Management of Inventory Systems
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Lecture - 20
Static Inventory Problems under Uncertainty (Contd.)

During this week we have discussed till now, a number of issues related to Static Inventory Problems under Uncertainty. And now, during this the last lecture session in this week we will just refer to certain important points that you must the keep in mind while you consider the static inventory problems under uncertainty.

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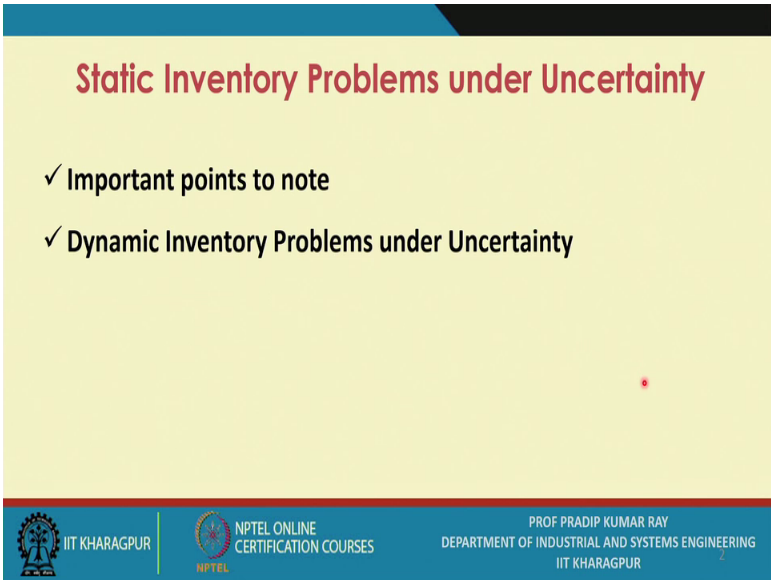

So, this important points we will we will elaborate, we will discuss and the next what you try to do that means, we will extend our you know the discussion to the dynamic inventory problems under uncertainty.

Now, while you formulate the dynamic inventory problems under uncertainty there are many you know the common aspects related to common aspects you need to consider or would we have already considered for the static inventory problems. But what are the, you know the changes you have to consider while you formulate dynamic inventory problems under uncertainty, this I will explain I will highlight in specific terms.

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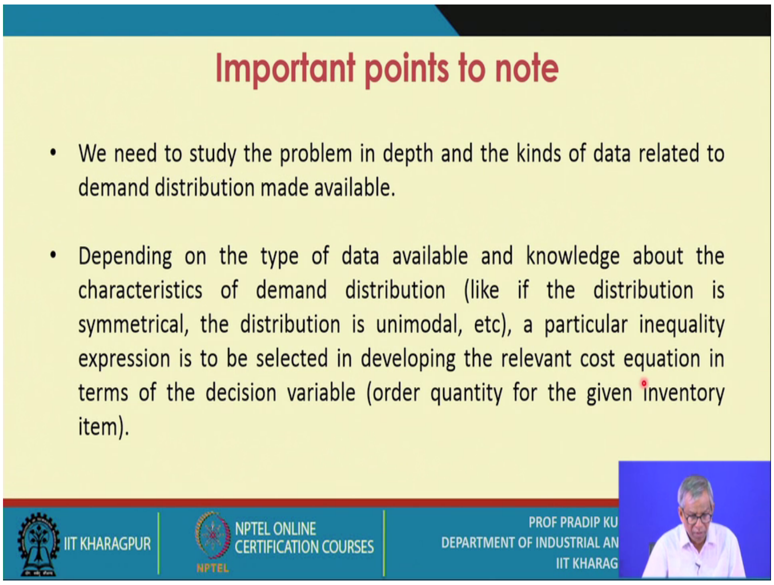

So, now, let us first consider that what are the you know the important points you should keep in mind while you formulate the problem whether it is a static inventory problem or dynamic inventory problem under uncertainty. We need to study the problem in depth that is the first objective and the kinds of data related to demand distribution made available. That means, here when you consider the case of uncertainty that means, you have not the sufficient amount of data with which you can define the demand distribution or the lead time distribution either in empirical form or in standard form.

So, but even if with the existing the data you can do several kinds of analysis which may not be that the results may not be that perfect, but without getting this results how can you run the system, is it ok. So, that is the first point you need to remember that is in depth analysis is also possible even the demand distribution is not known or the lead time distribution is not known.

Depending on the type of data available and the knowledge about the characteristics of demand distribution this point is to be noted, that means characteristics of the demand distributions. So, their knowledge you must have. Like if the distribution is symmetrical or the distribution is unimodal etcetera and so on, there are many such characteristics to how many such characteristics you are aware of. A particular inequality expression is to be selected in developing the relevant cost equation in terms of the decision variable order quantity for the given inventory item. That means, here what as we have already referred to the Tchebycheff's inequality and what you might have noticed this inequality expression is made available in different forms under different conditions, ok.

So, I have try to include or try to point out all the important or the relevant inequality expressions. And, but you also must know that under what situation for what kinds of say the characteristics of the demand distribution which particular inequality expression is to be used and valid.

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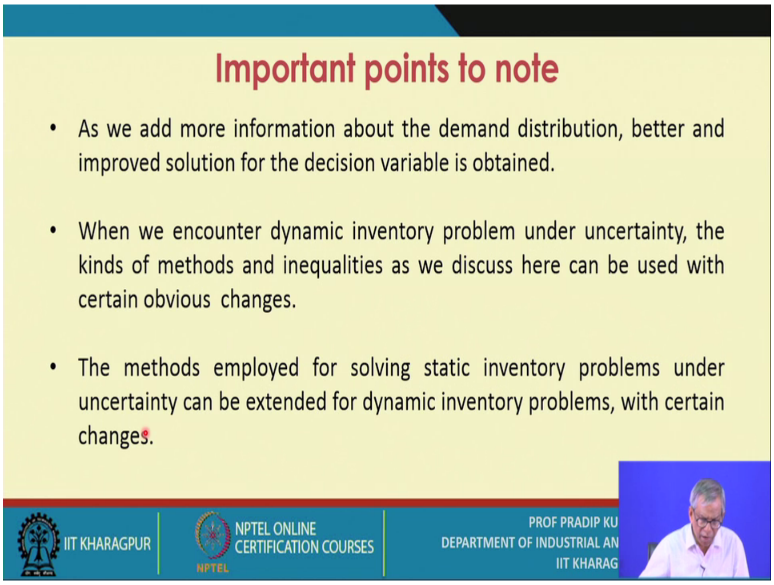

As we have more information about the demand distribution better and improved solution for the decision variable is obtained, is it ok. That means, what you say that supposing in the starting point you just you do not know whether the demand distribution is symmetrical, but still you can get a solution. But as soon as you add the information related to its say symmetricity that means, whether if you assume it to be symmetrical then it is expected that that you change that inequality expression and you consider this particular this information and you will get better solution, then the results will be better.

When you encore encounter dynamic inventory problem under uncertainty that means, more than one order. The kinds of methods and inequalities as we discuss here can be used to with certain obvious changes. Already we have discussed all those inequalities expressions or the static inventory problem, but what we are we mention that even for dynamic inventory problems almost all those you know the inequality expressions we can we can use with obvious certain obvious changes, that we will notify, we will discuss.

The methods employed for solving static inventory problems under uncertainty already you are aware of this methods some of this methods. Now, these methods can be extended for dynamic inventory problems with certain changes, ok. So, these are our comments.

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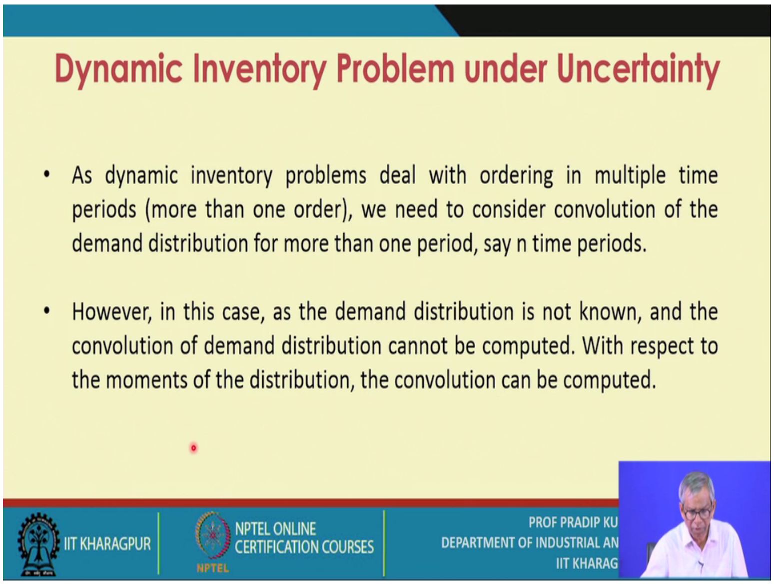

Now, as dynamic inventory problems deal with ordering multiple ordering in multiple time periods more than one order we need to consider convolution of the demand distribution for more than one period, say n time periods, ok. So, may be the distribution is given for one week whereas, you need to consider several such weeks n number of weeks. So, in many a time what you need to do that means, you need to convolute n number of times the original distribution.

However, in this case, as the demand distribution is not known, not known and the convolution of demand distribution cannot be computed. So, this is one restrictions you have with respect to the moments of the distribution the convolutions can be computed. In majority of the cases what you find that up to the fourth moment, you can complete. So, what you need to do, that means for the dynamic inventory problem as an as the demand distribution is not known so obviously, you unable to convolute the original demand distribution. So, what you can do as an alternative, the next best approach that you can convolute the moments of the distributions, ok. So, for that you need to use several kinds of formulations.

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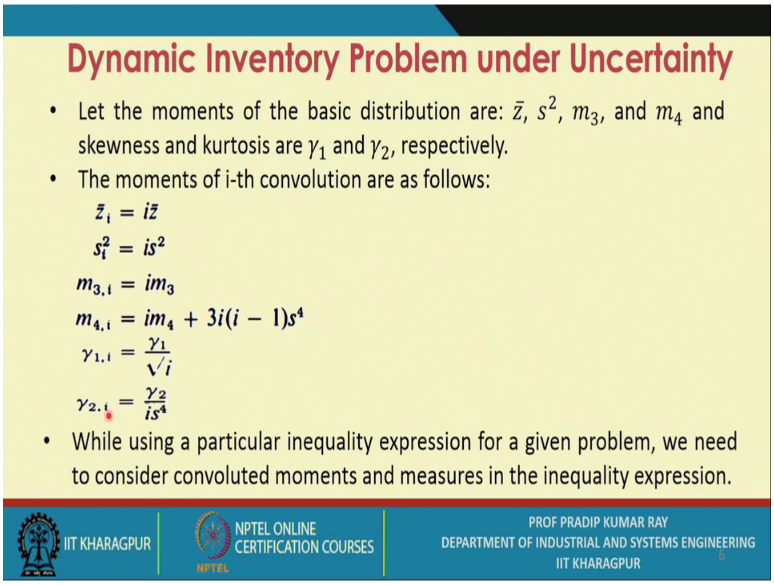

So, this right now, I am presenting that how to what are the how do you convolute say the different the moments of the distributions. Let the moments of the basic distributions are that is z bar, that is the second moment that, ok; first moment z bar. Next one is the variance s square, m 3 is the third moment and the fourth moment is m 4. And you have the information related to the skewness as well as the kurtosis of the distribution and the notations are gamma 1 and gamma 2.

So, if you consider the ith convolution of the moments. So, what will be the ith convolution of say z bar that is i into z bar? What is the ith convolution of the variance that is s i square the notation we have used and this is i into s square, ok. So, this is s square is the actually you know the variance of the original distribution. Similarly for say the ith convolution of m 3, ith convolution for m 4 and you have these expressions. So, you just note these, the expressions, and similarly ith convolution for say the skewness and you have the expression for the ith convolution of the kurtosis, ok. So, please remember these expressions for the ith convolution, for several kinds of moments as well as the skewness and the kurtosis of the distribution.

While using a particular inequality expression for a given problem say, Tchebycheff's inequality we need to consider convoluted moments and measures in the inequality expressions, ok. So, we will take up a number of problems numerical problems and this the point will be made very clear.

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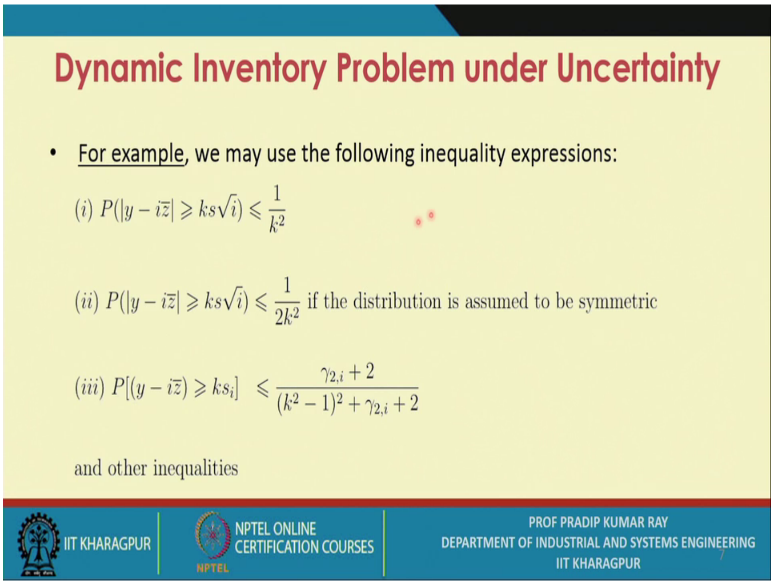

Now, now, this is the examples you may use the following inequality expressions for the dynamic inventory problems. So, what you try to do? That means, this is you know you refer to just original as the Tchebycheff's inequality and here what are the changes that means, this is basically ith convolution of the mean and this is essentially the ith convolution of the standard deviation, ok. So, this changes you have to make in the original say the Tchebycheff's inequality.

You have these expressions also like when the distribution is assumed to be symmetric. That means, the left hand expression remains same, with the convoluted expressions, but the right hand expression is 1 upon 2 k square. So, this logic already we know we have we have used and similarly this is the another expressions of the inequalities, but here what you try to do that means, this is the kurtosis gamma 2 that is you know this is ith convolution of gamma 2 and so that you need to consider, is it ok. And similarly the other inequalities you need to change, as per the convolution of a particular moment.

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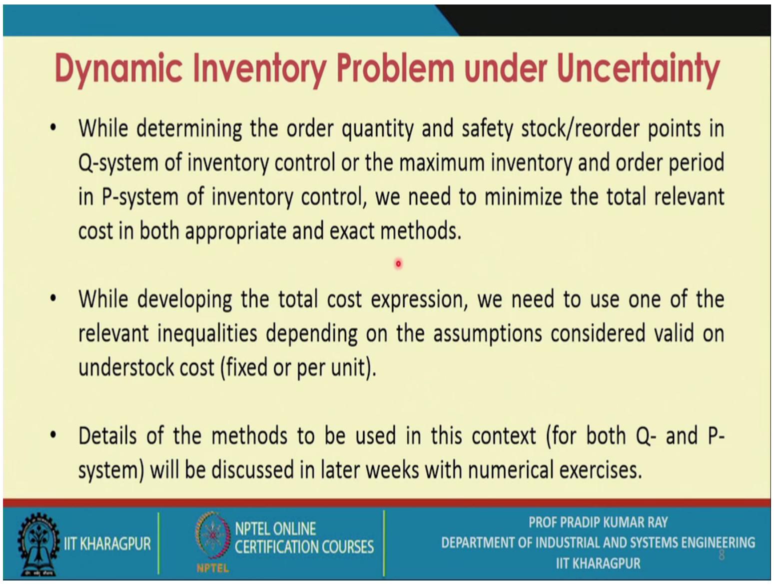

While determining the order quantity and the safety stock for the reorder points say for the Q-system of inventory control or the maximum inventory and the order period in P-systems of inventory control. So, these are the you know the inventory control parameters and so we need to minimize the total relevant cost in both appropriate and exact methods. So, this point already we have elaborated.

While developing the total cost expression, we need to use one of the relevant inequalities depending on the assumptions considered valid on the understock cost, fixed or per unit. That means, what you need to do? You need to go through the problem statement and you must understand that what are the conditions specified and what are the restrictions, up to which moment the information is made available, what is how many times you have to convolute the moments the value of i. So, all these information you note down and accordingly from the list of so the inequality expressions you select the most appropriate inequality expressions. So, this is this is considered to be a systematic approach.

Now, the details of the methods to be used in this context for both Q and P systems will be discussed in the later weeks with numerical exercises, ok. So, we will differently take it up.

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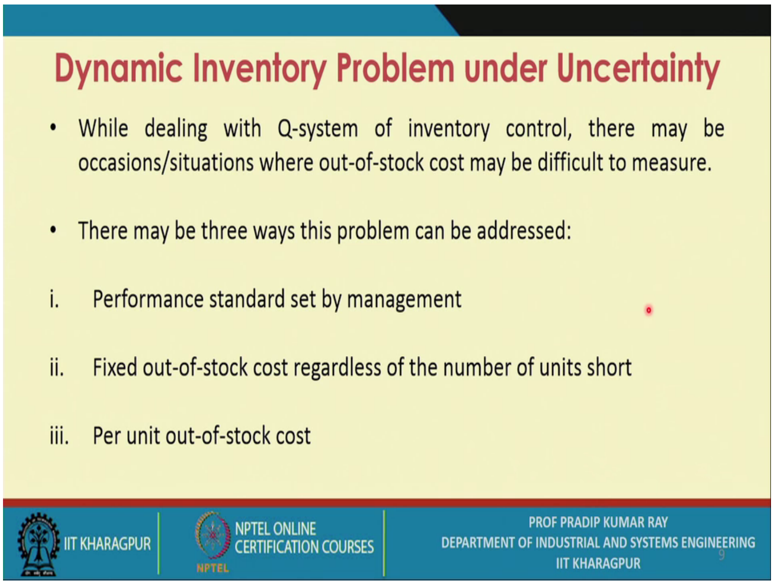

Now, right at this moment what will try to do that means, in dealing with the Q-system of inventory controls you also must know that as an alternative to say the knowledge of the distribution or the demand distributions, so: what are the next based alternatives you may have. And basically you are you are modelling the problem or you are formulating the problem under uncertainty, under uncertainty conditions.

So, while dealing with the Q-systems of inventory control let us first take up the Q-systems of inventory control and then we move to the P-systems of inventory controls there may be occasions or situations where out of stock cost may be difficult to measure, ok. Now, this problem everyone faces in fact so you have to search for the alternatives, and later you in the you know in the subsequent the lecture sessions you will find that there are many approaches you can adopt or you can use for estimating out of stock cost, ok. And you have to select that particular method which provides an estimate of the out of stock cost with the minimum error.

Now, there may be 3 ways this problem can be addressed. So, what is the first one? First one is the you said the performance standard that means, we are saying that the performance standard said by management. The second one is you may assume that the out of stock cost remains fixed irrespective of the number of units short. So, many a time while you formulate the problem we find that these assumptions is valid that means, the fixed out of stock cost regardless of the number of units short and the last case we consider that is out of stock cost you can estimate per unit basis.

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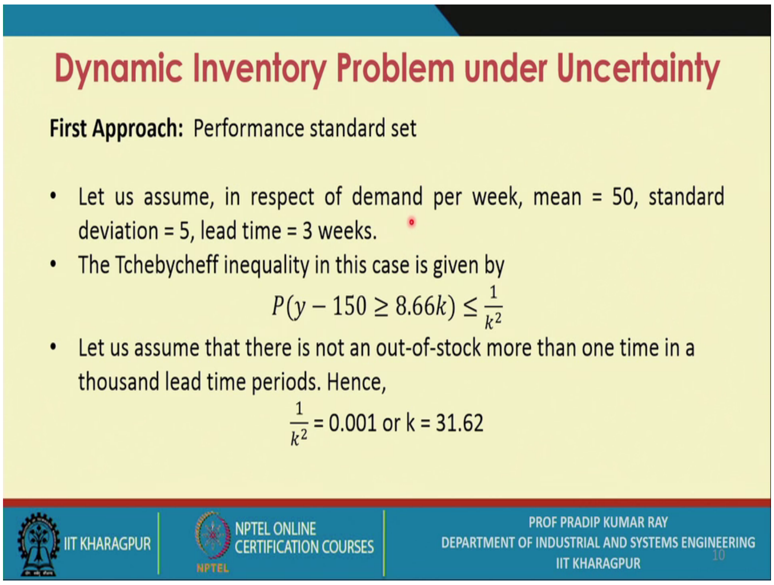

Now, let us take all the 3 alternatives one by one. Now, the first approach is while you deal with dynamic inventory problems under uncertainty that is we will consider the performance standards said by management. So, how do you consider this? Now, let us assume in respect of demand per week suppose the demand per week for the given item we have these really good estimates like the mean is equals to 50, the standard deviation of demand is 5 and the lead time is 3 weeks, and this the mean demand is weekly basis that means, this is the demand per week, so the mean demand during week.

Now, the Tchebycheff's inequality in this case is given by, so how do you get this one? That means, Tchebycheff's inequality in one form that is the probability that y minus. Now, this is 150 that means, this is z bar, but this is i into z bar that means, how many how many times, you have convoluted, that means there is a possibility of going out of stock during the lead time only because this is the Q system and so the average demand during lead time is 50 into 3 because we are assuming that there will that the lead time is 3 weeks.

So, that is why it is 3 into 50, 150 greater than 8.66 k. Now, this is also you know the 3 times convoluted that means, this is the standard deviation that means, k into s. Now, the standard deviation is how do you compute 8.66, that means, here the standard deviation for one week is 5, for 3 weeks that means you consider the lead time or 3 weeks it has to be 5 into root over 3. So, that is why 5 into root over 3 is 8.66. So, that is why it is considered 8.66 k less than equals to 1 upon k square that is the right hand side.

Now, let us assume certain standard performance standards. So, what is the performance standard? Performance standard is there is not an out of stock more than one time in a thousand lead time periods, so that means, in a thousand order cycles for Q-systems of inventory control so for each order cycle there is a lead time and so out of say 1000 lead time periods only in one lead time period the stock out is allowed. So, that is the standard you have you have set. So obviously, 1 upon k square there is the probability of going out of stock is 1 upon 1000 that is 0.001, and if you and from this you get a value of k that is 31.62 is very simple once the standard is known.

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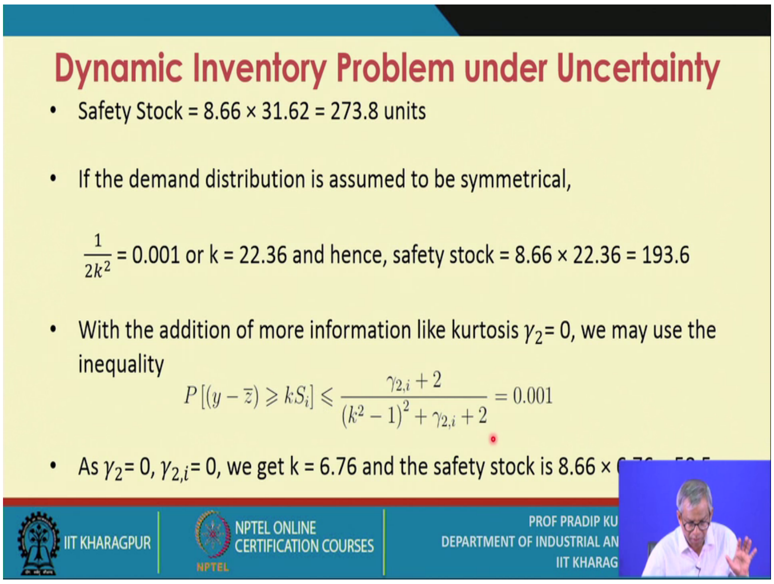

So, what may be the safety stock? Safety stock is obviously, you know that is the standard deviation for the lead time for the lead time. So, that is 8.66 into 31.62 so that is 273.8 units that means, k into s i. So, value of k is 31.62 and s i is equals to that is i into, root over i into s that is that is 8.66 already we have computed. So, this has to establish 73.8 units.

Now, if the available information we assumed that the demand distribution is symmetrical. So, what you try to do? You just make this the change that means now, 1 upon 2 k square equals to 0.001 that means, with the added information the results are improved. So, the value of k is 22.36 and hence the safety stock is 8.66 into 22.36 that is 193.6 so that means, it has been reduced from 273.8 to 193.6.

Now, with the addition of more information like kurtosis gamma 2 equals to 0 we may use the inequality that means, this is the inequality expressions how did we have we have shown this expression and so in terms of say the kurtosis, kurtosis is gamma 2 and this is for the lead time period that means, gamma 2 i, ith convolution of gamma 2. And this value what the standard we have said? That is 0.001.

So, as gamma 2 equals to 0, gamma 2 i also will be 0. So, what we get? So, if you solve this problem, if you solve this, inequal; equality that means, the maximum probability of occurrence. So, we get a value of k as 6.76. And so what will be the safety stock? Safety stock will be 8.66 into 6.76 that is 58.5. So, this is basically the change the standard deviation that means, standard deviation of lead time.

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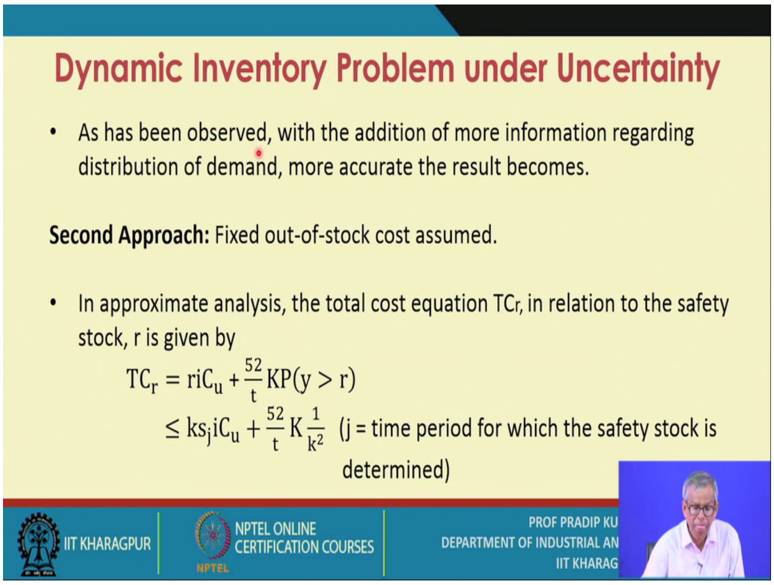

As has been observed with the addition of more information regarding distribution of demand more accurate the result becomes. So, this is your conclusion, this is your say you know this is your observations and so that is why even if you set a standard it is important, it is imperative that you try to get more information about the characteristics of the distribution.

Now, when you opt for the second approach here the method is slightly different. So, in the second approach we have assumed fixed out of stock cost already I have mentioned. So, now, an approximate analysis as we have already pointed out is there could be two types of analysis, the first one is the approximate analysis and the second one is the exact methods of analysis. In approximate method of analysis the total cost equation TCr that means, r stands for the reserved stock or the safety stock in relation to the safety stock r already I have mentioned.

Now, this equation, so total cost equation is given by riC u that means, this is the carrying cost for the safety stock and this is opposed by out of stock cost. So, how many to the cycles you have, order cycles per year? That is 52 by t, assuming there are 52 weeks in a year. Capital K is a fixed out of stock cost multiplied by probability that y is greater than r, obviously, during the lead time. So, ultimately you have these expressions and you know this is r k into s j into i into C u, is it ok. So, what you try to do? That means, you try to determine the expression for small k. So, the j is the time period for which the safety stock is determined that means, it is the lead time period, ok, for the Q systems of inventory control.

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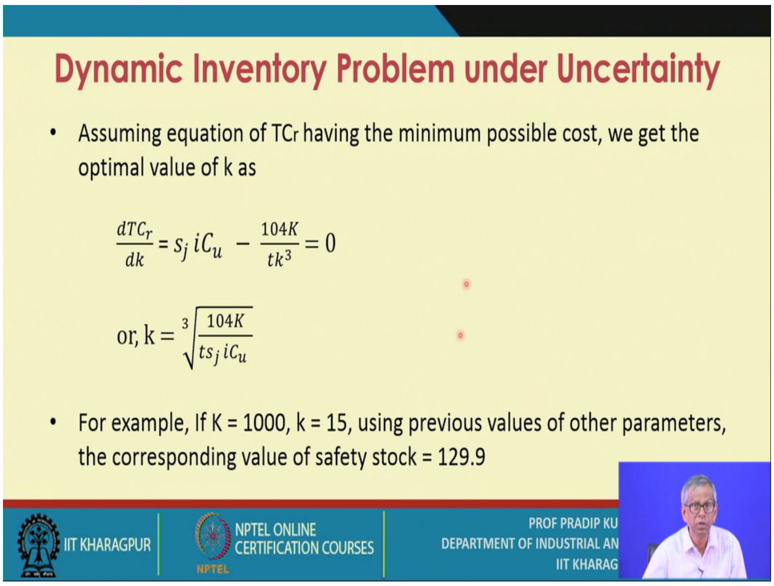

Assuming equation of TCr having a minimum possible cost we get the optimal value of k as that means, we take the derivative first derivative with respect to small k of the total cost equation and when we get this expressions. So, ultimately the expressions for small k is this that means, cube root of 104 into capital K, so the capital K value will be specified and then t into s j into i into C u, ok.

Now, for example, if capital K is 1000, small k is equals to 15 using the previous values of other parameters. So, you just refer to those values and you use those values the corresponding value of the safety stock becomes 129.9. So, the results are getting improved, ok.

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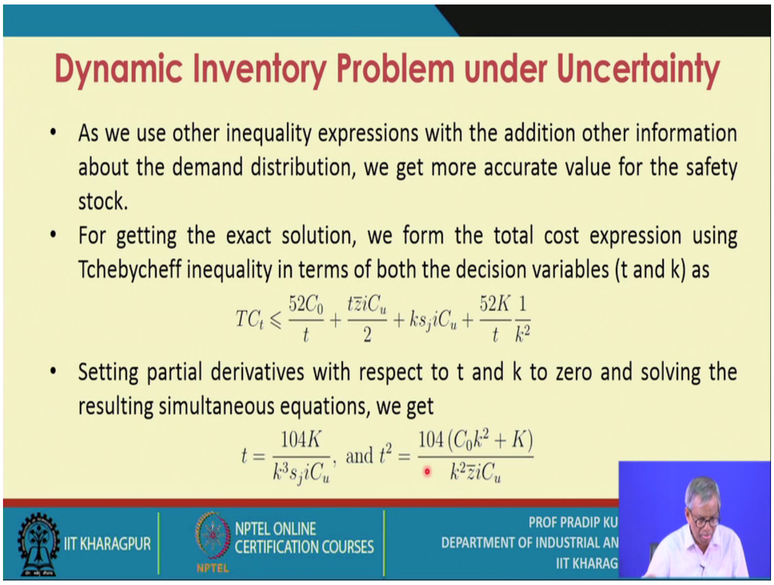

As we use other inequality expressions with the addition of other information about the demand distribution we get more accurate value for the safety stock. Now, this point is to be noted. So, I repeat as we use other inequality expressions with the addition of other information about the demand distribution we get more accurate value for the safety stock. So, you keep on you know the collecting more relevant information regarding the demand distribution.

For getting the exact solution we found the total cost expression using Tchebycheff's inequality in Tchebycheff's inequality in terms of both the decision variables t and k. So, now, we have moved from approximate method to the exact method of analysis. So, what you try to do? That means, we have this two decision variables t and small k and in terms of small t and small k we found the total cost equation, is it ok. So, this is the expressions.

So, the first part is referring to the ordering cost or meeting the average demand, the second one is the inventory carrying cost or the average demand, the third one is the inventory carrying cost for the safety stock and the fourth one is that the out of stock cost where you have used this particular expression 1 upon k square that is the probability of going out of stock. And this is the maximum probability and this here you use the Tchebycheff's inequality. So, again setting partial derivatives with respect to t and k to 0 and solving the resulting simultaneous equations we get this expressions. So, please go through this expression.

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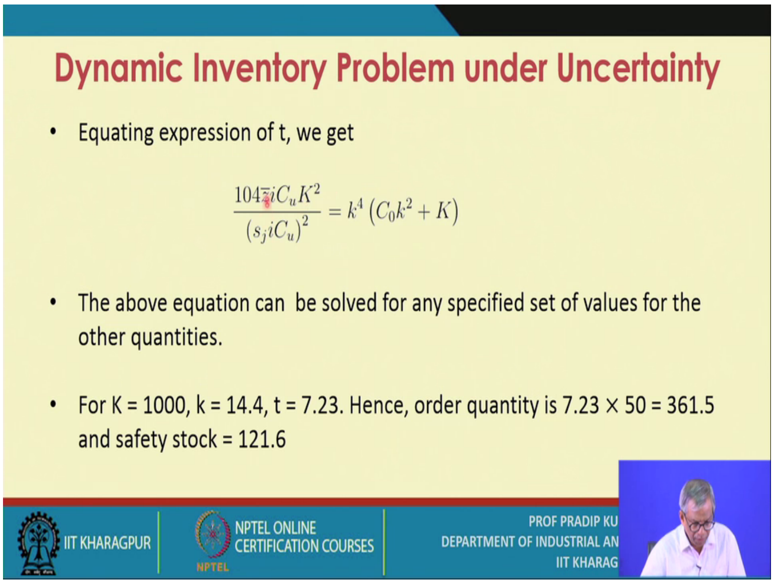

And then we have these equations over here in terms of small k and terms of capital K and other parameter values. Now, this above equation can be solved for any specified set of values for the other quantities, ok.

So, for example, for capital K equals to 1000, small k equals to 14.4, t equals to 7.23 that is the order interval hence the order quantity is 7.23 into 50 that means, 7.23 weeks and 50 is basically the mean demand for 1 week. So, that is why it is 7.23 into 50 that is 361.5 is order quantity and the safety stock similarly you have defined, you have determined.

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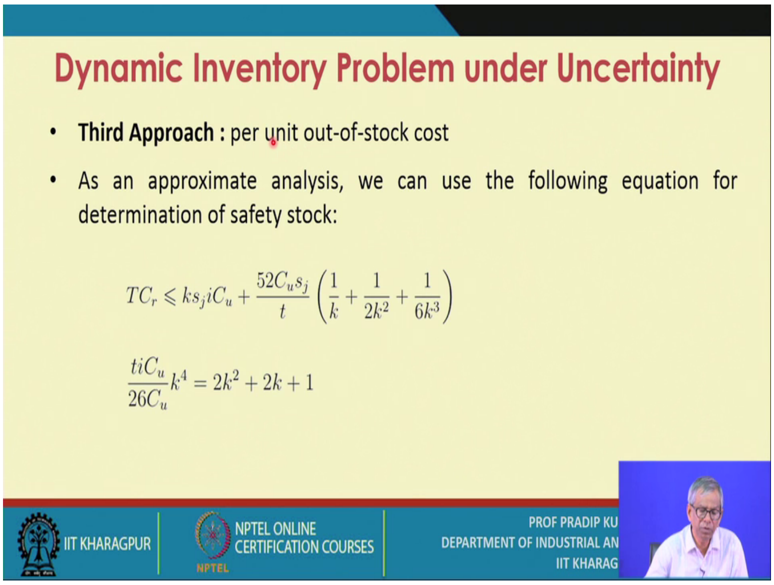

And the third approach what you do that means, per unit out of stock cost you consider. So, as an approximate analysis what we do we can use following equation for determination of the safety stock. So, you have this two terms the first one is this term is basically is related to the inventory carrying cost for the safety stock and this one is the out of stock cost, ok.

So, in the other lecture sessions we have already you know the identified this particular say inequality expressions which is exclusively used for say per unit out of cost case, out of stock cost case. So, ultimately when you simplify these expressions you have these equation terms of k and we determine the value of small k.

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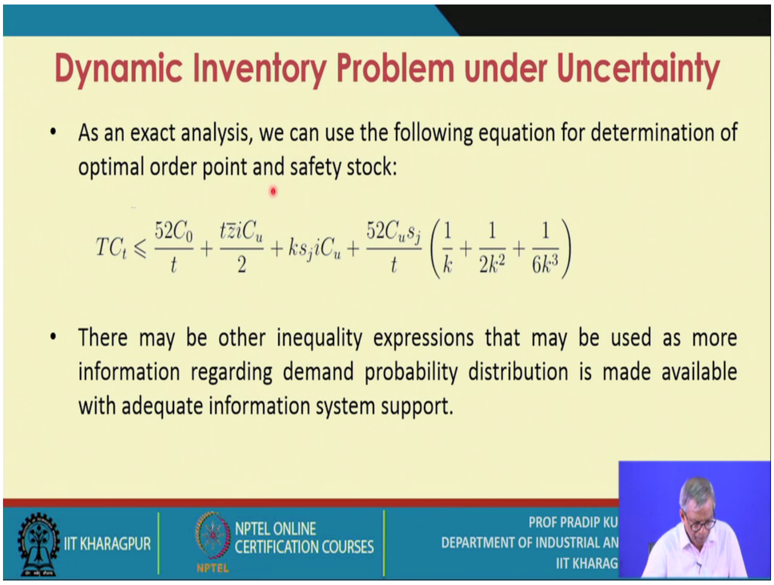

As an exact analysis we can use the following equation for determination of optimal order point and the safety stock. So, the optimal order so the period and the safety stock. So, here in terms of t and in terms of the safety stock so, we write down the total cost expression. So, we have 4 terms like in the previous case. And so now, you follow the usual approach that means, taking the partial derivative with respect to t as well as with respect to small k, you actually you get the say and setting them equals to 0 partial derivative, you said that equals to 0 you get two simultaneous equation.

And solving them you get the expression for t that is one of the parameters and you get the expression for say the small k, one small k is known obviously, k into s j that is also known and once this is known that means, the safety stock is known. So, there may be other inequality expressions that may be used as more information regarding demand probability distribution is made available with adequate information system support.

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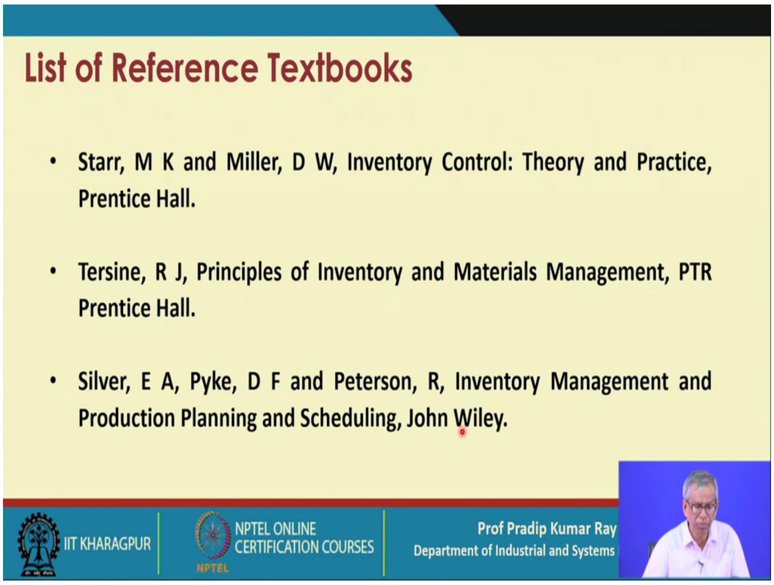

So, during this week we have thoroughly discussed the many issues related to say the static inventory problem under uncertainty. And as we do not have any this is specific topic on so the dynamic inventory problems under uncertainty, so we have extended our discussion from say the static inventory problem to dynamic inventory problem.

But the, but one advantage is that you even if you are deal with dynamic inventory problems under uncertainty. Obviously, the kinds of inequalities we are supposed to use the same that inequality expressions are also valid for dynamic inventory control systems with inventory control problems with just one change that means, many a time those the parameters those expressions are changed considering the convolution of say the moments of the distributions. So, we conclude this particular the discussions on this particular important topic.

Thank you.