Management of Inventory Systems
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Lecture – 11
Static Inventory problem under Risk

During this week, this is a third week we will be discussing at length the Static Inventory problem under Risk.

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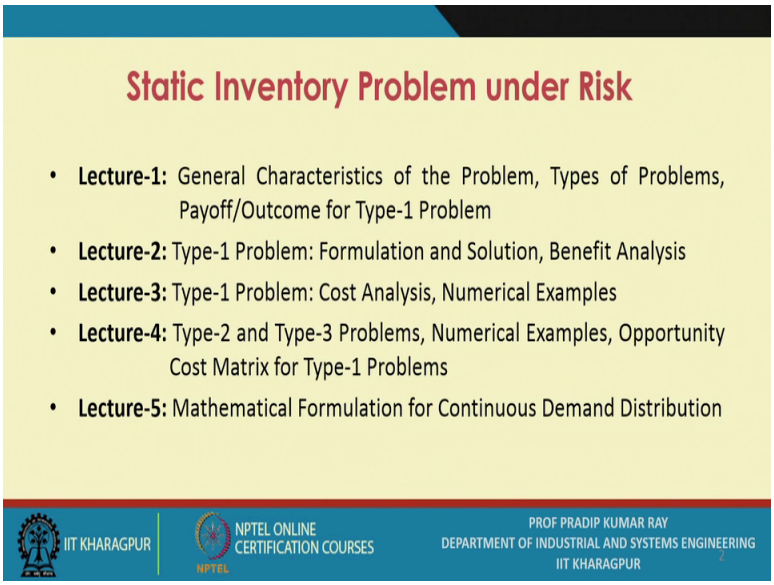

Now, during the 5 lecture sessions the topics which I would like to cover are as follows. In the first lecture; that means, at this point in time, I will be lecturing on the general characteristics of the problem, inventory problem obviously. So, we must be aware of what are these, what are the are the typical characteristics a problem may have you should be aware of that is the starting point. The types of problems already in the in the very first week we have we have come to know that what are the factors affecting an inventory problem, and we have a classification scheme for the inventory problems.

So, we will be refereeing to this the types of problems, and each and every type of problem we will take up. And we will try to the formulate the problem, and then we will we will propose the solution steps. And then obviously, you know the approach or the solution steps we will explain with respect to numerical of the data or the numerical exercises. So, types of problems one important issue to be discussed that is the payoff or

the outcome. I have mentioned for type one problem, but essentially for any type of problem, this payoff or the outcome you should be aware of. So, later on when we take up for the formulation of the problem as well as the numerical exercise, this point will be made very, very clear to you.

The next lecture will be concentrating on type one problem formulation and solution in in detail we will discuss. And we will also refer to one particular aspect called benefit analysis, ok. So, this is benefit analysis we will do, during third lecture session again we will be referring to type one problem. Cost analysis will specifically will be dealing with and the a few numerical examples we will discuss.

During lecture 4, type 2 and type 3 problems essentially what we will find here; that means, when we refer to the static inventory problem under risk. This where the problem can be of say the 4 types. The first problem we do not need to say or refer to you do not need to consider, or I will tell you what is the reason for the remaining 3 problems you need to consider. So, all this 3 problems are to be formulated and their solutions the steps must be known,

So, during lectures 4th lecture session, type 2 and type 3 problems will be dealing with a numerical examples, and the concept of opportunity cost matrix for the type one problems we will be we will be discussing at length. And during lecture 5 session, mathematical formulation for continuous demand distribution; that means, mathematical formulation of the inventory problem under this category static inventory problem under risk. But what will assume that the demand distribution is a continuous type, ok. So, continuous demand distribution is a very specific case so, we will be dealing with this case along with the numerical exercises.

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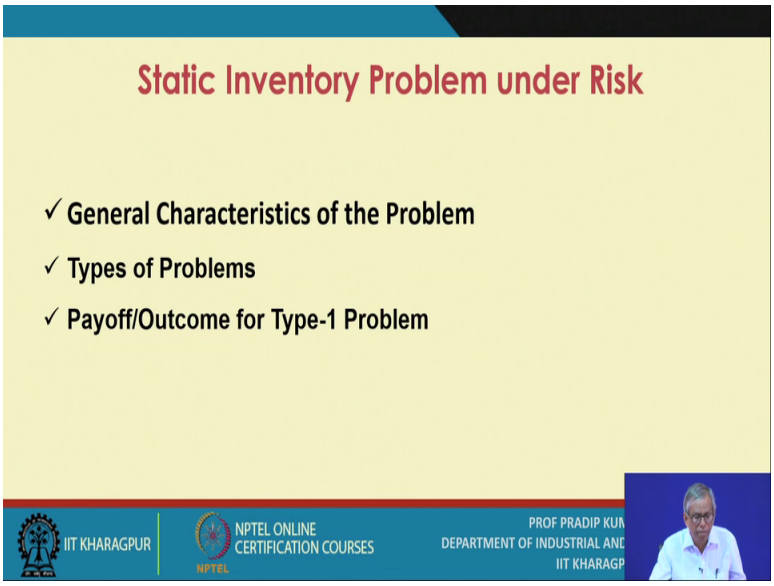

So, this is our the plan for this week. And let us first talk about the 3 important issues one by one, first one is the general characteristics of the problem, then the types of the problem we will we will discuss. And the payoff or the outcome matrix or outcome the determination for type one problem we will be dealing with, ok.

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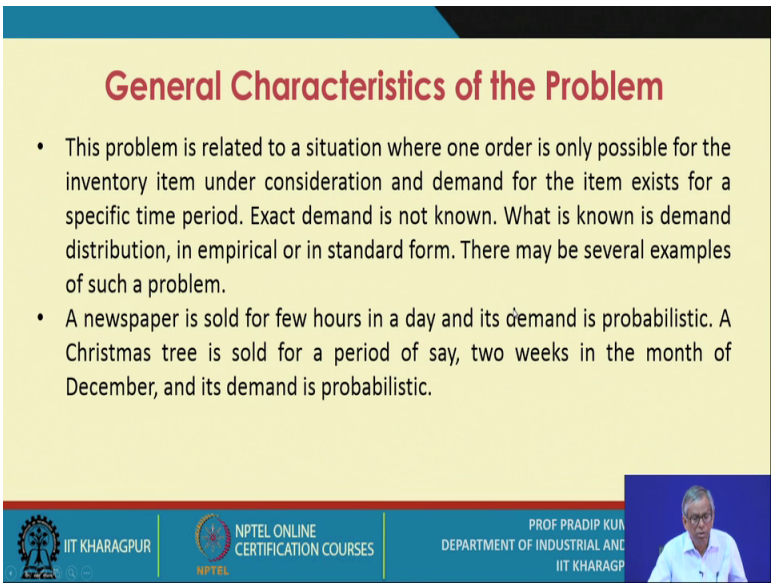

Now, here let me just tell you that we are referring to at this point in time static inventory problem. If you refer to our the week the lecture sessions, you will find that is the most simple classification of inventory problems, we refer to first to the static inventory

problem, and then the dynamic inventory problem comes. What is the static inventory problem? That means, just it is a onetime occurrence of demand. And as it is one-time occurrence of demand, you can have logically just one order. So, the one order is sufficient to meet the demand of say onetime occurrence for a given item.

So, that is basically the static case. Whereas, you know there are items particularly for the running production system, those items you were need to say the procure on a on a regular basis is continuing the production is continuing. And so, so also the requirement of a particular item so that particular item is to be procure on a regular basis; that means, one order specific time period, the next time period you need another order so, and this process continues.

So, this is essentially a dynamic situation. So, that is why it is referred to as the dynamic inventory problem. Next as we have already mentioned that now we refer to the demand so, demand may be known with certainty, or the known with risk or known with uncertainty, is it ok? Now for the static inventory problem under say if the demand is known with certainty; obviously, you know the decision variables are known and everything is known.

So, you no need to formulate the problem we never search for the solution. So, that is why the static inventory problem under so the certainty; that means, the order will be known one order quantity also will be known and so obviously you know you know everything. Only when you have you deal with multiple items and so, then possibly know you can it is definitely not an this not coming under the inventory control problem, but if you deal with say the multiple items, it may be a problem related to the scheduling, ok.

So, we will not be discussing; obviously, we do not need to discuss static inventory problem under certainty. This point we have already emphasized at the demand for the item exist for a specific time period. So, a one week or one month, or even one day or even for few hours exact demand is not known. That is why it is called the means demand is known with risk. What is known is demand distribution, this point already we have highlighted, the dealing our say the lecture sessions in week one.

So, demand distribution is known either in empirical or in standard form, ok. Empirical means is a table form is given, essentially the possible demand levels and corresponding

the probabilities, is it ok? With the so this values are given. So, these are referred to as empirical form whereas, it could be in a standard form. Like, the demand may be poison or the demand may be normal etcetera, etcetera. There may be several examples of such a problems you come across,

So, that you need to find out so when we propose the assignments. So, you may be asked to so identify a case, which falls into this particular category, ok. So, that you know the during the assignment the sessions, you will come to know. So, typical examples 2 examples I am giving you. One is a newspaper is sold for few hours in a day. And it is demand is probabilistic, is it ok? So, sometimes this problem is refer to as the news by problem, similarly a Christmas tree is sold for a period of say 2 weeks in the month of December.

And it is this and this demand is probabilistic, that is why it is this problem is also refer to as you know the Christmas tree problem, ok. So, I repeat the problem is refer to as in many text is a news by problem or news vendor problem or news by problem or Christmas tree problem, ok.

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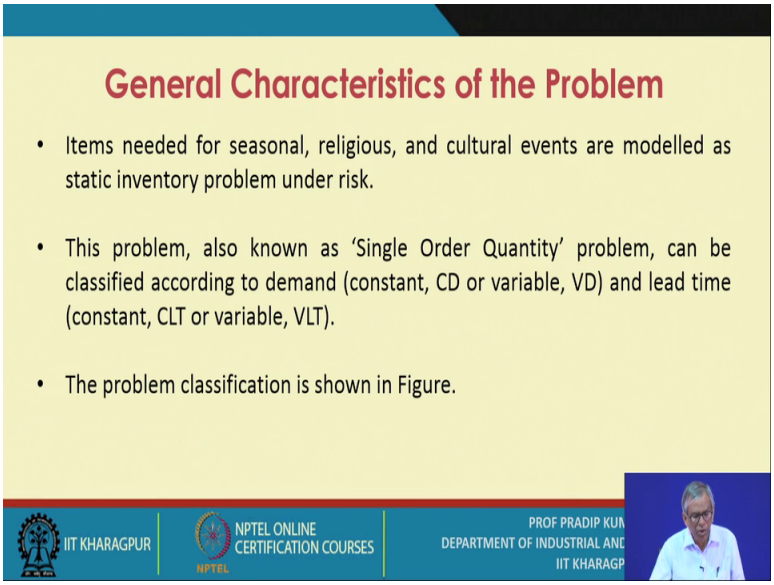

Items needed this point is to be noted, items needed for seasonal, religious and cultural events are modelled as static inventory problem under risk, is it ok? This problem so also known as the single order quantity problem, right if you go through the text, if you go

through the case studies many a time this is referred to as a single order quantity problem. Now this problem can be classified according to demand,

That means, there are 2 important or the factors, the first factor determining a problem or the type of problem determining the problem, first one is the demand. So, the demand could be constant demand you may assume. So, CD or demand could be variable demand the notation is VD. And the second important factor that is the lead time or the lag time, either it may be assumed to be constant CLT constant lead time, or it may be assumed to be a variable lead time VLT, ok.

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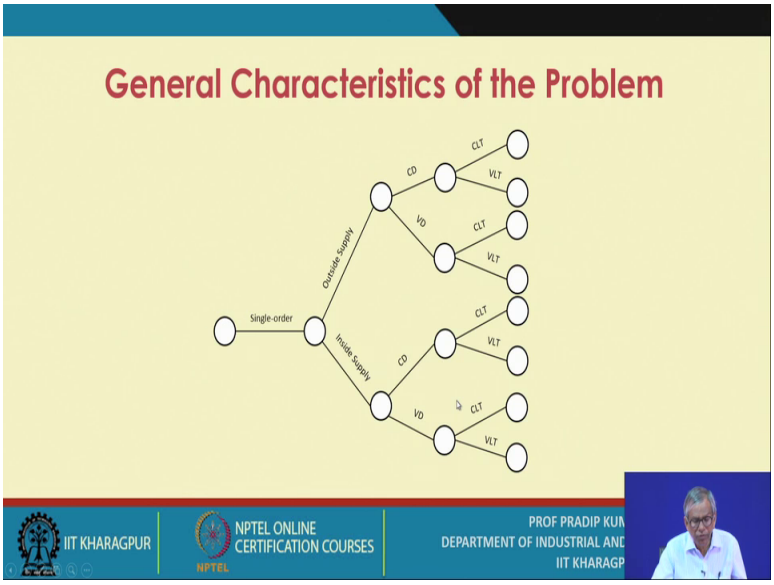

So, the problem classification is shown in the following figure, ok. So, just look at this figure is very clearly we have drawn this diagram. So, what you find here there is a single order, model single order just a one or really sufficient to meet the demand. Then it could be the outside supply or it could be the inside supply as you know that if you refer to the original or the classification of problems this diagram are made for say multiple orders also the dynamic situation.

So, here it is a static situation single order outside supply inside supply. Now if it outside supply there could be constant demand or variable demand. We need to explore all sorts of possibilities. And then if against constant demand there could be constant lead time or the variable lead time.

Similarly, against variable demand there could be constant lead time or the variable lead time, ok? If it is an inside supply case, later on we will we will determine you know we will both the cases, while you determine the order quantity, and the important say so the inventory control the parameters in a given system or a given system. So, both outside supply case as well as the inside supply case you need to you need to consider. So, that means, you need to determine the order quantity or sometimes this is refer to as the production quantity,

So, if it is production quantity is inside supply, if it is an order quantity it is an outside supply. For outside supply what you try to do? You place a purchase order, whereas, for the inside supply, that means, from one say the production unit 2 another unit, ok. So, suppose you need this item in the machine shop, and this item is to be fabricated, right and then you need to place or work order to the fabrication shop. So, that is a inside supply so this is inside supply and obviously, I do not you do not place as a purchase order, you place the work order, is it ok?

Similarly, if it is a inside supply there could be constant demand or variable demand, against constant demand there could be constant lead time or the variable lead time. And similarly against variable demand, there could be constant lead time or the variable lead time. So, if you look at this diagram, what you find? That we have a slowed all source of possibilities; that means, you follow a particular path, you get one type of problem.

You change this path all possible paths you have 1, 2, 3, 4, 5, 6, 7, 8. So, 8 possible paths you have, that means, 8 types of problems you might files; that means, for each path you need to formulate the problem. You need to go for formulation, and then depending on say so the type of formulation, or the characteristics of the formulation, you search for the solutions or the solutions stays, ok.

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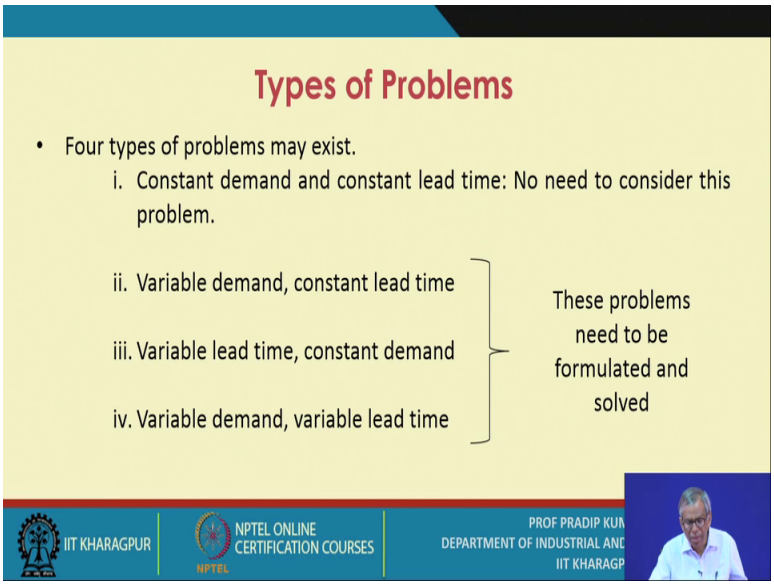

So, so you may find that when you consider the both consider both demand and lead time as a variable one. That may be treated as a most complex problem, is it ok? Whereas, when you consider both or so other extreme, when both demand as well as the lead time, and consider both as a constant. So, it may be treated as a simple problem so, always we will find if you look at a particular path and looking at particular path you can get an idea about the complexity of the problem, ok. So, the 4 types of problems may exist constant demand and constant lead time. I have already pointed out no need to consider this problem for a single item case, ok.

So, here we are referring to single item case. There could be an extension of single item case to the multiple item case, right? Now we are not discussing that, ok. So, if you refer to multi item case and for each item, you have this condition constant demand and constant lead time, then it is a problem related to the scheduling, but not related to inventory control, ok. Next problem is that variable demand constant lead time; so, this problem will be taking up, variable lead time constant demand, is it ok?

So, this problem also you come across and you have to formulate this problem. And of course, the 4th one that is the variable demand, and variable lead time, it is a complex problem definitely. So, we will be dealing with this problem also ok. So, this problems need to be formulated and solved, this points I have mentioned.

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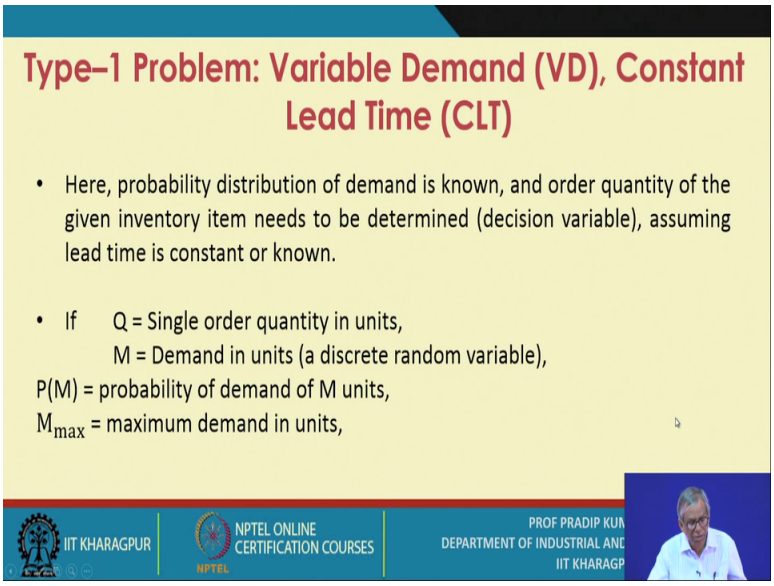

Now, let us talk about the type one problem, ok. So, what is the type one problem? That is the variable demand and constant lead times. So, let us discuss this problem in detail, and while you discuss this problem, you come to know actually you know the basics of. So, the static inventory problem under risk, ok. So, here whenever we say that the problem is under risk; that means, the probability distribution of demand is known, ok.

Now, these demand; that means, there is a random say random variable, ok. And so, so, obviously, you know this demand could be a discrete random variable or it could be a continuous random variable. So, the probability distribution that you search for it may be a standard one, if it is suppose it is a it is a discrete random variable; obviously, under discrete random variable, normally you know if you refer to the textbook if you refer to the case studies, you will find 3 or 4 types of you know standard probability distribution.

You may assume for the demand depending on the situation; obviously, we are assuming that the physical basis behind say selecting a particular say the standard or the distribution, the physical basis is known. And based on so you have a physical basis and then only you search for or you assume a standard distribution under given level of significance.

So, here the probability distribution demand is known. So, these details we will discuss later on, but right now just you keep in mind, ok. So, probability distribution of demand is known. Now just I will give you one you know one exercise that you start thinking of

that how many ways or which method you follow to specify the probability distribution of demand, for a given item. So, what sort of data you need to collect? Order quantity of the given inventory items, needs to be determined. This is your decision variable, assuming lead time is constant or not so, this is your problem.

Now, what you try to do? If say a certain notations you have to use. So, what is the first notation; that means, Q is a single order quantity in units, is it ok; that means, here what we are assuming if you say in units, and suppose it is a discrete units, ok. So, obviously it is a discrete say random variable, or say not random variable it is a discrete order quantity. Whereas, the demand in units a discrete random variable I have already mentioned, because this order quantity is discrete variable, ok.

Now, what is P M? P M is the probability of demand of M units, actually it is probability mass function P M, f for discrete random variable the corresponding probability of a given the value is known as the probability mass function so, that is given as P M, ok. So, these notations we have used, what is M max? That means, the maximum demand in units, is it ok? Is a discrete case. So, was the maximum demand is 50 units. So, that value you must be able to specify.

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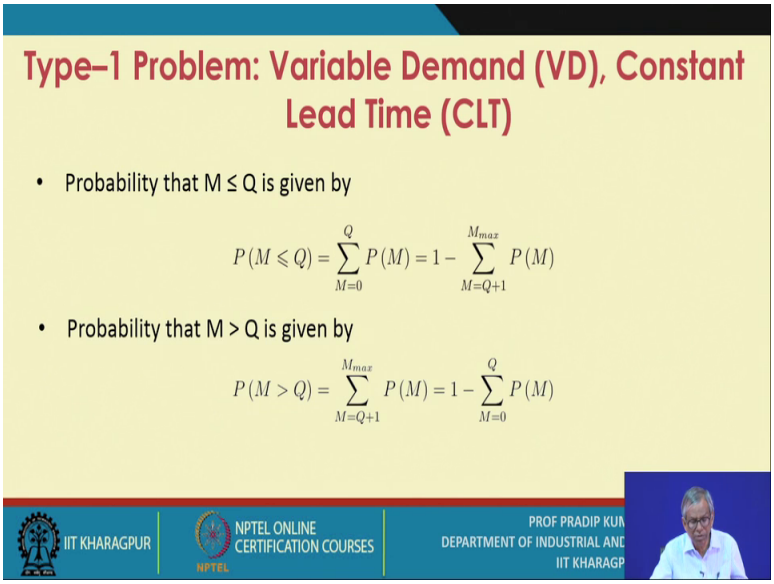

Now, here as you come to know that ok, as you come to know that you know there are 2 sorts situations. One is you know against a particular demand, the demand is not known with certainty, there is a it is, but the demand is known with risk; that means, a

probability distribution of demand is known. So, the probability that M; that means, a demand is less than or equals to Q; that means, order quantity is given by this notations you need to use, the probability that M is less than equals to Q.

So, how do you specify this is a later on when we formulate the problem. So, you just go through all these the notation the way, we are you know representing this probability expressions, ok. So, you please followthese notations, and these are basically this one of the concept you must have while you formulate the problem. So, that means, this is M equal to 0 to Q; that means, a demand could be 0 and demand could be up to Q, ok. So, that means, less than equals to Q, it is clear? That means, it will never the actual demand will never cross already more than Q, that is why up to Q probability that that there is a demand for M units P M,

So, this is 1 minus M; that means the demand Q plus 1 more than Q to M max. Maximum value these are discrete case P M is it ok? So obviously, what you find that the sigma M equals to 0 to M max P M; obviously, it will be 1 right? Probability that M is greater than Q is given by probability M is greater than Q; that means, this part basically; that means, M equals to Q plus 1 to M max, P M equals to 1 minus M equals to 0 to Q P M, ok. So, this is the standard notation so, the expressions for the probability we will be using later on.

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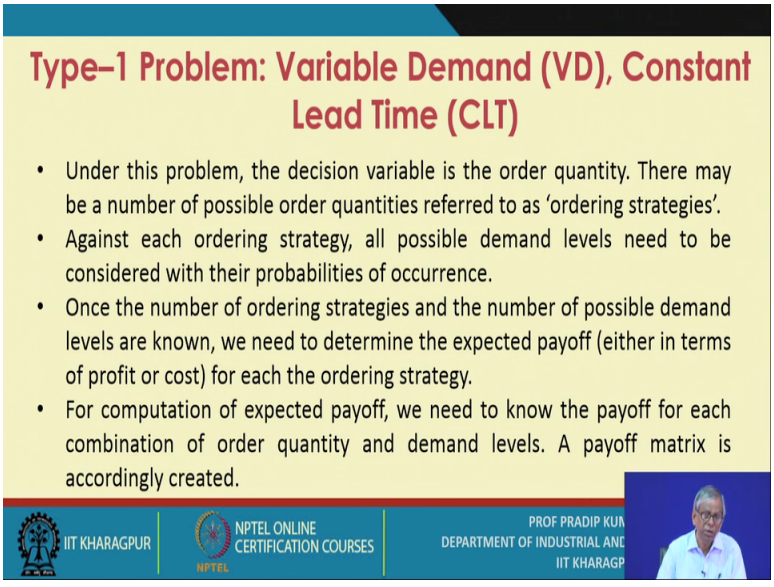

Now, when you deal with this particular problem, I will be just adding few important all the points related to this particular problem. The decision variable is the order quantity, and when you decide this order quantity, what you are assuming that there is there is a time period for which the demand occurs during this time period the demand occurs say one week or 1 hour. And all and the order quantity is sufficient to meet the demand during a specified time period, this is the core issue,

So, the decision variable is order quantity, there may be a number of possible order quantities refer to as ordering strategies; that means, what we are saying the first you look at the problem, and you just to try to find out that what are the possible demand levels suppose this is a discrete case, right. For a given item the item is. So, the number of say a certain items and their demand is known. Suppose the demand for one-unit demand for 2 unit demand up to say 10 units so, you have 10 possible demand levels. So, 1 to 10 for now you collect data, pass data, analyse the data and you say that against the demand one, the probabilities 0.05 against demand of 2 units the probabilities say 0.15 like this.

And similarly against say the demand of. So, the 10, the probability is 0.05, is it ok? So, what you must have that is the precondition, that is you must have the frequency data; that means, your preference should be that these probabilities must be objective probabilities. In many cases you know suppose it is a new situation and this is a past data are insufficient. So, you cannot get you know the objective probabilities, but you have to solve the problem you know it is a it is a probabilistic demand, ok. So, what you need to do you might get the expert opinion, and then from the experts from the experts opinion analysing experts opinion you can have also the probability values against a particular demand level, but in those cases this probabilities are essentially the subjective probabilities.

So, in many cases what you find that you start with the subjective probabilities when you know the past data are insufficient, and as you proceed; that means, as you get the data. So, in course of time may be after it is a few months may be after one year or 2 years, now we have the sufficient data. So, that means, a subjective probability can be replaced with the objective probability, this is the point to be noted so whenever you will be using the probability values we may not be you know explicitly telling you whether it is objective probability or the subjective probabilities, ok. So, when you discuss the

probability aspects so this 2 issues must be discussed in detail; that means, what is the basis must of determining the probabilities must be the known.

Now, so, this so, suppose you have 10 possible demand levels; obviously, you must have the 10 possible you know the ordering strategies, is it ok? So, the ordering can could be 1, 2, 3, 4 up to 10 against each ordering strategy all possible demand levels need to be considered with their probabilities of occurrence, is it ok? So, that is there are suppose there are 10 levels of the demand; that means, and suppose you know one possible order quantity is one; that means, when you place order one you place an order quantity of 1, then against the order quantity there could be the demand of one or the demand could be 2, demand could be 3 like this, is it ok?.

It may continue up to demand so, up to demand of 10. So, against each ordering strategy all possible demand levels need to be considered with their probabilities of occurrence, is it ok? So, the first condition is the probabilities occurrence of the demand levels against the demand levels, you determine or you identify the possible demand strategies, so order strategies.

Once the number of ordering strategies and the number of possible demand levels are known, we need to determine the expected payoff either in terms of profit or cost for each of the ordering strategies, is it ok? For computation of expected payoff, we need to know the payoff for each combination of order quantity and demand levels; that means, suppose your order quantity is 1 and the demand level is 1 with a probability say 0.05. So, what is your expected payoff? Similarly, so, suppose your ordering quantity is 3 your demand is say 5,

So, in these conditions 3 5 combinations what is the expected payoff? So, you will have say suppose you have 10 possible demand levels 10 possible ordering strategies. So, how many the combinations we will have? It will be 10 into 10; that means, 100 combinations you will have. And for each combinations you have to calculate what is the expected payoff; that means, what is the return you get.

And then what you do against each ordering quantity against each ordering quantity or ordering strategies considering all the possible demand levels and their probabilities, you need to determine the expected payoff. So, if you have say 10 possible order strategies.

So, you will have 10 expected payoff values. And out of this 10 expected payoff values which one you will select.

Now, here one point you must keep in mind, let is this payoff you know which you calculate, the payoff can be either in terms of the profit or in terms of or in terms of the cost. If it is in terms of the profit or more specifically contribution to profit, you just note this point that actually sometimes we say it is a profit, but truly speaking it is contribution to profit. So, if it is in terms of contribution to profit, you select that particular ordering strategies, for which the expected payoff is maximum. Whereas, if the payoff matrix is created in terms of the cost only. So, you select so, that particular ordering strategy for which the expected payoff is minimum, ok.

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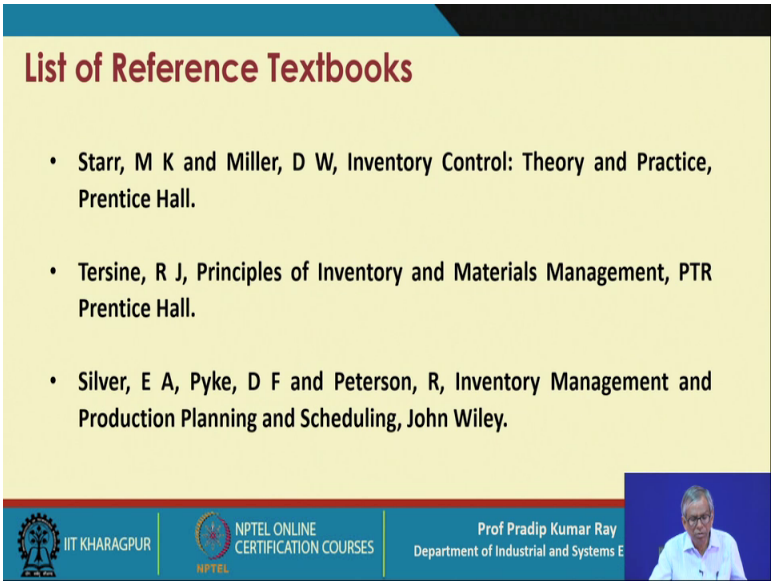

So, I conclude these sessions in the next sessions, the other aspects of the static inventory problems we are going to discuss.

Thank you.