Management of Inventory Systems
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Lecture - 17
Static Inventory Problems under Uncertainty (Contd.)

So, during this session on Static Inventory Problems under Uncertainty, I will be referring to two important issues one is the Bayes criterion and the second one is problem formulation with partial information.

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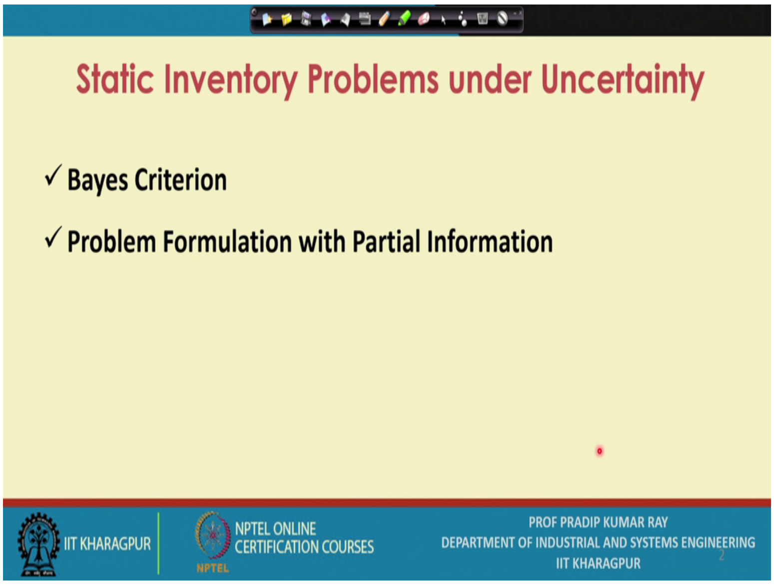

As you are aware we have already mentioned that for this kind of problems static inventory problems under uncertainty in majority of the cases will be dealing with partial information. And so the partial information related to the demand of the inventory item can be expressed in several ways. So, during this lecture session, I will be referring to certain methods of which you may apply to represent or start in measures which you should be aware of with which will get an idea about the level of partial information. So, this is very important.

And our main focus is you know how to get this partial information’s or as many information as possible relevant information’s as possible regarding the characteristics of the demand for the given inventory item.

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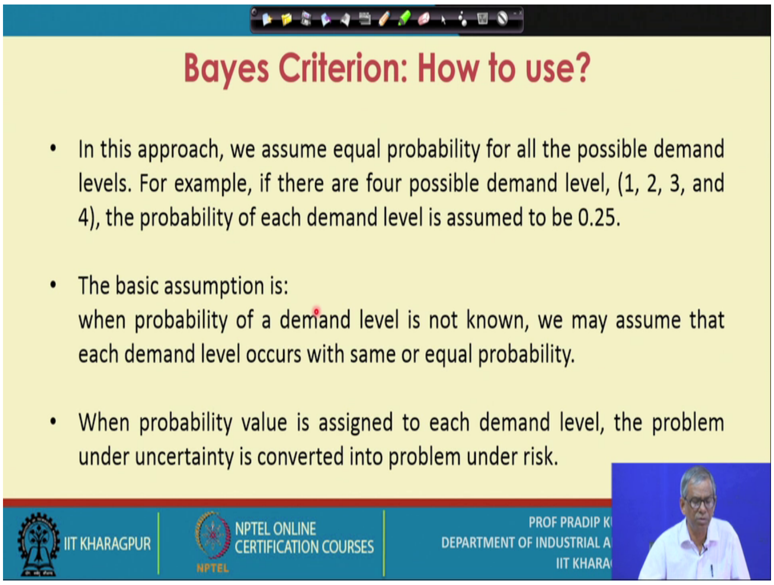

So, let us the first discuss on the Bayes criterion. How to use the Bayes criterion in this case? Now, you know I have already mentioned that there are 3 criteria decision criteria you may adopt and the probability values are not there, but you have the payoff matrix. So, only the missing point is the probability value.

So, in the payoff matrix so you have to adopt some other some other means and to get a solutions and in these case we will just discuss, so the Bayes criterion. Bayes criterion is also known as the equal probability criterion, where in this approach we assume equal probability for the all possible demand levels. That means, equal probability sometimes made if it is equal probability case that means, it is we are directly or indirectly referring to a particular distribution called uniform distribution.

So, just make a note that a many a time when in the probability distribution is not known your the starting point could be for analysis it could be just to assume that that in the equal probability distributions or uniform distributions. And as you are aware that uniform distributions you can use or so the discrete random variable or the continuous random variables; so, for both the cases you may use the uniform distribution. So, then sometimes the uniform distribution is also referred to as, so the distribution with maximum ignorance that means, you have really ignorant of say the possible the demand levels in this case.

Now, here we are saying that if the probability against a particular demand level is not known so why do not you assume for each possible demand for all possible demand levels the same probability. So, that is the basic assumption. Now, these assumptions may be constant, but in many cases, so you know there is a logic behind assuming equal probability for so the each the demand level. So, we will go by this logic and what is this logic? That means, if there are 4 possible demand levels, so 1 2 3 and 4, ok. So, these are the, so, either 1 unit, 2 units, 3 units or 4 units, ok.

The probability of each demand level is assumed to be 0.25 that means, when the exact probability are known let it be assumed to be the same that means, 1 by 4 that is 0.25. The basic assumption is when probability of a demand level is not known we may assume that each demand level occurs with the same or the equal probability that means, the we are assuming that the frequency comes remains same for each event. When the probability value is assigned to each demand level the problem under uncertainty is converted into problem under risk.

And then what we will do? It is very simple that means, against each order strategy you calculate the expected return or expected payoff, and either in terms of profit or in terms of the cost. So, you select that particular order strategy for which the expected profit is maximum or expected cost is minimum this two cases always.

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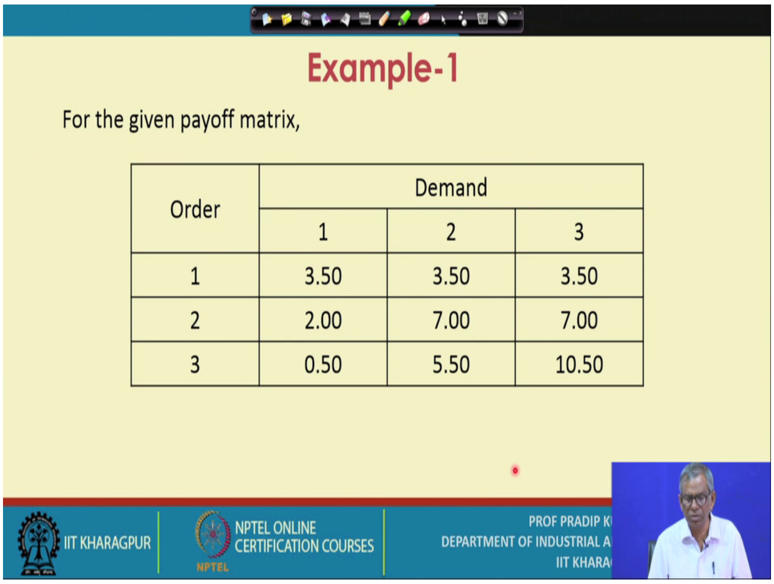

We highlight for the given payoff matrix we are already if are familiar with this particular payoff matrix, so 3 by 3 payoff matrix, 3 demand levels and 3 order quantities. So, you have this tables I do not know to explain it once again. So, once these values are known from what you try to do that comes against each demand level demand level is 1, ok. So, what is reverse probability? There are 3 possible demand levels so obviously, for each possible each demand level.

So, 1, 2 or 3 the corresponding probability will be 1 upon 3 that means, 0.33. So obviously, you write it here the corresponding probability so that probability against 1 0.33, probability against 2 0.33 and probability against demand level 3 again 0.33. So, this matrix the payoff matrix is converted into con converted into a payoff matrix for the static problem static inventory problem under risk.

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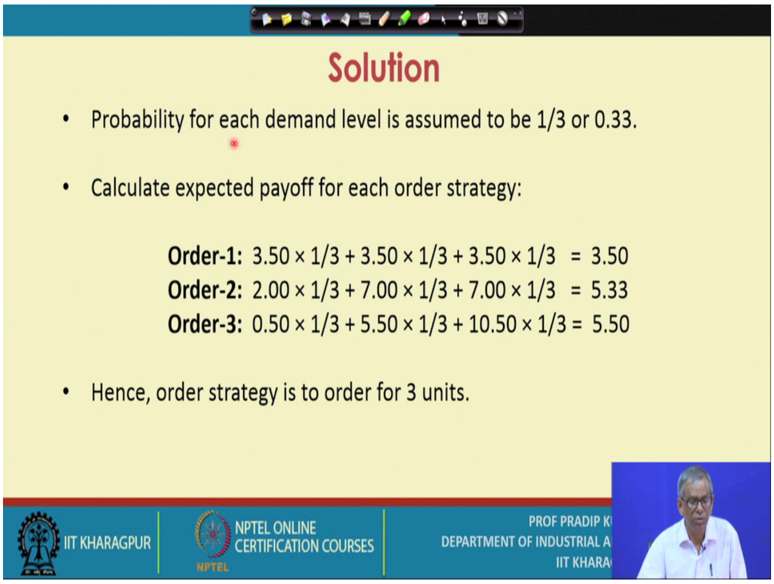

So, the probability for each demand level is assumed to be one-third or 0.33 already this point is highlighted. Calculate expected payoff for each order strategy that means, for order one obviously; you say 3.5 into 1.3 plus 3.5 into 1.3 plus 3.5 into 1.3. So, typical calculations we do for determining the expected the payoff, so it is 3.5.

Similarly for the order 2, this is a return 2.00 when the demand level is 1 and corresponding probability is one-third this is the return when the order quantity is two and demand level this 2 and the corresponding probability is one-third. And similarly for the third demand level that means, when the demand level is 3 units and order quantity is

2 the corresponding the written is 7 with the probability of one-third. So, the total is 5.33. So, the same approach we follow for calculating the expected payoff expected return against order 3, so these values 5.50. So obviously, the order strategy is to order for 3 units because 3.5, 5.33 and 5.50. So, against order 3 the expected payoff is maximum that is 5.50 in this case now, ok.

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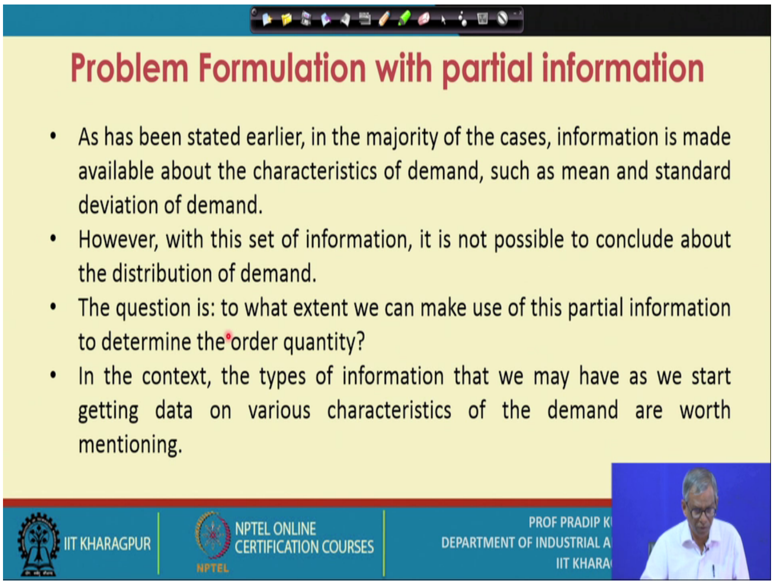

So, this is this 3 decision criteria already were explained. That means, the first one is the Wald criteria you just check whether the Wald criterion is suitable in your case or not. If it if you find it is suitable you apply it. If you find its not suitable then means you do not want to say one to adopt to a conservative approach, so you may opt for the savage criterion that means, the regret matrix and then go for it and you get the solutions. But, if you come to make the whole system of taking so, determining the order quantity is simple one our solution should be why do not you go for equal the probability the criterion that means. So, the uniform distribution is functioned, ok. So, that has been proposed by Bayes and the Laplace.

Now, when you are trying what happens that this is a this extreme situation what we have explained, but in many a time what happens that you are running say the inventory control systems with respect to one particular inventory item. And as you are say working with the inventory control system so obviously, you know you are becoming aware of the levels of the demands or the given inventory item and as the situation becomes the stable and so you can get an idea or obviously, you can get the idea about the possible demand levels, is it ok.

So, the first when the you start the system as the time passes you start collecting. Now, the relevant data related to the demand of the given inventory item in a systematic manner and so you will be building up this information based for the for the item demand over period of time. So, at any point in time we may find that we will be dealing with partial information.

It is not that at that point in time at any point in time suppose if someone ask you about the demand distribution and the type demand distribution you say that, yes I do not know: what is the type of demand distribution or I can tell you certain information related to the demand. Like I might tell you what is the average, what is the mean demand or what is the standard deviation of the demand, what could be the possible the range, the maximum value, the minimum value. So, many such the information in you may have related to the demand.

So, this aspect is referred to as the partial information. And the first attempt should be that why do not you try to formulate the problem with whatever the information as the given are related to the demand of the distribution. So, that is why we have named it problem formulation with partial information in this particular topic.

As has been stated earlier in the majority of the cases information is made available about the characteristics of the demand. So, the demand pattern say, such as mean and standard deviation of demand is it, that means, this may be known. However, with this set of information it is not possible to conclude about the distribution of demand they are already aware of. The question is to what extent we can make use of this partial information to determine the order quantity. I hope that you have understood this problem that means, forget about the demand distribution now, for the time being.

Whatever the information we have collected let us do this information be used to formulate the problem, and while you formulate the problem obviously, there could be certain say that criterion you try to done it and in millimeter of the cases you try to minimize the cost or more specifically try to minimize the total relevant cost is it, and the decision value is known that means, the order quantity in this case. So, in this context the types of information that we may have as we start getting data on various characteristics of the demand are worth mentioning. So, this the point we are highlighting now.

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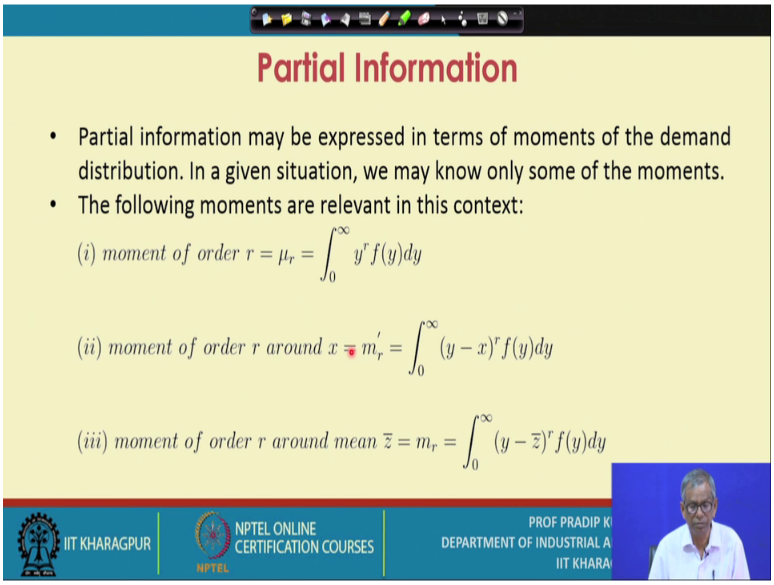

As we are all aware of you know the concept called moments of the distribution moments and so this partial information may be expressed in terms of the moments of the demand distributions. The students were not aware of the moments of the distributions so say the please refer to any textbooks on the probability on statistics and just go through them and I am sure that you will you will have proper understanding of the moments of the distributions, ok. So, again we will be referring to the basics of the moments of the distributions in this particular lecture situation, lecture session.

In a given situation you may know only some of the moments is it, ok. So, what are the and the common say the set of moments about the distributions or the demand distribution in this case should be aware of. So, how do you define moment of order r? Moment of order r this notation is mu r and f y is the probability density function, y is the demand, or is a it is basically the continuous random variable. So, the y against y the probability density function is f y. So, it is integration 0 to say infinity, y to the power r it is this order r f y dy this is the basic definition of moment of order r.

Now, the next extension of moment that is mu r is moment of order r around x. So, the standard notation is m r dash which is defined as integration 0 to infinity y minus x to the power r, that is why it is around x y minus x to the power r f y dy. So, this is the moment of order r around x.

Now, this particular x may be the mean and the mean notation is z bar. Please note it down that we are using a notation z bar for mean. So, moment of order r around mean z bar, so the standard notation is m subscript r m r 0 to infinity integration y minus z bar to the power r f y, the same definition same definition we have followed except in your x is replaced with z bar. So, you call it moment of order r around mean z bar. I think it is clear.

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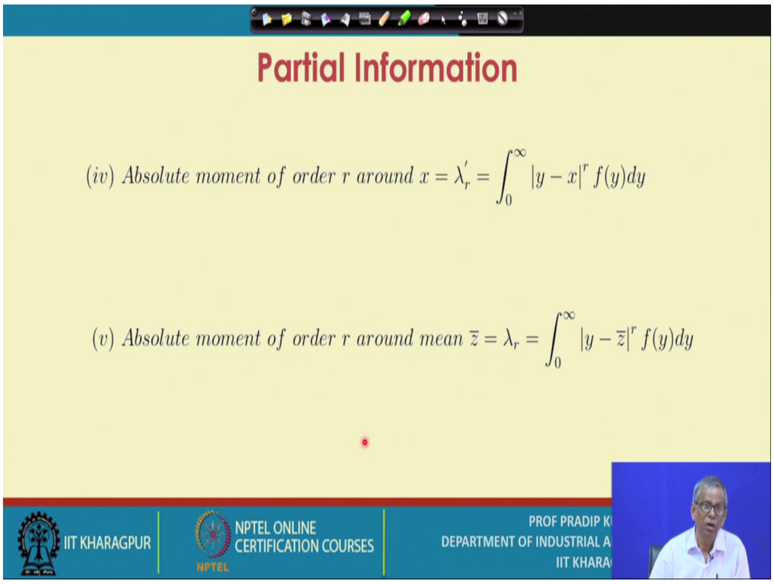

Next one is next stage you go for say defining absolute moment of order r around x. So, the standard notation is lambda r, lambda subscript r dash. So, it is integration 0 to infinity absolute y minus x, absolute to the power r f y dy because this is the absolute moment of order r around x and this x could be the mean z bar. So, if you want to have an expression or absolute moment of order r around mean z bar, ok. So, these this is the note standard notation is lambda r that is integration 0 to infinity y minus z bar absolute to the power r f y dy.

Now, why we have you know proceeded up to the absolute moment of al around order r around min z bar? Now, there is a there is a reason for this one, you will find that to solve the, this kind of problem that means, the static inventory problem under uncertainty that means, we partial information and with the help of distribution free analysis.

So, if you want to the determine the order quantity in this case now, you are supposed to use as the researchers have pointed out have suggested are a number of you know the inequality expressions. And this inequalities which will find in the probability theory, this were this have been proposed by the great mathematicians in a situation where you know the exact form of demand distribution is not known that means, for any kind of distribution these inequalities hold or these inequalities are valid.

So obviously, you know in this case when you are not aware of the type of distribution for the demand. So, why do not you use any one or more of these inequality expressions. Now, one important inequality expression is referred to Tchebycheff's inequality I repeat this inequality is of what is it Tchebycheff's inequality. And this, the Tchebycheff's inequality has close reasons with the absolute moment of order r around mean z bar that is lambda r, is it clear.

So, I hope that that this point has been made very clear because we need two units very soon you will come to know that for solving such a problem you will be, so using many types of inequalities expression, and this inequalities particular Tchebycheff's inequality and other inequalities also as I have very close to resembles with absolute moment of order r around mean z bar that means, lambda r. So, that is why we are specifically the mentioned the expression for lambda r.

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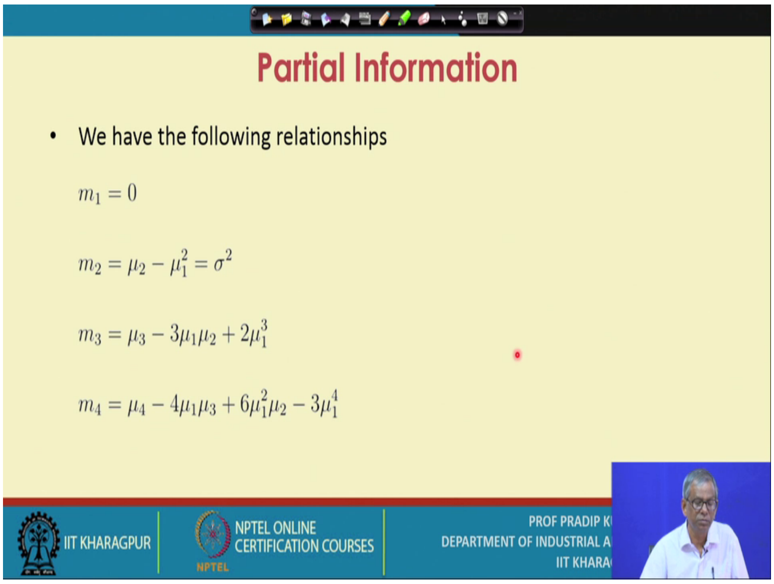

Now, so with this information partial information's what you have? We have the following relationships first one is m 1 equals to 0, you can compute why m 1 equals to 0, ok; m 2 is equals to mu 2 minus mu once that is the sigma square. It means what? That means, it is a essentially the second moment, the second moment around mean. What is m 3? m 3 is mu 3 minus 3 mu 1 mu 2 plus 2 mu 1 cube. And what is m 4? That means, mu 4 minus 4 mu 1 mu 3 plus 6 mu 1 square mu 2 minus 3 mu 1 to the power 4. So, I suggest that you as an assignment, why do not you go you go to the basic textbooks on statistics probability and statistics and you get these expressions, verified right.

So, this is the first moment, this is the second moment, this is the third moment and this is the fourth moment. That means, suppose you have a situation where only the information up to the first moment is known in many cases we come across such a situation. In many you know say the factories, in many workplaces is it, whether is the manufacturing or the service organizations in many cases what we find that the data they are referring to, that is only you know some average value or the mean value. In many cases even they just say that average is known. But you ask down sir can you get me some idea about variability or say you know the measure of dispersion. So, they might not tell you anything.

So, the standard deviation if we get the value up to the standard deviations of the variance it is, one of the measures of dispersion. So, we might say that, the information is available up to the second moment and as this situation as the time passes and the system becoming more stable and all. So it is natural that you will be getting more the data about the demand, demand patterns, demand distribution and ultimately you may you know the compute the third moment with the given data set, and in certain cases you may also be able to calculate m 4 that means, the fourth moment.

Now, you know if you calculate normally what we say that you may get a particular inventory item has demand this concerned. Suppose in a given situation in a work place you find that you know you can get an estimate up to the fourth moment you say that you know the maximum possible you know the data are say the volume of data or the types of data are made available.

So, up to so the m 4 what we may conclude that if the data are made available with which you can calculate up to fourth moment there is enough in (Refer Time: 24:52) of the cases.

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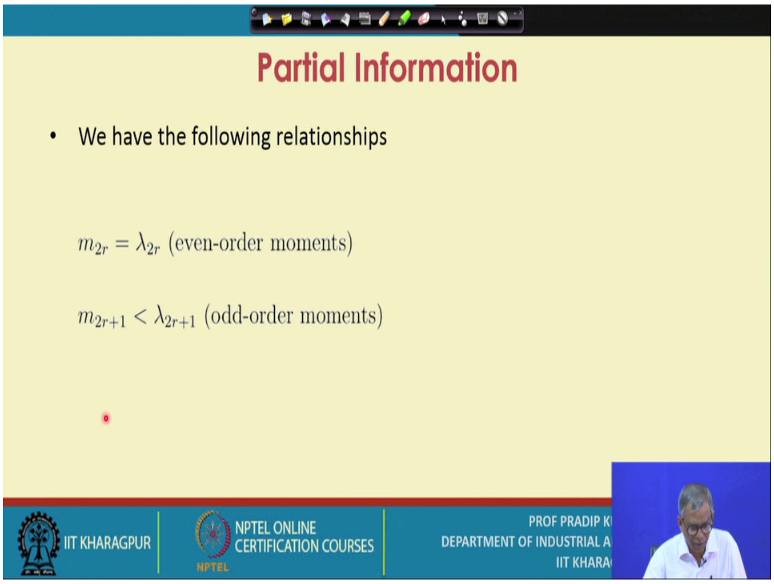

So, this point you just keep in mind. And then you have the following relationships the means m 2 r equals to lambda 2 r that means, moment even order moments you need to create if this is the absolute moment, this is the moment and this is absolute moment, ok.

So, what is the relationship m 2 r equals to lambda 2 r? That means, even order moments when you delay it and m 2 r plus 1 is less than equals to lambda 2 r plus 1 that means, for the odd order points that means, less than lambda 2 r plus 1. That means, absolute moment used al is you know the greater than the moment of the distribution and for the odd order moments. So, these two relationships you just remember.

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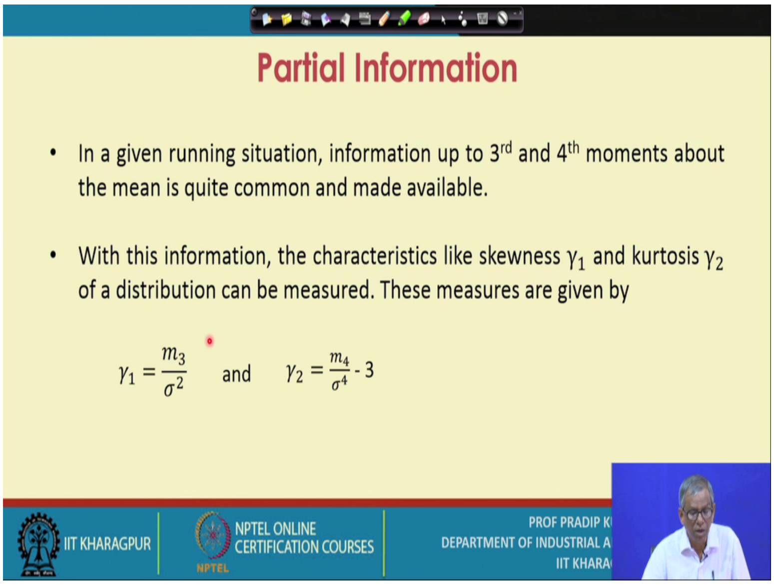

And in a given running situation as we have been referring to the information up to third and fourth moments about the mean is quite common and made available, ok. So, this point already I will elaborated and with this information the characteristics like skewness is the gamma 1, this skewness that notation is gamma 1 and kurtosis as the notation is gamma 2.

I think you have the aware of a distribution can be measured these measures are given by gamma 1 that is this skewness there is m 3 by sigma square and gamma 2 is equals to m 4 by sigma to the power 4 minus 3. Now, minus 3 this particular value comes because of say a normal distribution. Now, here one can give you know must keep in mind that is whenever you say that this is this is the skewness, this is the skewness gamma 1 it means what that means, it is essentially as a the demand distribution is symmetrical one or not whether it is symmetric or asymmetric.

So, if this skewness is 0 the gamma 1 is 0 what does it mean? If the gamma 1 is positive what does it mean? If the gamma one is say positive or some other values, or say positive values and negative values what does it mean? Similarly the kurtosis means actually the peakedness of the distribution, peakedness.

So, many a time you know you say that I can calculate gamma 1 and when you get the value of gamma 1 you can get an idea about the shape of the distribution, shape of the distribution. And when you can calculate so the gamma 2 that means, information up to m 4 is available, so then only you can calculate gamma 2 that means, the peakedness of the distribution also you can get an idea of. So, this two are very important gamma 1 and gamma 2 that means, what actually you are referring to that means, first you start with m 1, m 2, m 3, m 4. Then first you check whether with the given dataset, calculation of m 4 is possible or not if it is possible then what you do you go for calculating gamma 1 and gamma 2. So, you will get lot of information about the demand distribution.

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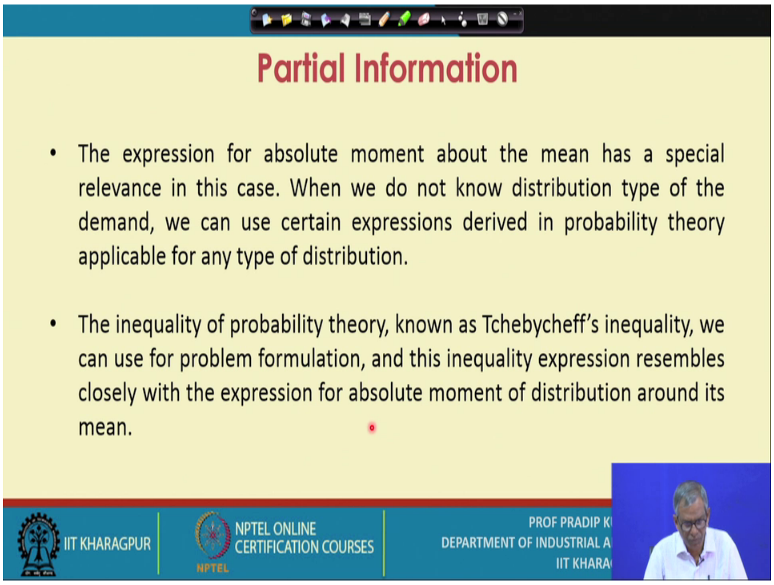

The expression for absolute moment about the mean as a special relevance in this case. When we do not know distribution type of the demand that is the case that is why the problem under uncertainty, we can use certain expressions derived in probability theory applicable for any type of distribution.

So, that is the greatest advantage you have when you use this inequality expressions as given in the probability theory. The inequality of probability theory known as Tchebycheff's inequality we can use for problem formulations and this inequality expression resembles closely with the expression for absolute moment of distribution around its mean. That is why specifically we are referred to the absolute moment of distribution around mean is it that is lambda r.

And in the next lecture sessions when we talk about the Tchebycheff's inequality, so will get in to the details and will find that to what extent this is dependent on this particular the expressions for the absolute moment of a distributions.

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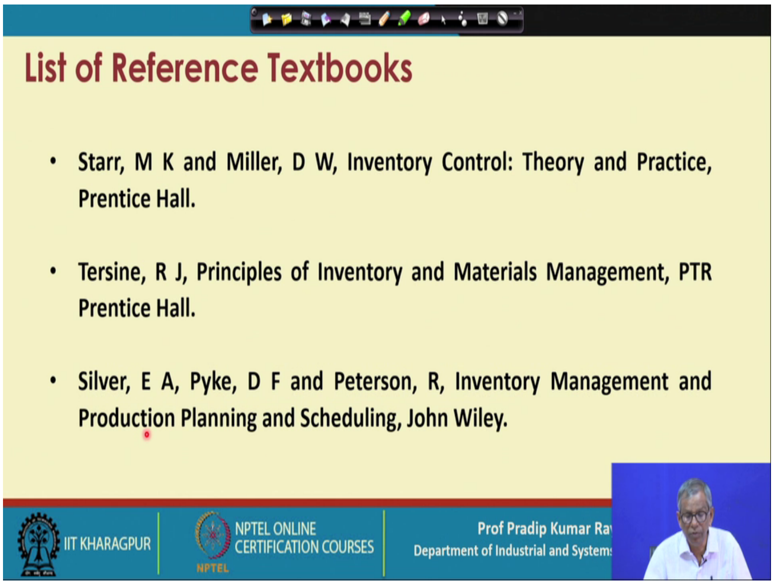

So, I close this session and in the next sessions we will be referring to the Tchebycheff's inequality, ok.

Thank you.