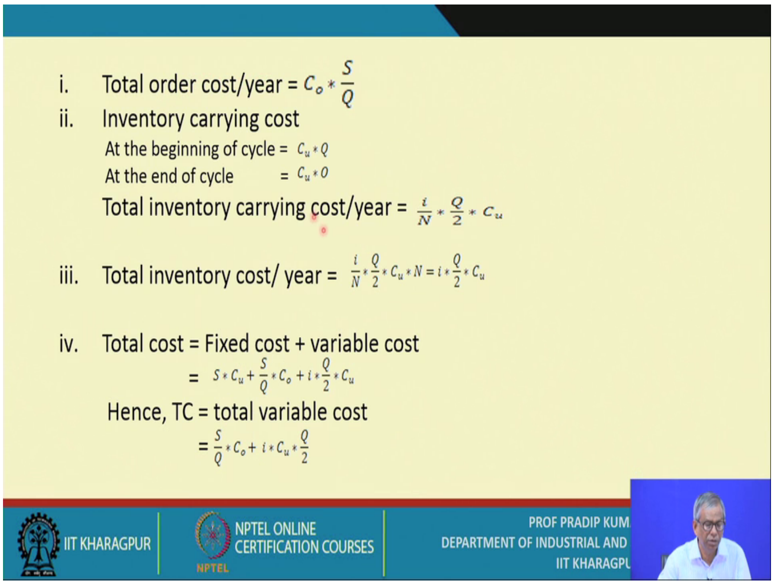

So, these please write this one. Now, the total ordering cost per year is C o into S by Q all these formulations are given. Similarly, we have calculated the inventory carrying cost. So, all the states we have mentioned the total inventory carrying cost per year is i by N into Q by 2 into C C u.

And the total inventory say inventory cost per year will be this one inventory carrying cost per year will be this one. And the total cost will be the fixed cost plus the variable cost. So, we have all the terms over here. So, please go through all the steps I am sure that you will have the proper understanding. And hence, we use the term called TC, TC stands for the total variable cost. So, this is S by Q into C o plus i into C u into Q by 2.

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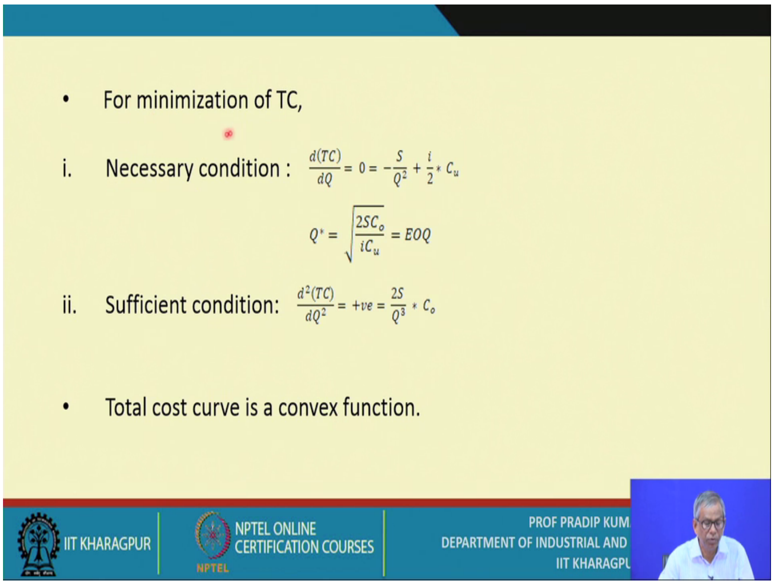

So, the for minimizations, what you need to do, you try to determine the order quantity such that that the total cost is held at a minimum level. So, for minimization of the total variable cost, necessary condition is the first derivative set to 0. And you have these expressions that is Q star root over S C o by i C u. And this particular expression is referred to as the economic order quantity.

And the obviously, for the sufficient condition for minimization, so all we will already aware of; that means, second derivative with respect to Q must be positive. So, it is 2 S by Q to the power 3 into C o. So, you can prove that this is it remains always positive. So, the total cost curve is a convex function.

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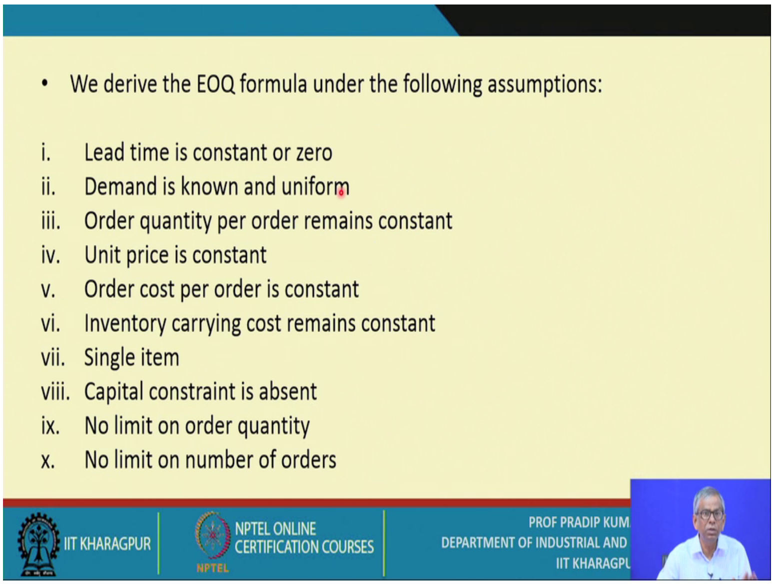

And so what are the just before I close I will just tell you, what are the assumptions we have when we derive the EOQ or the formula or the Wilson's lot size formula. There are ten some assumptions, I will just tell you one by one. So, the first assumption is the lead time is constant or zero; demand is known and uniform; order quantity per order remains constant; you please note them down unit price is constant does not change.

Whatever may be the order quantity, order cost per order is constant; inventory carrying costs remains constant; is the single item case capital constraint is absent that means, here the capital investment that means, you might say that average inventory which you hold in monitory terms; no limit on the order quantity it could be any value; and no limit on the number of orders. So, these are the ten the assumptions we have while you formulate the EOQ formula.

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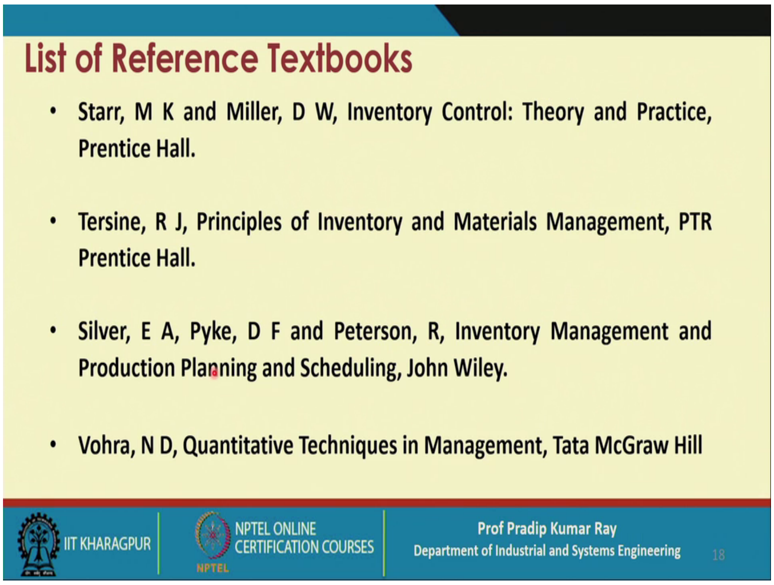

We will explain all these aspects later on in our subsequent lecture sessions. So, the first lecture session is over.

Thank you.