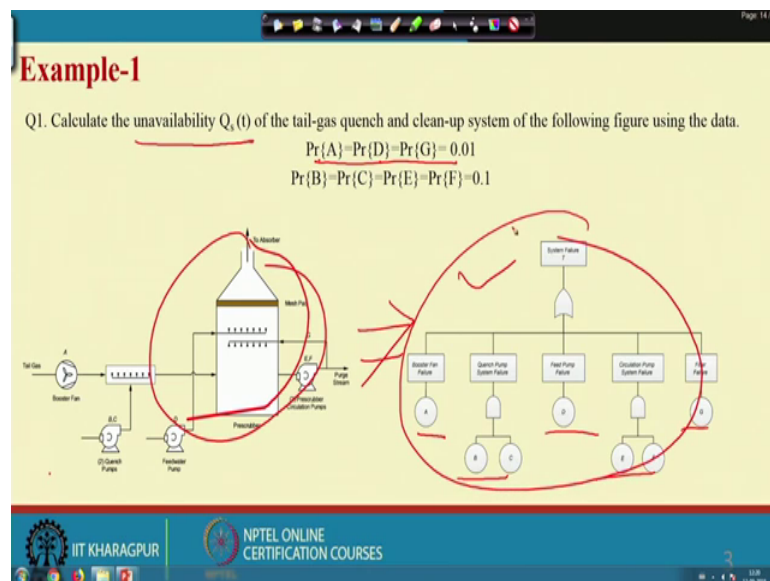


**Industrial Safety Engineering**  
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**Lecture – 40**  
**Systems Safety Quantification : Tutorial**

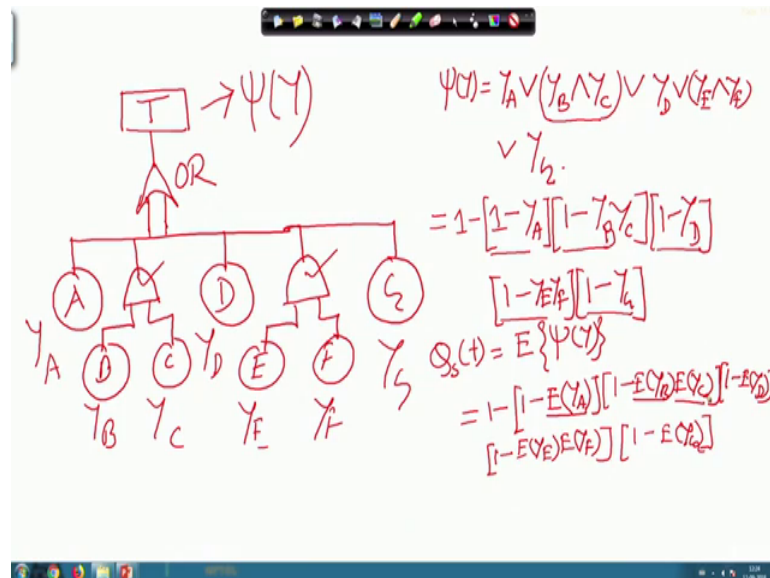
Hello a very good day to you all. So, before going to the today's session, let me first introduce myself, my name is Souvik Das, currently I am pursuing PhD in the Department of Industrial and Systems Engineering IIT Kharagpur, under the supervision of Professor J Maiti. So, today I will conduct an tutorial session on System Safety Quantification. So, this session is not only useful for better understanding, the system safety, but also it will also help for your upcoming examination. So, now I am straightaway going to the question.

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The question is calculate the unavailability of the tail gas quench and clean up system, this system you have already seen, this is the tail gas quench system and the fault tree of this tail gas quench system is given. Now we have to find the unavailability, the basic event probability is given here, the probability of A, probability of D all the things are given here. So, I will show the step by step procedure that, how to calculate unavailability given this fault tree.

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So, first I will draw the fault tree, suppose this is my top event then, there is one AND gate sorry OR gate, this is OR gate, now the basic event A then input is AND gate and the basic events are B and C. Now this is another basic event D, again another AND gate inputs are E and F another basic event that is G.

So now from this fault tree, we want to write the structure function of this fault tree. So, how to write? The indicator variable of this top event let us take phi Y the indicator variable of this basic event  $Y_A$ , this is  $Y_B$ , this is  $Y_C$ ,  $Y_D$ ,  $Y_E$ ,  $Y_F$  and  $Y_G$ . Now if we write phi Y that will be  $Y_A$  union as this is OR gate, it will be union. So,  $Y_A$  into then the input is this AND gate and the input of this AND gate is  $Y_B$  and  $Y_C$ . So,  $Y_B$  intersection  $Y_C$  as this is AND gate.

Now again union  $Y_D$ , again union  $Y_E$  intersection  $Y_F$  as this is AND gate, again union  $Y_G$ . So now, in algebraic form, we can write 1 minus 1 minus  $Y_A$  into 1 minus this is this input is  $Y_B$  intersection  $Y_C$ . So, this will directly be multiplied. So,  $Y_B Y_C$  then 1 minus  $Y_D$  then 1 minus  $Y_E$  and  $Y_F$  then, 1 minus  $Y_G$ . So now, we want to calculate the unavailability. So, we know this unavailability is nothing, but the expected value of the structure function.

So, using this formula we can write 1 minus as all those input are independent in nature, as all the things are appear only once not in twice that is why this term 1 minus  $Y_A$  1 minus  $Y_B Y_C$  1 minus  $Y_D$ , this all terms are independent in it is nature. So we can write

1 minus expected value of YA into 1 minus expected value of YB into expected value of YC 1 minus expected value of YD into 1 minus expected value of YE into expected value of YF and 1 minus expected value of YG.

So, now the expected value of YE, YB, YC these are nothing, but the probability that is given in the question. So, if we go to the question, we can see that probability of A, that is 0.01 D G are same then probability of B C E F 0.1.

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**Example-1**

The structure function

$$\psi(Y) = Y_A \vee (Y_B \wedge Y_C) \vee Y_D \vee (Y_E \wedge Y_F) \vee Y_G$$

Algebraic form:

$$\psi(Y) = Y_A \vee (Y_B \wedge Y_C) \vee Y_D \vee (Y_E \wedge Y_F) \vee Y_G$$

$$\psi(Y) = 1 - [1 - Y_A][1 - Y_B Y_C][1 - Y_D][1 - Y_E Y_F][1 - Y_G]$$

Unavailability:

$$U_s(t) = E[\psi(Y)] = 1 - [1 - E\{Y_A\}][1 - E\{Y_B\}E\{Y_C\}][1 - E\{Y_D\}][1 - E\{Y_E\}E\{Y_F\}][1 - E\{Y_G\}]$$

$$= 1 - (0.99 \times 0.99 \times 0.99 \times 0.99 \times 0.99)$$

$$= 1 - 0.951$$

$$= 0.049$$

Handwritten calculations on the slide:

$$1 - 0.01 = 0.99$$

$$0.99 \times 0.99 = 0.9801$$

$$0.9801 \times 0.99 = 0.970299$$

$$0.970299 \times 0.99 = 0.96059601$$

$$0.96059601 \times 0.99 = 0.9509890499$$

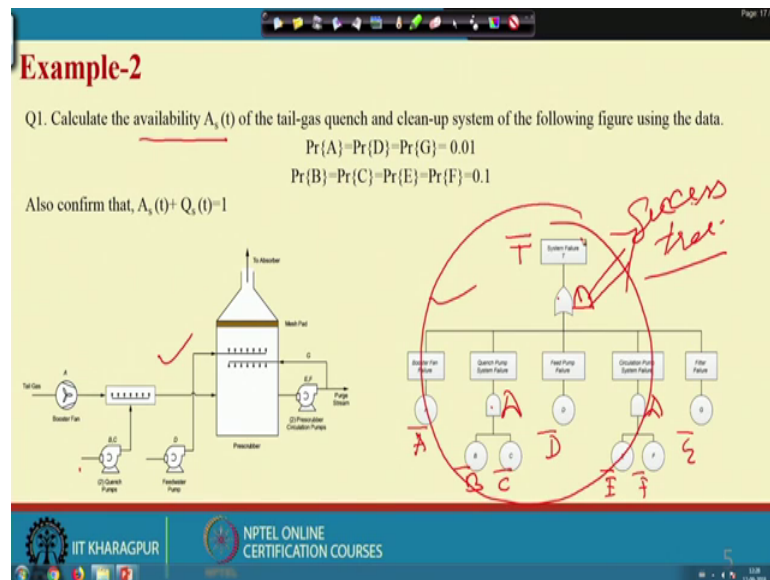
$$1 - 0.9509890499 = 0.0490109501$$

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So, you can see that this is my accepted value that we have got, sorry structure function. Now, this is my unavailability and this is the thing that, we have got last that accepted value of Y, YA, YB, YC all the things, now we have to just put the probability value of this probability of A is given 0.01. So, expected value 1 minus expected value of YB will be 1 minus 0.01 that will be 0.99. So, we have written 0.99 then for this YB and YC, it is given that probability of YB is 0.1 and YC is given 0.1. So, 0.1 into 0.1, it will be 0.01, again 1 minus 0.01, that will be 0.99.

So, again 0.99 for this value then 1 minus YD 0.99. Similarly all the things, we have calculated. Now, after calculating these we have got that the unavailability is 0.049. So, in this way we can calculate unavailability from the structure function approach and for that we need to know that the probability is nothing, but the expected value of all the basic event. So now, we will go to the second problem.

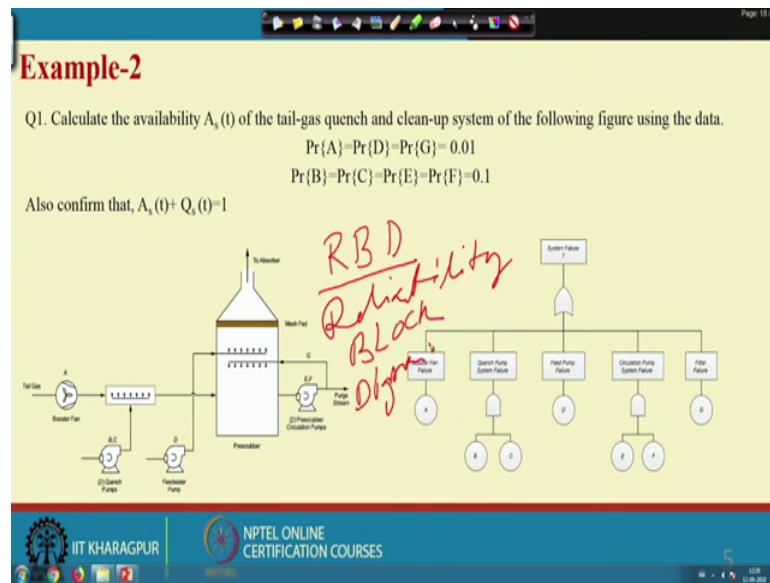
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In second problem, it is asking that we have to calculate the availability; we have to calculate the availability of that tail gas quench system, the same system is there and the fault tree is there. Now we have to calculate the availability. So, for calculating availability there are 2 approach, one is we have to convert this fault tree into success tree. So, how to convert this fault tree into success tree? Just you have to replace all the OR gate into AND gate and AND gate into OR gate and all the basic event, those will be A it will be its inverse, that is A bar, B bar, C bar, D bar, E bar, F bar, G bar and this will be OR this will be OR this will be AND and this will be T bar.

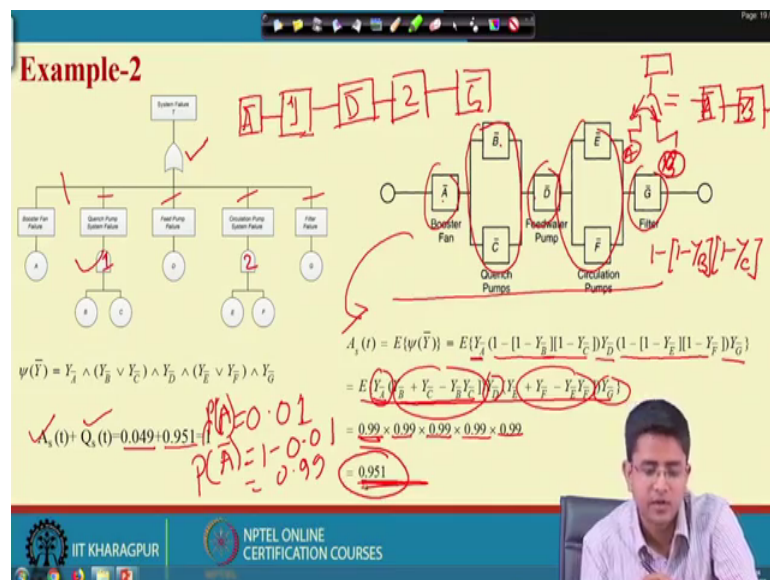
So, if you follow the same procedure, I have shown in the previous exam that, how to construct such structure function and from that how to get the unavailability, you will get the same manner that the unavailability of T bar; that means, availability of T.

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So, in this way you can find out the availability, but here I will show you another approach that is RBD approach Reliability Block Diagram.

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So, you can see that how to form the reliability block diagram from the fault tree. So, look it is OR gate as it is OR gate and this, the inputs are A then this AND gate suppose this AND gate is 1, I have naming it then, this is C, this is 2 then this is G. So, these are the input. So, as this is OR gate, we can write the RBD, this will be A bar in series, it will be 1 as the from the property of OR gate, we know that if this is OR gate and this is A

and B, then the corresponding reliability block diagram is A bar and B bar, in the same manner all the things will be series and the total things will be in bar.

So, A bar into 1 then, this will be D bar, then this will be 2 then, this will be G bar. So now, 1 is nothing, but B and C and it is AND gate, that is why it is in parallel. So, B bar and C bar. So, you can see that A bar in series with B bar and C bar is parallel then, again D bar is in series E bar and F bar is parallel then G bar in series. So now, we have got the reliability block diagram.

So, now from the reliability block diagram, we can calculate the structure function of it. So, how to calculate the structure function of it? As B and C, this is in parallel that the structure function of only this portion, if we want to write it will be nothing, but 1 minus 1 minus YB into 1 minus YC and as all the things are in series, this input A, this input, this, this, this all the things are in series, we have to multiply all the things. So, just like here that YA bar is the indicator variable of A bar, then for this we have write 1 minus 1 minus YB bar into 1 minus YC bar then for this YD bar.

For this portion again, this is parallel 1 minus YE bar into 1 F bar then again YG bar. So, if we further dig down, it we will get this type of equation and just put all the as this is YA, this is YB all the things are independent in nature, we can put the values that is given in the question that YA bar is nothing, but the probability of the A bar. So, the probability of A is given as 0.01 probability of A. So, you can write probability of A bar, is nothing but, 1 minus 0.01 that will be 0.99. So, that is written here.

Similarly, all the things you have to calculate, this YB bar plus YC bar minus YB bar into YC bar then, YD bar then, this portion then YG bar. So, ultimately we have got that availability 0.951. So, from this we can show that my availability 0.951 and my unavailability is 0.049. So, if we add those 2, we will get 1. So, we can show here that availability plus unavailability is always 1. So, in this way you can calculate the unavailability portion. So now, I will go to the third question. (Refer Time: 15:39)

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**Example-3**

Q1. Calculate the unavailability  $Q_s(t)$  of the tail-gas quench and clean-up system of the following figure using the data.

$\Pr\{A\}=\Pr\{D\}=\Pr\{G\}=0.01$   
 $\Pr\{B\}=\Pr\{C\}=\Pr\{E\}=\Pr\{F\}=0.1$

Use Minimal cut and minimal path representation.

The diagram shows a process flow for a tail-gas quench and clean-up system. Tail gas enters a quench tower, then goes through a feed pump, a quench pump, and a filter before being sent to a scrubber. The fault tree for the system failure (Y) is shown to the right. The top event is 'System Failure Y'. It is connected to five intermediate events: 'Booster Fan Failure A', 'Quench Pump System Failure 1', 'Feed Pump Failure D', 'Circulation Pump System Failure 2', and 'Filter Failure G'. The 'Quench Pump System Failure 1' event is further decomposed into 'B' and 'C'. The 'Circulation Pump System Failure 2' event is further decomposed into 'E' and 'F'.

Yeah continue.

So, now, I will show you the third example, calculate the unavailability of this system and now we have to use the minimal cut and minimal path representation. So, again this is the fault tree. So, how to calculate the minimal cut and minimal path?

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**Minimal cut generation: MOCUS algorithm**

The slide shows the same fault tree as in the previous slide. It illustrates the MOCUS algorithm for generating minimal cuts. A matrix is shown with rows representing the system failure (Y) and columns representing the events (A, B, C, D, E, F, G). The matrix is filled with 1s and 0s, indicating the presence or absence of each event in each cut. The minimal cuts are identified as {A}, {B,C}, {D}, {E,F}, and {G}. The formula for the system unavailability is given as:

$$Q_s(t) = 1 - \prod_{i=1}^n (1 - K_i(Y))$$

where  $K_i(Y)$  is the probability of the system failure given the occurrence of event  $i$ . The formula is also written as:

$$Q_s(t) = 1 - (1 - K_1(Y)) (1 - K_2(Y)) (1 - K_3(Y)) (1 - K_4(Y)) (1 - K_5(Y))$$

The minimal cuts are listed as: {A}, {B,C}, {D}, {E,F}, {G}.

So, you can see that this is my fault tree. So, for getting the minimal cut of this fault tree, we have to use the MOCUS algorithm. So, what is MOCUS algorithm? Suppose this is my matrix, 2 3 ok. So, the input to this OR gate, are A then 1, then D then, 2 then G. So,

as this is OR gate and when we go for minimal cut it will increase the number of rows. So, we will write here A first input then 1002C second input is D, third input is 2 fourth input is G, now from this we can get another matrix, that a will be alone then for A is the 1 for 1 that is the AND gate and the input is B and C.

And we know for minimal cut generation using MOCUS algorithm for AND gate, we have to increase the number of column. So, as this is 1 instead of 1, we will write B and C then, again for 2 instead of 2, we will write E and F then G. So, ultimately my cut sets are a then BC D, EF G, A BC, D EF and G. So now, the cut structure if we write in terms of indicator variable, we can write that the first cut structure is nothing, but YA second cut structure is nothing, but YB into YC then, this is YD this will be YE into YF this will be YG.

So, now if we go for the structure function, we can write easily that as this is OR gate, this all the total cut structure will be in like this, like  $1 - 1 - K_1 Y$  into  $1 - K_2 Y$  into  $1 - K_3 Y$  into  $1 - K_4 Y$  into  $1 - K_6 Y$ . So, in  $K_1 Y$ , we will write YA in  $K_2 Y$ , we will write YB into YC. So, like that we will get this structure function.

And now I have already shown that how to calculate unavailability from this structure function. So, you just have to take when you will calculate unavailability, you just have to take the expected value of structure function and you know all the probability that is nothing, but the expected value of this indicator variable related to that basic event. So, all the probability you can calculate and you can get the unavailability.

The only thing we have to see that all the thing that,  $1 - Y_A$ ,  $1 - Y_B$ ,  $Y_C$   $1 - Y_B$ ,  $1 - Y_E Y_F$ , all those things must be independent to each other, if those are not independent to each other means, some variable, some indicator variable appears more than once then, you have to go to at last level you have to dig down fully so, that you can avoid the error that you have seen in the last lecture.

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**Minimal path generation: MOCUS algorithm**

$\rho_1(Y) = 1 - [1 - Y_A][1 - Y_B][1 - Y_D][1 - Y_E][1 - Y_G]$   
 $\rho_2(Y) = 1 - [1 - Y_A][1 - Y_C][1 - Y_D][1 - Y_E][1 - Y_G]$   
 $\rho_3(Y) = 1 - [1 - Y_A][1 - Y_B][1 - Y_D][1 - Y_F][1 - Y_G]$   
 $\rho_4(Y) = 1 - [1 - Y_A][1 - Y_C][1 - Y_D][1 - Y_F][1 - Y_G]$

$\psi(Y) = \rho_1 \rho_2 \rho_3 \rho_4$

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So now, we will go for minimal cut. So, you have seen the minimal path. Now, I will go for minimal path. So, in minimal path for calculating the minimal path set there are again 2 option, first option is you have to just convert OR gate in to AND gate and AND gate into OR gate and then you just take the same way that, you have done in minimal cut representation means, you have to find out the minimal cut of this conversion fault tree.

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**Minimal path generation: MOCUS algorithm**

$\rho_1(Y) = 1 - [1 - Y_A][1 - Y_B][1 - Y_D][1 - Y_E][1 - Y_G]$   
 $\rho_2(Y) = 1 - [1 - Y_A][1 - Y_C][1 - Y_D][1 - Y_E][1 - Y_G]$   
 $\rho_3(Y) = 1 - [1 - Y_A][1 - Y_B][1 - Y_D][1 - Y_F][1 - Y_G]$   
 $\rho_4(Y) = 1 - [1 - Y_A][1 - Y_C][1 - Y_D][1 - Y_F][1 - Y_G]$

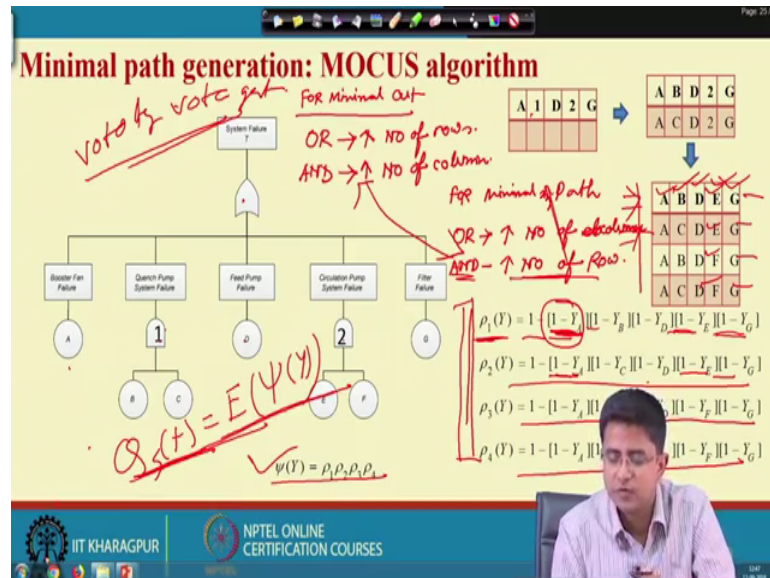
$\psi(Y) = \rho_1 \rho_2 \rho_3 \rho_4$

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So, when you will put here AND gate suppose, there is 1 AND gate and now OR gate and you take the minimal cut representation of this fault tree, convert converted fault tree, then you will get the minimal path, but I will show another the same MOCUS algorithm, during minimal path generation. So, we will not convert our fault tree, it will

be as it is. Now the inputs are, A this is 1, this is D this is 2, this is G. So, as this is OR gate, when we are going for minimal cut, we are increasing the number of rows, but when we will go for minimal path, we will increase the number of column, when there will be OR gate. So, for OR gate increase the number of column, but for AND gate in increase the number of rows.

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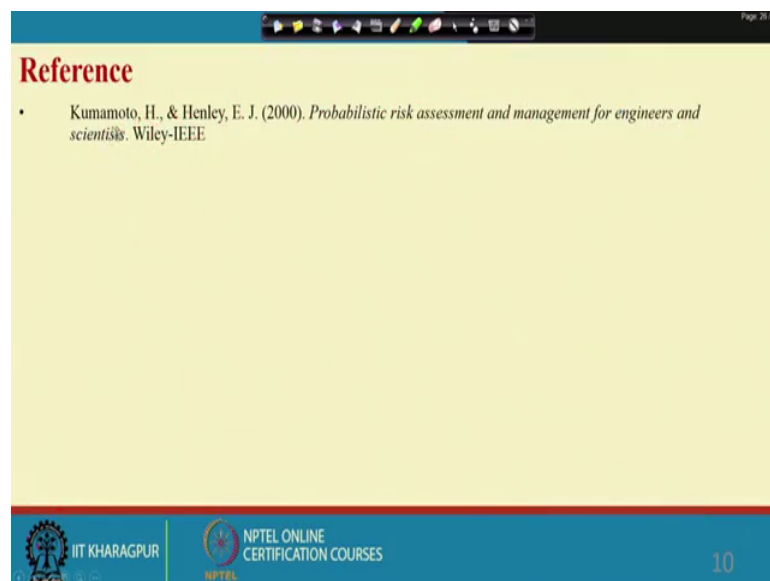
So, this is the difference between path representation and cut representation. So, we can write for minimal cut, if there is on OR gate increase number of rows and if there is AND gate increase number of column and for minimal path, if there is OR gate, increase number of column and if there is AND gate, increase number of rows, just inverse of it. So, we can see that as it is OR gate and the input are, A this is 1, this is D 2 and G. So, we will write in this manner. So, we have increased the number of column again, now this is 1 and the 1 is nothing, but AND gate. So now, for AND gate, we have to increase the rows. So, for 1 we have written A, AB and C.

So, you have increase, the rows then again for 2, we have written E F E F. So, in this manner you will get the path set. So, these are all path set. So, when you will get the path set, you can write that your row 1 this will nothing, but 1 minus 1 minus YA, 1 minus YB for every path set, 1 minus YD 1 minus YE 1 minus YG. Then for second path set, you write this for third path set, you write this for fourth path set, you write this then, you have to calculate the structure function.

How? Multiplication of all the structure path structure, when you will get this value, you can just for calculating unavailability, you just take the expected value of this and you will get the value, but one thing you have to notice here, that here in the path structure the term that is 1 minus YA in the second path structure again, 1 minus YA is coming, 1 minus YE is also coming, 1 minus YG is also coming means, all this value appears more than once in cut structure.

So, as these are appears more than once, you have to dig down at the last position for 2, otherwise you will fall in mistake. That you have seen in the last lecture that, when you are calculating the vote by vote method, vote by vote gate then, you have got that there are 2 unavailability value, because of this term appearing more than once in everywhere. So, for that you calculate and you dig down more. So, after that you put all the expected value of all the basic event then you calculate the unavailability.

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So, I think this lecture, this session, tutorial session basically will help you for your upcoming examination. And I can show you that this all the example, we have taken from this book Kumamoto and Henley Probabilistic risk assessment.

Thank you.