Management of Inventory Systems
Prof. Pradip Kumar Ray
Department of Industrial and Systems Engineering
Indian Institute of Technology, Kharagpur

Lecture – 15
Static Inventory Problem under Risk (Contd.)

Now, while we discuss the Static Inventory Problem under Risk in the previous lecture sessions. So, you might have noticed that we have the classified the problems under four categories.

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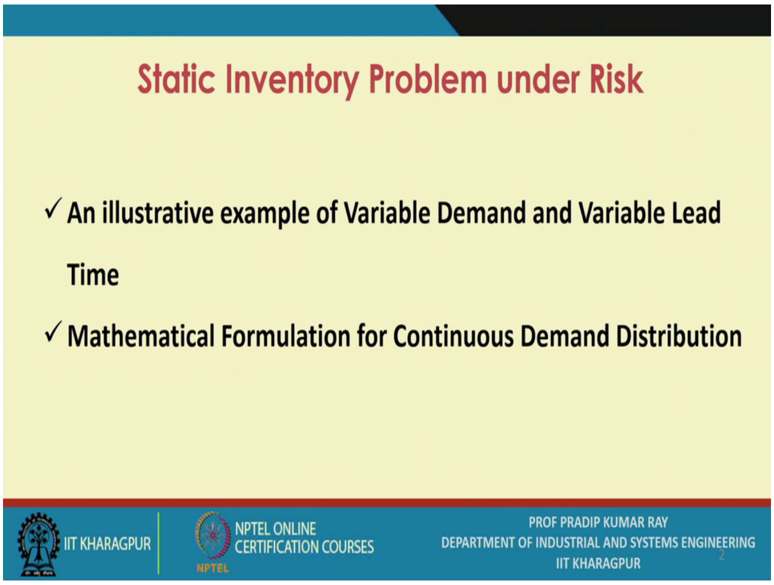

In the first category what we what we considered that is you know the constant demand and the constant lead time. So, usually you know the for the static inventory problem it is it is not coming under risk type of problem because both the important where you say considered to be constant and is and this is not considered as an inventory problem per say. If at all it is considered to be a problem, then maybe you are dealing with multiple items and if you deal with multiple items and for each item you have this constant demand and constant lead time situation. So, it is essentially a scheduling problem.

Now, the other the two problems so we have already we have already discussed that is we have already mentioned that is variable demand, constant lead time and then the constant demand variable lead time ok. So, essentially you know again when you try to formulate both the problems, we will find there are lot of similarities between similarities in this you know in this two types of problems. Now we move to the fourth one that is when you consider both you know say the demand as well as the lead time as a variable.

So, during this lecture session I will give an illustrative example of variable demand and variable lead time and the next what we try to do. That means we will refer to the types of mathematical formulations which you may adopt which you may use in case the demand distribution is of continuous type like say normal distribution or say exponential distribution.

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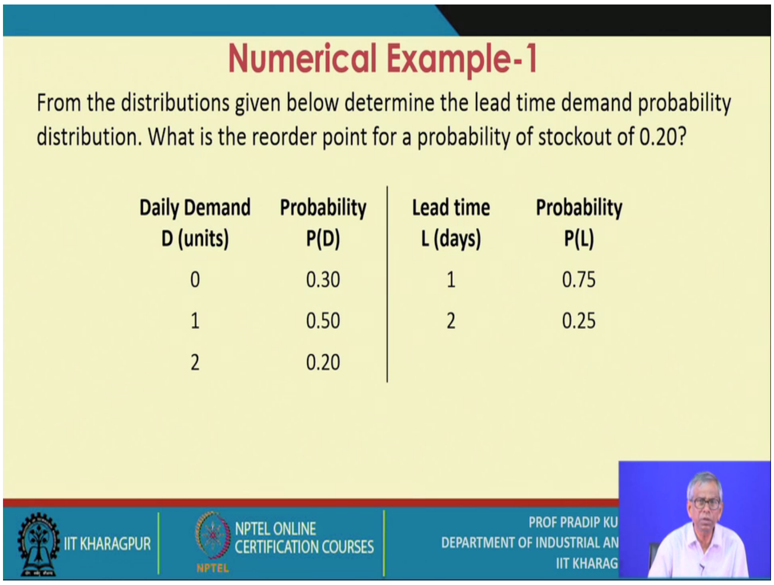

So, let us first take up one numerical examples the related to variable demand and variable lead time. So, what you need to do actually; that means, you need to the provide or the joint probability distribution like if it use a q systems of inventory control; obviously, you know you need to consider the demand during lead time. So, during demand during demand during lead time is considered to be a separate you know a one the random variable and so, you must know that the knowing the distribution of demand and the knowing the distribution of lead time separately, how can you determine the distribution of a demand during lead time. So, this is also referred to as a as one example of joint probability distribution.

So, let us first talk about this numerical example. Now from the distributions given below determine the lead time demand probability distributions ok. So, that is your goal what is the reorder point for a probability of stock out of 0.20 ok. So, when is the joint probability distribution is known, then can you answer to the second question that is what is the reorder point for a probability of stock out of 0.20.

Now, here what you find that for the given item, the daily demand in physical units the notation is capital D. So, the daily demand could be 0. That means no demand at all, there could be demand for 1 unit or there could be demand for 2 units ok. So, this is just an illustrative example. So, at this stage you must thoroughly understand the concepts behind say the inventory control systems the formulation and solution problem formulation and solution.

So, we are proceeding the step wise and from the past data; that means, we through analysis of the past data or from the frequency counts we specify the probability values for all the demand units. So, we are assuming at this stage that this probability values you have obtained from the frequency counts; that means, these are considered as objective probabilities

So, the demand is 0 corresponding probabilities 0.30; that means, 30 percent of the time there is no demand. Against one unit the probability is 0.50 and similarly against the 2 units your or the probabilities 0.20. Another assumptions we are making that this is an exhaustive list; that means there is no probability that the demand will be say the greater than 2 it is clear. So, when you add this 3 probability values, you get a value of 1. As far as lead time is considered, the possible the lead time in days could be either one day or 2 days; it cannot be just 3 days or say 4 days or other days. So, it is restricted to say the just two values 1 or 2. Against one day lead time, the probability is 0.75. That means, even to 5 percent of the cases you may assume the lead time could be one day whereas, remaining 25 percent of the cases the lead time could be just 2 days. So, against 2 days lead time, the corresponding probability is 0.25 ok.

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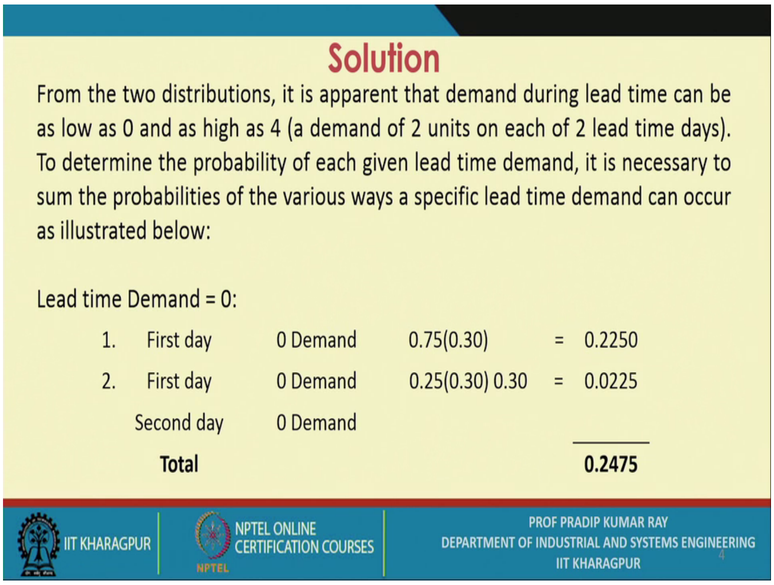

Now from the two distribution, it is apparent that the demand during lead time can be as low as o and as high as 4. That means, supposing the demand is say 0 and the lead time is 1. So, what is the demand during lead time is 0. Suppose demand in both the days; that means, maximum the 2 days lead time you have and there is a possibility there is a probability that in both the days the demand is 0.

So, so it should be or are at the maximum level both the days of say the lead time; that means, the first day there could be a demand for 3 units as well as in the second day there could be a demand for 2 units; that means,. So, so the 4 days the 4 units could be the maximum demand during lead time. So, the demand of 2 units on each of 2 lead time days that is 4 and the demand of 0 units on each of 2 lead time days. So, it is varying between 0 and 4.

To determine the probability of each given lead time demand, it is necessary to sum the probabilities of the various ways a specific lead time demand can occur as illustrated below. That means, we are actually we are using or our say the method of calculating the possible the demand levels as well as the corresponding the corresponding probabilities considering the basic say the events and from the first principles.

Now, the lead time demand is 0, there is a possibility. So, how many ways the lead time demand can be 0; the first day there could be demand, there is a possibility of 0 demand and first day 0 demand there is a possibility and the lead time is just1 day. So, what is the possibility that the lead time is 0.1 day that is 0.75 and what is the probability that the demand on a particular day is 0 unit that is 0. 30. So, it is 0.75 into 0.30 that is 0.2250. So, we need to consider all possible events leading to this particular condition.

Now, there could be you know the first day, there could be 0 demand there is a probability and there could be say the 0 demand on the second day. Now why do you consider the 2 days for the demand because the lead time could be the 2 days. So, what is the probability that the lead time will be 2? There is 2 days that is 0.25 already you would have you refer to the table, you get this values and in the first day the probabilities 0.30 and the second day is also 0.30. So, you get a value of 0. 0225; 0.0225. So, you add these to you get a value of 0.2475.

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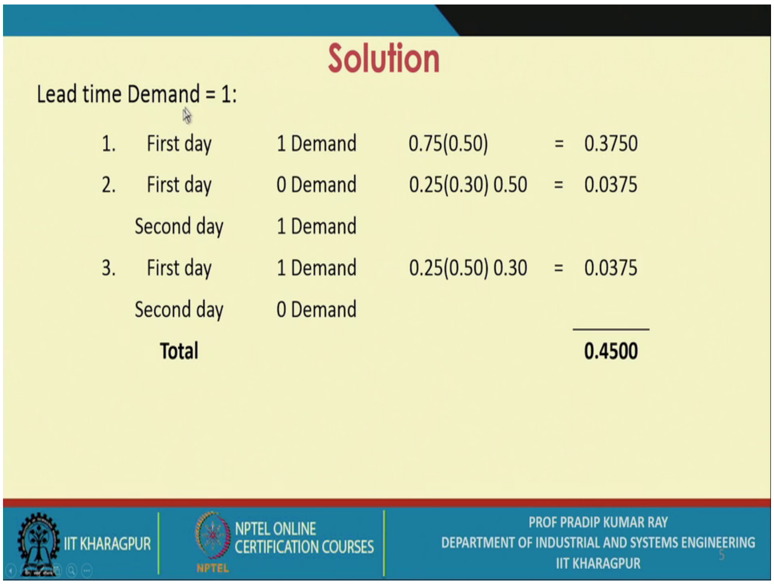

Next one lead time is equal lead time demand is equals to 1. So, we are we try to compute its probability. So, the first day in 3 ways might happen; that means, the first day 1 demand that is all. So, 1 day lead time that is probability is 0. 75 and what is the probability the demand will be 1 unit that is 0.50. So, it is 0.75 into 0.50.

Now, the other 2 possibilities are the first day 0 demand the second day 1 demand. So; obviously, 2 days lead time that is the probabilities 0.25. So, first day the demand is 1 probability is 0.30, the second day the demand is say 0 that is that is the first day it is 0 and the second day it is 1. So, this is 0.3 into 0.5. So, the total will be 0.0375 or alternatively first day there could be 1 demand and the second day there could be 0 demand and accordingly you know you have this probability values. So, the 0.25 into 0.50 into 0.30; that means, 0.0375 so, the total is 0.400.

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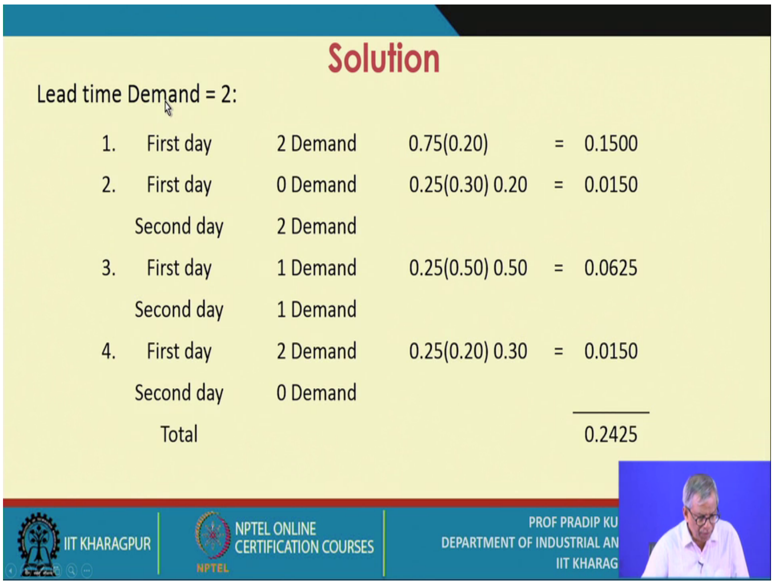

Then the lead time demand is 2 this is the possibility. So, how many ways it can it can happen; that means, just the first day that is the first possibility, there could be demand for the 2 units. So, this is your lead time. So, lead time is 1 day and during this day the demand is point the probability for the 2 units demand is 0.20, so it is 0.1500.

Now, the other possibility is are first day 0 demand, second day 2 demand or first day 1 unit demand, second day 1 unit demand or the 4'th possibility is the first day 2 units demanded the second day obviously, 0 units demanded and the you know this is this these are the probabilities. That means, the lead time is that what is the probability that lead time is 2 days that is 0.25. So, these are 2 days everywhere we consider 2 consecutive days. So, corresponding probabilities are 0.25 and then, you know that the first day 0 demand 0.30 and the second day the demand is say the 2, 2 units that is 0.20.

So, similarly we consider all the relevant probability values for 1 unit demand or the 2 units demand and then when you add all the individual probabilities, ok then you get the probability as 0.2425. So, you please follow the steps it is explicitly we have tried to elaborate all the steps and I hope there is no confusion yourself.

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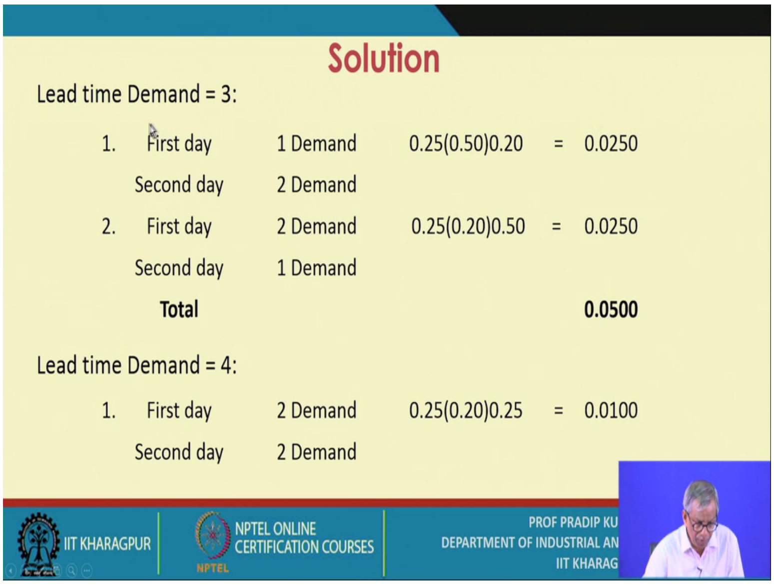

Similarly, the similar method we have applied to calculate or you may apply to calculate the lead time probability for the lead time demand of 3 units. So, in the 2 ways it can happen; first day1 demand, second day 2 demand, first day 2 demand, second day 1 demand; that means, the lead time will always be 2 days. So, the first day what is the probability that in the first day that the demand will be 1 unit that is 0.5 0 and in the second day what is the probability that the demand will be say the 2 units that is 0.20 and similarly similar similarly on the second day, what you find that on the first day so, there will be the demand for 2 units corresponding probability is 0.20 and on the second day there is a probability of the demand of say 1 unit; so, that probability is 0.50.

So, when we consider both the possibilities you get a value of 0.05; 0.0500 as the probability and similarly the lead time of the demand 4 corresponding probabilities how do you calculate that is the first day 2 units demanded the second day 2 units demanded. So, this is the possibility that is 0.0100 0.2 5 into 0.20 into 0.25.

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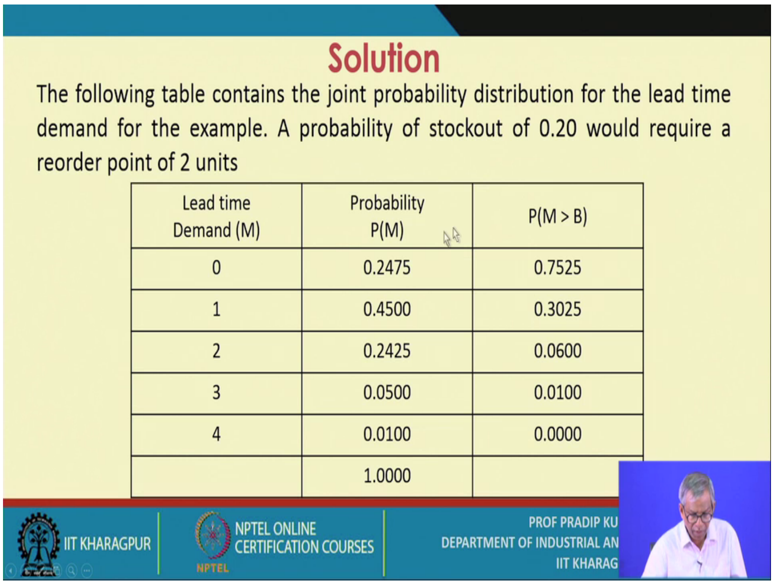

So, this way we calculate you know the lead time or the demand the probabilities. So, first we consider what are the possible lead time demand the units. In this case, there are this demand lead time demand may marry from 0 to say the 4 and it is a discrete case. So, that is why it could be 0 or 1 or 2 or 3 or 4. So, that is why the lead time demand we have used this notation capital M, it varies from 0 to 4 and you may have a lead time demand of 1 unit or 2 units or 3 units also, and the corresponding probability that is the lead time demand probability that also we have computed. So, when you add all this 5 values of the probabilities. So, it becomes 1 and then we add 1 say the column in this table.

So, what is this column? It is column is essentially probability that M; that means, lead time demand is greater than B. So, what is B? B is basically the reorder points. So, this notation we use. So, so you add this column like here the probability that the lead time demand is greater than the reorder point. So, this is a q system. So, the q system is having 2 important parameters; one is the order quantity and the second one is the reorder point and depending on the current level of demand you either you place the order or you do not place the order. When you place the order, the order quantity remains same. So, that is essentially the q system, but at what point in time you are required to place the order that is that is not known, it all depends on say how much the demand is occurring as well as what is the current inventory position ok.

So, during the lead time if you find that the demand is greater than the reorder point right. So obviously, there could be a stock out situation demand and so, this is the probability of going stock out. So, what is this probability; that means, that the lead time demand is say there will be no say they do not demand and the corresponding probability is 0.2475, but there is a probability that there could be demand greater than 0; so either 1 or 2 or 3 or 4. So, what is that probability; obviously, 1 minus 0.2475 that is 0.7525.

Now, from 0.7525 if you subtract 0.4500, you get a value of 0.3025; that means, when the lead time demand is 1 or greater than 1 what is this probability? Probability is 0.3025. So, same say the rule you follow the same logic you follow and ultimately against each lead time demand level corresponding the probability that the lead time the probability that lead time is greater than the reorder point. So, that value you can calculate against each lead time demand level. So, these values are given obviously if you have say if you have say the safety stock as 4 units. So, what is the probability that there will be stock out probability this is a stock out probability. So, the stock out probability is; obviously, will be 0 whereas, if you keep a say the say the safety stock as 3 units, there is a probability that the probability of 0.01 that that there could be a stock out ok. So, this is the interpretation

So, here the problem says that the probability of stock out of 0.20 would require a reorder point of 2 units is; obviously, 0.20 is it 0.20; that means, 0.20 these values is lies between 0.30 and 0.06. So obviously, you prefer the lower value; that means, against this value you get the lead time demand of 2 units. So, that should be your say the reorder point ok.

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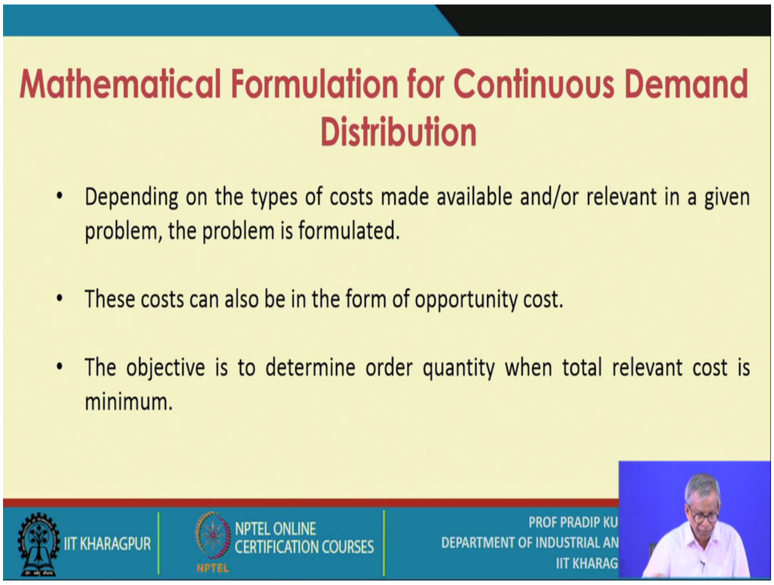

Now, you can use the different types of say the mathematical formulations when you deal with continuous demand distributions, like normal distribution like say exponential distribution or say a continuous uniform say distribution for the demand or the lead time. The depending on the types of costs made available and or relevant in a given problem the problem is formulated. I repeat. That means, depending on the types of costs made available. So, many a time we are restricted by this condition and or relevant in a given problem the problem is formulated. These costs can also be in the form of opportunity cost. So, concept of opportunity cost already we have defined. So, you please refer to those say the lecture sessions where with examples, we have you know by discuss the concept of opportunity cost, we have formed the opportunity cost matrix.

Now the objective is to determine order quantity when total relevant cost is minimum, it is in any inventory kind of problem say you got inventory problems of any type always we try to minimize the total relevant cost ok. And so, there could be different types of formulation when you deal with continuous demand distribution. So, we will just they refer to say some important formulations in this particular case.

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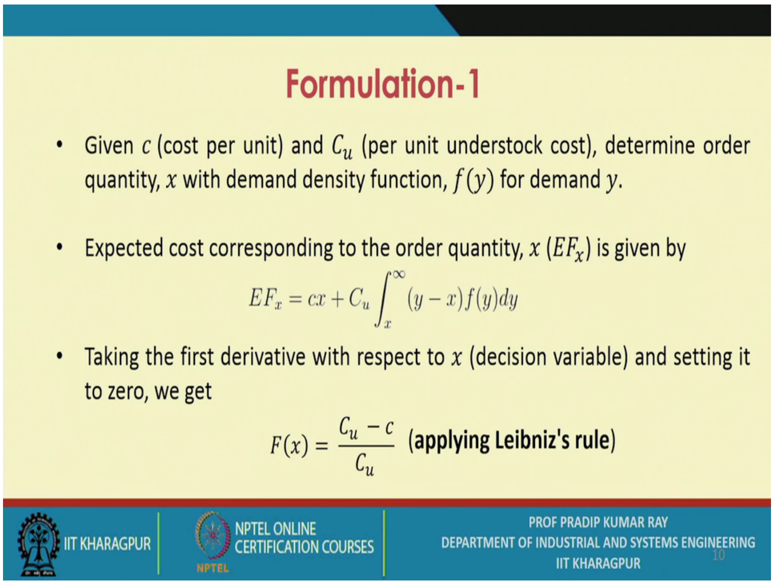

So, the first one is the formulation 1; given c the cost per unit; that means, the item is in the system item is to be procured or item is to be produced. So, you must know that what is the cost incurred per unit. So, this value is given to you this value is known. So, given c that is small c cost per unit and C u per unit under stock cost. So, this is the notation we have used.

Determine order quantity x with demand density function f y for demand y so; that means, there is a density function and the demand is y the demand the notation is y. So, y is a random variable. Expected cost corresponding to the order quantity x. So, that is EF subscript x is given by EF x equals to c into x plus C u integration x to infinity y minus x f y d y. So, this is the expected number of units short under stock out multiplied by say thus stock out cost per unit or sometimes it is referred to as the under stock cost per unit.

Taking the first derivative with respect to x, x is the decision variable and setting it to 0 we get these expressions ok. So, this is cumulative distribution function for x capital F x equals to C u minus c by C u. So, how do you get it? You need to apply the Leibniz rule; that means, you need to differentiate a an integral function. So obviously, you know you need to use the well known the Leibniz rule.

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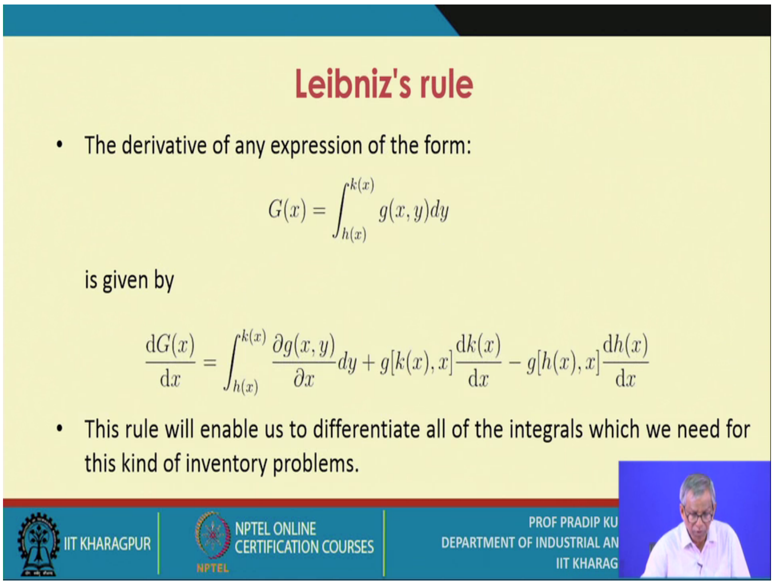

So, what is Leibniz rule? The derivative of any expression of the form; so this is the form; that means, capital G x is equals to integration h x to k x; so, the lower limit of integration and the upper limit of the integration; that means, g x y d y. So, this is the function. So, this is we are explaining the Leibniz rule in general form. So, the derivative is given by; that means, d x of g x is equals to this expressions. So, you just go through you study this expression and you can take up several numerical examples which will help you in applying the Leibniz rule

So, this is the partial derivative; this is the first arm particle derivative with respect to x and this is the changes you make with respect to in the function of say in this function g x y in respect of k x as well as in respect of h x and then, you take the partial derivative with respect not partial derivative you take the derivative of say k x with respect to x as well as the derivative of say h x with respect to x. This rule will enable us to differentiate all of the integrals which we need for this kind of inventory problems.

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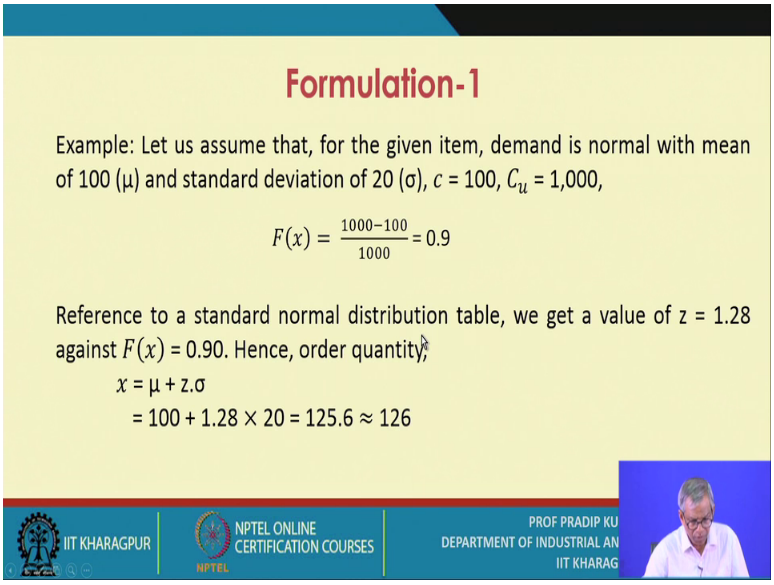

So, this rule is widely used. Now this is just one example to understand the say the applicability or so, understand how the Leibniz rule is used. Let us assume that for the given item demand is normal with mean of 100 that is the notation is mu and the standard deviation of 20 and the notation of standard deviation is sigma c equals to 100 and C u is 1000. So, what is F x capital F x is 1000 minus hundred by 1000 that is 0.9 ok. We have already derived this expression.

So, reference to a standard normal distribution table. So, I suggest that you pick up any textbook on the say probability and statistics and or any textbooks on say the inventory management systems which we have already we have suggested and definitely in the appendix you will find that this the standard normal distribution table is given ok. So, you must be able to say read at this table and how to get this value. So, against so from this standard normal distribution table you get a value of z equals to 1.28 against capital F x of 0.90. Hence order quantity is x equals to mu plus z into sigma and this is 100 and this value you get that is value of z from the standard normal distribution table that is 1.28 into 20 that is 125.6 approximately it has 126 units.

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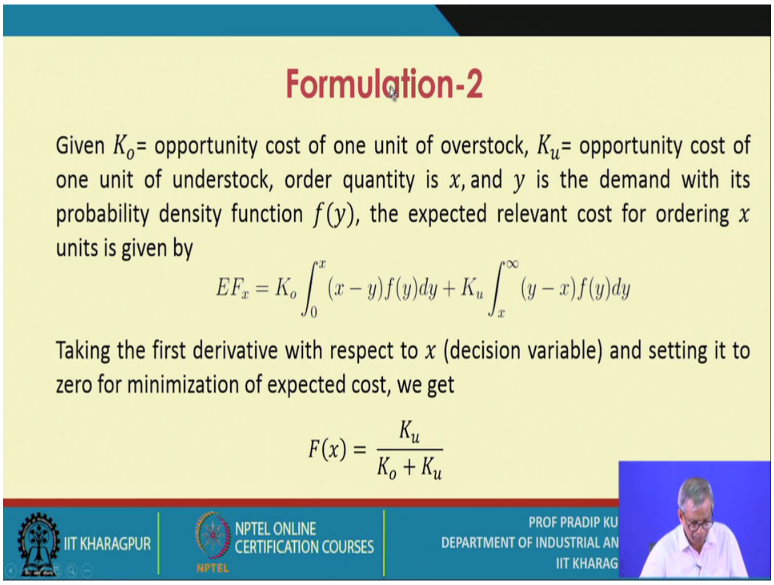

This is the next formulation; that means, given K 0 opportunity cost of one unit of over stock and K u is opportunity cost of one unit of under stock as already you have explained that entire you know the problem can be explained in terms of the opportunity cost also, order quantity is x and y is the demand with its probability density function f y like in the previous case. So, the expected relevant cost for ordering x units is given by; that means, in terms of the opportunity cost. So, this is the opportunity cost for 1 unit of over stock and this is the opportunity cost for 1 unit of under stock. So, this is 0 to x minus y f y d y and similarly for this one this integration x to infinity y minus x f y d y. So, this is the case of under stocking and this is the case of over stocking.

Taking the first derivative with respect to x decision variable and setting it to 0 for minimization of the expected cost, we get this expressions. That means, a capital F x equals to K u by K o plus K u.

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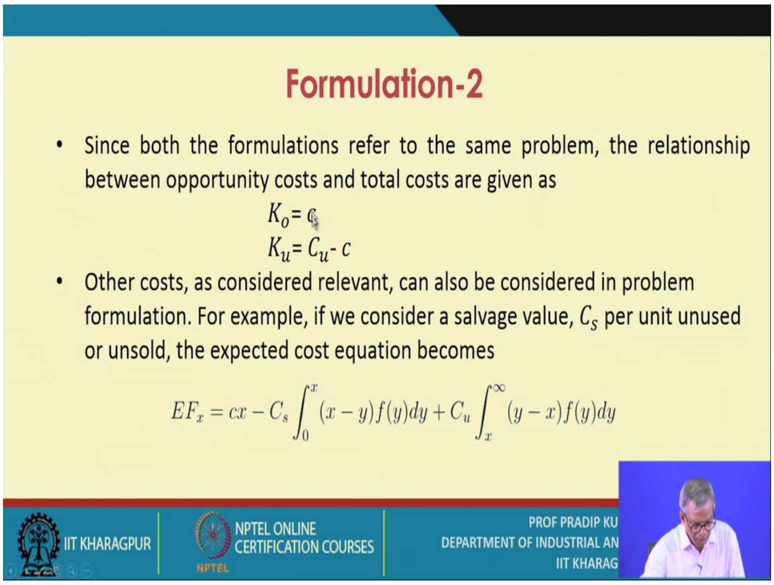

You can definitely derive this expression and then what is the equivalent; that means, the relationship between the opportunity cost and the total costs are given below; that means, K o equals to small c and K u equals to C u minus c. So, what you can do that while you formulate the total cost expression you can also add other relevant cost like say, you know you can add the salvage value ok. So, so the C s; that means, per unit basis if you consider the salvage value cost or the salvage value it is not a cost actually it is basically a revenue per unit unused or unsold.

The expected cost equation becomes EF x equals to C S. So, you need to the cost equation that is why it is a negative term because it is opposite to cost. So, C s into 0 to x x minus y this is the over stock amount. In fact per unit over stock you sell it with reduced price obviously. So, that is basically called the salvage value. So, it is basically you know this is the inflow of cash not the outflow of not outflow of money this is inflow money that is why it is negative one and this is under stock cost.

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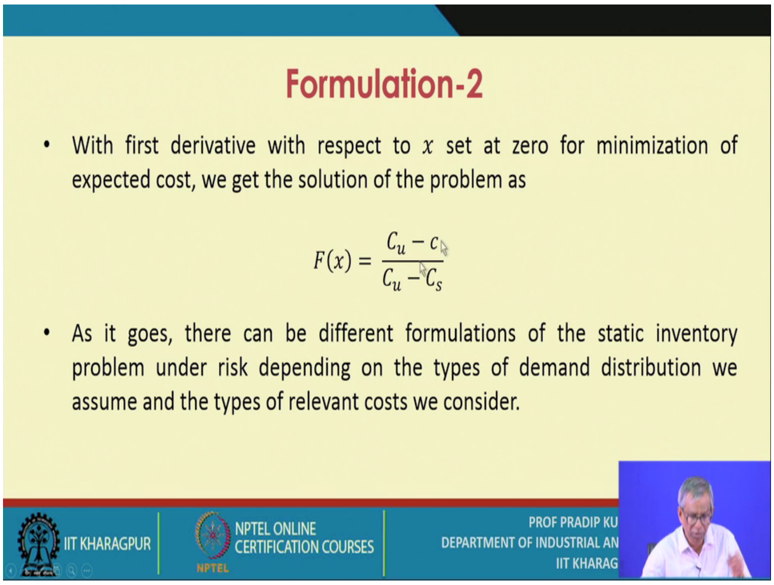

So, when you apply the Libniz rule you get an expression like this ok, with the minimum the expected cost. So, F x equals to C u minus c by C u into C s C s. As it goes there can be different formulations of the static inventory problem under risk depending on the types of demand distributions we assume and the types of relevant considers relevant costs we consider.

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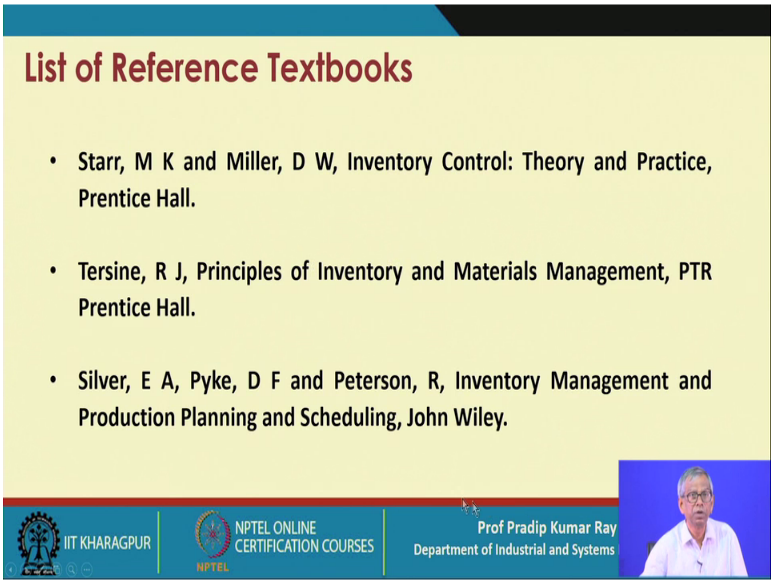

So, with this we conclude the session on say the static inventory problem say under risk. So, the both you know the discrete distribution as well as the continuous distribution you need to consider. And I am sure that with those the illustrative examples the concept which you which you develop, it will help you in the formulating the problem and there could be different others. So, the inventory problems we will be dealing with and this knowledge will help you in the formulating or the difficult types of so, the inventory problems.

Thank you.