Management of Inventory Systems
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Lecture - 16
Static Inventory Problems under Uncertainty

If you remember during the past 3 weeks of our lecture sessions, we refer to different kinds of inventory problems and these were inventory problems are appropriately classified. In the last week that means, during the third week of our lecture sessions we referred to static inventory problems under the risk. During this week that means, during this fourth week of our lecture sessions we will be exclusively dealing with the Static Inventory Problems under Uncertainty ok.

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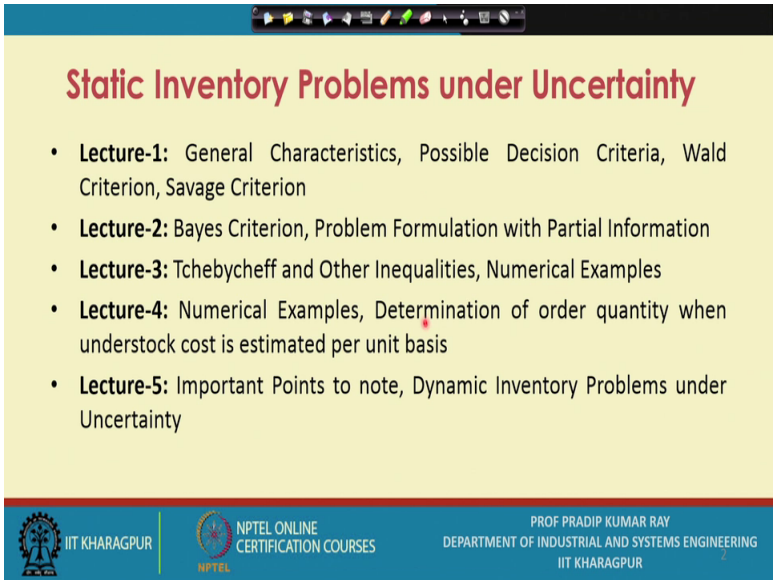

And, during this week all this 4-5 lecture sessions we will highlight you know that the different aspects of static inventory problems under uncertainty. Now, this sort of problem in inventory management we frequently come across. And so obviously, you know when you come across such a problem first thing you need to do that is you need to formulate the problem. And then once the formulation is over you suggest the appropriate solution techniques. So, this is the general you know the rule will follow. Now, lecture-1 we will be discussing general characteristics of the problems, you should be aware of the possible decision criteria under uncertain condition.

The two specific decision criteria we will discuss, the first one is Wald criterion and the second one is the Savage criterion. During lecture-2 we will be referring to Bayes criterion and then we will discuss the problem formulation with partial information. During lecture-3 the well-known Tchebycheff and other inequalities as you find in the probability theory, we will be referring to we will discuss with numerical examples. In the next lecture session we will be again continuing with the numerical examples, you should be aware of I mean different kinds of formulations, and against which formulation against all this formulations, you also should be aware of the numerical problems.

So, we will be continuing our discussion on numerical examples and then a specific you know the formulation we will discuss in detail that is the determination of the order quantity when the understock cost is estimated per unit basis. So, this is just one class of problem and we will be specifically dealing with this problem. During the last session of this particular the topic, what are the important points you should remember. So, we should summarize all these important points and we will refer to the dynamic inventory problems under uncertainty. This is just an extension of the static inventory problems under uncertainty and we will refer to this kind of problem also ok.

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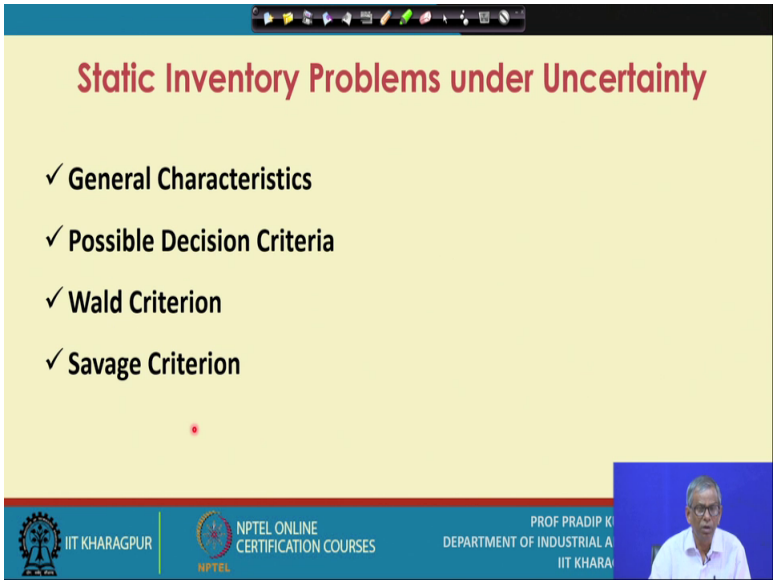

So, now during this lecture session I will first refer to the general characteristics of the problem, static inventory problems under uncertainty. Then what are the possible decision criteria, then two specific that criteria we will be referring to with we will

discuss then with specific numerical examples. The first one is the Wald criterion and the second one is the Savage criterion ok. So, this will be our coverage.

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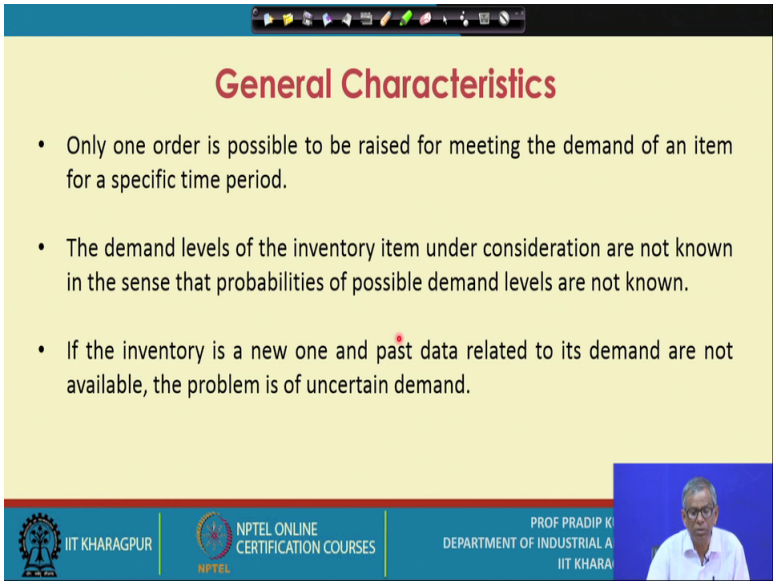

Now, as per the general characteristics of this problem is concerned you may note down several important points. Now, as it is a static inventory problem you are aware that only one order is possible and this order is to be raised for meeting the demand of an item for a specific time period. If you took that means, the demand is occurring and for a specific time period is a limited time period and what we are assuming that one order is sufficient to meet the demand. So, this is the first characteristic. The second one is the demand levels of the inventory item under consideration are not known in the sense that the probabilities of possible demand levels are not known.

So, you know what we have mentioned when we refer to static inventory problem under risk, we are saying that against the demand of an item the corresponding probability distribution of demand is known and then only we are saying that this is the problem under risk. Now, here in this case when we are saying that the problem exists under uncertainty, we are assuming and that is the assumption that the probability distribution of demand is not known. There could be some you know some information related to demand, but with that information you cannot conclude anything about where the distribution of demand.

So, that is why we are saying that the demand levels of the inventory items under consideration are not known and corresponding the probabilities. If the inventory is a new one and past data related to its demands are not available. Obviously, if it is a new situation so, you do not have any the past data related to the demand of the item; that means, forget about the objective probabilities of demand even you may not have the subjective probabilities against the demand. So, this problem is of uncertain demand right.

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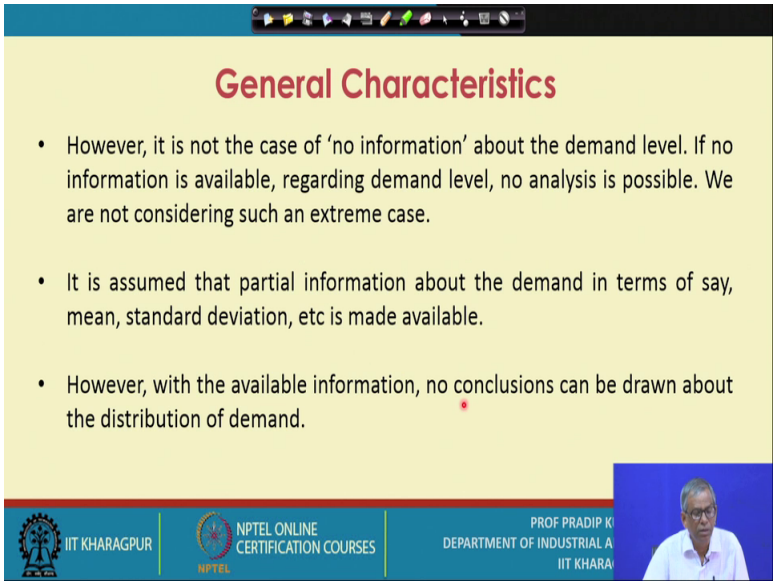

Next one is what we might say that we might conclude in many cases that if the demand distribution is not known, you might conclude that there is no information. But that is not the case that is why we are mentioned it is not the case of no information about the demand level. If no information is available that is nothing is known about the level of demand or the characteristics of demand obviously no analysis is possible. So, we are not considering such an extreme case.

So, what we are saying that you have some partial information related to the demand of the item and to what extent you can use this partial information logically, scientifically so that you know the other quantity you can determine. Our basic purpose is under this situation are you in a position to determine the order quantity for the given inventory item. So, that is our decision variable.

It is assumed that the partial information about the demand in terms of say, mean, standard deviation, etcetera is made available. Later on we will come to know that how do you represent this partial information ok. So, usually this partial information is expressed in terms of the movements of the distribution. So, we will explain it in detail and the later section of this particular presentation. However, with the available information, no conclusions can be drawn about the distribution of demand. This point we have already emphasized.

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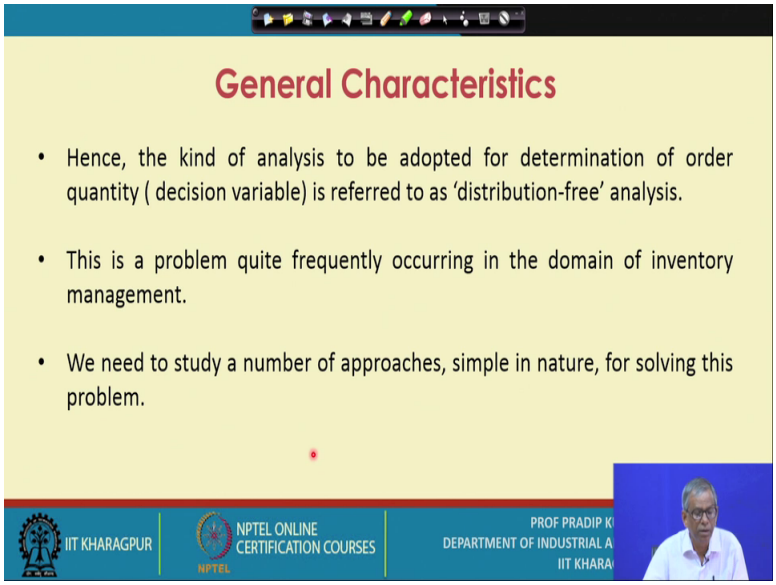

Hence, the kind of analysis to be adopted for determination of the order quantity that is the decision variable is referred to as 'distribution-free' analysis. So, in the in the textbooks in the many cases you come across such a situation; that means, can you go for so, the distribution-free analysis to determine the order quantity that is the decision variable ok.

So, please remember this particular point; that means, here whenever we are saying that the problem exists under uncertainty in respect of the demand of the inventory item; essentially will be using distribution free analysis. This is a problem quite frequently occurring in the domain of inventory management, this point already we have emphasized. We need to study a number of approaches, simple in nature for solving this problem. That means, under distribution free analysis a number of approaches you

should be aware of and one by one in a systematic manner we will be discussing all these important approaches.

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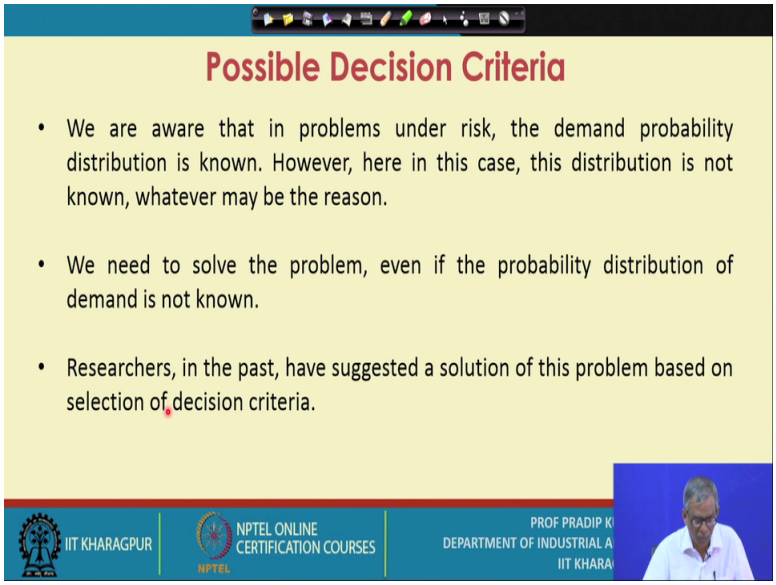

So, we have to take a decision related to say the order quantity. Now, obviously it is suggested that why do not you use certain decision criteria. And if you go through the literature what we will find that the researches, the practitioners they have been suggesting a number of decision criteria. And first you should be aware of what are the possible decision criteria and you also must know that in a given situation which particular decision criterion is appropriate.

So, we will first discuss what are the possible decision criteria that you may adopt or that may you may use for solving static inventory problem under uncertainty. Now, few comments were made we are aware then in problems under risk the demand probability distribution is known or specified either in empirical form or in standard form. However, here in this case this distribution is not known, whatever may be the reason right.

So, it is the point we underlined, that is whatever may be the reason. We need to solve the problem even if the probability distribution of demand is not known; that means, we cannot say that that as the demand distribution is not known how can there be any say the inventory control systems for a given item. So, we cannot say so and that is why we have to think of other or appropriate ways and means to solve the problem. To solve the problem means that you need to determine the order quantity ok. So, the researchers in

the past have suggested a solution of this problem based on the selection of the decision criterion. So already it is there.

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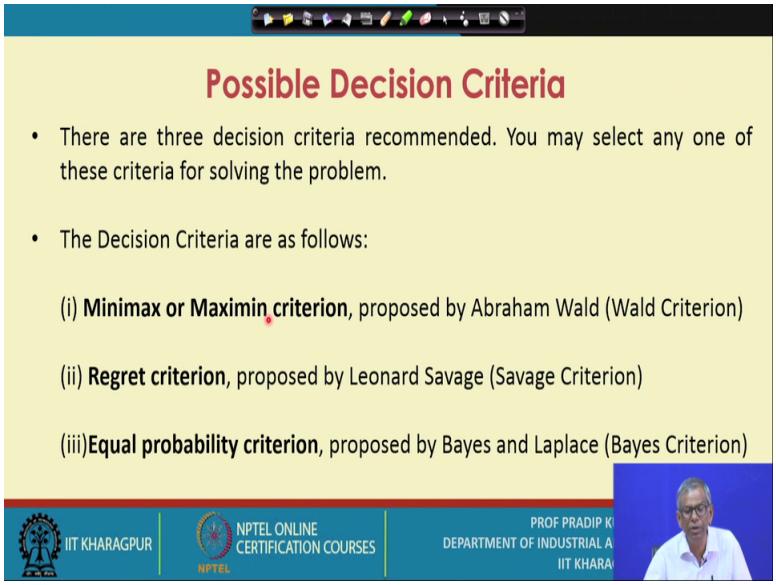

Now, if you go through the literature you will find the mostly one of the three important decision criteria you may use. So, very soon you will come to know; what are these three decision criteria. So, that is why they are saying that there are three decision criteria recommended, you may select any one of these criteria for solving the problem. So, what are these this three decision criteria? The first one is the so called the minimax or maximin criterion proposed by Abraham Wald and that is why it is referred to as the Wald criterion.

So, we will explain that which how do use this Wald criterion for solving the problem. The second one you may opt for that is referred to as the regret criterion proposed by Leonard Savage, that is why many a time it is referred to as the Savage criterion. And the third one is the equal probability criterion is proposed by Bayes, but extensively used by Laplace ok; two great mathematicians. So, that is why it is referred to as the Bayes criterion.

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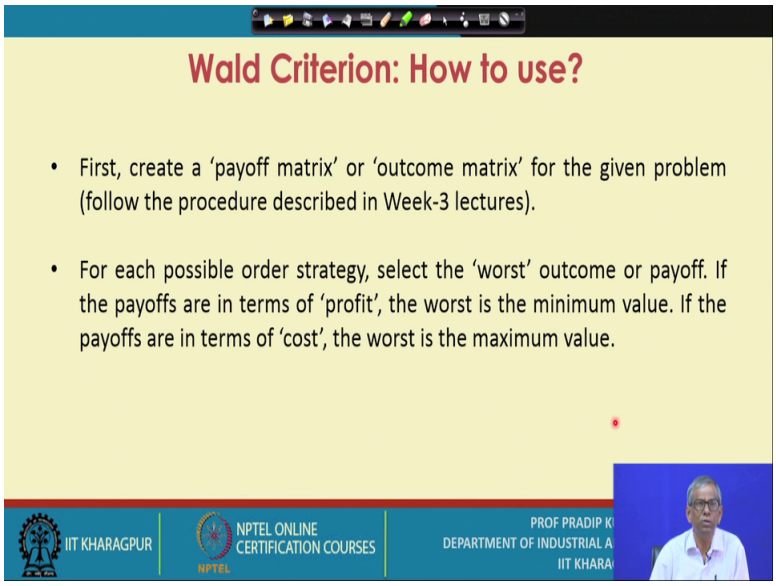

Now, let us first talk about Wald criterion. How to use, how do you use Wald criterion is very simple. If you if you refer to our previous discussions of the last week so, or the previous weeks so, the discussions on inventory problem what you are aware of, you are aware of the payoff matrix ok. So, when so, the demand levels are known with the probabilities so; obviously, you know we need to identify or we need to know that; what are the possible say order strategies.

So, possible order strategies versus the possible demand levels with that probabilities these are known. And for each combination of order strategies and the demand level you need to calculate the outcome or the payoff, either in terms of profit or the contribution to profit or in terms of say the cost. So, these exercise already we have carried out in the previous lecture sessions. So, the first what you need to do here also when you try to apply the Wald criterion to solve the problem under uncertainty, the first you have to create the payoff matrix or outcome matrix ok.

So, this procedure is known to you; for the during problem follow the procedure described in week-3 lectures, I have already mentioned. So, please refer to the week-3 lectures you will get the details. For which possible order strategy; that means, order quantity possible order quantity, select the worst outcome or the payoff ok. So that means, against a particular order strategies there could be the different the possible

demand levels. Now, against to which particular demand level you get the worst outcome or the payoff. Is it ok?

So, if it is in terms of profit obviously, you search for that particular demand level for which you get the minimum payoff whereas, that means, the minimum profit. Whereas, if it is in terms of the cost that means, the payoff matrix you have created in terms of the cost. That means you select that particular given level against the particular order strategy for again for which the outcome is all in terms of the cost is worst; that means, the maximum cost value. If the payoffs are in terms of 'profit', the worst is the minimum value. If the payoffs are in terms of the 'cost', the worst is the maximum value. Is it ok?

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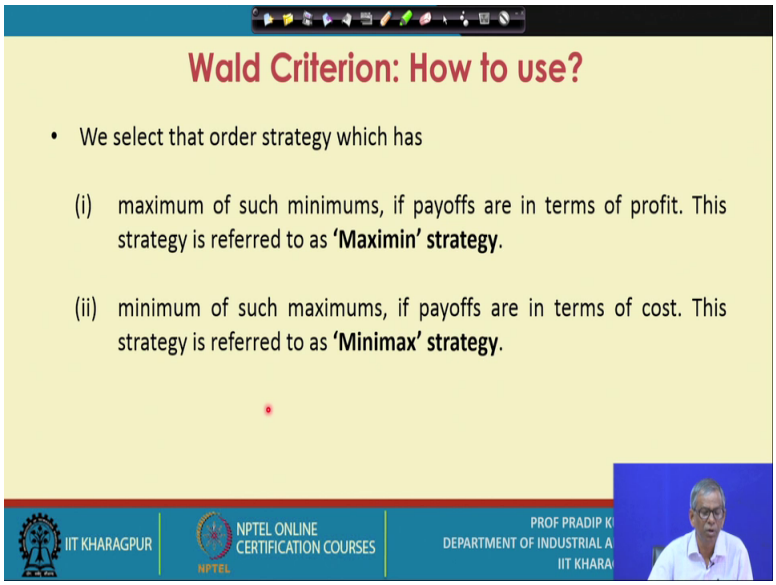

So, it is well understood we select that order strategy which has maximum of such minimums, if payoffs are in terms of profit. This strategy is referred to as 'Maximin' strategy. So, I repeat we select the maximum of such minimums; that means, against all the demand levels you get the say is all order strategies you get the minimum min minimum values. And now, you try to maximize these minimum values; that means, the maximum of such minimums, if payoffs are in terms of profit. That is why it is referred to as maximin strategy, first we go for the minimum one and then you try to maximize it.

So, in the maximum of the minimum that is why it is referred to as maximin strategies. Whereas, if the payoffs are in terms of the cost what you do, you select the minimum of such maximums; that means, this strategy is referred to as minimax strategy. That means,

first you select the maximum value. That means, the maximum cost that is the worst value and then out of all possible say maximum values you search for the minimum one that is why it is referred to as the minimax strategy. So, you just remember the two strategies, the first one is the maximum either it could be maximin strategy or it could be say the minimax strategies.

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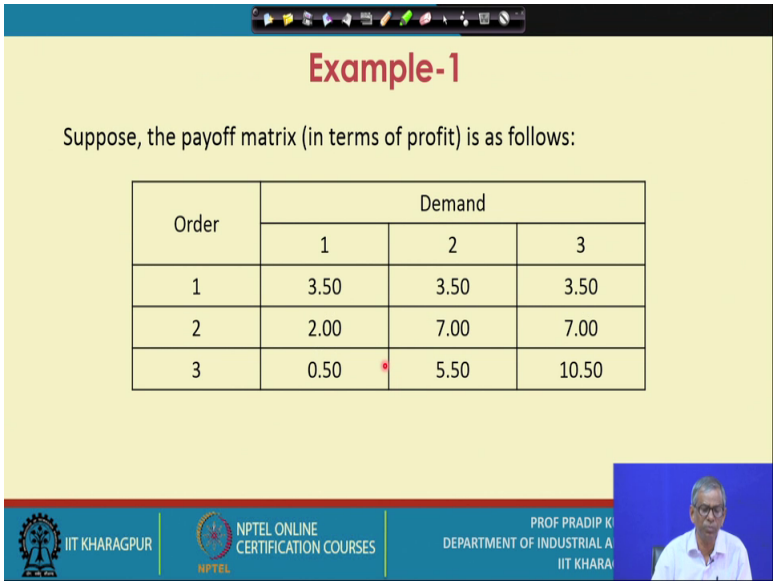

Now, here is an example suppose, the payoff matrix in terms of profit. So, it is 3 by 3 matrix. So, the demand as occurring in 3 levels either, 1, 2 or 3. So, the order quantity could be either, 1, 2 or 3. So, there is just an example that the cost the values are given obviously, and as well as the cell values are given in a given numerical problem.

So, with the given data related to the cost as well as the cells so, you can calculate the payoff. Is it ok? That means, if the order quantity is 1 and the demand is also 1, what is the corresponding payoff. So, the payoff is say 3.5 in monetary terms and similarly for all other you know the combinations like say if the order quantity is 3 and the demand level is 1; that means the corresponding payoff is 0.50. Similarly, if the order quantity is 3 and the demand level is also 3 the corresponding say the payoff, is it at this combination that payoff is 10.50 ok. So, this way we create a this payoff matrix in terms of profit.

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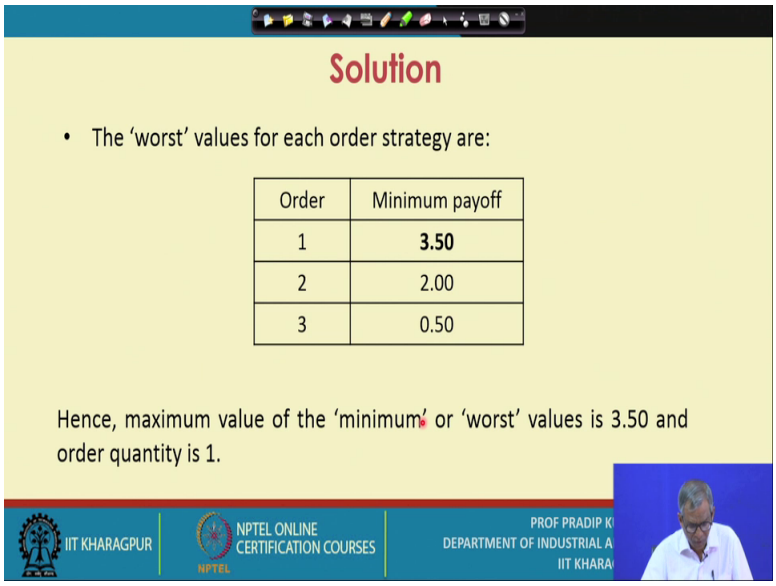

Next what you do the worst values for each order strategy? So, you select the worst value; that means, if you refer to this particular say you know by the table what you find that against order 1 the worst value is 3.5 because, in this case is a very particular case all the values are remaining same so, it is 3.5. For order quantity 3 with the worst value is 2 obviously, this is the worst values and for order quantity order quantity 2 the worst value is 2 and for the order quantity 3 the worst value is 0.50 ok.

So, next what you find that means, we have identified against each order strategy the worst value. So, order 1 3.5, order 2 2.0 and order 3 the minimum payoff is 0.50. Hence, maximum value of the minimum or the worst value is 3.5, and hence the order quantity is 1. So, this is a very the simple approach and I hope that you have understood, what is the basic the idea.

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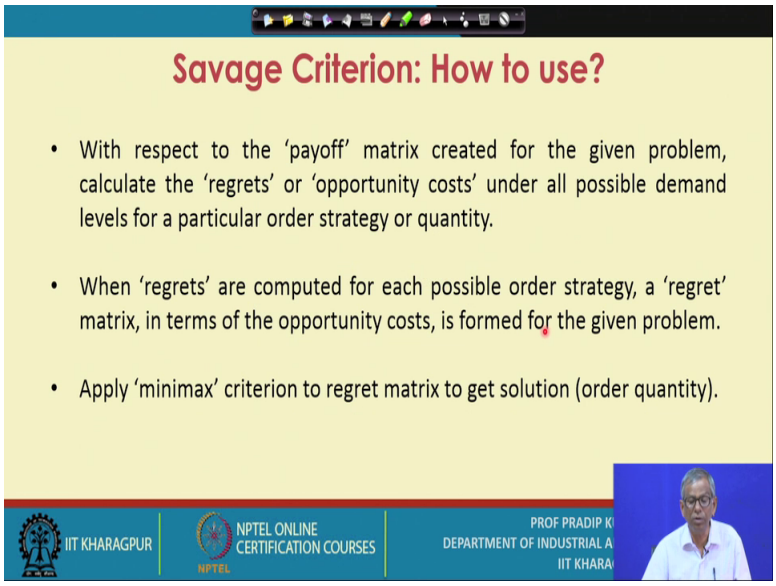

Now, just keep in mind that when you opt for a Wald criterion actually it is referred to as conservative you know you become conservative; that means, in the sense that you first try to protect yourself against worst possible situation. Is it ok? So, this approach is a conservative one, but in many cases when you try to avoid risk means first you must have you must get an assurance that what is the minimum say the payoff, you should be assured of.

So, if you try to say create a situation where the minimum payoff is assured obviously, you will opt for Wald criterion. Now, the alternatively you can also use the Savage criterion. Now, in a Savage criterion so, how do you use the Savage criterion? With respect to the 'payoff' matrix created for the given problem, calculate the 'regrets' or the 'opportunity costs' under all possible demand levels for a particular order strategy or the quantity. Now, in the previous lecture sessions we are referred to the opportunity cost matrix.

So, how do you create these opportunity cost matrix? Such the methods are the method to be used is also very clearly explained. So, the first you create the payoff matrix and this convert the payoff matrix in to asy the opportunity cost matrix ok. So, and obviously, you will find that when you get the opportunity cost matrix you will find that all the diagonal elements they are they do not have any value; that means, all the diagonal elements are 0's.

So, that means, when the demand is exactly matching with the order quantity that is the best possible decision. And obviously, you do not have any regret; that means, there is no the cost associated with loss of opportunity, you are not losing any opportunity. So, that is why there is no loss associated with or the lost opportunity so, that is the best possible strategy.

Whereas, for all other combinations where the demand is not matching with the order quantity; either the demand is greater than the order quantity or the less than the order quantity, what you will find that you are you are incurring certain cost and this cost is referred to is the opportunity cost. So, so, the when or the demand is more than the order quantity; obviously, it is a stock out situation. Whereas, if the demand is say less than the order quantity it is then it is in a worst stock situation.

So, you have already completed that what is the cost associated with the worst stocking per unit basis or what is the cost associated with say understocking unit basis. So, when we refer to the say the opportunity cost matrix, you will get two such important information. So, with respect to the payoff matrix created for the given problem, calculate the regret so, the opportunity cost is it that is the first thing you have to do; the first exercise you carry out under all possible demand levels for the particular order strategies or the quantity.

Once it is done that means, when regrets are computed for each possible order strategy, a regret matrix in terms of the opportunity cost is formed for the given problem. This point already we have elaborated. Apply ‘minimax’ criterion to regret matrix to get solution that is the order quantity. That means, here obviously, it will be minimax, because if you remember that if you were refer to the payoff matrix now, the payoff matrix could be in the form of the cost. So, if the payoff matrix is you create in terms of the cost obviously, you have to apply the minimax strategy. So, the minimax strategy you apply over here because this is a table, this is a matrix you have created the in terms of the opportunity cost.

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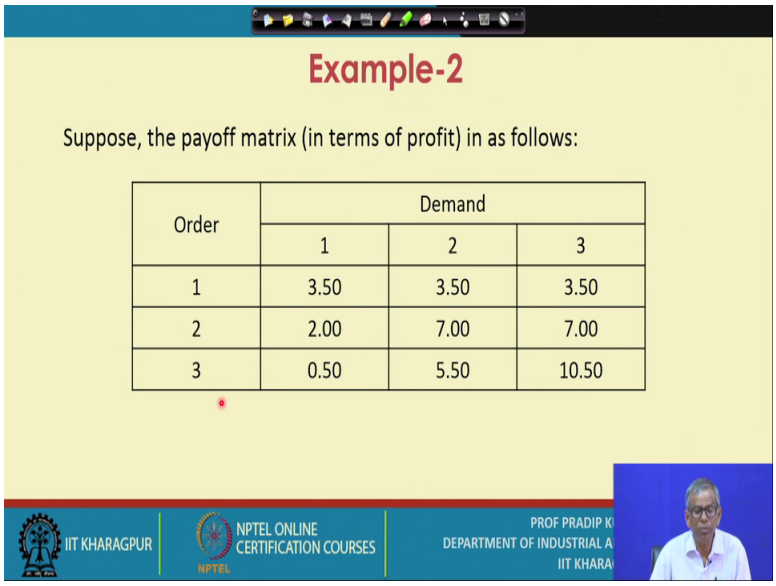

So, here is an example. Suppose the payoff matrix in terms of profit is as follows; that means, again the same table you refer to same table like you have 3 demand levels, this is just for illustration purpose. So, that you have proper understanding of a particular criterion and how do you use this particular criterion for solving the problem.

So, suppose this is again 3 by 3 matrix; that means, the demand occurs at 3 levels in one of the 3 levels. And similarly so, there could be 3 possible order quantities 1, 2, 3, demand is 1, 2, 3. So, you have computed the payoff so, in terms of the profit that is 3.5 3.5 3.5 and all other combination right. So, so, this is your the profit matrix.

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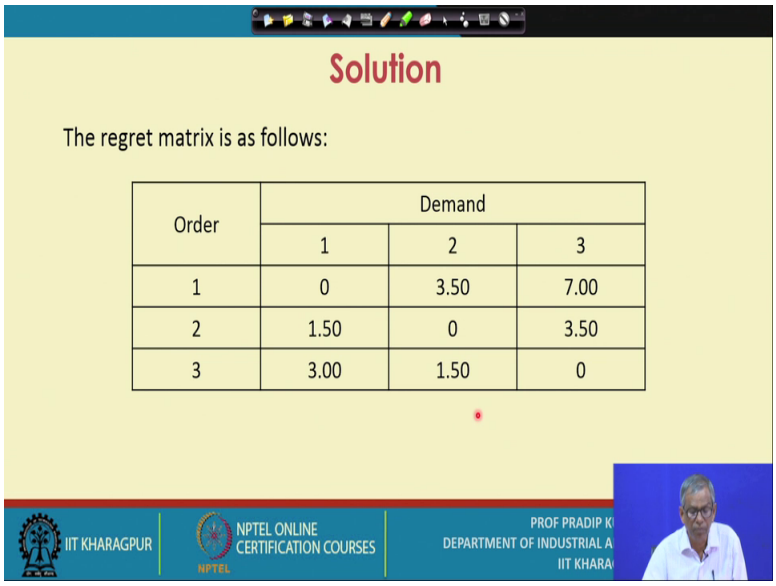

Now, what you do you apply the regret matrix. So, how do you compute the regret matrix? It is essentially the opportunity cost matrix. So, just we refer to this particular table, what you find that when the demand is 1; if you order for 1 you get a value of 3.5. If you order for 2 you get a value of 2.00 and if you order for 3 units you get a value of 0.50. So obviously, this payoff matrix is in terms of profit; that means, the higher values are higher values are always preferred. So, the 3.5 is the maximum one so; obviously, if you go by these the column values you will find that against so, the demand level 1 and against the order quantity 1.

So, you do not have any regret; that means, 3.5 is the maximum and maximum value is 3.5. So, how much is your regret? How much is the opportunity cost? That is you subtract the maximum from the value which actually have obtained. So, the maximum is 3.5 and how much you have obtained that is 3.50. So obviously, you do not the value is 0; that means, there is no regret, there is no opportunity cost. When you get a return of 2 that means, outcome is 2 obviously this 2 should be subtracted from the maximum value that is 3.5.

So, what is your regret? What is your opportunity cost? That is 3.5 minus 2 that means, 1.5 and when you order for 3 you are getting a return of 0.50. So, how much you are losing from the maximum that is 3.5 minus 0.50 that means 3. So, same value you apply for the second column, what you find when the demand is 2 and so, if you go by the

column values you will you will find that against order quantity of 2 you get the maximum value that is 7.

So, when you get 3.5 again the order quantity of 1, how much you are losing that is you subtract 3.5 from the maximum that is 7 minus 3.50; that means, here it is 3.50 will be your opportunity cost. When the order quantity is 2 obviously, the opportunity cost will be 0, 7 minus 7 and when the order quantity is 3 we are getting 5.50. So, how much you are losing that is 7.00 minus 5.50 that means, 1.50. So, same logic you apply to calculate the opportunity cost for the elements in the third column ok.

So, we have this regret matrix so obviously, we will find these all these diagonal elements are 0's and all other elements like say when the demand is 1 the order quantity is 2. So, how much you know the regret you have that is 1.50. When the demand is 1 order quantity is 3 so obviously, their opportunity cost is 3.00 ok. So, similarly when the demand is 2 order is 1, it is not a perfect match obviously, we are using something so, that is the opportunity cost this is 3.50. When it is 2 versus 2, there is no opportunity cost and when it is against 2, when order quantity is 3 again value is in 1.50.

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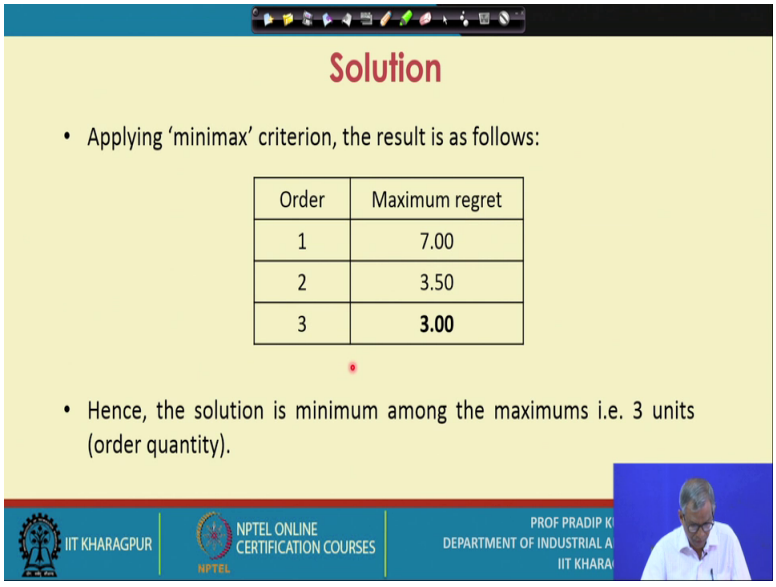

That means, here what you find if you look at this table, here you get a regret matrix. So, regret matrix, opportunity cost matrix. Now, what you do? This payoff this is another kind the payoff matrix essentially that in terms of the opportunity cost. So, you have to apply the minimax criterion. So, what is the result? That means, maximum regret against

order quantity 1 will be 7, against order quantity 2 3.50 and against order quantity 3, it is 3.00.

Hence, the solution is minimum among the maximums. So, that is the minimax that means, first you go for the maximum value and then we try to minimize. So, minimum among the maximum that is the 3 units that means, the order quantity is 3. The minimum value is 3.00 and against 3.00 the order quantity is 3; that is why your decision variable order quantity is 3.

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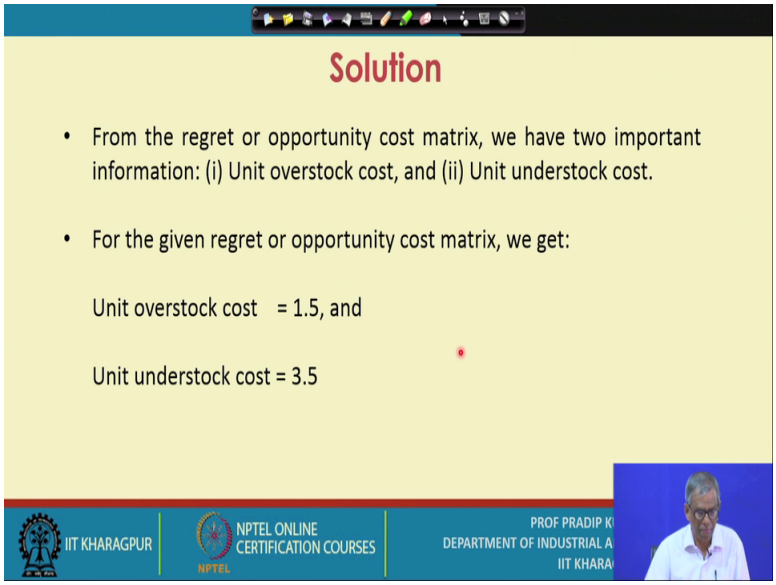

Now, from the regret or the opportunity cost matrix we get two important information; the first one is unit overstock cost and the second one is the unit understock cost. So, if you refer to the regret or the opportunity cost matrix, already developed ok. What we get? Unit overstock cost is equals to 1.5 in monetary units and unit understock cost is 3.5. Now, here one important points to be noted you know whenever we try to access you know the quality of any inventory control system against the particular item.

Now, we have many kinds of say the performance measures; here we are reflecting the quality of inventory control system. Now, one is that at any point in time against a particular say also during a period of time how much over stock you carry. And simultaneously are you know the facing a situation, where for the same item you face an understock situation. Now, in most cases when you go for empirical analysis or empirical

study, what you find that we need to calculate say overstock cost per unit basis, that is preferred or you need to calculate or need to estimate the understock cost.

Now, in almost all the cases what you will find that understock cost is always significantly more than worst stock cost situation. So, if you have a choice between say minimization of understock situation or understock cost or minimization of a worst stock cost. Obviously, we have first preference should be that minimization of understock cost.

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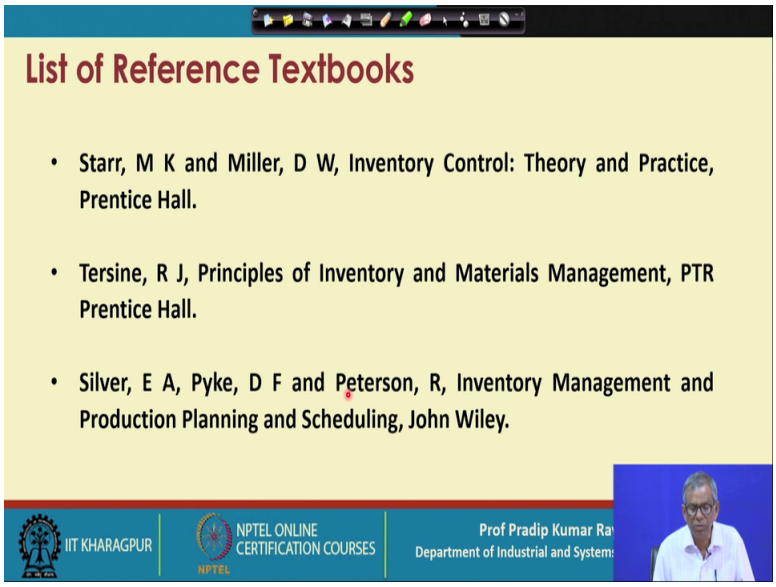

So, with this the reference I conclude this session. And in the next sessions I will be starting with another decision criterion called equal probability decision criteria.

Thank you.