Management of Inventory Systems
Prof. Pradip Kumar Ray
Department of Industrial and Systems Engineering
Indian Institute of Technology, Kharagpur

Lecture – 14
Static Inventory problem under Risk
(Contd.)

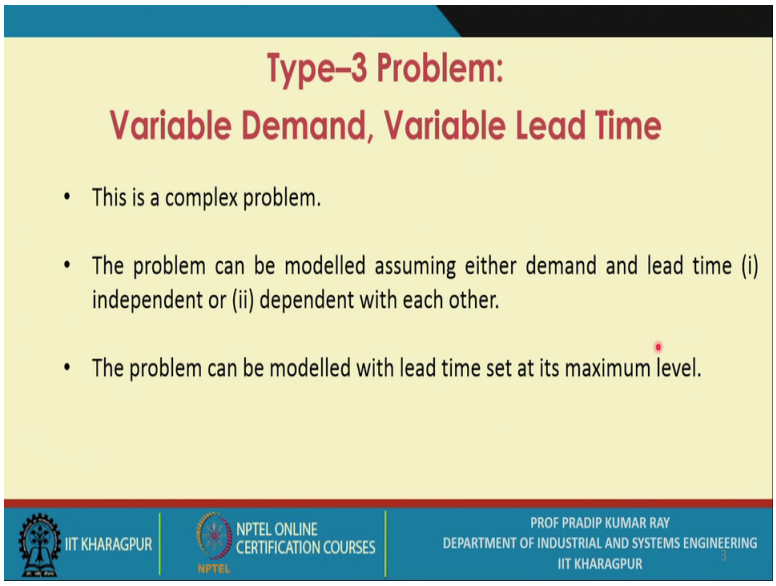
So, during this session on Static Inventory Problem under Risk I am going to discuss the type 3 problem.

(Refer Slide Time: 00:20)

Already we have discussed say the type 1 problem and the type 3 problem, type 2 problem. So, now, I will be referring to type 3 problem and then an important issue we are also going to discuss during this session that is the opportunity cost matrix and this opportunity cost matrix concept I intend to discuss with refer to with referring to. So, the type 1, type 1 problem, ok.

And opportunity cost is also known as the regret matrix and what we will get at different say the idea about the about the stock out or over stock situation and for both the cases you know you need to calculate or need to get an idea about the opportunity the cost due to lost of opportunity. So, this is an important concept to be to be used or to be understood while we go for say developing an inventory control systems. There will be few numerical problems also we are going to discuss.

(Refer Slide Time: 01:52)

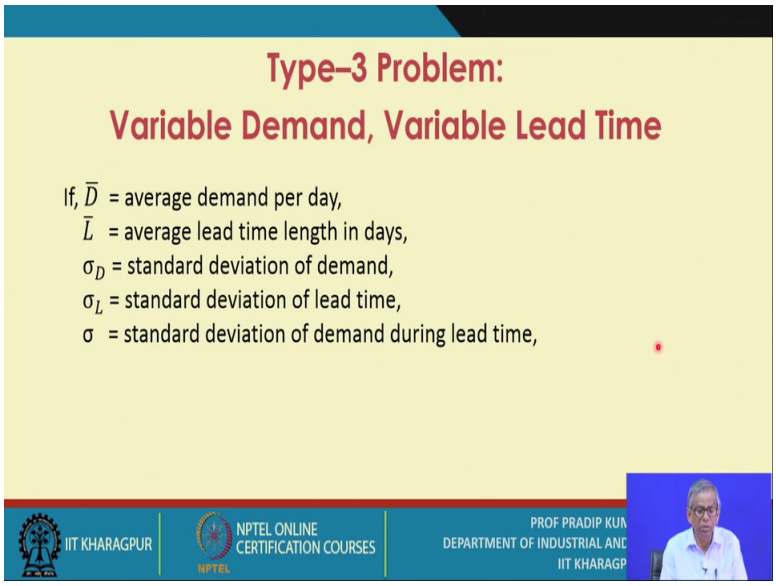


Now, let us first talk about the type 3 problem. As I have already mentioned that the type 3 problem in type 3 problem the demand is considered variable and so also the lead time. So, it is a variable demand and variable lead time case.

So, this is a complex problem right and the problem can be modelled assuming either demand and lead time, independent or dependent with each other, ok. So, many a time you know the mathematical, even if you go for mathematical the modelling or such a problem but you may not get a solution in closed form. So, many many a time you opt for the next based alternative that is a simulation approach, ok. So, in course of time we will be discussing the simulation approach in inventory control modelling. So, the that point in time will be discussing these aspects also.

So, this problem can be modelled with lead time say that is maximum level. So, so that is just one type of compromise we make many a time, that means, you say the variable lead time so why do not you collect the data on the lead time and you model the problem as per the maximum on the lead time. So, that is just one approximate solution you might get, ok.

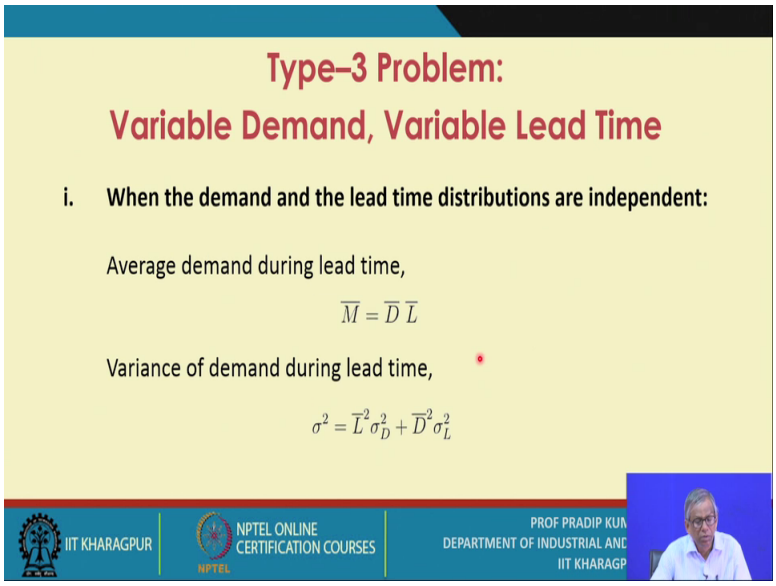
(Refer Slide Time: 03:43)



Now, so let me say first you know explain this problem. And there will be certain say the parameters certain variables we will be referring to while we model this kind this particular problem we are referring to as a type 3 problem variable demand variable lead time.

If D bar is the average demand per day, ok, L bar this average lead time length in days, ok, so that means, the lead time period actually. The number of say the measuring unit is a day. Sigma D is a standard deviation of demand and sigma L is a standard deviation of lead time, ok. So, and sigma is the standard deviation of demand during lead time, ok.

(Refer Slide Time: 04:46)

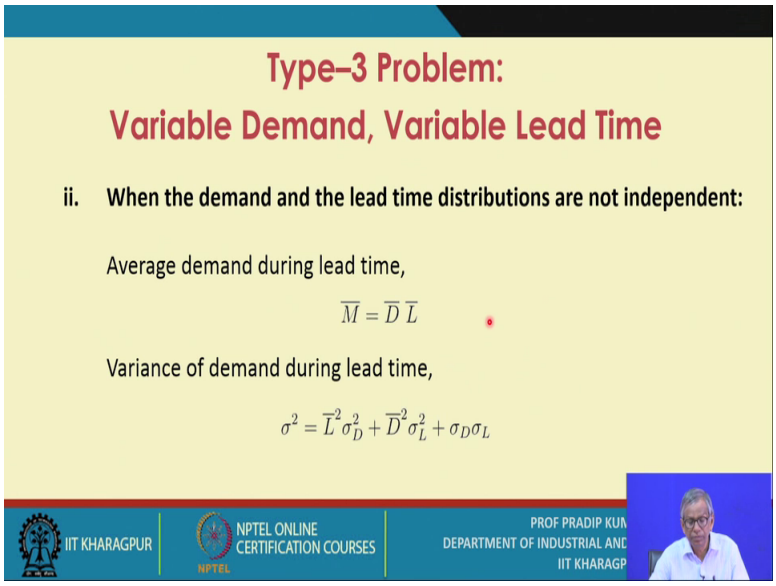


So, when the demand and the lead time distributions are independent, this is just a this is the first assumption we are making. So, this is if this is the case average demand during lead time, so the notation is M bar, ok. So, the M bar equals to D bar into L bar, ok. So, this is the average demand and this is the average lead time. So, if you multiply them that means, you multiply one with the, another you get the average demand during lead time.

But then how to calculate the variance of demand during lead time; so, the notation is sigma square and this is the formula we use like L bar square sigma D square plus D bar square into sigma L square, is it ok. So that means, this is the average the lead time square of average lead time into the demand you know the variance of the demand and this is the square of average demand and this is the variance of the lead time, is it ok. So, this formula we use when the demand and the lead time distributions are independent, ok.

So, we are we will be considering actually that whether your inventory control system is is suitable or its it perfect one or or it is working perfectly during the lead time is it, ok. So, that is why so the average demand during lead time is an important factor to be considered and the variance of demand during lead time, is it ok.

(Refer Slide Time: 06:54)



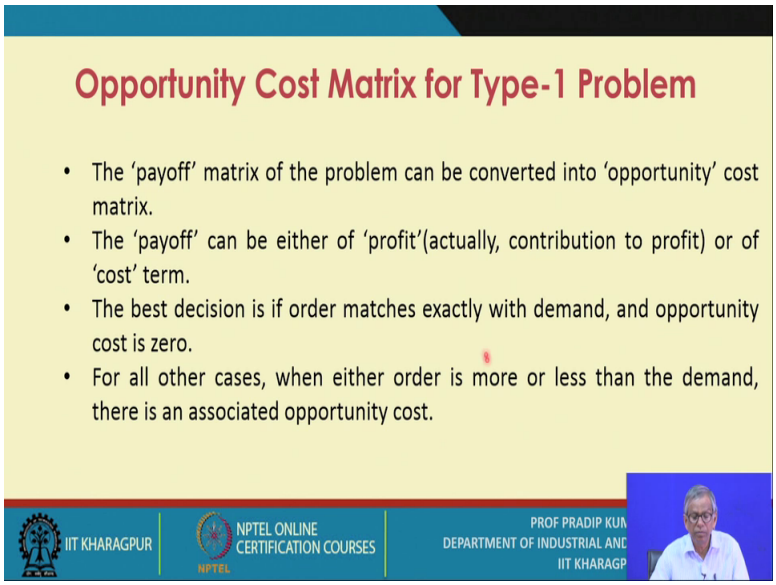
So, as I have already mentioned that you know whenever you try to propose an appropriate inventory control system make sure that the during the lead time there should be there should not be any stock out situation. So, the stock out situation and as you may be knowing that there is a possibility of the stock out occurs because of the two reasons that means, the fluctuations of demand and during lead time as well as the fluctuations of lead time itself, is it ok.

So, this is the case that means, if there is a variable demand as well as the variable lead time, so you need to say while you design in inventory control system you have to keep some extra stock and this extra stock of item is referred to as the safety stock or the buffer stock. So, the whole exercise actually boils down to the determination of the safety stock is it, ok, determination of the safety stock. Later on we will take up these issues in much the detail.

Now, the second case could be when the demand and the lead time distributions are assume to be assume to be the dependent or not independent with one another with each other. Average demand during lead time is M bar equals to D bar and L bar like in the previous case whereas, the variance of demand during lead time it will have an added component that is sigma D into sigma L. So, the sigma square that means, the variance of demand in lead time these notations we have used that is L bar square into sigma D sigma D square that means, is the variance of demand plus you know the square of

average demand into the lead time variance is it, plus the standard deviation of demand into standard deviation of the lead time, so ok. So, you refer to the notations we have used and I think that that there will be no problem in understanding the expressions which we need to use.

(Refer Slide Time: 09:21)



Now, so, this is the problem type 3 and frequently we come across in the real world we come across problem type 3, ok. So, later on we will be referring to this case in we and we will discuss it in much detail.

Now, at this point in time I like to bring in the concept of I like to discuss the concept of opportunity cost, ok. So, as you already aware that the given a problem, with all the required data given you need to construct the payoff matrix. So, the concept of payoff matrix is made very very clear it is well understood. Now, from the payoff matrix when many a time you make the opportunity cost matrix or the original payoff matrix can be converted in the opportunity cost matrix.

Now, let us first take the type 1 problem that means, type 1 problem means that the variable demand and constant lead time, and we will discuss the opportunity cost matrix concept for the type 1 problem. Now, the payoff matrix of the problem can be converted into the opportunity cost matrix. So, given a type 1 problem what you need to do? First you construct the payoff matrix. So, that your payoff matrix can be either in the form of say the profit or in terms of say the cost. So, whatever it is the payoff matrix of the

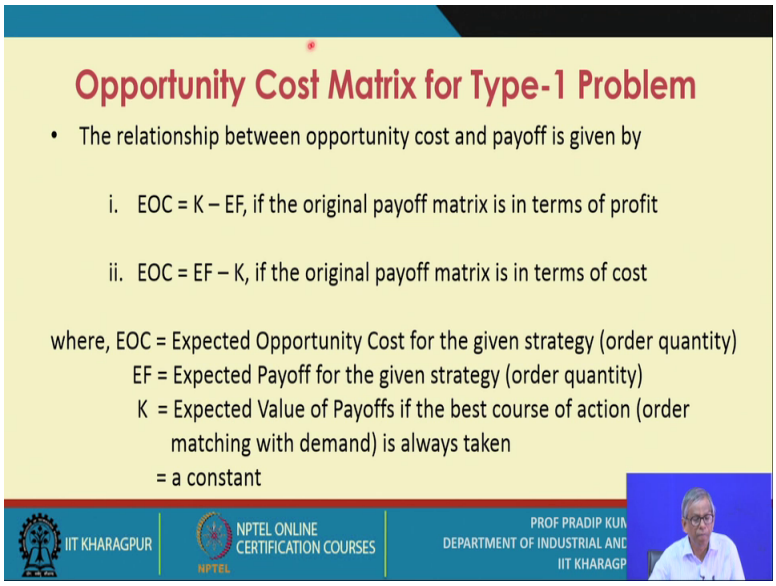
problem can be converted into the opportunity cost matrix how we will convert it I will tell you in detail.

The payoff can be either of profit, actually contribution to profit this point already have mentioned or of the cost term depending on the type of problem, is it ok. So, if it is an outside the supply case the sales is involved then obviously, you know the payoff you compute or the payoff or the outcome in terms of the profit. But if it is an inside supply case only the cost related you know the terms are involved so obviously, the payoff or the outcome you define in terms of the cost.

The best decision is if order matches exactly with the demand and opportunity cost is zero. So, what is this opportunity? That means, whenever say suppose the order quantity matches exactly with the demand level so obviously, it is the best possible in the situation. And what do you expect that say against that that particular combination? You get a payoff or the cost, is it ok. So, it is just or the best possible matching and obviously, there is no opportunity the lost. So, the opportunity cost is assumed to be zero, or all other combinations when suppose the demand is more than say the order quantity or the demand is less than the order quantity, over stock situation. So, always there will be some the loss due to due cost due to loss of opportunities. So, opportunity cost will be there in any other combinations.

So, for all other cases when either order is more or less than the demand this point already I have mentioned there is an associated opportunity cost, ok. So, when we take up the numerical problem, so this will be made this concept will be made more clear, ok.

(Refer Slide Time: 13:44)



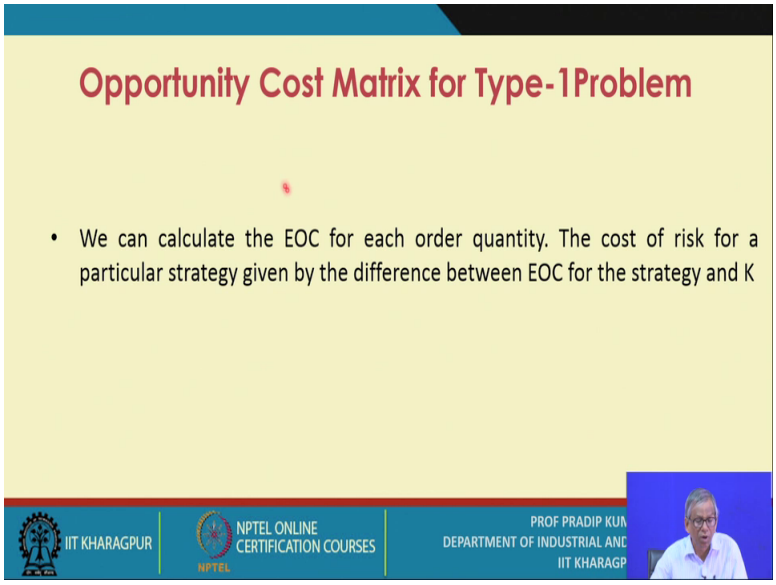
Now, the relationship between opportunity cost and payoff is given by that means, given a problem like say suppose the problem say the benefit analysis you have to do and it is the it is the payoff matrix and the payoff matrix is in terms of profit, ok. So, that is the problem type you have taken up.

So, in that case the expected opportunity cost for a given strategy that means, for a given order quantity will be K, capital K, capital K is a constant minus EF. EF, what is that? That means, for the given strategy say strategy is I order quantity is I. What is the expected? Payoff is it ok. So, this is the relationship you can very easily you can prove that EOC is equals to K minus EF if the original payoff matrix is in terms of profit whereas, if the original payoff matrix is in terms of cost is it, then the relationship is EOC expected opportunity cost for a given strategy is equals to expected payoff in the cost terms obviously, expected payoff in cost terms for that particular strategy minus the constant that is capital K.

So, this we have elaborated we have explained that means, EOC is expected opportunity cost for the given strategy that is the order quantity. EF is the expected payoff for the given strategy, given strategy means essentially the ordering quantity or ordering strategies. And capital K is the expected value of pay payoffs if the best course of action order matching with the demand is always taken that means, essentially it is what we will find that the payoff the values in the in the diagonal in the diagonal of the matrix, ok. So,

the best possible the situation so obviously, this is the maximum or the mini minimum value you get and that is essentially constant, ok. So, the K is a constant. So, when you take up the example it will be made more clear.

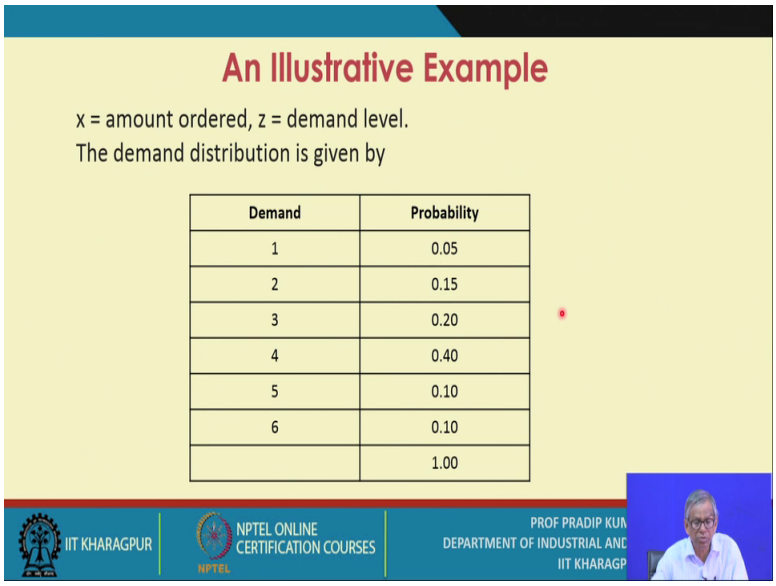
(Refer Slide Time: 16:00)



So, we can calculate the EOC expected opportunity cost for each order quantity. The cost of risk for a particular strategy given by the difference between EOC for the strategy and K, so there is a concept called cost of risk right that means, you are you place an order quantity and this may not be the best possible decision, is it ok.

So, always there is a risk and there is a risk and the corresponding cost of risk you need to calculate. So, essentially the difference between say the K and the expected the payoff expected payoff is referred to as actually the cost of risk , is it ok. So, the cost of risk for a particular strategy is given by the difference between EOC for the strategy and the K, is it clear. So, so this way we interpret the cost of risk.

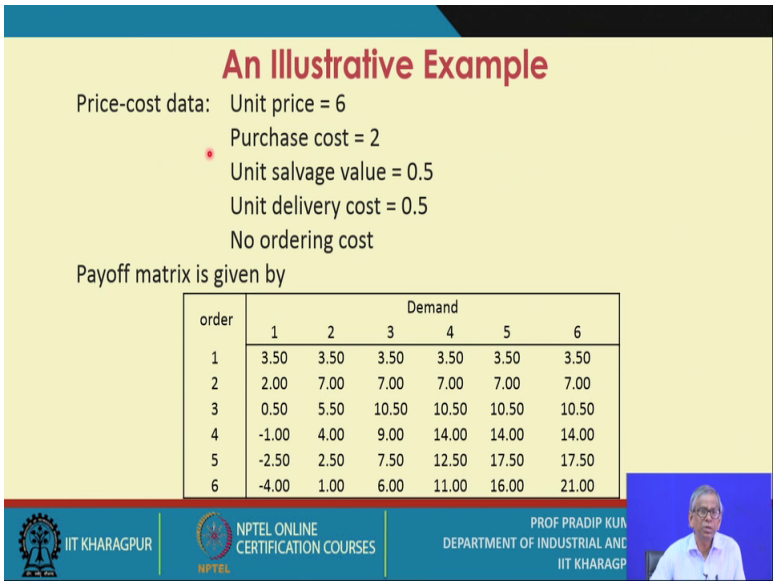
(Refer Slide Time: 17:21)



Now, here is an illustrative example. Now, x is the amount order, z is the demand level, for the given the inventory item, right. So, we are considering one item normally, is it ok. So, the all these analysis all these concepts we are trying to explain with respect to one or the single inventory item. The demand distribution is given by say as you find in this table that means, the possible the demands demand levels are 1 2 3 4 5 6, 6 possible demand levels.

So, the corresponding probability assuming it to be objective probability that means, the frequency based on the frequency counts. So, you have determined this values and so against the demand level one the corresponding probability is 0.05 against demand level two 0.15 and so on is it, for other levels. So, against at the demand level of 4 the probability is 0.40 that means, 40 percent of the cases the demand level will be 4 units, ok. So, the total is just 1, that probability, ok.

(Refer Slide Time: 18:51)



Now obviously, with the given the problem you will you will come to know the what certain data will be given are related to the price and the cost. So, the unit price of the item is considered to be 6 ok, purchase cost is 2 ok, unit salvage value 0.50, unit delivery cost 0.50 again is it, ok. So that means, whenever the demand occurs and you make a you make a supply and for each supplied item there is a delivery cost is it ok, for each supplied item and that cost is 0.5 dollars.

Similarly if there is an over stock situation that means, you are unable to select. So, you can sell it with reduced price and you have some salvage value. So, salvage value per unit is assumed to be 0.5 dollars. The cost of say the purchase cost per unit purchase cost is 2 dollars and unit price is 6 dollars that means, that this price you can sell it.

And what we are assuming there is no ordering cost or the ordering cost is negligible. We know there is a static inventory problem so that is why just one order is to be placed and suppose the ordering cost is ordering of the process or the procedure you follow for placement your order; suppose this procedure is very very simple its, hardly any cost you are incurring is a right. So, we may assume that the ordering cost is negligible or say no ordering cost.

Now, with these values so payoff matrix you must create first as we have we have done before. So, what are the ordering strategies? The possible ordering strategies are either 1

2 3 4 5 or 6. So, all possible orders strategies you consider and with respect to or when you consider the possible demand levels. So, the possible demand levels are 1 2 3 4 5 6.

Now, the payoff matrix you construct that means, here just a few examples I will tell you that how do you how do you know a calculate this payoffs against a particular combination. Suppose the order quantity is 1 and you demand is 1 that means, it is exactly matching, ok. So, as such there is no opportunity cost similarly this.

Suppose the order quantity is 2 and you demand level is 2 so it is exact match and that is why there is no opportunity cost. Similarly for all other combination all this combinations 3 versus 3, 4 versus 4, 5 versus 5, and 6 versus 6, is it ok. So, these are all diagonal elements diagonal elements. So, if you if you look at these diagonal elements, so when the demand matches with the order quantity exactly there is no opportunity cost. For all other cases when the demand does not match with order quantity there will be some loss and this is referred to as the opportunity cost, is it ok. So, first you construct the payoff matrix.

Now, here for the order quantity 1, and the demand is 1 obviously, you are selling, because there is a demand for one unit how much you will be getting 6 1. And for ordering how much cost you have incurred? For purchasing, that is 2, so 6 minus 2 equals to 4. And how many units you have supplied? That is just 1 and for each unit you supply there is a delivery cost that is 0.5. So, it is 6 minus 2 minus 0.5 that is 3.5, is it ok.

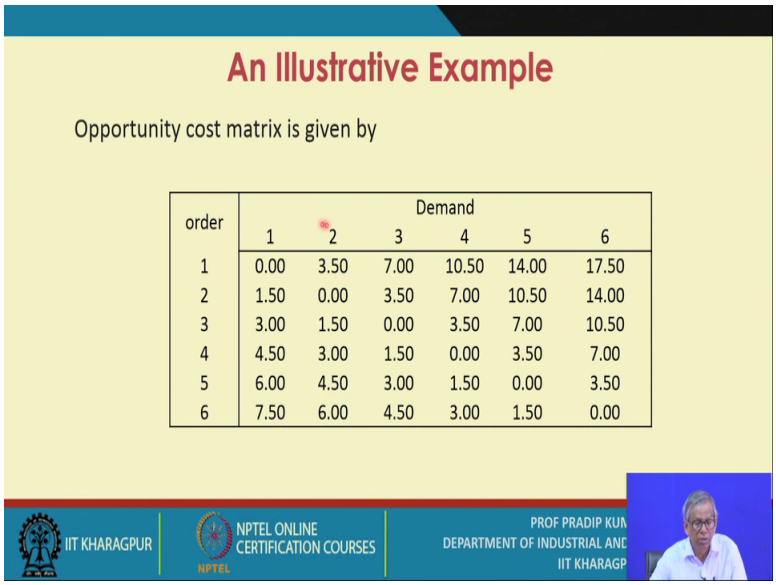
Now, for the second case when the demand level is 2 obviously, the how much the outcome it will be? It will be just 3.5 because the other unit you cannot sell is it ok, so the 3.5, 3.5, 3.5, 3.5, it is clear. So, it is unfulfilled demand right so obviously, the delivery cost just for 1 unit and the price unit price that you get that is for just 1 unit that is 6 minus say the 2 4, the 2 because we have just purchased one unit, so it remains at 3.5.

Now, the next one is the order quantity is two. So, how much you have spent that is 2 into 2, 4 and the 6 units you can sell just 1. So, 6 minus say the 4 the 2 and the 2 units. So, 1 unit what you can say, that you can you can sell extra with 0.5, but again the unit delivery cost per one unit because the demand is 1 unit you have all you need to spend 0.5. So, the 0.5 minus 0.5 that means it is 0. So, it becomes 2.

So, if there are 2 units of demand and the 2 units of the order quantity. So, how much you are spending? So, the 2 into say 6 that is sales price is say 2 into 6, 12 minus 2 into 4, so 12 minus 4 that is 8. Now, what you are trying to do? That means, the 2 units that means, unit delivery cost 2 into 0.5 that is 1. So, 8 minus 1 that is 7 and for all other demand levels it remains are 7, is it ok.

So, this procedure you have followed and you have ultimately for all other ordering strategies for all other demand levels. So, you get the end of this payoff matrix. I hope that the logic is made very very clear and you just you know you just verify each and every the payoff say the value given in this particular table, ok.

(Refer Slide Time: 25:32)



So, now, how do you calculate the opportunity cost matrix? That means, original payoff matrix as I have already pointed out that when the demand matches exactly with the order quantity that means, there is no opportunity cost, is it ok. So, all this diagonal elements you will find in the opportunity cost matrix table this all this diagonal elements are 0 that means, there is no opportunity cost.

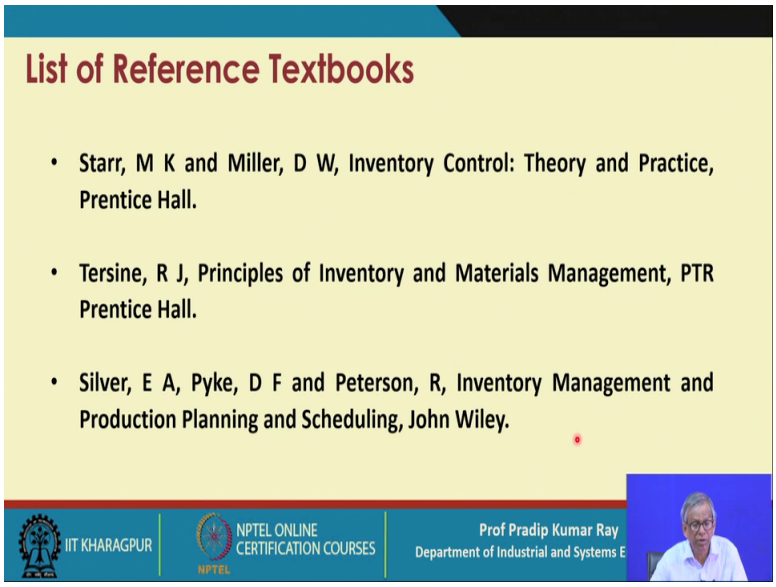
Now, supposing you are your demand is 2 your order quantity is 2, is it ok. So, you are you are getting an opportunity cost of 1.5 that means, what you try to do that means, this is the maximum that means, you just select one particular column against a particular demand level. Now, this is the payoff. Now, first you check that: what is the maximum value you get. So, in this case it is 3.5, 3.5 whereas, if you opt for say order quantity 2,

how much you will be getting the payoff that is 2, ok. So, how much you will be losing? As you are not following the best possible ordering strategies is it ok, so that means, from the maximum value that means, you subtract that whatever the value you have got that means, it is 2 so that means, 3.5 minus minus 2 that is 1.5, it is clear.

Now, this value the payoff you will find it is 0.5, but the maximum you could have obtained that is 3.5 so that means, you are losing a value of 3, ok. So, similarly for all other you know the combinations, ok. So, you calculate the opportunity cost that means, here if you your demand is 1 and if you place an order quantity of 6 that means, you will be losing 7.5, 7.5 say the dollars. So, that is the opportunity cost. So, the same logic you follow. Here in the for the second column what you find that this is the problem 2 into 2 that means, demand level is 2 and the order quantity is 2 you get the maximum possible return that is 7 is it, whereas, if the order quantity is one you get a value of 3.5.

So, how much you will be losing? That is 7 minus 3.5 that is 3.5. So, this logic you follow right. That means, from when the original payoff matrix is in terms of say the profit, so go to a particular column you select the maximum value and you subtract the actual value from the maximum value and you get against a particular column all the opportunity cost values, ok. So, this logic you follow I suggest that for all other columns you verify this logic and you must know that how you are getting each and every value, is it ok.

(Refer Slide Time: 28:38)



So, this way you convert the original payoff matrix into the opportunity cost matrix, ok. So, what we try to do essentially, that either you use opportunity cost matrix table or you can also use the payoff matrix, payoff matrix table and against each say the order quantity or the ordering strategies you calculate, ok. You go a row wise you calculate the expected payoff is it, ok

So, expected payoff and you select that particular strategy where the expected payoff is maximum if the original payoff matrix is in terms of the profit whereas, if its original payoff matrix is in terms of cost that means, again you calculate against each ordering strategy the expected payoff in terms of the cost. And you select that strategy for which the expected cost is minimum whereas, if you convert the original payoff matrix in the opportunity cost matrix like through we have done here in this particular case. What you need to do? That means, you refer to the opportunity cost matrix against each ordering quantity you calculate expected opportunity cost. Like for this one the first row you calculate the expected opportunity cost against each demand level there is a probability associated with it, is it ok.

So, you calculate the expected opportunity cost and you select you know that particular say the strategy where the expected opportunity cost is minimum, is it ok. So, this way you know you have understood that already that given a particular problem in this category static inventory problem under risk how to construct the payoff matrix, how to convert the payoff matrix into the opportunity cost matrix and how to you know get a solution.

Now, here you know your the criterion which you are you are you are using for selecting the best possible order strategy that is the expected payoff. So, give there could be many other say the cases where this, the criterion may not be the expected profit or expected cost or expected opportunity cost. So, later on we will be referring to those problems where for selection of the best possible ordering strategy we need to use say some other criteria. So, we explain in the next cases the next lecture sessions we will be discussing the continuous distribution demand distribution case, ok.

Thank you.