Management of Inventory Systems

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Lecture – 13
Static Inventory problem under Risk (Contd.)

In last session related to Static Inventory problem under Risk, if you recollect, we have referred to the benefit analysis.

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Now; obviously, if there is a benefit analysis when the sales of the product is involved related to the production of an item within a manufacturing system and when you know the cost based analysis is needed, what is important is that related to type 1 problem for that matter, any type of problem, you must go for cost analysis.

So, during this session, 3 specific issues we will be discussing. First one is the type 1 problem and why cost analysis is needed for type 1 problem. We will highlight this case and with a numerical example and then we will refer to type 2 problem and the type 2 problem also, we will discuss with a numerical example so, for the next half an hour. So, this will be our coverage.

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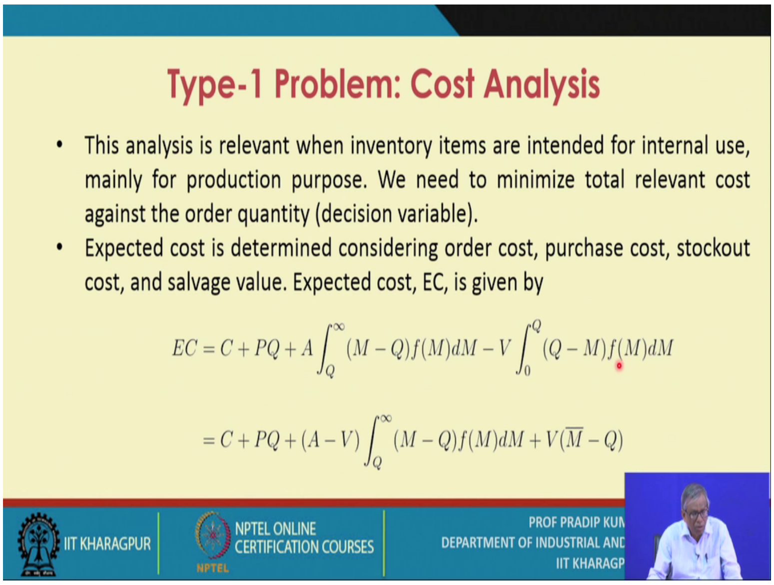

Now, so, let us talk about type-1 problem. Already, we have stated; what is a type-1 problem, when the demand is a variable and the lead time is constant. So, so, this is just one combination and we will be dealing with the cost analysis. Now the certain points you should keep in mind.

Now, the cost analysis is relevant when inventory items are intended for internal use mainly for production purpose, as we have been mentioning all the time that the kinds of activities which we carry out within the production systems, within the manufacturing systems, all these activities are having a bearing on the cost. So, those are basically the cost related activities. So, whenever the inventory items are intended for internal use, like for example, say inside supply case, you know, what is an inside supply case, so; obviously, you must go for cost analysis; that means, if once you have the cost equation, you establish the cost equation and you determine the order quantity in such a way that the total relevant cost is held at the minimum level.

So, we need to minimize the total relevant cost against the order quantity and why I am saying total relevant cost because you know if you go through a particular problem, you will come to know that, there are certain cost elements which are referred to; whereas, there could be many other cost elements which may not be relevant or with respect to a given problem; that is why you need to identify only those cost elements which are referred to as relevant.

So, what you need to do once you identify this cost elements. So, total relevant cost equation; you have to establish. So, expected cost is determined considering ordering cost purchase cost, stock out cost and the salvage value, we have already referred to all these different types of cost elements like we have defined; what is order cost. So, you are placing an order is a static inventory problem and that is why say one order is placed; single order. So, for placing an order, you must incur certain cost. So, that is referred to as ordering cost. Normally, this is estimated on order basis, then you have the purchase cost; that means, the purchase the price or say the production cost per unit must be known. So, this cost element also we must consider.

Then we have stock out cost and we also have the salvage value; that means so, unused items; what you do with the expiry of say you know its use with the expiry of the time or the order or say of the season, what you do? You need to you can to what extent; you can the salvage its value; that means, can you dispose it off with a reduce price. So, this is referred to as the salvage value. So, this 4 types of cost elements you need to consider.

So, what will be the expected cost expression that is given as EC? So, this is given by C plus P Q. So, C is the ordering cost the P Q; that means, Q is ordering quantity and P is the purchase price. So, P into Q will be the total purchase cost and capital A is per unit stock out cost and M minus Q f M d M. So, what is M is the demand and Q. So, M minus Q is basically the demand is more and the order quantity is less. So, this M may vary from Q to infinity.

So, this is a stock out case. So, so, this is the expected number of stock out units; so, M minus Q f M d M; so, what is f M? F M is against a particular demand M, what is probability density function? Now this is the cost, this is added that is why it is positive. Now the certain you know the quantities which are considered to be excess ok, after the expiry of the selling season or what may be considered to be excess and these excess amount, you can sell, you can sell with a reduced price that is why it is referred to as a salvage value.

So, salvage value per unit is given as capital V and this is the excess amount; that means, the order quantity Q minus M; that means, when the you know the actual demand varies between say 0 and Q so; obviously, this is a excess amount; so, Q minus M f M d M; so, expected number of units excess.

So, this is the total expression, we have simplified this expressions, we have maniple after going manipulations, we have these expressions C plus P Q plus A minus V integration Q to infinity M minus Q f M d M plus V into M bar minus Q. So, this is the average demand ok. So, this is the expression.

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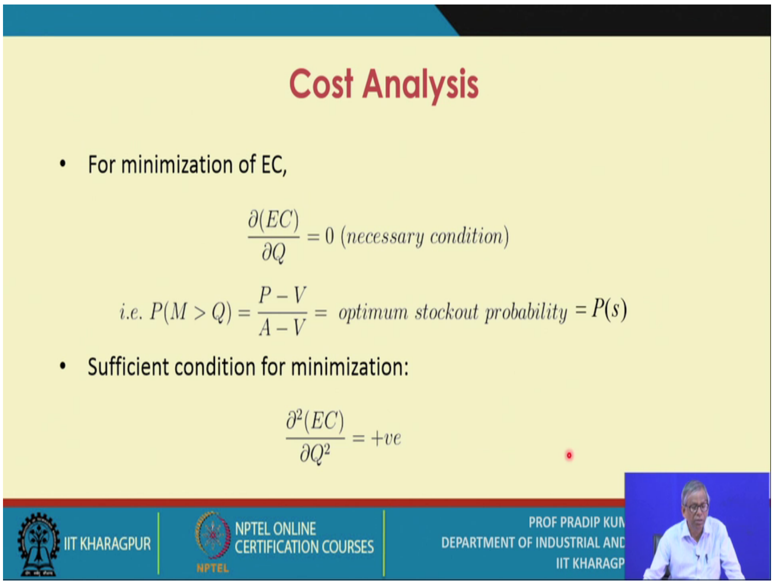

Then what you do; obviously, you determine the order quantity Q in such a way such that that the expected cost is minimum. So, for minimization of EC, what you do? You take the partial derivative with respect to Q del del Q of EC and set it equals to 0. So, this is the fulfilment of necessary condition. So, ultimately; what you get? You get probability that M greater than equals greater than Q; that means, this is the probability of stock out.

So, what is the expression for the optimum stock out probability? That is P minus V divided by A minus V. So, I suggest that you take the partial derivative with the of the expected cost expression. So, it equals to 0 and you carry out the exercise. So, that ultimately you get these expressions. So, this is the defining relationship which is referred to as the optimal stock out probability this notation also sometimes we use that is P s; P s stands for probability of sock out.

So, the sufficient condition for minimizations, as you already aware of that is the second derivative partial second partial derivative with respect to EC partial derivative with respect to Q that is of EC you will be is positive. So, this is you can also check that whether the sufficient condition for minimization is fulfilled or not ok.

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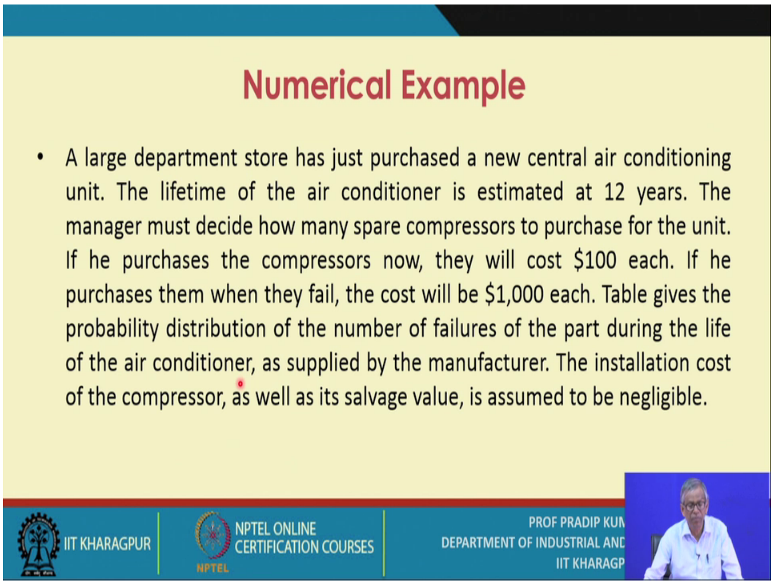

Now, so, once you have established this equation, you know, what is a decision variable and you determined the value of the decision variable in such a way that expected cost is held at minimum level. So, now, we can refer to a particular example.

So, let me first elaborate the what is this numerical example. A large department store has just purchased a new central air conditioning unit. The lifetime of the air conditioner is estimated at 12 years, the manager must decide; how many spare compressors to purchase for the unit. If he purchases the compressors, now they cost; they will cost 100 dollars each, if he purchases them when they fail, the cost will be 1000 each.

The table gives the probability distributions of the number of failures of the part during the life of the air conditioner as supplied by the manufacturer; that means, this the table the related to the probability distribution of the number of failures. So, this table is given to you and then we have other data the installation cost of the compressor as well as its salvage value is assumed to be negligible ok. So, this is the assumptions.

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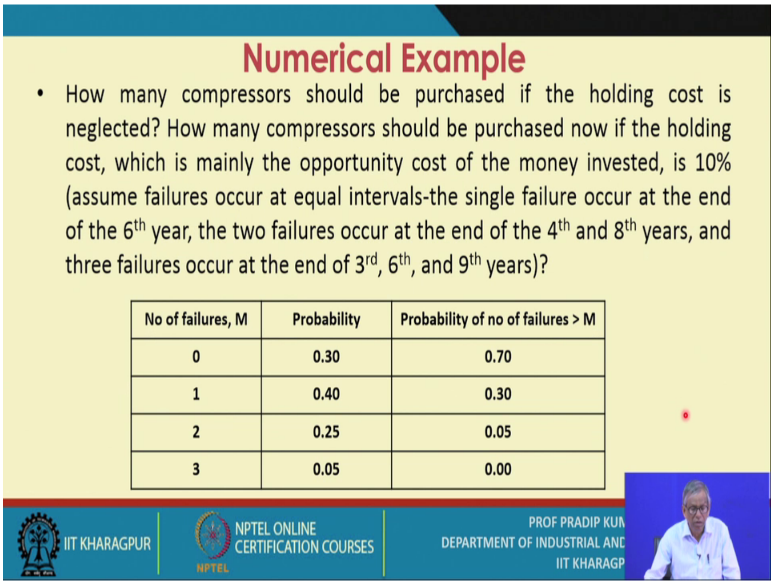

Now, what is what is to be determined; how many compressors should be purchased if the holding cost is neglected ok? So, this is so, the you need to determine the number of compressors, how many compressors should be purchased? Now if the holding cost which is mainly the opportunity cost of the money invested is 10 percent ok.

So, as I have already mentioned in the very first week that there are that the different types of the cost, you need to consider for inventory analysis or for determining the inventory control policy; so, one such the cost is the holding cost or say inventory carrying cost. So, usually the inventory carrying cost is estimated as a percentage of the average inventory which you hold ok.

So, that is why; so, here what is mentioned that in this particular problem that the holding cost is 10 percent is just 10 percent and so, so it is considered to be reasonable. So, 10 percent holding cost assume failures occur at equal intervals; that means, the single failure occur at the end of 6th year; that means, the total the lifespan of the compressor is 12 years. So, if say one failure occurs we will assume that.

So, the once in 12 years; that means, at equal intervals; that means, single failure occur at the end of 6th year, if say the 2 failures occur, they will be occurring at the end of fourth and at the end of eighth year ok. So, this simple rule, we will be following and you suppose, for example, the 3 failures occurring; that means, this failure failures will occur at the end of the third, sixth and ninth years. So, these assumptions we are making ok

So, now this is the table given; that means, against the number of failure what is the corresponding probability. So, so, based on the you know the frequency counts so, we have this data; that means, if number of failures is 0 corresponding probability is 0.30 ah; that means, 30 percent of the cases, there will be no failure at all; in with one failure the probability is 0.42 failures, the probability is 0.25 and 3 failures, the probability is 0.05 just 5 percent of the cases, there will be 3 failures ok.

Now, we have added this column; that means the probability of number of failures greater than M ok; that means, if 0 is for against 0 failure the probability is 0.30. So, the probability that the failure will be greater than 0; obviously, it could be 1, 2 or 3 so; obviously, it is it is 1 minus 0.3; that means, 0.70. Similarly the probability of number of failures greater than one; that means, it will be 1 minus 0.3 plus 0.4; that means, 0.30 and similarly, we calculate that the probability of number of failures greater than 2, this will be 0.05 and the probability of number of failures greater than 3; obviously, it will be 0 ok. So, we have added this column, please, you know try to understand the meaning of all these values ok

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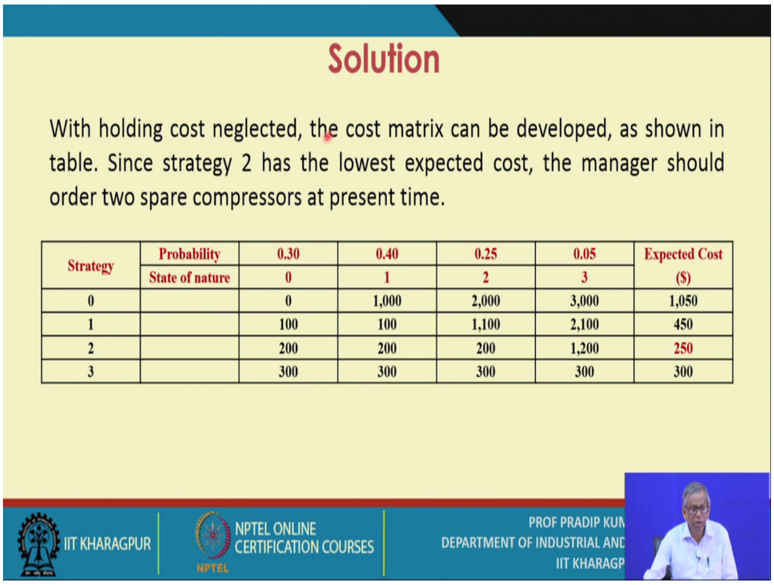

Now with holding cost neglected the first case, we will assume that there is no holding cost for this particular numerical problem and in the second case, we will realize this assumption, we will assume that the holding cost is 10 percent ok. So, with holding cost neglected first case, the cost matrix can be developed as shown in the tables. So, this is we have the ordering strategies ordering strategy depending on say the how many the how many possible failure levels you have. So, that is referred to as a state of nature ok. So, either there could be 0 failure, 1 failure, 2 failures or 3 failures and corresponding probability, already it is known as given in the table previous table. So, the ordering strategies could be 0.

That means there will be no ordering for the compressor or there could be 1 unit ordering or 2 units ordering or 3 units ordering. So, this is all possibilities, we have explored. Now against suppose the your ordering strategy is 0 and your the demand is also 0 so; obviously, there is no cost involved, is it ok, if suppose you do not place any order, but there is one failure so; obviously, your cost will be 1000 will be 1000 and if suppose, there is you do not place any order and there will be there are there are there are 2 failures; obviously, you need to purchase immediately 2 compressors with a total cost 2000 and similarly, if this is a 3 failures the total cost is 3000. So, we follow this logic, but suppose you have just purchased one compressor. So, you have spent right now 1000 dollars, but there is no failure ok.

If it is; so, the if it is one failure you have already 1. So, the cost will be 100 dollars, but if it is the, if the 2 failures, already you have spent 100 and for the second one, immediately, you have to purchase 1 more compressor. So, the total value will be 100 plus 1000; 1100 dollars and similarly, if there are 3 failures. So, the 2 more compressors you have to purchase with a cost of say 2000. So, 2000 plus 100 already you have spent. So, it is 2100.

So, similarly for the, this third strategy; that means, ordering for 2 compressors. So, it will be 200 dollars against 0 failure, against one failure again 200, against 2 failures 200 and if it is if there are 3 failures; that means, one more compressor you have to purchase with a price of 1000 dollars. So, the total the money spend will be 1200 dollars and if there is a ordering for 3 compressors. So, already you spend you have purchased 3 compressors. So, irrespective of the number of failures from 0 to 3 your the cost will remain same that is 300; 300, for the third one 300 and for the fourth one, again it will be 300.

So, this is the total cost matrix we have we have developed and then what you do against each ordering strategy, you calculate the expected cost so; obviously, if the ordering strategy is 0, the expected cost will be 0 into say 0.3; that means, 1000 into 0.4 plus 2000 into 0.25 plus 3000 into 0.05. So, if you add them, you will get a value of 1050. Similarly, for the other 3 strategies, following the same rule, you calculate the expected cost.

So, against ordering strategy one the expected cost is 450 against ordering strategy 2, the expected cost is 250 and against ordering strategy 3 the expected cost is 300. So, these are the 4 expected cost values and out of this 4, which one is the minimum is; obviously, 250. So, against 250, your ordering strategy is 2. So, that is the solution ok. So, this way you get a solution.

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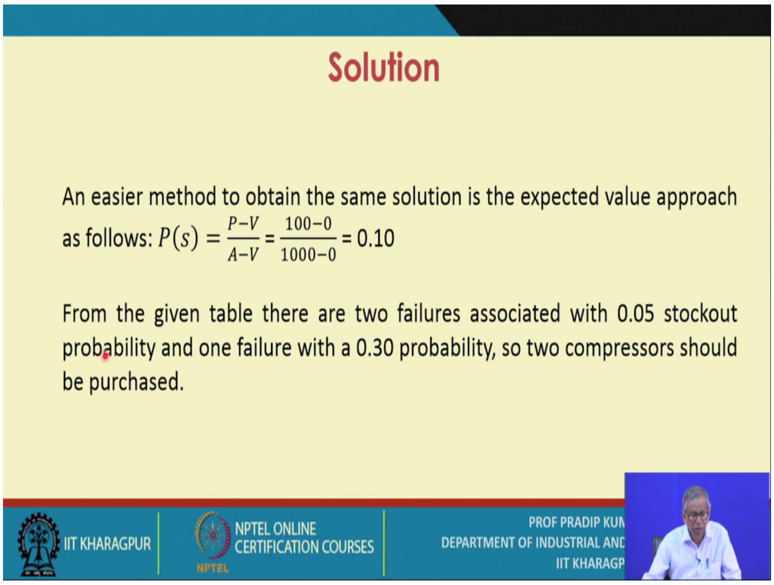

Now, what so, this the cost matrix, if you form, you get the solution when the probability values are known and, but and easier method to obtain the same solution is a expected value approach already, we have explained what is that expected the value; that means, the probability of going out of stock.

So, we have the expression for optimum probability of stock out stock out probability optimum stock out probability that is given as P s and this value is P minus V divide by A minus V. So, this is the purchase price P and V is assume to be 0 in this case, this is the stock out; that means, the demand is occurring, but you do not have at that point in time when the demand occurs when the failure takes place there is there is no stock of the compressors. So, immediately you have to purchase and that purchase price is 1000 dollars so; obviously, it is 100 minus 0 divide by 1000 minus 0.

So, this value is the optimum stock out probability that value is 0.10 from the given table, there are 2 failures associated with 0.05 stock out probability and one failure with 0.3 probability. So, 2 compressor should be purchased ok. So, that is so, we have already you know already we have mentioned that the solution is 2 out of this cost matrix.

But an easier solution approach is why do not you get the value of optimal the stock out probability and you refer to the probability, probability distribution table in empirical form and you get the solution ok.

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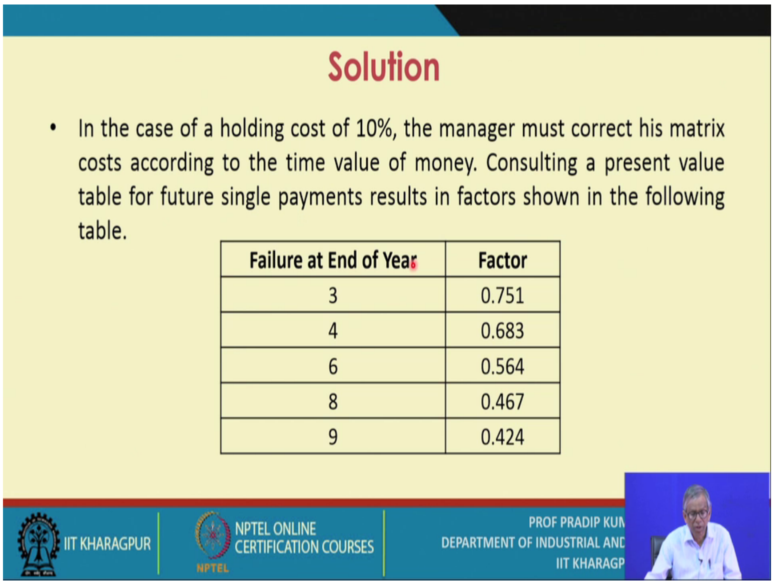

So, now, the second part is that the second part of this numerical problem is related to the holding cost assume to be 10 percent ok. So, in case of a holding cost of 10 percent, the manager must correct his matrix cost; that means, the cost matrix according to the time value of money; so, the consulting a present value table for future single payments results in factor shown in the following table ok.

So, what you try to do; that means, you have to refer to this; that means, suppose the failure at the end of year; that means, there could be say 3 failures, 4 failures, 6 failures, 8 failures, no failure occurring at the third year ending, fourth year ending, 6 years, sixth year ending, eighth year ending, ninth year ending ok. So, if so, there could be different occurrences there could be one failure there could be 2 failures there could be 3 failures ok.

So, if suppose there are 3 failures. So, what will assume the at the end of third year, there will be first failure at the end of sixth year, there will be second failure at the end of ninth year there will be third failure similarly if there are 2 failures, we will assume that given the lifespan of the compressor 12 years so; that means, if there are 2 failures at the end of fourth year there will be first failure and the at the end of eighth year there will be the second failure.

And if suppose there is just one failure within a time period of 12 years we will assume that this failure will occur at the end of sixth year. So, now, with the 10 percent holding cost; that means, at that point in time suppose at the end of third year you need to purchase one compressor with the say 1000 dollars; obviously, now; obviously, its present value you should calculate; that means, as of now. So, it this value will be 1000 into 0.751.

Similarly suppose at the end of the fourth year you need to say the purchase a compressor with a value of 1000 dollars. So, what is his present value? So, for press present value computation what you need to do you need to use this particular factor that is 0.683 into 1000 ok. So, the same rule we will follow for failures occurring and different time periods ok.

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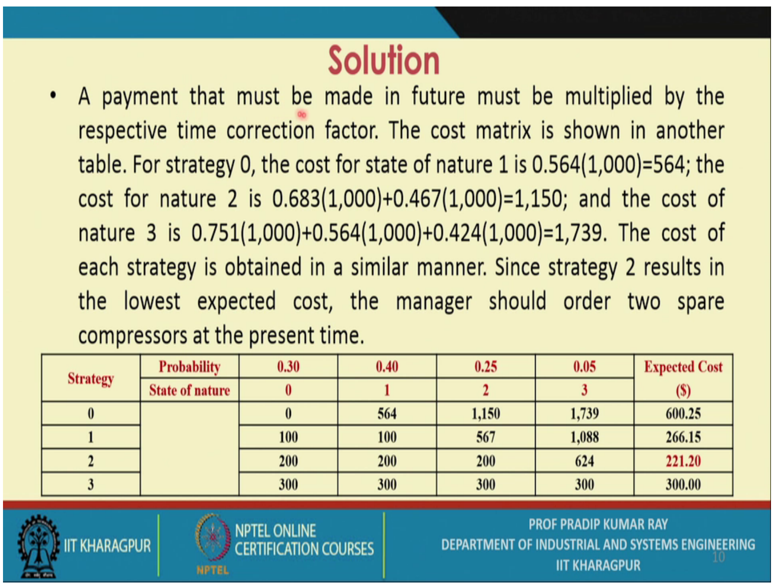

So, so, I will just explain the steps which we are following a payment that must be made in future must be multiplied by the respective time correction factors. So, already in the previous table, this time correction factor is given the cost matrix is shown in another table.

So, this is the cost matrix, we are showing for strategy 0, the cost for state of nature one that I have already mentioned; that means, 0.64 into 1000; now this 0.64; that means at the end of the sixth year, at the end of the sixth year. So, we have we have this value that is 0.564. So, if there is one failure we are assuming that it will fail at the end of sixth years. So, this factor you need to consider; that means, at that point in time at the end of the sixth year. So, you have to purchase a compressor with a price of 1000. So, its present value will be 1000 into 0.564.

The cost for nature 2 is 0.683; 1000 that is at the end of third year and this is at the end of say this is at the end of fourth year and this is then at the end of say the 8 year this corresponding value is 0.683 into 1000 that is the first say the compressor you need to purchase and this is the second compressor you have to purchase with the price of 1000 at the end of 8 year. So, corresponding factor is 0.67. So, the total value is 1150. So, the same logic, we follow for calculating the cost against say the state of nature 3 that is 0.751 into 1000 plus 0.564 into 1000; 0.424 into 1000 that is 1739.

The cost of each strategy is obtained in a similar manner since strategy 2 results in the lowest expected cost the manager should order 2 spare compressors at the present time; that means, there are 4 possibilities; that means, ordering quantity could be 0 or one or 2 or 3. So, again 0; that means, if say suppose failure is 0; that means, this is 0 if it is 1; that means, at the end of. So, the sixth year it fails. So, corresponding cost is 564 and if there are 2 failures; that means; obviously, 1150 already it is clear if there are 3 failures; that means, a value will be 1739 ok.

So, the same logic we will apply, suppose ordering strategy is one. So, this is 100, this is 100 against to failure 1, against 2, it will be 567, is it ok? So, 100 plus the second failure; that means, just one failure as occur; that means, this is say the 567 and this is 1088. So, same logic you follow you will get the answers ok. So, there will be 2 failures. So, the one failure occurring at this one 564; so, this is 564 and so, this is 567; 1088. So, like this you calculate like 624 you calculate; that means, third a failure is occurring ok. So, accordingly you calculate; what is the corresponding value; so, 624 that is 221.20; the expected cost against ordering strategy of 2 ok.

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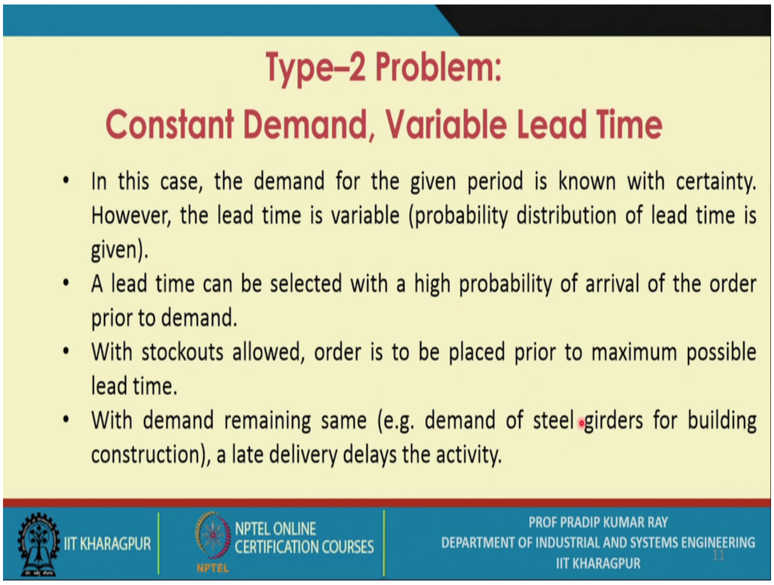

So, so again we know what is the solution when the inventory carrying cost is assumed to be 10 percent. So, I suggest that you follow all the steps meticulously. So, that you know you understand the logic fully.

Now, just before I close, I will just refer to the type 2 problem as already we have mentioned that the type 2 problem is a situation where there is constant demand is constant demand that does not change its known with certainty whereas, the lead time is a variable. So, in this case, the demand for the given period is known with certainty; however, the lead time is a variable probability distribution of lead time is given ok, a lead time can be selected with a high probability of arrival of the order prior to demand, is it ok, with stock outs allowed, order is to be placed prior to maximum possible lead time ok, with demand remaining same; that means, demand say for example, demand of steel girders for building construction a late delivery delays the activity, is it ok?

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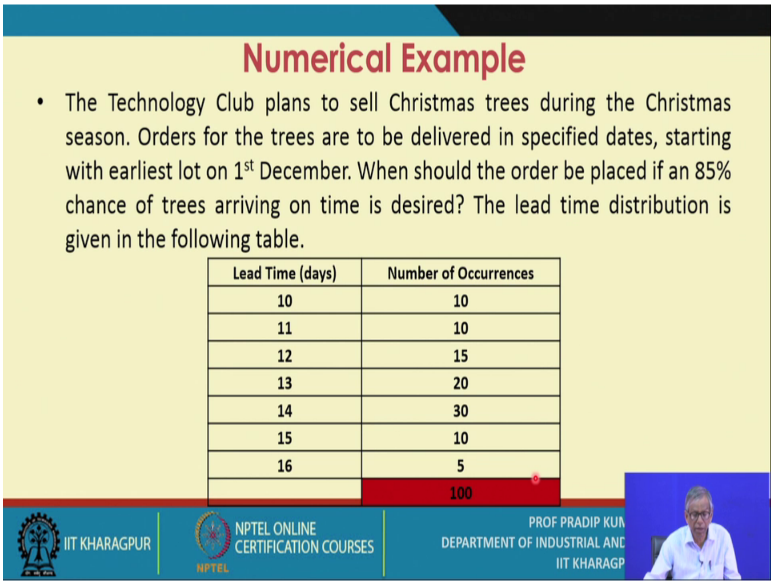

So, there are several cases. So, this is just one example like. So, the technology club plans to sell Christmas trees during the Christmas season orders for the trees are to be delivered in specified dates starting with earliest, lot on first December when should the order be placed, if an 85 percent chance of trees arriving on time is desired. So, this is given 85 percent chance, the lead time distribution is given in the following table. So, these are the possible lead times; that means, from 10 days to 16 days corresponding the number of occurrences are given; that means, the corresponding you know the frequency distribution table and assume it to be assume to be the probability distribution are given.

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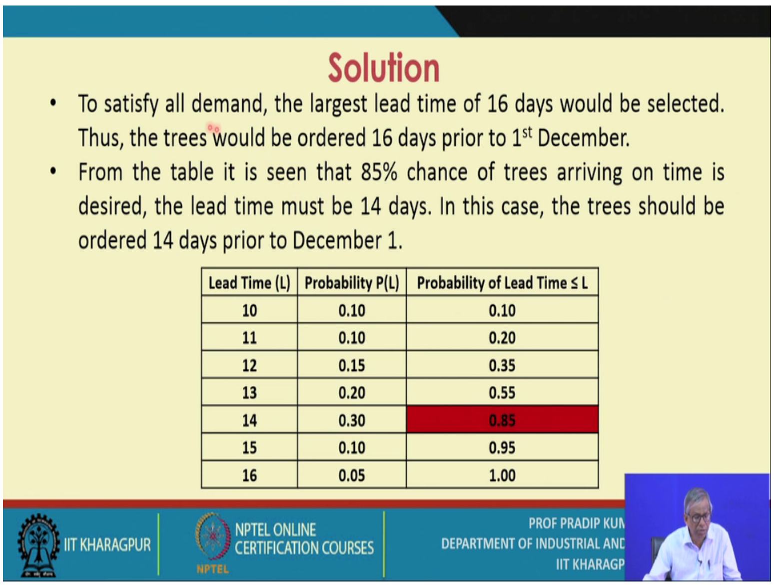

And this is the solution to satisfy all demand. The largest lead time of 16 days would be selected. Thus the trees would be ordered 16 days prior to the first December; that means, there will be no ah; that means, there will be no stock out situation; however, from the table it is seen that 85 percent chance of tress arriving on time is desired the lead time must be 14 days. So, so 85 percent of the chance there will be no you will get the get the item on time ok.

So, so, in this case, the trees should be order 14 days prior to December 1, with 85 percent chance of arrival. So, this is a solution; that means, we have just to add we have added one more column over here probability of lead time less than equals to l. So, if the probability for 10 days lead time is 0.10 this will be 0.1 against 11, it will be 0.20. So, this way you calculate. So, when it is 14 days lead time. So, the probability of lead time less than equals to l will be 0.85. So, this is the solution ok.

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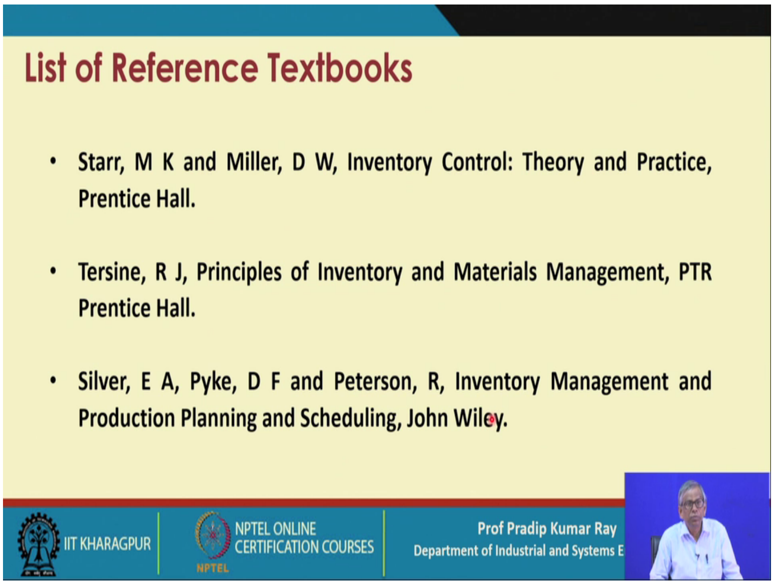

So, these are the reference textbooks.

Thank you.