Management of Inventory Systems
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Lecture-18
Static Inventory Problems under Uncertainty (Contd.)

During this lecture sessions on static inventory problems under uncertainty, I will be now referring to the Tchebycheff and other inequalities.

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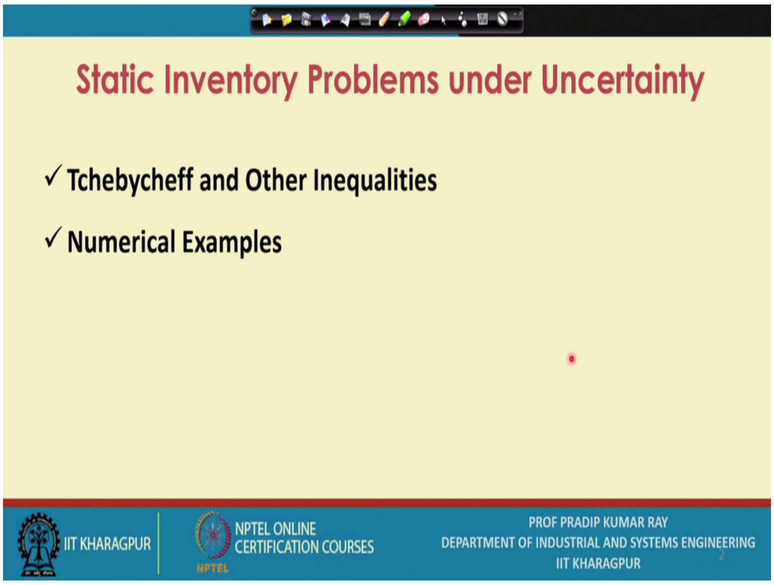

And I will explain all application of all these inequalities, for the given static inventory problems with number of numerical examples.

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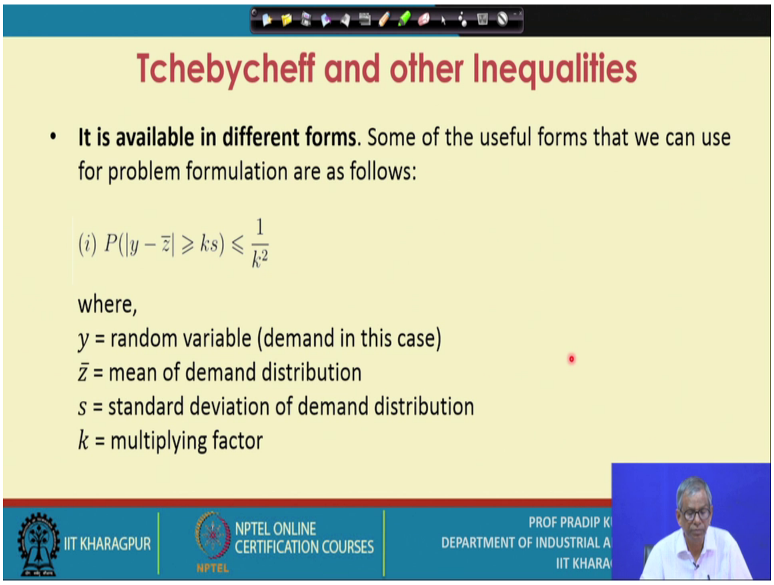

We have already mentioned the causes of using such inequalities and this inequalities are well known and are used for many kinds of the problems. And, as you are aware that under inventory management systems, you come across various kinds of the situations constraints and as I should say that these are binding constraints and all and.

So, and many a times, you come across a situations which are really uncertain; that means, the unpredictable are said; say, the occurrence of many such events and which would directly affect the performance of the inventory control systems. So, many a time this factors are considered as the noise factors or so; the innocence variables. So, there could be many reasons in that; why you face an uncertain situations.

So, this is very very important topic; that means, whenever you try to formulate a problem when; that means, we are trying to do the formulate a real word problem, the so called real world problem. And as you may be knowing that though we are saying that the there must could be some assumptions related to the type of distribution, but it is always stated that, the 2 distribution is never known, I repeat, 2 distribution is never known. What is known is, some sort of say the assumptions related to the type of distributions under given say the level of significance.

So, keeping in mind this concept, we might say that almost all the problems we face are of say, the problems under uncertainty and that is why, we have no other alternatives to use inequality expressions. Even if we know that the results may not be that perfect, but in a given situation that is imperfect at least some decisions you can take.

So, now, how this given, so, this background information. Now, I am going to discuss the Tchebycheff's inequality and other inequalities also. Now, Tchebycheff's inequality, you will find if you go through literature, you will find that this inequalities is available in different forms.

Some of the useful forms that we can use or problem formulation in our case are as follows, ok. So, the first one is probability that the difference between y that means, a particular value of you know, you say random variable discrete or continuous random variables, the difference between a particular value of the variable and the mean that is z bar is greater than or equals to k times s. That is s? s is the standard deviation.

So, the probability that these difference y, difference between y and z bar is greater than or equals to k times s is always less than or equals to 1 upon k square, ok.

So, the small k square where y is the random variable demand in this case, z bar is the mean of demand distribution, s is the standard deviation of the demand distribution and k is the multiplying factor. So, this is you know, this is referred to as the Tchebycheff's inequality but it is not you know in a general form; in the sense that the information of 2 z bar and s; that means, up to the variance or the standard deviation is made available.

So, what do you do and beyond those, we do not have any information related to a demand distribution. So, what do you do? You use this inequality for problem formulation and subsequently, the determination of the decision variable. So, I repeat probability that the difference between y minus z bar greater than equal greater than or equals to k times s is less than equals to 1 upon k square, ok.

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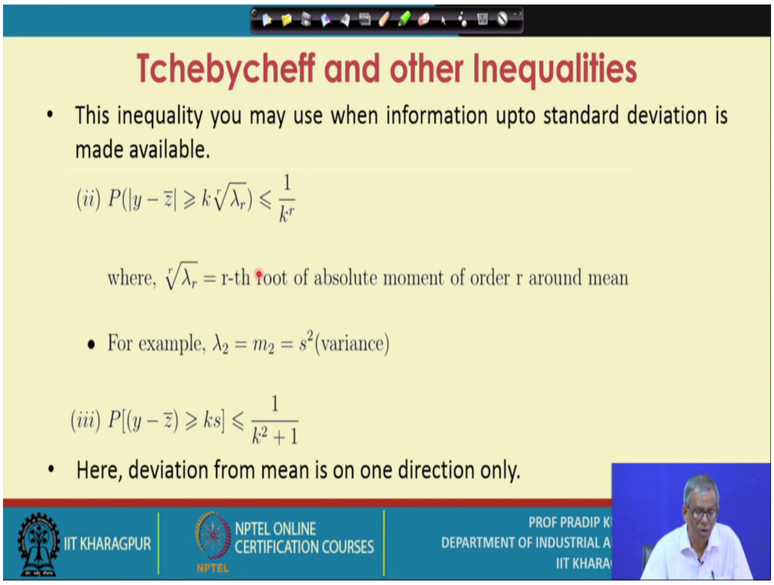

So, the maximum value of this probability is 1 upon k square. This inequality you may use when information up to standard deviation is made available. This point already I have mentioned. So, you are proceeding systematically one by one; that means, first you try to get the value of n and then you try to get the value of standard deviation and then you are in a position to use the Tchebycheff's inequality.

That means, the expression one, the next expression of the Tchebycheff's inequality is like this, probability that the difference between y and z bar is greater than or equals to k times r-th root of lambda r is less than or equal to 1 upon k to the power r. So, you just make a note that this is the Tchebycheff's inequality expression and most general form.

So, now; obviously, lambda r you know, lambda is the absolute moment of order r around the mean. So, already you know the expressions for lambda r so it is the r th root of lambda r; that means, absolute moment of order r around mean. And already, we have referred to the expression for this particular absolute moment, is it ok; in the previous lecture session. So, as I have already mentioned, that this, the inequality Tchebycheff's inequality has a close resemblance with the absolute moment of order r around mean, is it ok.

So, you have already understood the absolute moment of order r around x and then automatically you can explain or you can interpret the Tchebycheff's inequality expression. For example, it is just an example lambda 2 equals to m 2 equal to s square.

Here is the 2 is the even number so the moment and the absolute moment around mean so they are same. Because, for the even order for the even order moments and this is the standard notation we use, that is, the sample variance s square.

So, this is the second expression is the most real expressions. The third expression is probability that y minus z bar greater than or equals to k times s is less than or equal to 1 upon k square plus 1. Just you make a note this is just to difference, it is not the difference between y and z bar; that means y is what is y minus z bar ok. So, that is just the one way you can say that the difference work computed it is not the positive or negative, ok.

So, here the deviation from mean is just on one direction only. So, in that case, this inequality expression; that means, on the right hand side of the inequality right hand side at the inequality changes to 1 upon k square plus 1. So, this is the third expression then inequality. So, the first one you can use, the second also you can use, third one also you can use so, we will sight several numerical examples.

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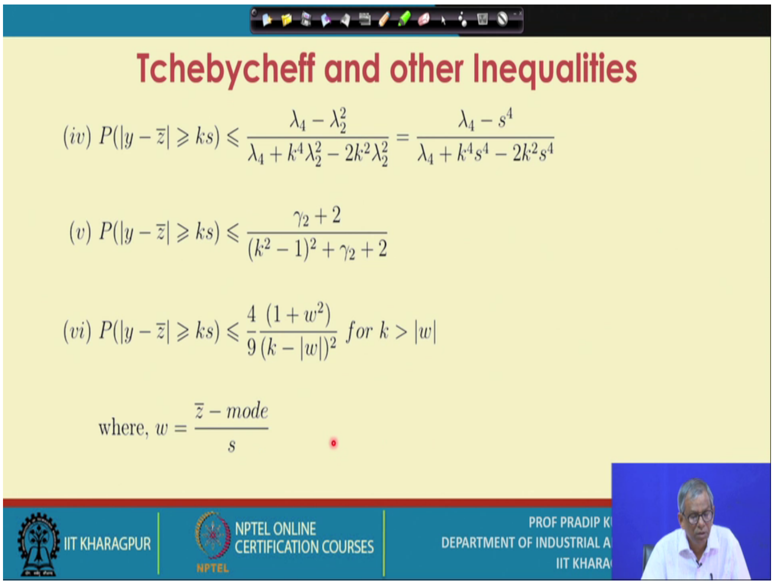

Then the fourth one is preliminary that the absolute value of y minus z bar; that means, the difference between y minus z bar probability that the difference between y and z bar is greater than or equals to k times s is lambda 4 minus lambda 2 square divided by lambda 4 plus k to the power 4 lambda 2 square minus 2 k square lambda 2 square which is same as lambda 4. This is s to the power 4 because, lambda 2 is into is equals to s square.

So, that is why, it is s to the power 4 and this is lambda 4 k to the power 4 lambda 2 square with s 4, s is the standard deviation, s to the power 4 minus 2 k square s to the power 4.

So, either you use lambda 2 square or you can use s to the power 4 when the standard deviation is known. That means, here you must know the m 4; that means, information up to m 4 is made available with you and when the m 4 is known; obviously, lambda 4 also you can compute. And when s square is known, s to the power 4 also you can compute, it is clear.

Now, the next expression is the fifth one; that is probability that absolute y minus z bar absolute is greater than equals to k s is less than equals to gamma 2 plus 2 divided by k square minus 1 whole square plus gamma 2 plus 2. So, what is gamma 2? Gamma 2 is essentially, what is gamma 2 is the kurtosis of the distribution and for computing kurtosis you need to have m 4, ok.

So, when m 4 estimate is there, you can calculate gamma 2 and when gamma 2 of 2 gamma 2 you have the information why do not you use this particular inequality, ok. And the next stage, what you try to do? That means, now we bring the concept of mode. So, what is mode you know this is just one of the measures of the central tendency on the given dataset. Now this, there are 3 measures of central tendency, the first one is the mean, second one is the median and the third one is the mode. That mean mode is value with the maximum frequency.

So, this you know the definition is known to you. Now, when the z bar is known and the mode is also known and s is of s is also known, now how do you define w? w is z mode by minus mode that could be difference between say the mean and the mode and this difference is expressed in standard deviation units. So, that is why, it is z minus mode divided by s. So, this difference between z bar and mode, we express in standard deviation units

So, now, when you have this expression so, in terms of w, you can again express this inequality Tchebycheff’s inequality. So, what is this? That means, the probability the absolute y minus z bar. The difference between y minus z bar probability that y the difference between y and z bar is greater than equals to ks is 4 by 9 into 1 plus w square divided by k minus absolute w whole square for k greater than equal greater than absolute w, ok.

So, this these are the different expressions. Obviously, you may there is the question that are how do you get these expression. Obviously, my suggestion is that there is all derivation of this particular inequality expression this derivations, all this derivations are there in the textbook. So, I have referred to a number of textbooks ah. So, please go through them and you try to get there the proofs.

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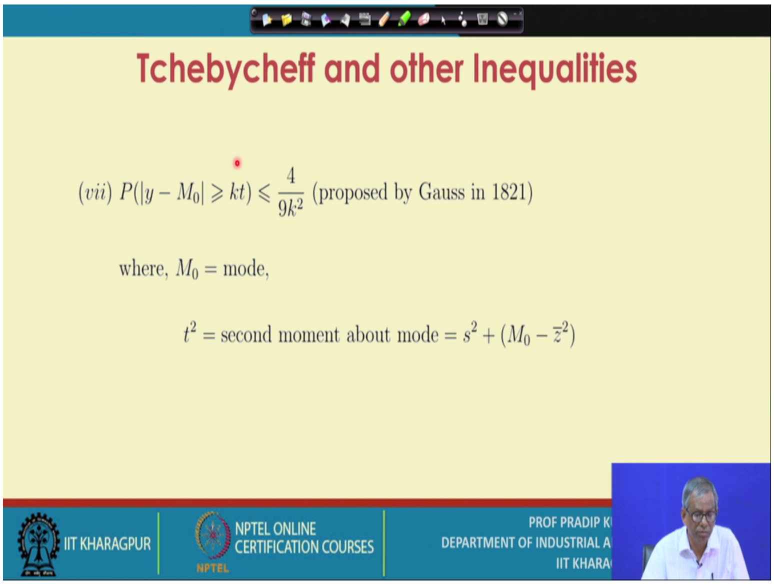

Last one; that means a seventh one, again you may refer to now this particular inequality expression was proposed by Gauss, the great mathematician way back in 1821. So, this is the seventh one; that is the probability y minus M 0. Now, the here, he has choose the base. Till now, the your reference point was z bar so it is y minus z bar. Now, the reference point could be M 0. So, what is M 0? M 0 is the mode so am measuring central tendency with mode, so that is the case not by mean.

So, in that case, is the probability that the difference between y minus y and M 0 greater than equal to k times t; that means, it is the second moment around mode. Is it ok? It is not the second moment around mean. In that case, it is it is the variance, ok. But here, you have to change it to t square; that means your base is the moment not the mean.

So, you just cannot be k into t almost you have a notation that is k into t. Now, this probability is less than equal to 4 by 9 k square, ok. So, just we keep in mind that this particular inequality also is you can use where your reference point is the mode and t square is the second moment about mode. So, what is you know the relationship between the second moment around mode about mode is equal to the variance; that is equal to s square plus M 0 minus z bar whole square M 0 minus z bar whole square, is it.

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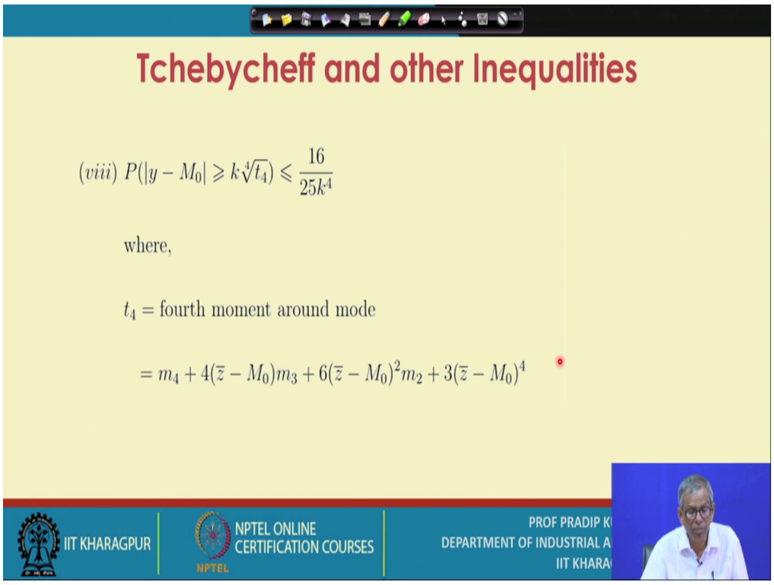

So, this is the expressions you remember and the eighth one is another inequality expression. That is also an extension of the previous one ok, as proposed by Gauss that is the probability that the difference between y and M 0 is greater than or equals to k times fourth root of t 4; that means, it is basically the fourth moment Around the mode fourth moment around mode less than equals to this probability will be always less than equals to 16 by 25 into k to the power k to the power 4. Where, t 4 equals to fourth moment around mode. And this fourth moment around mode is a relationship with m 4, m 3 m 2 and M 0 m 4 m 3 m 2 as well as M 0 and z bar.

So, this is the relationship you have, then this will define t 4, is it ok. So, there is a relationship between moment say the fourth moment around mode. Around the corresponding say is a function of m 2 m 3 m 4 z bar and M 0 and m m m and M o.

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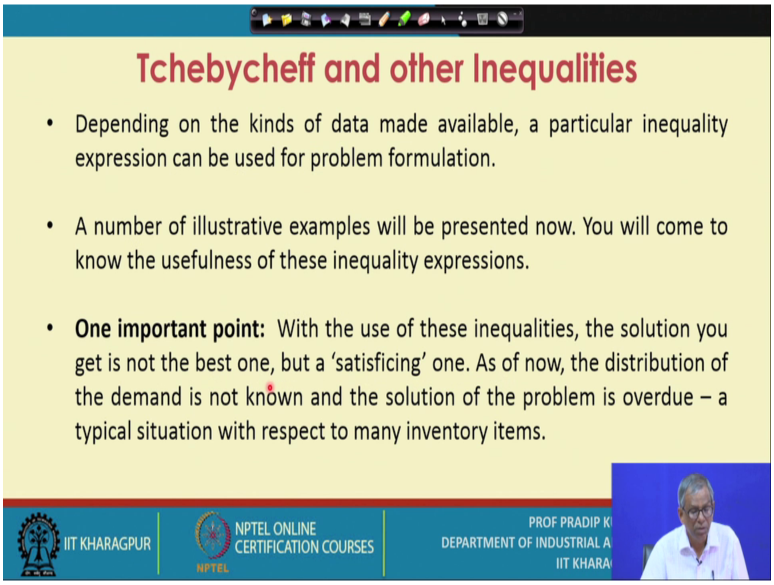

So, these are the 8 inequality say the expressions we have suggested or we have included. Now, what you need to do? That means, given problem first you need to go through the problems statement and then, you have to select a particular inequality expression. Now, obviously, they could be different types of problems and the depending on the type of problem and the type of data, you have you select a particular inequality expression.

So, depending on the kinds of data made available; that is, very very important a particular inequality expression can be used for problem formulation. So, the first, what we try to do? A problem should be clearly understood and then, you formulate the problem and while you formulate the problem you use one or more of these inequality expressions and then, when the problem is formulated, then you will suggest the methodology for it is solution a number of illustrative examples will be presented now, ok.

So, it is better that you come to know the application of all this inequality expression one by one so that I will do. So, we will come to know usefulness of these inequality expressions, is it ok. One important point I should highlight with the use of these inequalities the solution you get is not the best one, but a satisficing one. I repeat the solution you get is not the best one, but a satisfying one. This point already I have already elaborated that means, you need a solution.

I have already mentioned important of ah. So, the inventory management systems in any organization and essentially inventory management systems deals with the determination of a particular so the inventory control systems or inventory control, sometimes you order in policy for say for all sorts of items, inventory items as listed.

So, obviously, you know you cannot wait for getting the best possible solution. So, whatever the data you have you try to say use this data to get a solution. So, that is why, you know the solution in majority of the cases may not be best one, but definitely you may call it satisficing one it serves the given purpose. And as of now, the distribution of the demand is not known and the solution of the problem is overdue, is it ok? You cannot wait or you cannot say that first get me the first the distribution or the demand distribution say probability distributions of demand and then only you will get the solution. So, the problem is overdue a typical situation with respect to many many inventory items.

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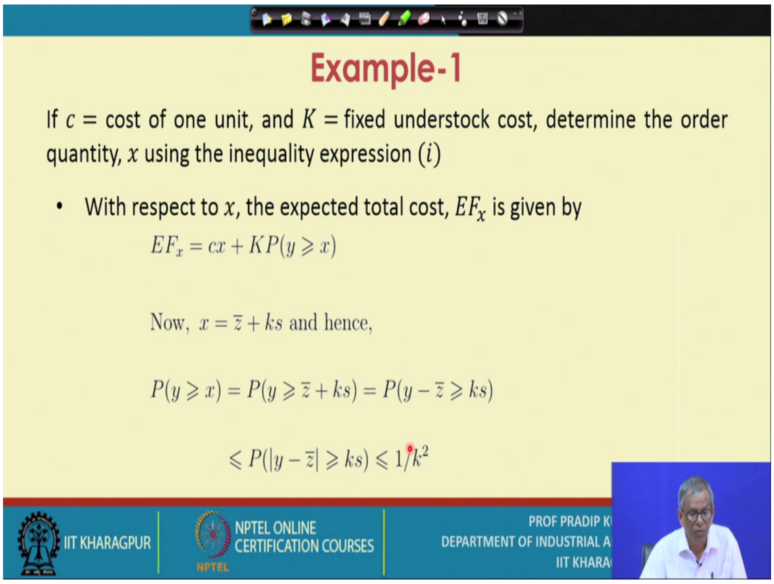

So, now I will be referring to example one, if c is the cost of 1 unit; that means, it could be a purchase cost ok; that means, the unit is purchased from outside or it may. So, happen that this could be the production cost; that means, it will be the insight supply case, you are already aware of the 2 situations. You come across either the insight supply case for the outside supply case for the given inventory items.

So, if c is cost of one unit and capital K, these notation we are using it is fixed understock cost, determine the order quantity, x using the inequality expression one. So, K fixed on understock cost; that means the cost of understock almost remains fixed; that means, irrespective of the number of units short. So, you have the data and you can prove that this condition holds. So, if you have, say the values for the small c and capital K, then what you do? You if you are order quantity is x.

So, with respect to x expected to total cost E F x is given by this notations. We have been using, E F x equals to c in to x plus K the probability that y is greater than equals to x; that means, this it is the understock probability; that means, if the actual or the demand is greater than greater than or equals to the order quantity. Obviously, it is a stock out situation.

So, what is this probability. Now, x equals to z bar plus k into x. So, k is the multiplying factor k times s. And hence, probability that y is greater than equals to x equals to probability is greater than y probability that y is greater than equals to z plus k into s; that means, x is replaced with z bar plus k s.

So, it is probability that y minus z bar greater than equals to k s which is less than equals to probability that the difference between y minus z bar greater than equals to k into s. And this is essentially the Tchebycheff's inequality in it is original form less than equals to 1 upon k square.

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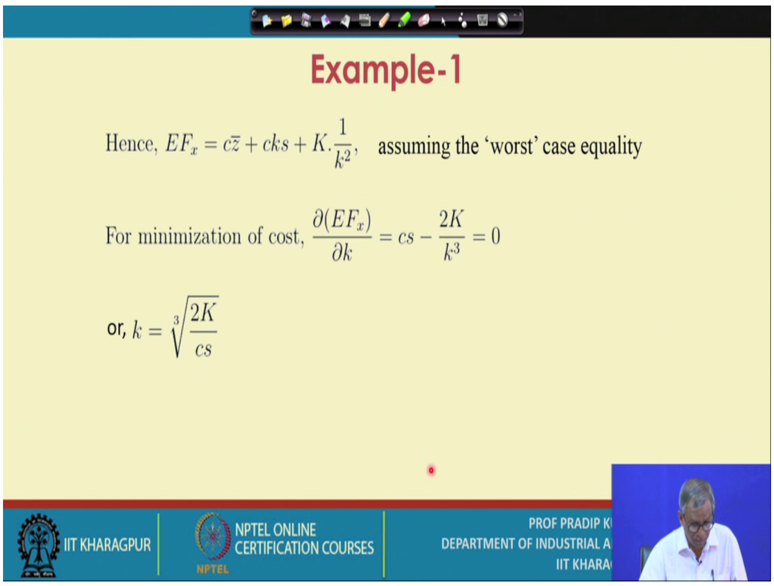

So, now what do you get? That means, that expression for E F x becomes z c into z bar plus c into k s c k s plus capital K and the probability that y is greater than equals to x is replaced with the use of Tchebycheff's inequality, 1 with replaced with 1 upon small k square assuming the worst case equality. Ok. So, it is essentially Tchebycheff's inequality less than equals to; that means, worst case is 1 upon k square.

So, for minimization of cost, del del k of EF x; that means, the partial derivative with respect to x, you set it equals to 0 this is the necessary condition. So, it becomes cs minus 2 K divided by k to the power 3. So or small k equals to say that root of cube root of 2 into K by cs.

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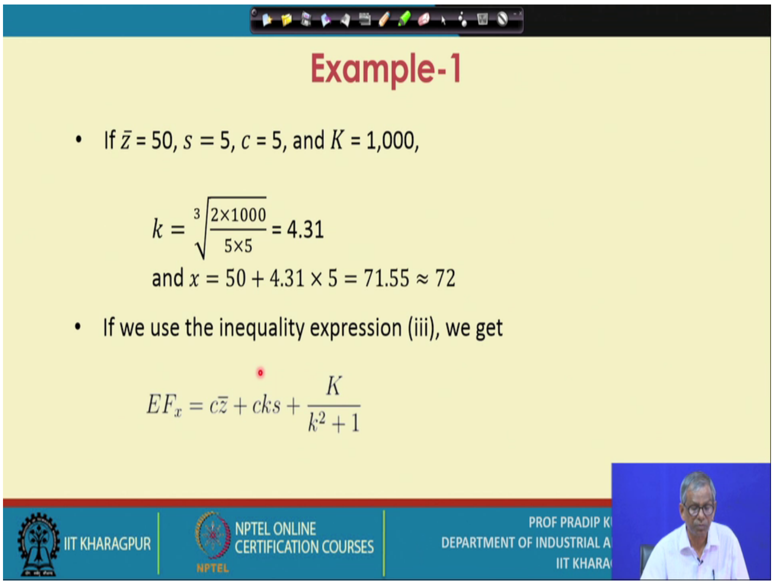

So, if z bar equals to 50, s equals to 5, c equals to 5 and K is 1000, is it ok. This is just an illustrative example. What is the value of k? So, that the value of k will be 4.31 and so, hence the order quantity x is 50 plus 4.31 into 5; that is, 71.55; approximately 72. If we use inequality expression 3, we get EF x equals to c z bar plus c k s plus k; capital K divided by k square plus 1. So, you please refer to the inequality expression 3; that means, the right hand side is 1 upon k square plus 1.

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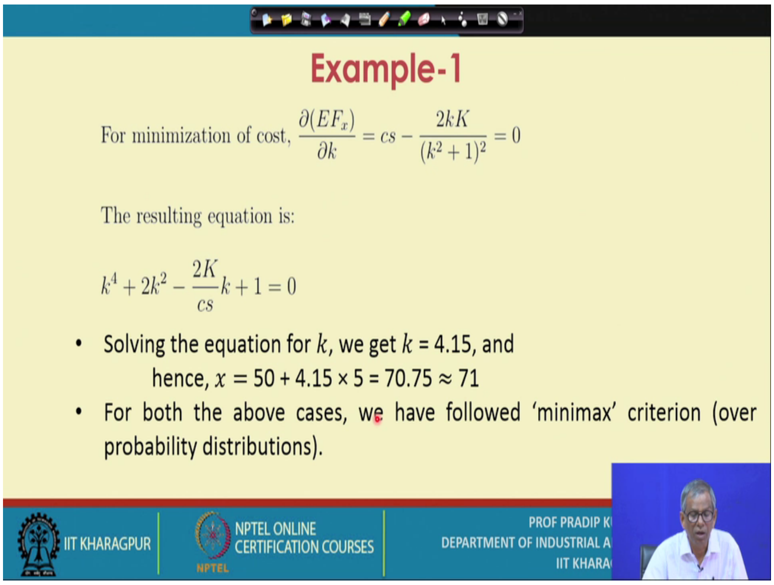

So, this expressions we have used and what you get if you go for minimization of cost? You take the partial derivative with respect to small k. Because, small k is the unknown and you determine whether value of small k. So, this is the expression you get so, it equals to 0.

So, the resulting equation is this. So, solving the equation for k we get k equals to 4.15. Now, my request is that to you is that try to get the solution procedure for this particular equation. So, we have followed the procedure and we have found that the value of k is 4.15 and hence, x equals to 50 plus 4.15 into 5; that is 70.75 that is 71. For both the above cases, we have followed minimax criterion, but over probability distributions.

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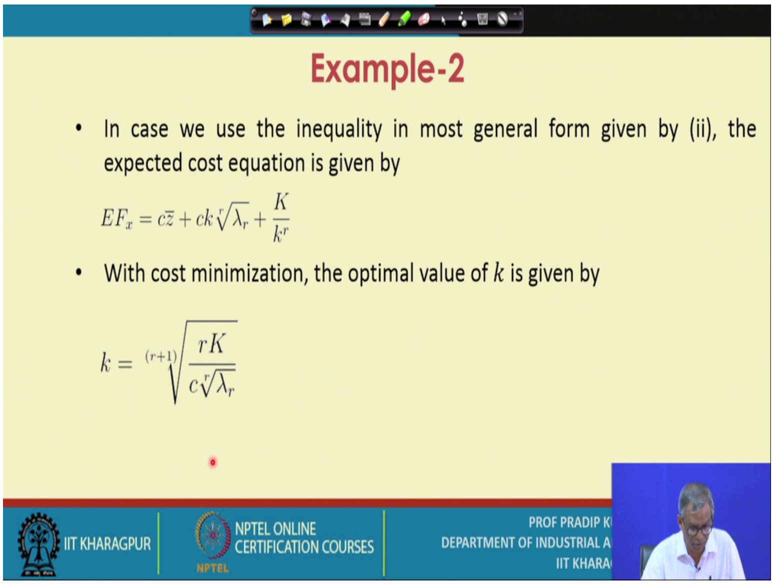

So, this is the example 2, in case we use the inequality in most general form. So, what you try to do? That means E F x equal to c z bar plus ck r th root of lambda r plus K by k to the power r.

So, same approach we follow and we get the cost minimization for with cost minimization we get the expression for the small k optimal value; that is r plus 1 th root of r capital K divided by c into say r th root of lambda r.

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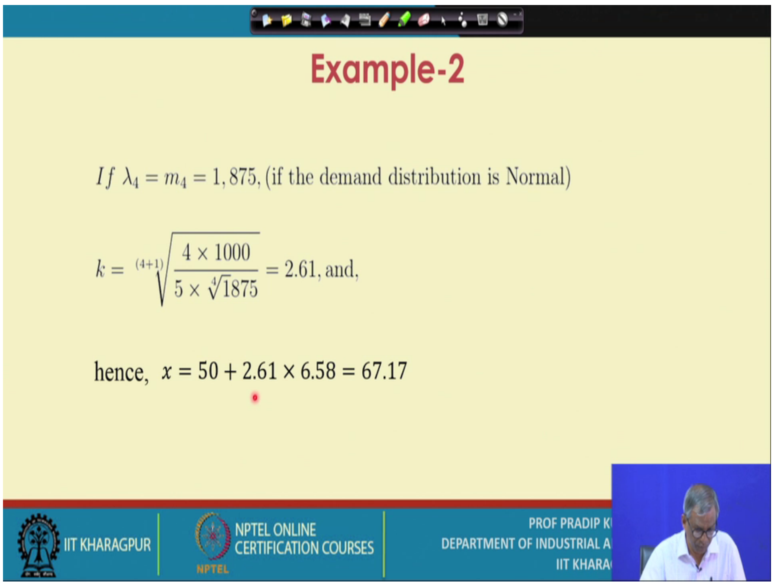

So, these are the values; that means, up to fourth moment the values are available. That is up to fourth moment 1875, suppose, if the demand distribution is normal, then we have the value of k, is it ok. So, the expression for small k is already known and hence order quantity is 50 plus 2.61 into 6 point into 2 point 2.61 into 6.58; that is 67.17, ok.

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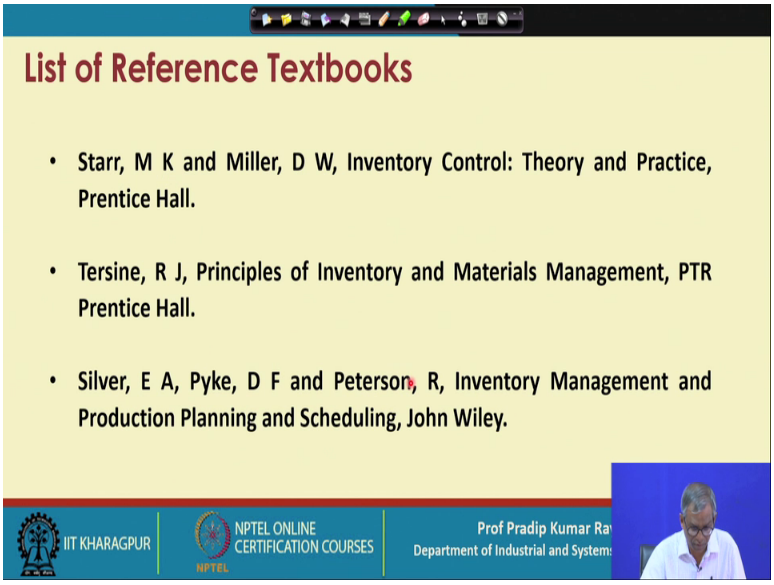

So, this is the expression or the expand 2. So, we are just explained few cases of application of Tchebycheff inequality. So, such examples we will provide in the subsequent lecture sessions also.

Thank you.