Management of Inventory Systems
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Lecture – 12
Static Inventory problem under Risk
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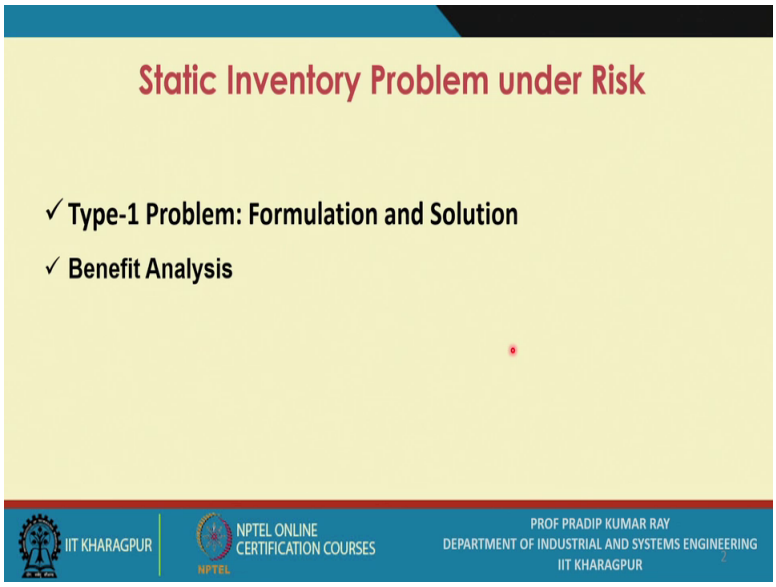

So, during this session on Static Inventory Problem under Risk you know, we will take a we will again discuss other issues or the formulation part of say formulation and the solution aspect of the type 1 problem. So, the formulation in detail, we will discuss. We will provide the formulation and we will also talk about the so called Benefit Analysis. This is very very important in fact.

In fact there are two kinds of analysis we do first one is either you know you go for benefit analysis or you also may opt for the cost analysis ok. So, during this session, I will again I will continue my say discussions on type one problem, followed by the benefit analysis.

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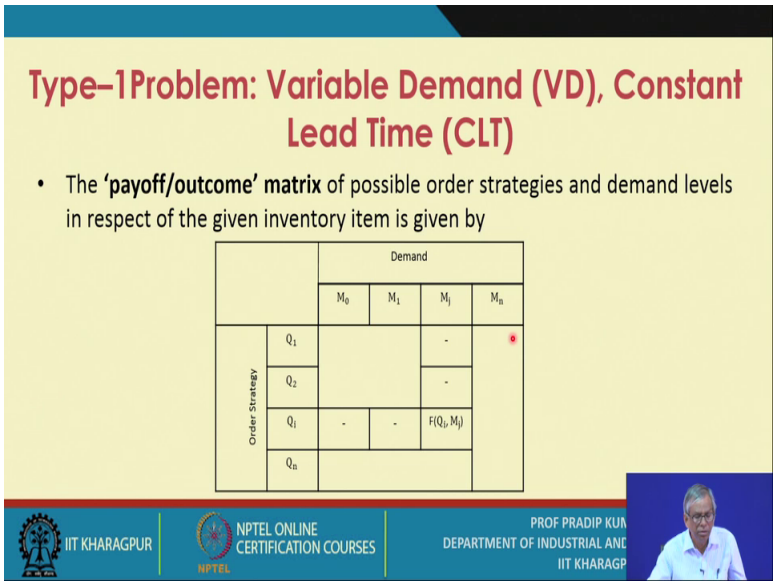

Now in the last session, I mentioned about the payoff or outcome matrix, mainly we have used a term the payoff or the outcome ok. So, against a particular order quantity and a demand level, is it ok. So, you need to determine the payoff or the outcome. So, when you consider all the possible demand levels. Suppose here is a case where n number n say that demand levels will possible from 0 to n; that means, the demand could be 0 units or 0, could be 1 unit or the demand could be the j th units.

So, demand could be n units. So, this is your demand and against this demand obviously, the corresponding probabilities are known, static inventory problem under risk. Now, the order strategies obviously, they could be n number of order strategies; that means, I am using this notations Q 1, Q 2 general notation is Q i and then you have the Q n. So, you form this matrix. So, this matrix of possible order strategies and demand levels in respect of the given inventory item; that means, is a single item case we are considering

So, what you try to do, like say in general, suppose order quantity is Q i and say the demand level is M j. So, you need to you know the specify you use this notations; that means, f of Q i M j; that means, with this combination what is your payoff or outcome, is it in monitory terms usually you need to determine first; that means, with all possible sales you first did determine F Q i M j, is it ok. So, I varies from 1 to n and j varies also from, i varies from 0 to n and j varies from say 0 to n right.

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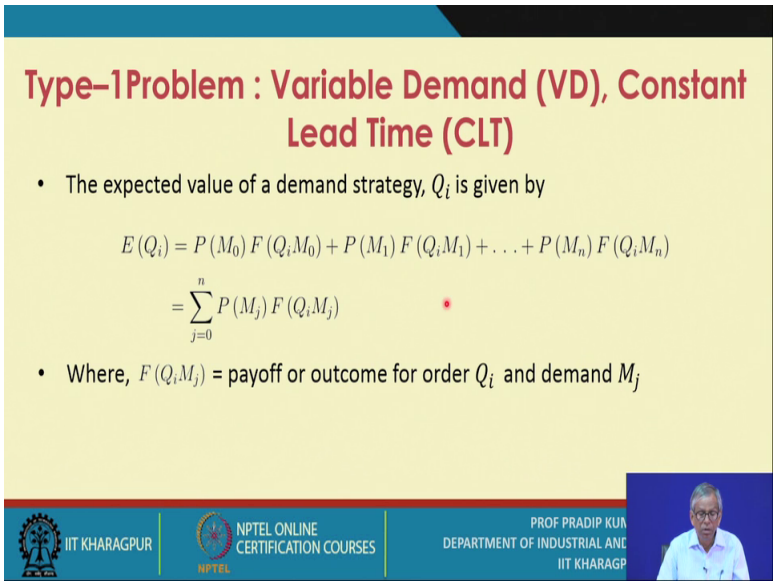

Now, we are considering variable demand and constant lead time case. This is a very you know is a standard problem. There could be few variations of this problem, but later on we will discuss those variations. Now, the expected value of a demand strategy Q i is given by; obviously, expected value or expected payoff with order quantity Q i, so obviously, this is the probability that the demand will be M 0 and then what is the expected payoff with a combination Q i M o, ok. Next, possibility is that the demand could be M 1 corresponding probability is P M 1 and then with the combination Q i M 1, what is the expected, what is the payoff, is it that combination.

So, this exercise will continue ultimately, you know you need to consider the probability of say the demand of n items n and corresponding say of the say you know the payoff. So, you have this summation j equals to 0 to n, P M j f Q i M j is it ok. So, I hope that you have clearly understood of the notations and their interpretations now where we have mentioned that F Q i M j; that means, payoff as I have already mentioned payoff or outcome for order Q i and demand M j, so, ok.

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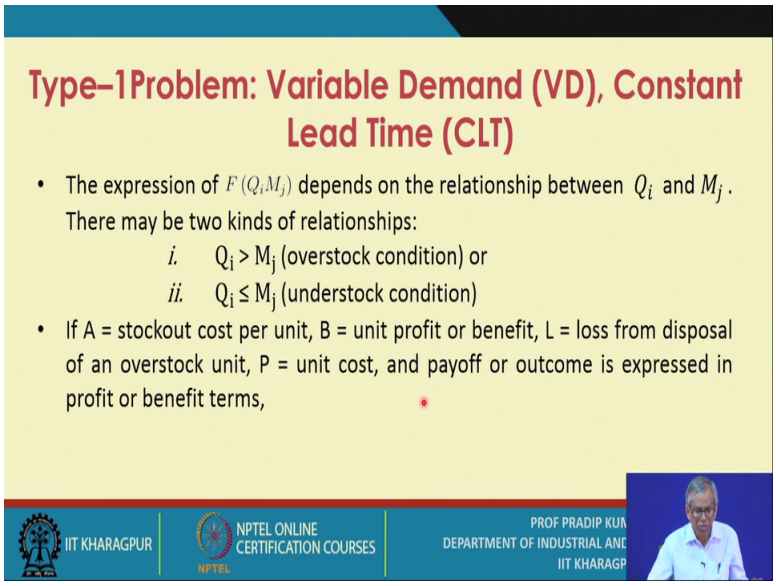

It is your starting equation. The expression for F Q i M j; that means, the payoff or the outcome depends on the relationship between Q i and M j, it is clear Q i and M j; that means, either is a Q i is greater than M j or the Q i is less than M j is possibility. And one case is Q i equals to M j is it that is the best possible case there may be two kinds of relationships Q i; that means, order quantity is greater than the demand overstock condition is; obviously, So, you just note it down that in any inventory problem, so say you need to deal with either overstock or under stock. So, in majority of the cases both overstock as well as the under stock, both are considered to be very very critical. So, first you consider the overstock condition.

But supposing Q i is less than or equals to M j; that means, order quantity is less than the demand; obviously, you are unable to fulfil certain demand and. So, it is an under stock condition ok. So, overstock condition is excess and under stock condition means you have insufficient stock to meet the demand, it is clear. If A, now what we will do we will you know the several cost elements you need to consider right and say the several cost related say you know the elements you need to consider as well as you know you need to consider the profit or also the benefit.

So, these estimates you must have. So, now, you have several notations. If capital A is equals to stock out cost per unit. So, this estimates you have later on I will tell you what is a procedure you might follow to estimate the stock out cost per unit; right now, we are

assuming that the stock out cost which we have we have estimated is a reliable one and it is it is accepted one and it is. So, the estimate is acceptable.

B is unit profit or benefit unit profit or benefit, L equals to loss from disposal of an overstock unit. So, overstock unit what you try to do; that means, if it is overstock unit. What you try to do? You try to dispose it of; that means, with a lesser price; that means,; obviously, this is a salvage value. So, what we are assuming that suppose your original price is 10 rupees, now when you dispose it off that means, with lesser price that means, you just you know you just dispose or say you sell it or salvage with it rupees 2 per unit. So, the per unit there is a loss 10 minus 2; that means, the 8 units or 8 rupees you are losing from disposal of an overstock unit. So, that we are considering P is unit cost and payoff or outcome is expressed in profit or benefit terms, is it ok.

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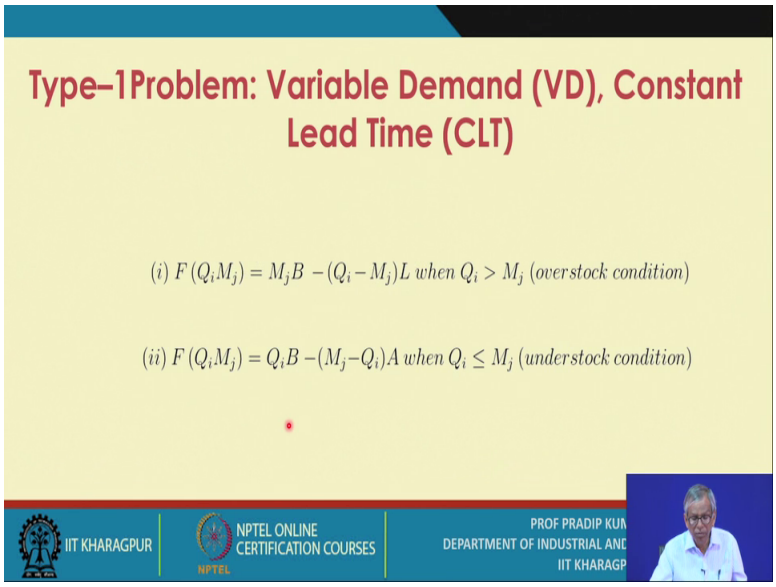

So, I have already mentioned that the payoff can be made either in terms of in terms of cost or in terms of profit or contribution to profit. So, here in this case, suppose so, what will be the equation for the payoff under overstock condition. So, F Q i M j obviously, is overstock condition; that means, under overstock conditions, whatever with the demand that means, M j only that demand level or the that demand also you can sell. So, the per unit selling you get a you say the profit of B. So, capital B into M j minus this you are unable to you know sell; that means, Q i minus M j that is the overstock is it because Q i is greater than M j. So, Q i minus M j overstock amount and for each one you are losing

an amount of L, is it ok. That means, the original price minus the salvage price is it ok. So, that is basically the l that you are losing per unit of overstock. So, when Q i is greater than M j is clear.

In the next case, you may also face the under stock condition. So, under stock condition the outcome is that Q i into B; that means, under stock conditions when the demand is more you have to procured less. So obviously, Q i units also only you can sell and for each selling you get a profit of B. So, it is Q i into B minus M j minus q; that means, this is this is actual demand you are unable to fulfil you know the entire demand. So obviously, M j minus Q I, so that is the under stock. So, this is the under stock amount and against each unit of under stock, you are incurring a cost of capital A. So, the estimate of A is reliable and it is a good estimate that we are assuming. So, it is Q i into B minus M j minus Q i into a where Q i is less than equals to M j, is it ok.

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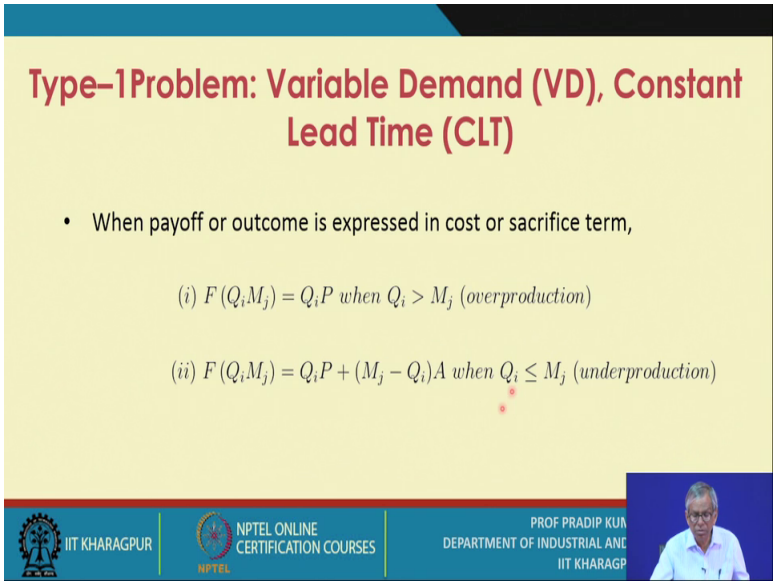

So, this is the both the conditions; that means, it is overstock as well as the under stock conditions we have considered. And it is essentially, you know we say that there is a possibility of selling; that means, the item is item you are procuring and item also you are selling is it ok. So obviously, whenever there is the inflow of cash or inflow of money. So, you try to calculate the benefit whereas, when all the activities you carry out within your manufacturing setup, within your, you know the organization and so, so within your production unit so; obviously, there is no question of getting the sales.

So, what you need to do; that means, you need to formulate the problem in terms of the cost elements only. So, the cost is also known as the sacrifice ok; is the cost or sacrifice term when payoff or outcome is expressed in cost or sacrifice terms. That means, you are dealing with the production related. So, related to the production. So, the first one is when Q i that means, order quantity order quantity is greater than M j. Now, this order quantity you have already produced. In fact, it is a production quantity Q i is a production quantity. So, this production quantity is greater than the demand. So obviously, you know this is a production cost per unit already you have produced. So, Q i into P is the cost, is it ok.

So, Q i F Q i M j equals to Q i into P when Q i is greater than M j, but if suppose there is under production that means, there is a shortage and the shortage cost per unit is capital A. So, what is that shortage is M j minus Q i, your requirement is M j, you have produced Q i so; obviously, the per unit you are incurring a shortage cost of A right. So, it is M j minus Q i into a plus Q i into p because already you know you have produced Q i units and against each unit you produce, you incur a production cost of P. So, this is the expression for F Q i M j when Q i is less than equal to M j under production.

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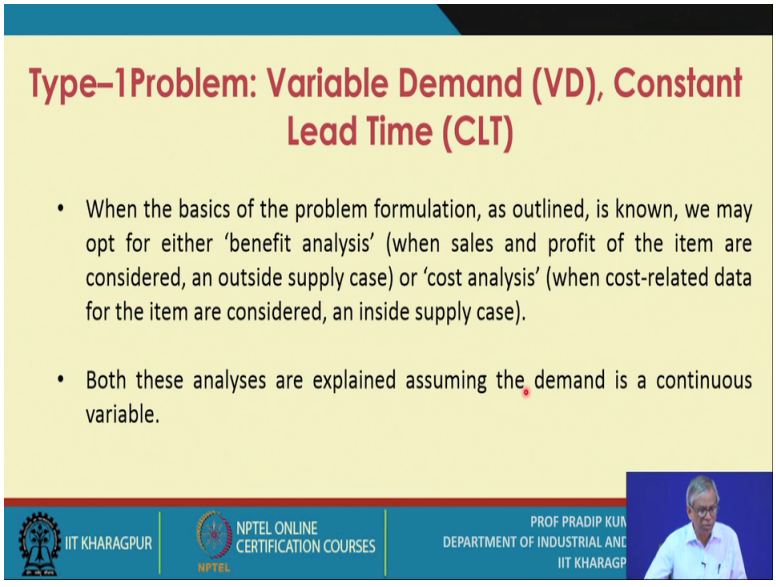

And over production, both the cases we have considered. When the basics of the problem formulation as outline is known ok, actually the basis of the problem formulation as outline is known, we may opt for either benefit analysis when sales and profit of the item

are considered and outside supply case ok. At this point, I have already mentioned or you may opt for cost analysis. When cost related data for the item are considered, an inside supply case as we have been repeatedly mentioning that in any inventory control problem either it is a outside supply case or in a inside supply case. If it is an outside supply case, you go for benefit analysis. If it is an inside supply case, you go for cost analysis both these analyses are explained assuming the demand is a continuous variable ok.

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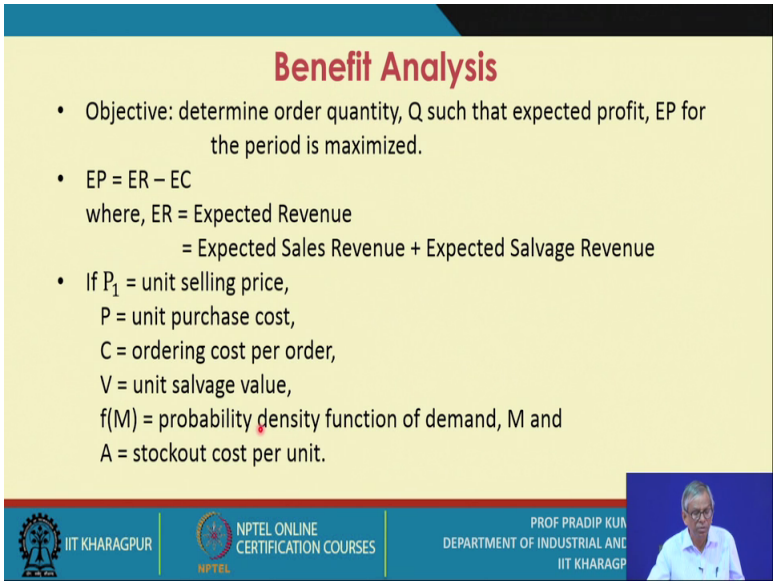

So, now if you go for benefit analysis what you do? What is the objective of benefit analysis? Determine order quantity Q such that expected profit notation is EP for the period is maximized ok. So, this is here the main objective of the benefit analysis and here, just one time you determine the order quantity because you are dealing with a problem called static inventory problem under risk ok.

Now, expected profit how do you get the expression for the expected profit? So, expected profit is essentially expected revenue minus expected cost where E R is expected revenue right. Now, in the given situation what do you mean by the expected revenue? Expected revenue means expected sales revenue; that means, out of the sales you are getting the revenue and out of the salvage of, so the excess quantity, you are also getting salvage revenue. So, expected sales revenue plus expected salvage revenue when there is a possibility of salvage. Now, these are the you know the notations we have used

like say P 1 is unit selling price C is the ordering cost per order, is it ok. The later on, we will tell you that how do you how can you estimate the ordering cost per order; that means, essentially when you place an order, there are many kinds of activities you carry out. It is the responsibility of the purchasing department usually and so, you carry out lots of activities and. So, before is purchased order is placed and after and when the order is executed that means, the time between placement of an order and receipt of the supply. So, before you receive the supply, you place order there are many types of activities you carry out and any activity you carry out, there is a cost associated with it.

So, you need to identify those activities in a given situation and you need to estimate corresponding costs and in a systematic manner, you get an estimate of the ordering cost and usually ordering cost is calculated per order basis. One order may consist of say one item or it could be more than one item. So, both the cases later on, we will discuss in detail. So, the C is the ordering cost per order, V is the unit salvage value is it ok. So, how to determine the salvage value later on when you when you go for excess stock modelling; So, we will refer to this then what is the how to determine the best possible salvage value in a given situation under conditions?

So, V is the unit salvage value f M is the probability density function of demand M; that means, is a continuous random variable, capital A is the stock out cost per unit. So, again you know the stock out cost is estimated and it is an acceptable good estimate that we are assuming.

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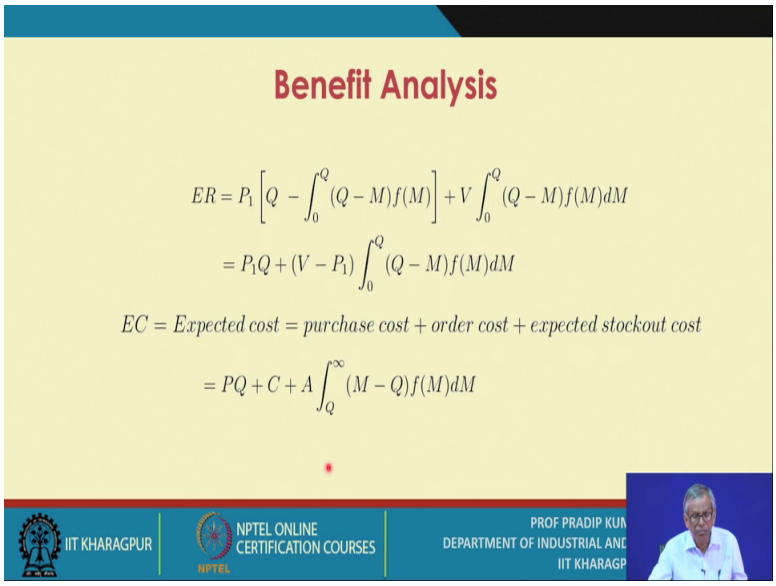

So, when you go for benefit analysis, so this is the expected revenue. So, how do you express expected revenue? So, expected revenue is essentially you know, but the Q is the you know the order quantity Q. So, the P 1 into Q, it is a sales you can do now it may. So, happen that the M that means, a demand may be less than Q. So obviously, the entire there is a possibility that the entire amount which you have produced or sold, you are unable to select. So, that is why we have said that what is probability the expected number of units remaining unsold; obviously, Q minus M f M when M varies from 0 to Q. That means, this is the expected number of units remaining unsold is it ok.

So, that is why Q minus this expression is it and P 1 is the sales price unit sales price now plus you know there will be a unsold amount and the unsold amount that is Q minus M f M d M and the demand varies from 0 to Q is the conditions. So, when that M varies from 0 to Q under this condition only, there is a possibility of excess stock. Now, what you do this excess talk you know the certain recovery is possible that means, excess talk you can sell with the salvage price. So, the salvage price per unit is V. So, that is why it is V into you know the integration 0 to Q, Q minus M into f M d M that to be a expected number of units unit salvaged. So, this is the simplified expression, so, P 1 Q plus V into P 1 0 to Q, Q minus M f M d M.

Expected cost is expected cost equal to expect purchase cost, ordering cost and expected stock out cost. There are three elements in this. So, what is the purchase cost that means,

Q units we have purchased and for each one, the purchase cost is P, C is the ordering cost already you know we have estimated, ordering cost per order one order you have placed because is a static inventory problem under risk. And this is the shortage one; that means, if suppose the demand is more than the order quantity that means, M minus Q. So, and the M varies from Q to infinity any value of M is a continuous case. So, M minus Q into f M d M into this is the expected number of units short and this is the shortage cost per unit. So, that is why it is multiplied with capital A.

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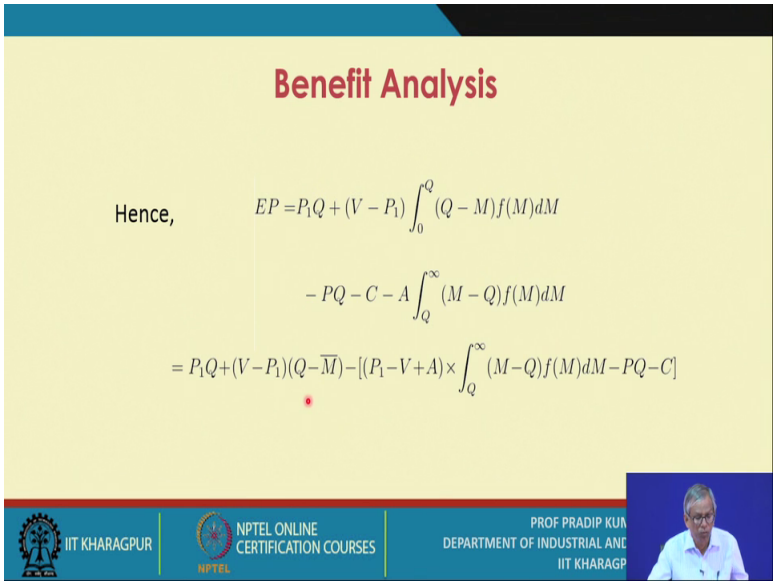

So now, what is the expected profit? Expected profit equals to this minus this that means, expected revenue minus expected cost, is it ok. So, when so, this is the expected revenue and this is the expected cost. So, expected revenue minus expected cost is expected profit. So, ultimately you get these expressions.

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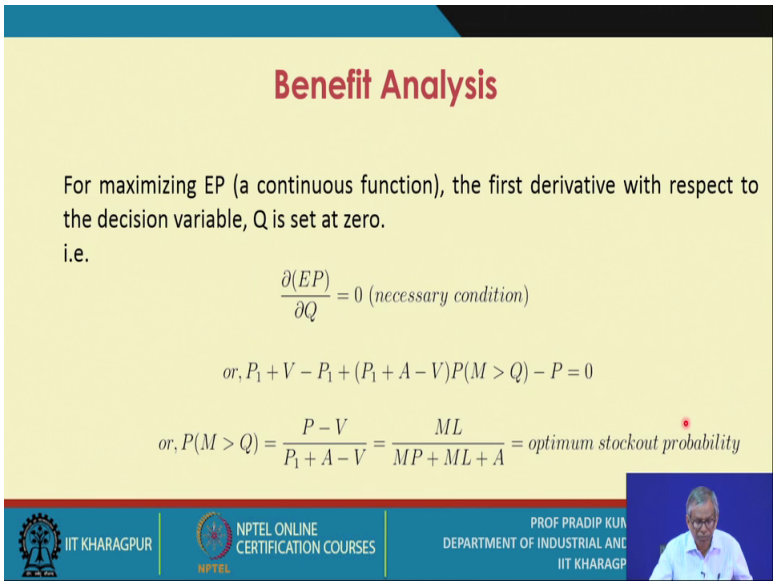

For maximizing expected profit E P, assuming it is a continuous function and that is why it is differentiable. The first derivative with respect to the decision variable Q, your decision variable is the order quantity is set at 0 is it ok. It is a first derivative you set at zero; that means, with the partial derivative del del Q of E P is equals to 0. This is the necessary condition right. So, first you satisfy the necessary condition. So, you follow the steps and ultimately what you get, you get this expression the probability that M is greater than Q what is M? M is the demand level is greater than Q is equals to P minus V divide by P 1 plus A minus V.

So, you just follow the steps, I am sure that you will we will we will understand each and every step, all these the details I have provided. So, this is P minus V divided by P 1 plus A minus V and with some you know the manipulation. This is this is you know change to with this particular ratio that is M L, marginal loss divided by marginal profit plus marginal plus plus A. So, this is referred to as probability that M greater than Q, this is basically the stock out probability and this is the optimum stock out probability is it because the, you are maximizing this condition is fulfilled, maximizations of expected profit. So, I we have derived the expressions for the optimal stock out probability. So, this is marginal loss that means, divided by marginal profit plus marginal loss plus A ok.

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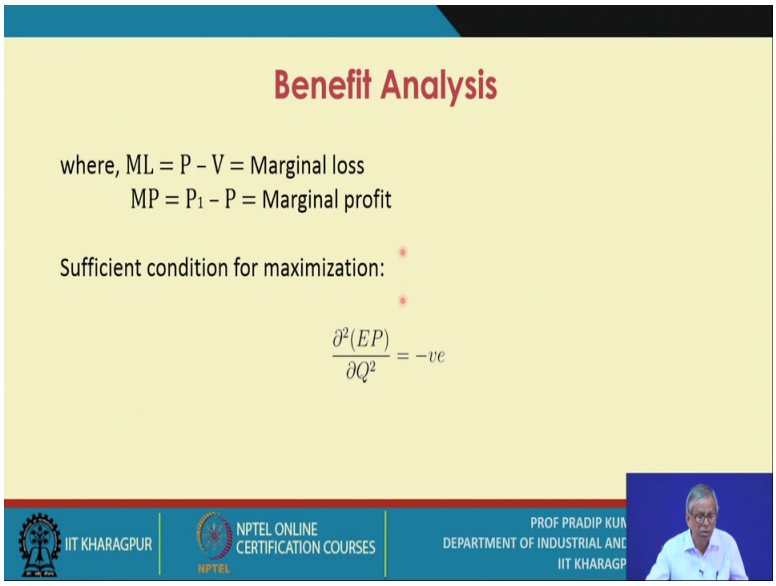

So, what is where marginal loss is equals to P minus V, what is P? P is basically the production cost, what is V? That is a salvage one is it ok. So, the P minus V is a marginal loss. You just make a note and what is marginal price that means, this is a self price minus the production cost that is called marginal profit, is it ok. Sufficient condition for maximization obviously, I presume that you know it that means, a sufficient condition is the second derivative partial derivative with respect to Q of E P is equals to must be negative, is it ok.

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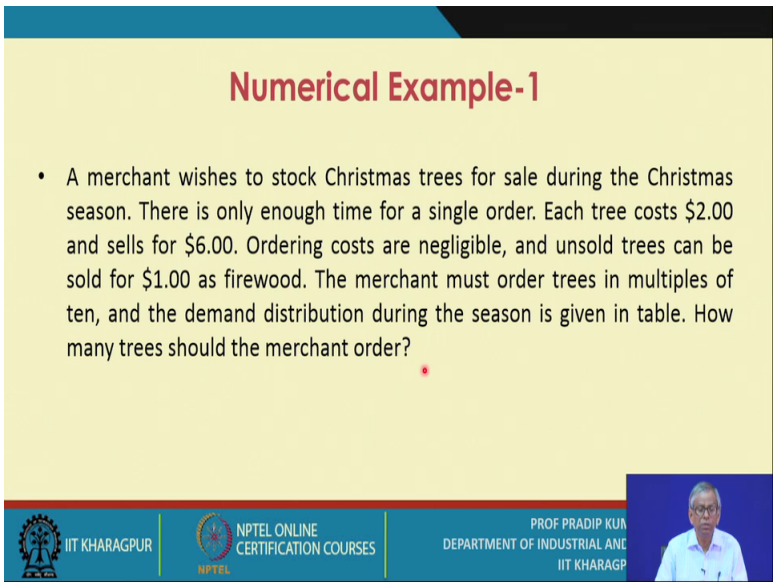

Now, here is one before I conclude this session, I will be referring to just one numerical example ok. A merchant wishes to stock Christmas trees as I have already mentioned that many a time this static inventory problem under risk is refer to, as a Christmas tree problem or say news by problem. So, here we are referring to the Christmas trees and the Christmas trees are sold exclusively for a specific time periods say one week or two weeks in the month of December each year.

So, a merchant wishes to stock Christmas trees for sale during the Christmas season. There is only enough time for a single order right. Each tree costs 2 dollars and sells for 6 dollars right ordering costs are negligible, we are assuming that are the C is absent ordering cost is absent and unsold trees can be sold for 1 dollar as firewood is it ok. So, that is a salvage price essentially V.

The merchant must order trees in multiples of 10 and the demand distribution during the season in given in table that means, you that means, while you place this numerical example; that means, you have collected the data, you have analysed the data. And so and the demand distribution in empirical form is you know in empirical form and that is presented in the table, now the question is how many trees should the merchant order; that means, what is the order in quantity?

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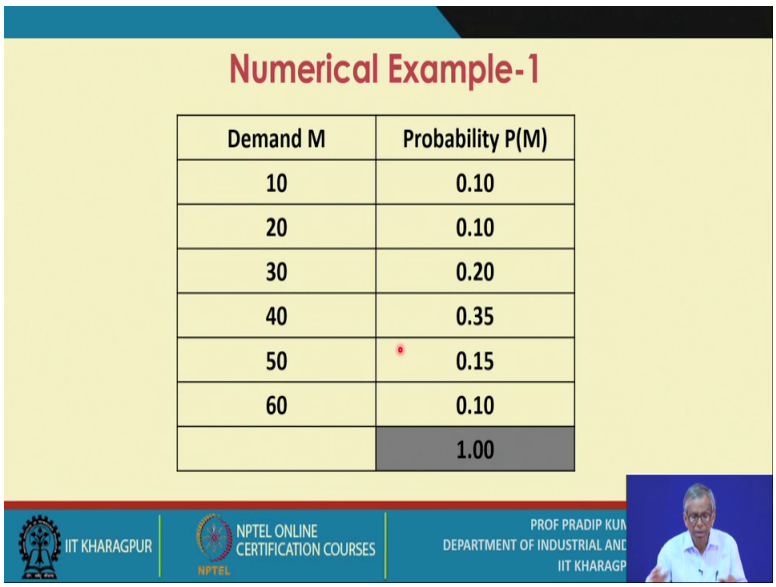

So, this is the demand level. So, in multiple of 10 that means, demand could be 10, 20, 30, 40, 50 and 60, these are possible that means, what we are assuming and these are

possible probabilities. So, add them one; that means, what we are assuming that there is no possibility of demand less than 10 and there is also no possibility of demand greater than 60; that means, this is when all possible demand levels that means, is an exhaustive situation which is mainly may not happen in almost all cases there is a theoretical possibilities. But here for the illustration purpose, we are assuming that all possible demand levels are known and the corresponding and the probabilities are also known.

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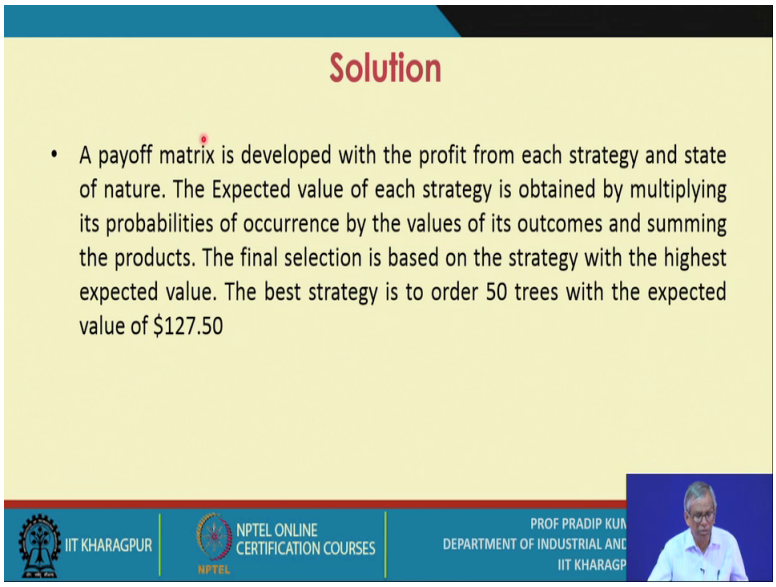

So, what you try to do? That means, now a payoff matrix is developed with the profit from each strategy and state of nature is it ok. So, the payoff, so the matrix you have to create, the expected value of each strategy is obtained by multiplying it is probabilities of occurrence by the values of it is outcomes and summing the products ok. All, the individual the product terms you add so, against a particular row you get the corresponding the expected value ok.

So, in this case, it is a sell situations. So, the expected value that means, in the payoff that means, in terms of the profit you get the expected say the payoff. The final selection is based on the strategy with the highest expected value. I have already pointed out. The best strategy is to order 50 trees with the expected value of 127.50.

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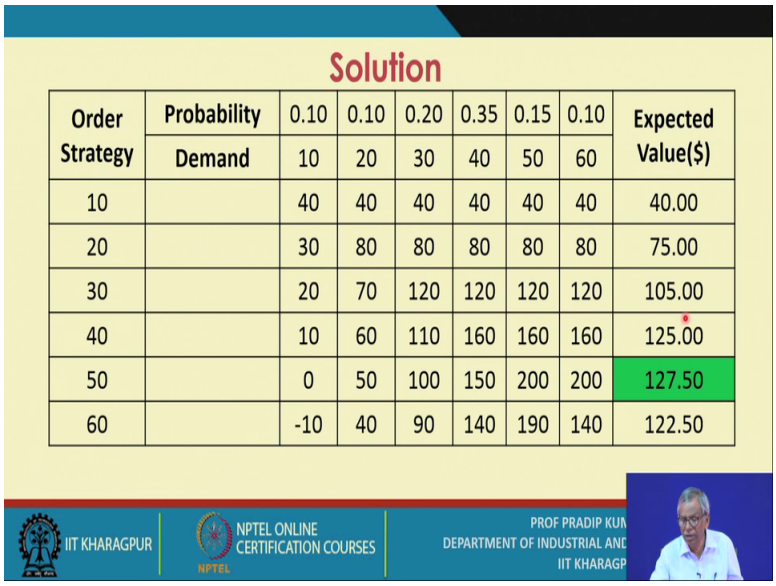

So, this is the table we have created. So, just you look at this table and how it is constructed, these values are given these values are given. So 0.10, probability that demand is 10; 0.10 is the probability the probability that demand is 20 like this. So, then 10, 20, 30, 40, 50 and 60, so the order strategy is obviously, will be from 10 to 60. So, 6 possible ordering strategies as there are 6 possible demand levels. Now when you have say order quantity 10 and your demand is also 10, so how much you will be getting; that means, 6 you are selling and you are ordering cost is around 2. So, so the purchase cost is 2.

So, for each unit, you are getting a savings of 4 the profit. So, it is 40, is it ok. So, 60 minus 20, that is 40 and here also you could sell all the 10 units. So your, you return is 40, 40, 40, 40, 40. So, you just look in to the calculations and how do you get what is the assumptions under which you get this values are 40, 40, 40 irrespective of that means, it remains at 40. The next one is the 20 that means, here that means, 10 unit shortage that means, what you do; that means, there will be excess of 10 if the demand is 10.

So, this 10 units you can sell as firewood and with the price of. So, the one so that is why it is 40 minus 10; that means, it becomes 30 this is 80 straight and for all other demand levels it remains at 80 when the order quantity is 20. So, like this, you calculate is it for all demand levels ok, what is the payoff against a particular order quantity. So this, the matrix is created.

Next what you do, against each ordering strategy you calculate the expected value. So, how do you calculate the expected value like for example, for the order strategy 10, this will be 40 into 0.1 plus 40 into 0.1 plus 40 into 0.2 plus 40 into 0.35 like this. It will be 40. For the second one whose ordering strategy is 20, it will be 30 into 0.1 plus 80 into 0.1 plus 80 into 0.2 like this. So, you add all this values individuals product terms and you get 75. So, like this you, you complete you do the calculations for all the rows and you select that row that means, which gives you the maximum expected value. So, in this case when the order quantity is 50, you get a return of 127.50 as a maximum. So, that is why you select an order quantity of 5 ok.

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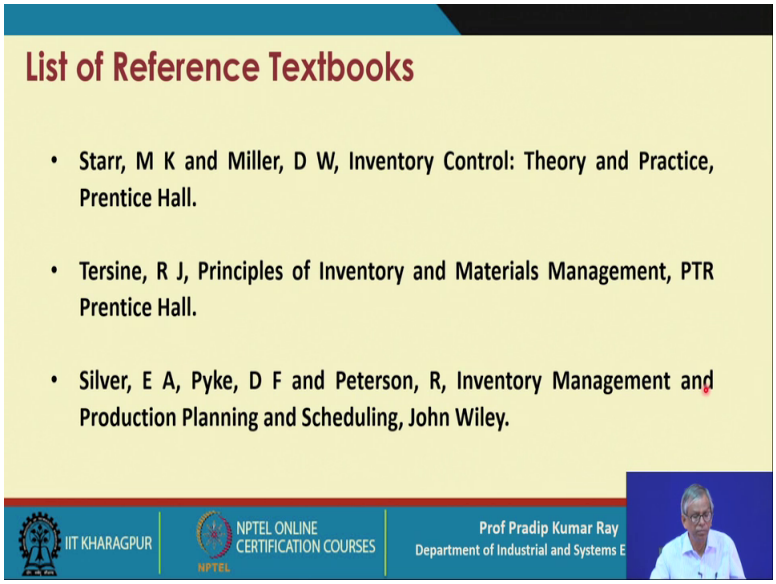

So, this is a simple exercise we have carried out. Later on, we extend this exercise in some other details in different forms under different conditions ok. So, I conclude this session.

Thank you.