Management of Inventory Systems
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Lecture - 19
Static Inventory Problems under Uncertainty (Contd.)

In the fourth session of this week, the fourth week under Static Inventory Problems under Uncertainty, I will be continuing with discussing a number of other, say numerical problems.

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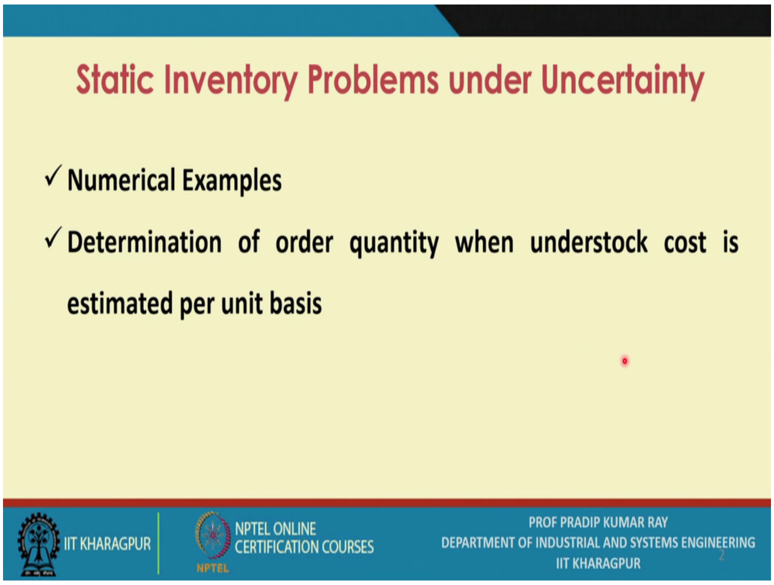

The first, we will refer to those numerical examples because, already we have referred to the several kinds of inequality expressions. And we will discussing all these numerical examples, illustrative examples; in such a way that you will come to know the application potential of all these inequality expressions.

So, that is the first part we will do and the next important topic are going to discuss; that is, the determination of order quantity when understock cost is estimated per unit basis. So, here also in this case, we will be using several kinds of inequality expressions.

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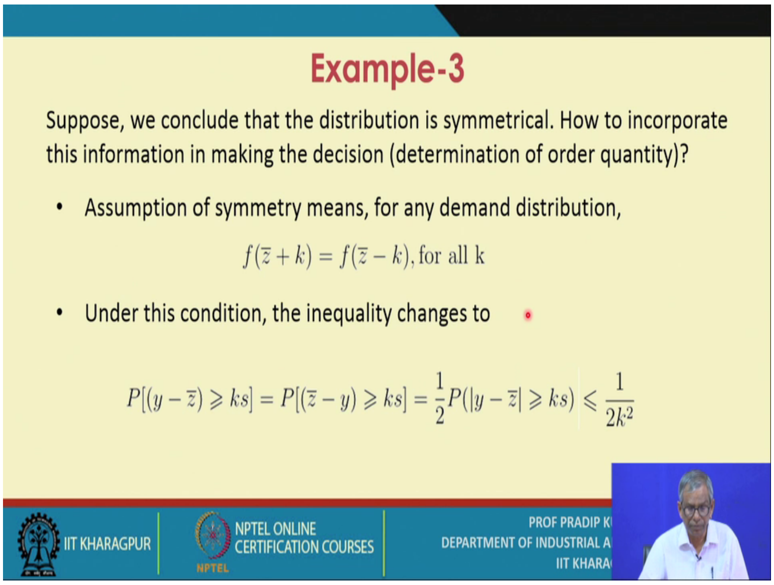

And, and you will come to know that how these inequality expressions are different from the original inequality expressions the proposed by Tchebycheff's and Gauss.

Now, all the in all the pre previous examples, in the last 2 examples, what we are assumed that that understock cost the remains fixed; that means, there is a situation where irrespective of the number units at for a for a given inventory item the understock cost the remains same. So, that is a fixed under stock cost situation.

So,. So, this is one case and we can use all these previously refer to inequality expressions. Now, the next example: example 3 illustrative example; suppose we conclude that the distribution is symmetrical; that means, how do you know that the symmetric the distribution is symmetrical? That means the skewness value is assumed to be 0; that there is no this you know the distribution could be you know the skew to the left and the skew to the right. If you say that the skewness is 0.it mean say it is perfectly symmetrical.

So, you have the information up to gamma 1 and with these information, you conclude that the distribution is symmetrical. How to incorporate this information in making the decision, determination of the order quantity static inventory problem under uncertainty assumption of symmetry means for any demand distribution f, small f z bar plus k is equals to; that means, f of z bar minus k for all k. That means, this represents the probability values; that means, z bar is essential point.

And the positive side; that means, there is value z bar plus k against which the probability is f z bar plus k. And on the other side; that means, at the same distance, but on the other side, the corresponding value is z bar minus k and against z bar minus k corresponding probability is f z bar minus k. Now, these 2 probabilities are same; that is why, this is referred to as a symmetric distribution and for all k this condition holds, is it ok.

So, this condition you remember. This condition for all k; that means, whatever may be the value of the of your say random variable with respect to; that means, the essential point is z bar the deviation from z bar on the positive side or on the negative side the corresponding probability is remaining same.

So, that is why, this is referred to as the symmetric distribution. So, you remember, this condition and if this condition holds, what you do? This is say one of the inequality expressions. We have inequality Tchebycheff's inequality expressions. So, this means this one; that means, y minus z bar means z bar minus y, ok. That is because, this is asymmetric distribution and this means, half of probability the difference the probability of the difference between y minus z bar greater than or equals to k into s.

So, this is the original. You know the Tchebycheff's inequality and this is less than equals to 1 upon k square multiplied by half. So, that is why, it is less than equals to 1 upon 2 k square.

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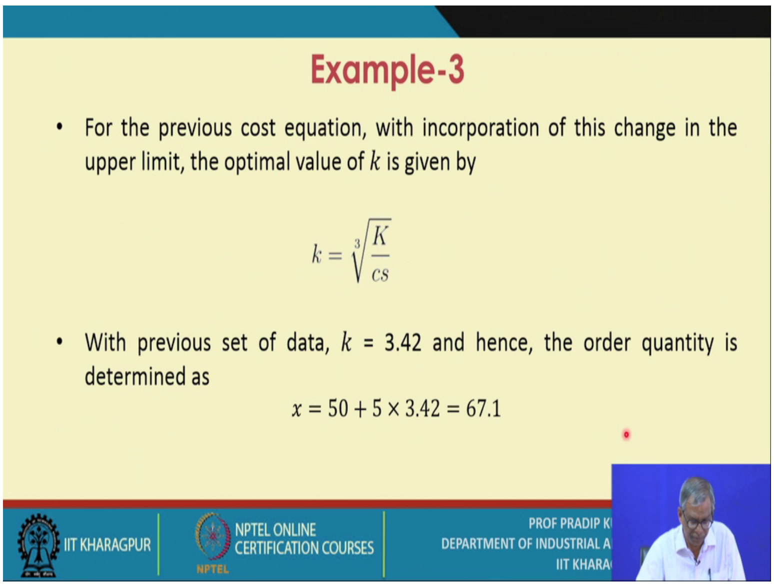

So, this the change you make and then, for the previous cost equation with the incorporation of this change, in the upper limit the optimal value of k is given by small k equals to cube root of K by c s. So, it was in the original expression when you do not know whether it is symmetric or asymmetric.

So, I use original expression; that means, the equality value that it is one upon small k square. So, you get a value of cube root of 2 k by c s. But here, when you assume the symmetric distribution symmetric, then the expression for small k changes to cube root of K c to s. K is the stock out cost, fixed stock out cost. Under stock cost, the c is the cost of the item and s is the standard deviation, is it ok.

So, with previous set of data where so, k equals to k becomes small k becomes 3.42. And hence, the order quantity is determined by x equals to 50 plus 5 into 3.42. S is 5. We have assumed and so, this becomes 67.1. So, as you might have noticed that, the values are changing. That means, and the values are improving in the sense that as you get more set of information.

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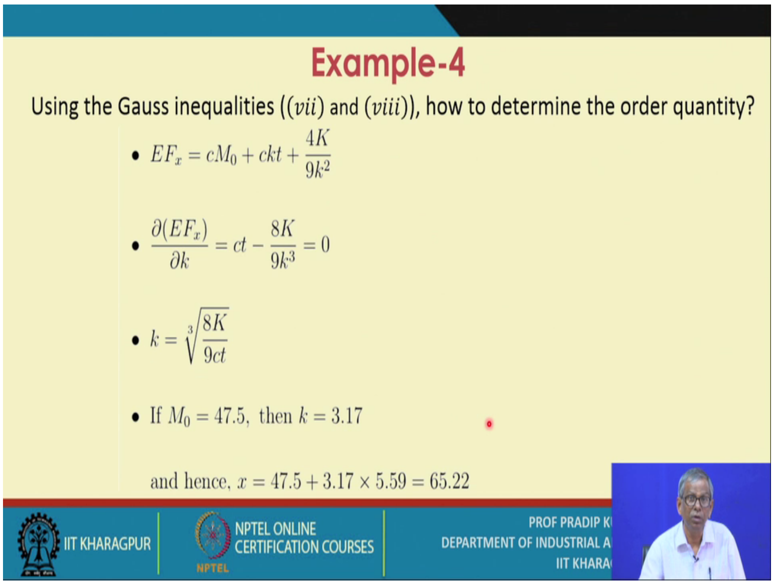

So, your decision becoming more perfect now use the there is a example for the next four example is using the Gauss inequality expression. So, the expression number 7 and expression number 8, ok. These 2 inequality expressions were proposed by Gauss way back in 1921; how to determine the order quantity? So, what you try to do here base is not the mean here the base is. So, which is the moment and the notation is M 0. So, how do you express your the order quantity x is essentially M 0 plus k t ok. So, c into x means c into M 0 plus c into k into t, where is t; t is the second moment around mode plus 4 y 9 minus into K by small k square.

So, you just refer to expression 7, you will get this one is the equality value is this one 4 K by actually k into probability of going out of stock. So, that expression is 4 y 9 minus small k square. So, again, you follow the same approach; that means, you take the derivative partial derivative with respect to small k. So, these becomes this ultimately you know set it equals to 0. The first derivative you set it equals to 0 and the value of k becomes say cube root of 8 into K by 9 c t. If M 0 is 47.5, you will if you remember, you have assumed z bar is equals to 50 and s equals to 5; that is that is the data we have assumed.

So; obviously, M 0 we have we have assumed to be 47.5 and then k becomes 3.17. Now, obviously, you need to calculate t. So, you have an expression for t square in terms of say, M 0 s and z bar. So, you use that use those values and that expression for t square to get the value of small t in this case. So, please note it down that, you need to calculate the value of t separately. So, for that, there is expression for t square is it ok; that means, second moment around the mode, ok.

So, please use that expression. So now, when the k is known; so obviously, when t is known; so we get the value of x 47.5 plus 3.17 into 5.59, is it ok; that means, c that is k and t t value is 5.59. So, please check whether you are getting a value of t as 5.59; 5 9 or not ok. So, your order quantity is 65.22.

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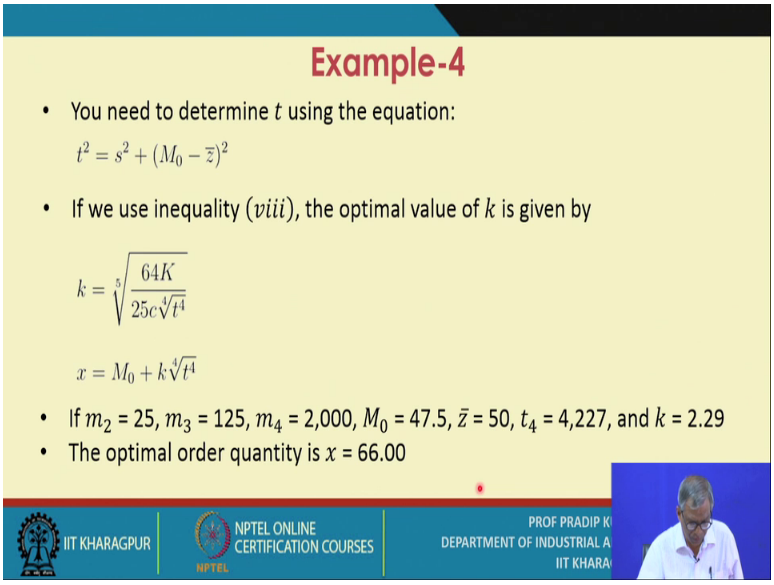

So, this way you calculate. Now, you need to determine t using the equation. As I have already mentioned, that is t square equal to s square; that means, this is the variance plus M 0 minus z bar. So, this is 47.5 and z bar is 50. So, so 47.5 minus 52.5 minus 2.5 square and s square is 5 square; so 25. So, you take the square root of t square. So, you get the value of t.

If you use inequality, say expression 8; the optimal value of k is given by this one, is it ok; that means, small k is the fifth root of if you square root of 64 K divided by 25 into c then the fourth root of t 2 the power t to the power 4,. So, this is x equals to M 0 k; that means, fourth root of t to the power 4.

So, if m 2 is 25 m 3 is 125 m 4 is 2000 and M 0 is 47.5 z bar is 50 t 4 is equals to 4.227 and k is equals to 2.29. So, my suggestion is, you just assume all this values m 2 m 3 m 4 m. M 0 also is given z bar is also given you calculate t 4, ok. So, once t 4 is known t to the power 4 is known, then you can calculate say fourth root of t 4, is it ok; so t t to the power 4.

So, the k is equals to you get a value of k equals to 2.29, all right. So, the optimal order quantity is x equals to 66 right.

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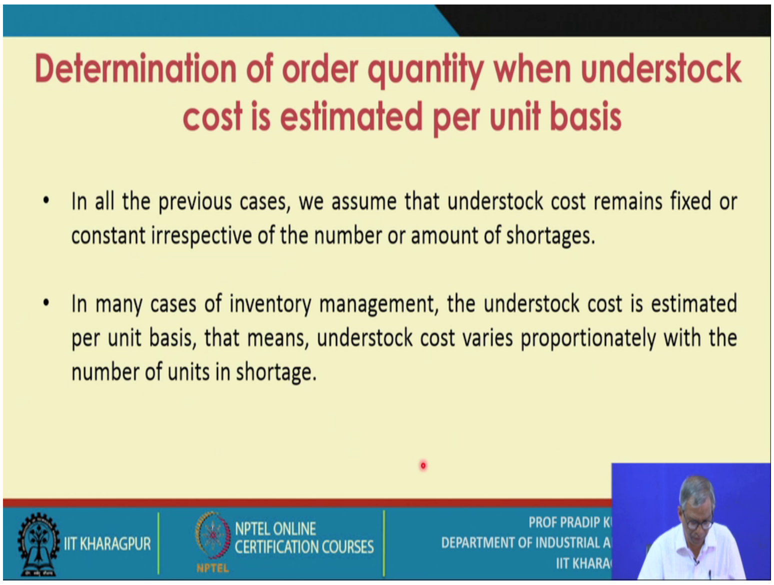

So, you get these values. So, the four examples we are provided. So, please go through them and we have just used out of say 8 inequalities expressions we have given. So, we have used just 4 4 or 5 expressions for it inequality expressions will be other numerical problems related to the remaining inequality expression.

So, in course of time, we will be discussing those inequalities. Also, like will be you know the discussing say that under uncertainty problems; that means, right now, we are discussing static inventory problems under uncertainty. And then, in course of triangle we discussing dynamic inventory problems. That means, the repeated orders over series of the time periods, but demand is a k known with uncertainty. That means, those cases will be using will be discussing and or the dynamic inventory problems under uncertainty the same sort of inequalities expression. We will again we will be using, but the problem formulations will be slightly different.

So, we will in course of time, we will discuss that part right; now, what we will be discussing? That is, how to determine the order quantity when understock cost is estimated per unit basis. Now, this is also case which you frequently come across. In the in the previous case, what we have assumed that the understock cost remains fixed; that means, irrespective of the number of units short the understock cost for is does not have much change and so, it remains fixed, but in many cases this the condition or this particular assumption may not may not hold.

And that is why, you first check whether you need to calculate or you need to estimate the understock cost per unit basis or per outrage basis to outrage basis. It means the fixed understock cost; that means, just one value will be there irrespective of the number units short.

So, in all the previous cases, we will assume that understock cost remains fixed or constant irrespective of the number or amount of shortages, ok. In many cases of inventory management, the understock cost is estimated per unit basis. That means understock cost varies proportionately with the number of units in shortage, ok. So, if you have this assumption, how do you formulate the problem?

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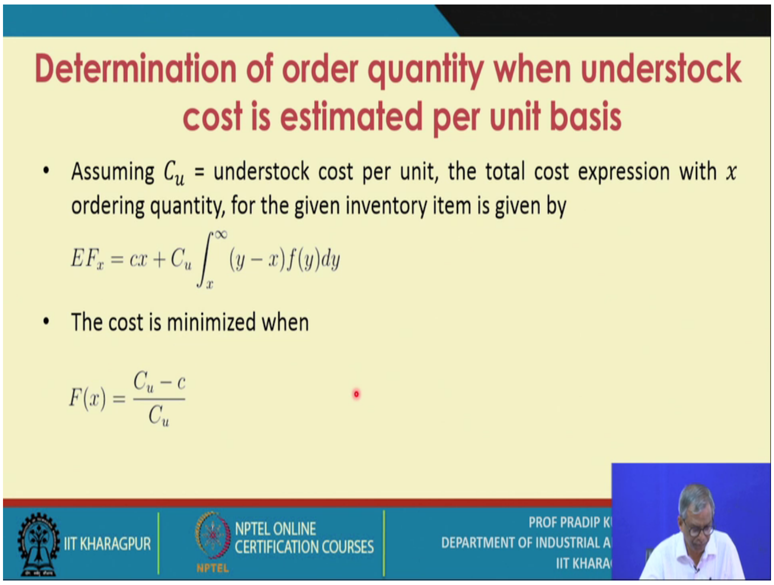

Now, we have been using this notation that is C subscript u; C subscript u; that is understock cost per unit. So, this notation we have used the total cost expression with x ordering quantity. Ordering quantity is the unknown and x say the value of value of x is to be determined.

So, for the given inventory item for the given inventory item the total cost expression or the total relevant cost expression is given by E F x into c x C u integration x to infinity y minus x f y d y. So, what is this expression? This is basically the expected number of unit short y is this actually your demand and x is ordering quantity. So, if the y varies between x to infinity; obviously, there is a shortage because, you have x units whereas, demand is greater than equals to x up to infinity and for each the value there is a probability of f y probability density function.

So, that is why this is the expression y minus x f y d y integration x to infinity I am sure that you have understood. The meaning of this expression the cost is minimized when f x equals to C u minus c by C u. That means, this is this is an integral expression and what you need to do; that means, you take the derivative of the integral expression.

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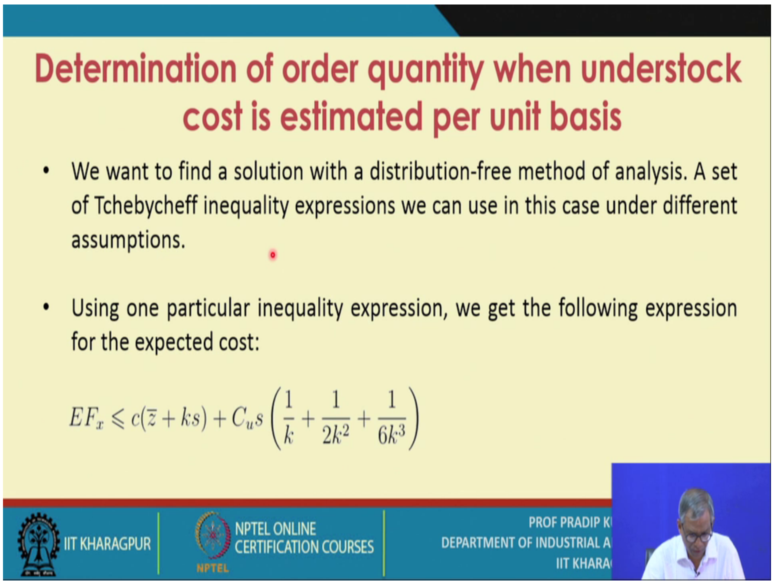

So, what the rule we have to follow the rule you follow that is essentially the Leibniz rule we want to find the solution with a distribution free method of analysis.

We want to find the solution with distribution free method of analysis a set of Tchebycheff's inequality expressions you can use in this case under different assumptions ok. So, this is the point to be denoted using one particular inequality expression, we get the following expression for the expected cost. That means, here one expression is E F x less than equals to c into z bar plus k s plus C u into s into 1 upon k plus 1 upon 2 k square plus 1 upon 6 k q k k 3 k to the power 3, ok.

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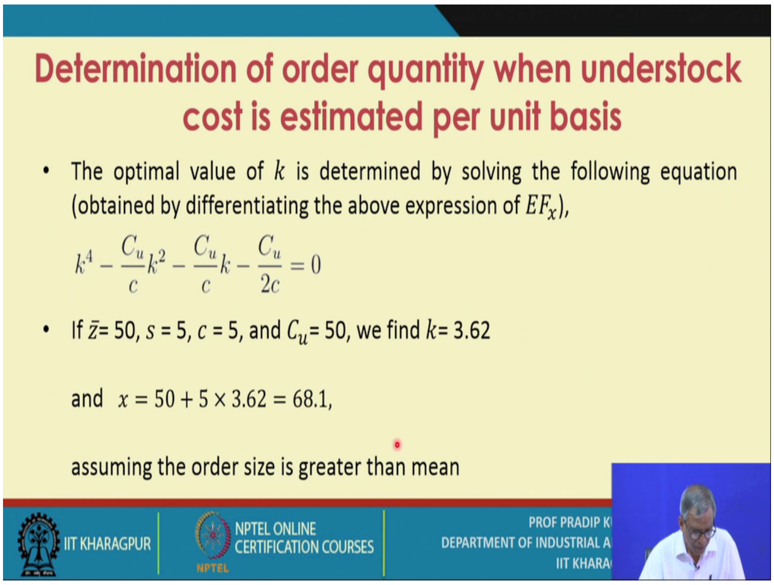

So, this is this is the expressions. You can get in this is you can use, and the optimal value of k is determined by solving the following equation, you just you know the follow the steps and you just check whether you are getting this the expression or not. And what you get? That means, you differentiate you take the derivative first derivative with respect to x of the expression E F x and ultimately, you get this equation, is it ok, order 4 in k small k.

So, k to the power 4 minus C u by c k square minus C u by c k minus C u by 2 c equals to 0 ok. So, this is the constant part. So, if z bar equals to 50 s equals to 5 c equals to 5. We have all assumed c equals to 50, we find k equals to 3.62 and x equals to 50 plus 5 into 3.62 that is 68.1, this is the solution assuming the order size is greater than mean; that means, what we are assuming that, we always say that x is z bar plus k into s; that means, order quantity is greater than z bar; that means, x minus z bar is always as it is ok.

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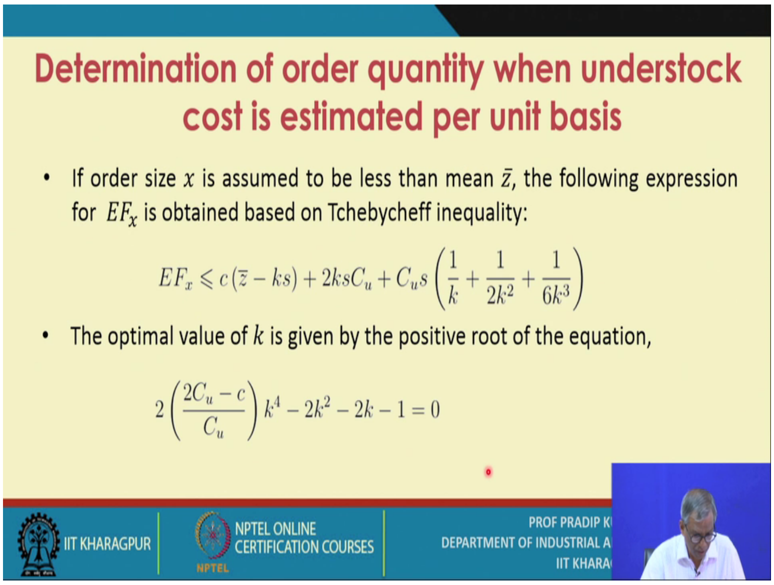

So, this is assumption. If order size x is assumed to be less than z bar; that means, what we are assuming? Suppose here the z bar is 50, we say that order quantity could be less than 50. So, less than z bar. So that, assumptions we have right now. So, if order size x is assumed to be less than mean z bar the following expression for E F x is obtained based on Tchebycheff's inequality. So, E F x less than equals to c into z bar minus k s plus 2 k s C u plus C u into s into 1 upon k plus 1 upon 2 k square plus 1; 1 upon 6 k cube.

So, the optimal value of k is given by the positive root of this equation. So, again, this is the equation you have to solve right to get the positive root of the equation.

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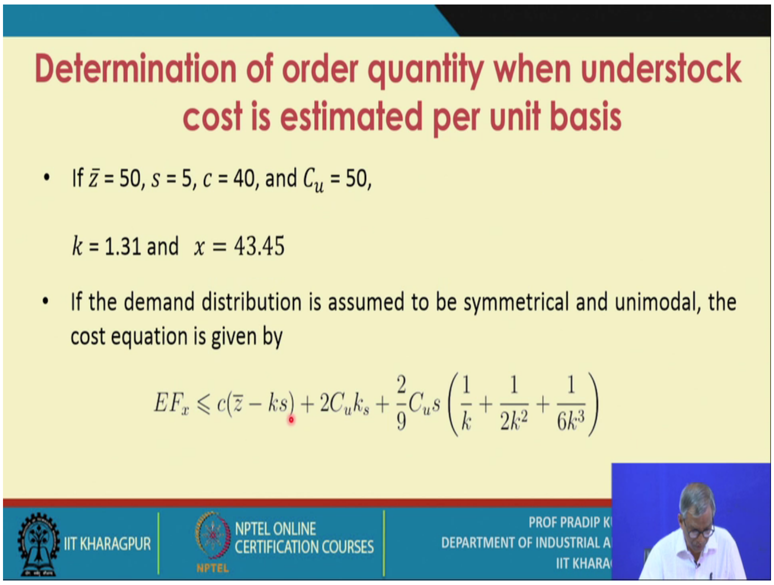

So, this positive root of the equation. When with respect to these values; that means, if z bar equals to 50 s equals to 5 c equals to 40 and C u equals to 50. So, these values will be specified. So, you get a value of k as 1.131 and hence the order quantity x is 43.45, ok. So, this is this way you calculate; that means, here the assumption is the order quantity is less than is less than the mean. So, here the mean is a 50.

So, what you find here order quantities 43.45. Whereas, in the previous case, what you find that the order quantity is 68.1; that means this is greater than 50, ok. So, the assumption holds ok. So, in this is the procedure if the demand distribution is assumed to be symmetrical and unimodal the cost equation is given by this one whether means E F x less than equals to; that means, this is expected the total relevant cost when the ordering quantity is x is less than equals to c into z bar minus k s.

Here again we are assuming that that the order quantity is less than z bar. So, c into z bar minus k s plus 2 C u to k into s k into s plus 2 y nine C u into s 1 upon k plus 1 upon 2 k square plus 1 upon 6 k cube. So, minimization of the cost results in the following equation for k. That means 9 into C u minus c by C u to k to the power 4 minus 2 k square minus 2 k minus 1 equals to 0.

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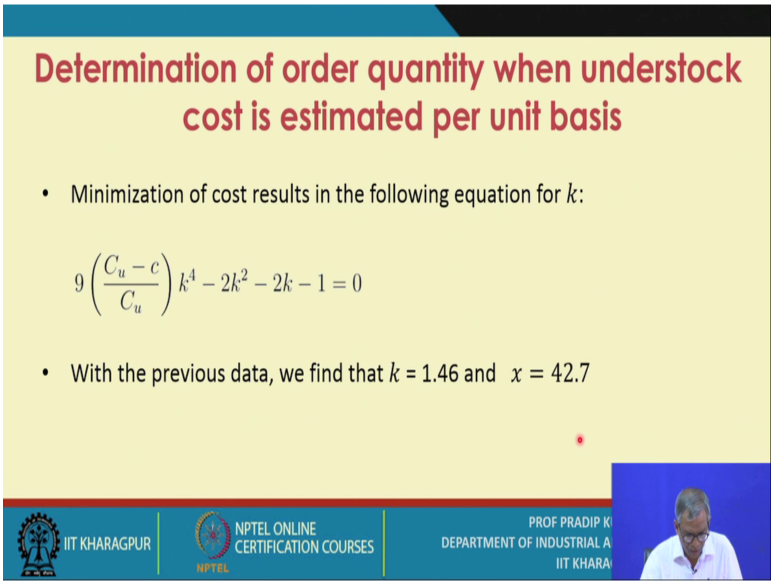

So, with the previous data you just refer to the previous data; that means, z bar equals to 50 s equals to 5 c equals to 40 and C u equals to 50, what you get? You get the value of k equals to 1.46 and x equals to 42.7. Now, the point to be discussed over here; that is what you need to do first thing that you will be first you must identify the relevant cost elements once this relevant cost elements are there like in this case, what we are assuming that the purchase cost unit purchase cost or the unit production cost is one of the relevant cost elements.

And the second important cost element that you have assumed. In the first case, we have assumed that it is fixed a understock cost, ok. So, you have the formulation alternatively you are you may assume that that understock of cost is to be estimated per unit basis. So, that is the different case.

So, one particular part is one particular you know the problem formulation related to the fixed understock cost or understock cost for outrage you just note down this particular term; that means, cost of cost of understock; understock per outrage or fixed understock cost of understock per unit basis.

So, these are the two cases; two different sets of formulation this is point number 1. Point number 2 is that, when you assumed the demand distribution. Now, for depending on the type of the inventory item, you deal with the demand distribution would be either or the discrete and distribution or it could be you know the continuous demand distribution;

that means, depending on the type of inventory item either you assume that this is a continuous random variable or discrete random variable. That is point number 2.

Point number 3 is that, while you develop while you develop the total you know the relevant cost equation. Now this total relevant cost equation must be returned in terms of x. What is x? X is the decision variable. Now here, as it is a static inventory problem, always there will be just one decision variable; that is the order and quantity. Now, in many cases, what you will find that when you refer to say the P system of inventory control or Q systems of inventory control. What you find that, always there are at least at least 2 parameters of any inventory control systems.

So, the first one in, suppose it is a Q system of inventory control. So, in the first case it is 2; that means, inventory; that means, of your say the first parameter is the ordering quantity. And the second parameter is obviously, a reorder point. Now the reorder point, in this case where you can you can determine, but as it is a static inventory problem.

So, there is no question of the reorder point whereas, if you extend it to say you know the dynamic inventory problem say under say uncertainty; that means, you must determine also along with the order quantity for the Q system of the inventory control. That means you know that continuously to inventory control system any to determine. So, the reorder point.

So, the further order quantity you will have one expression and for the reorder point you will have another expression, now. So, when you extend this the total relevant cost expression for the dynamic inventory problem case; that means, order quantity you can determine applying all this you know say inequality expressions. Similarly, you need to also determine the reorder point. Referring to or say another say inequality expressions or another cost expressions which is dependent on essentially one or more obvious inequality expressions.

So, this is very very important. That means, what are the even if the cost estimates are not; now the fourth important point remember that, even if there is the no reliable cost estimates, can there be any solution. Yes, there could be a solution, but in that case the alternatively you need to define the service level.

So, in course of time in the subsequent lecture sessions, what will be referring to? We will be referring to the cost based valuation and if the cost based solution is not visible; that means the cost estimates are not reliable. So, what you need to do? Alternatively, you must opt for the service level based solutions. So, both the cases, we will discuss this course of time, ok.

Thank you.