Industrial Safety Engineering
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Lecture - 39
Systems Safety Quantification : Structure Function

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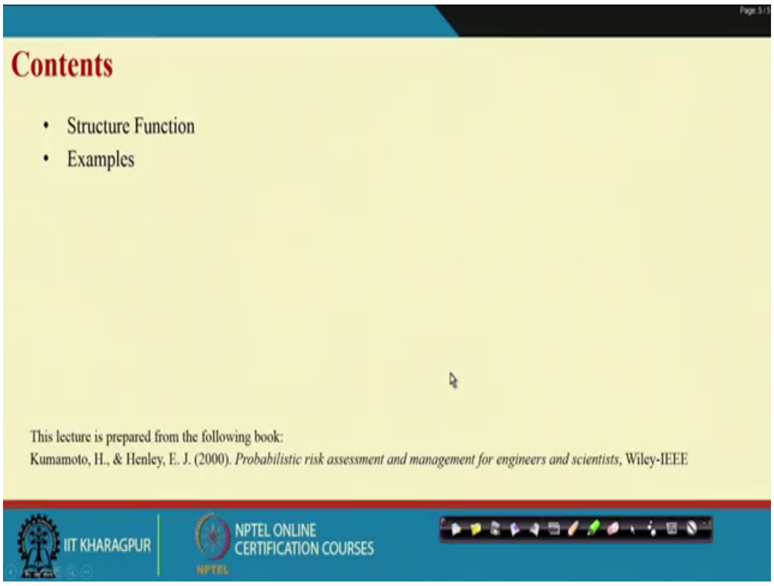

Hello everybody, good day today. We will discuss Structure Function; the topic is System Safety Quantification by Structure Function Approach. So, we will discuss the basic of structure function, and then several examples I will put forth, so that you will understand the structure function approach. And the lecture is prepared from the book Probabilistic risk assessment and a management for engineer and scientist, Kumamoto and Henley.

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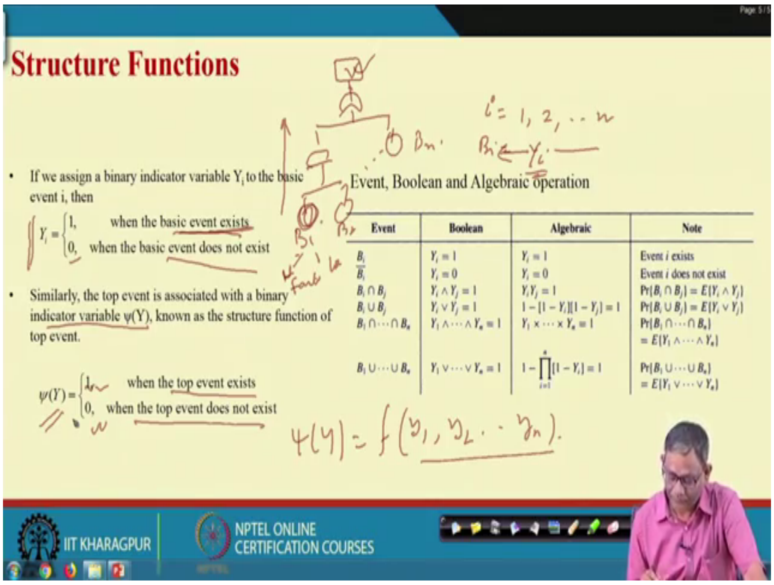

So, what is structure function? So far what have you seen? You have seen fault tree, and event tree, and then in fault tree, fault tree cut sets. Then we have shown you different quantification in terms of basic events, and then we started the safety system safety quantification, where we have defined that the system is available, if the top event does not exist; top event in fault tree.

So, then we have shown you that how to compute the system safety, particularly from fault tree quantification point of view, where the top every fault tree has a top event, and then probability of top event. If it is a failure event, then that is the unavailability of the system. And if the top event is a success event that we will ultimately tells you the availability of the system. And we also consider one top event at a time, not several top events.

Then and then I have we have seen that how reliability block diagram can be used for quantification. And we have seen that there are several methods for quantification, one of the methods is a structure functions, another one RBD - Reliability Block Diagram you already given to you, another one is the truth table that is also you have seen in my last lecture.

So, today our discussion will be on use of structure function in quantifying the system safety, particularly in terms of fault tree or success tree. When you go for cut set, we say fault tree; when you go for path set, we will basically talk about the that is the success tree. And today I will discuss that how the structure function will be defined, and it will be used with several examples.

So, essentially the what is the definition of structure function, you have seen in any fault tree, there is a top event and then there will be several bottom events several bottom events. So, suppose if we say this is B 1 and this is B 2 in this manner, so this is B n, there are n number of bottom events or basic events. We started thinking i equal to that 1, 2, n ok, so that mean n number of basic event, so then the B i is basically the ith basic event. So, we will relate a variable called Y i with reference to B i the basic event i in such a manner that this relations hold. What is the relation here, Y i equal to 1, if the basic event exists.

So, suppose this is basic event B 1, so B 1 either it is unavailable or available unavailable means, it is failed or it is working. So, now basic event exists means we are defining basic event let the failure of that particular component or that is what is the basic event and then basic event exists means, this particular ith component is actual state or 0 when the basic event does not exist means, ith component is working. So, but if you think that you will be used other way round that it is success or failure that 0 and 1, so that is also possible. But, for in terms of fault tree, we have defined the basic event as failure event.

So, now if I if I say Y i is the Y i is basically 1 or 0 depending on the basic event exists or does not exist and in the fault tree format, all those basic events basically leading to the top event. So, we will create another function for top event, which is indicator variable psi Y, so that is what is the psi indicator variable psi Y, psi Y talks about the top event point of view. So, top event if it occur exist, then it is 1; if top event does not exist, it is 0.

So, then essentially what happen in structure function, you have defined two indicator variables two types of indicator variables, one for basic event, another one for top event. Now, for basic event, the indicator variable is Y I, it takes value of 1, if the ith basic event exists, it otherwise it takes the value 0. And similarly, psi Y is another indicator variable, which takes value of 1, if the top event exist, otherwise 0. So, this is what is basically the definition.

Now, our sole purpose is that psi Y, psi Y the top event will be a function of basic events, so Y 1, Y 2, Y n function of basic events. So, this is what is our structure function, so that means, psi Y is a function of that a top event indicator variable is a function of basic event indicator variable that is our function. Now, using this, we will be able to find out the system unavailability, availability, and that is what is the primary concern in case of computation system safety computation.

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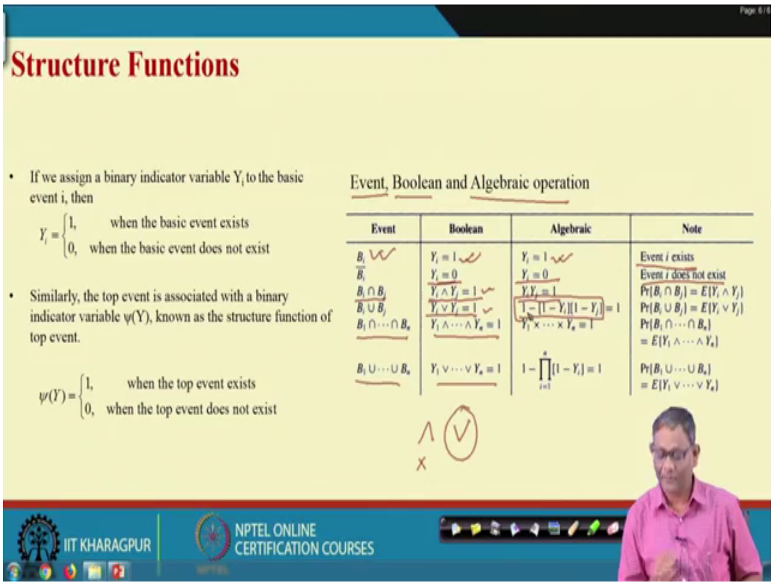

Now, you require little bit of little bit of that Boolean algebra and a Event Boolean form and algebraic operations that is knowledge you require. We say B i is basically the basic event ith basic event. And in terms of Boolean, suppose basic event exists, means Y i equal to 1. And in algebraic form also, we write Y i equal to 1 that means, note is that remark is that event i exist. When you write B i bar, we have seen earlier, so that means, event does not exist, so Y i is 0, and in algebraic form Y i also 0, so event does not exist.

Now, when we talk about intersection B i intersection B j, this is basically then we are basically talking about that Y i intersection is equal to 1 that mean that intersection exists kind of things. Similarly, union that also we in Boolean, we are writing that is 1. And when there are n number of basic events, so n n this n indicator variable will be interacting, so that interaction is 1. And when it is union, that union is also 1.

Now, in algebraic form what we are doing, the symbol you are what you are writing in Boolean, Boolean you are writing intersection in this manner union in this manner in this symbol. But, when you are writing in a algebraic form algebraic form for intersection, it is basically multiplication Y i Y j. But, when it is union form that is important, so Y i union Y j equal to 1, it means in case algebraic form this is the issue ok.

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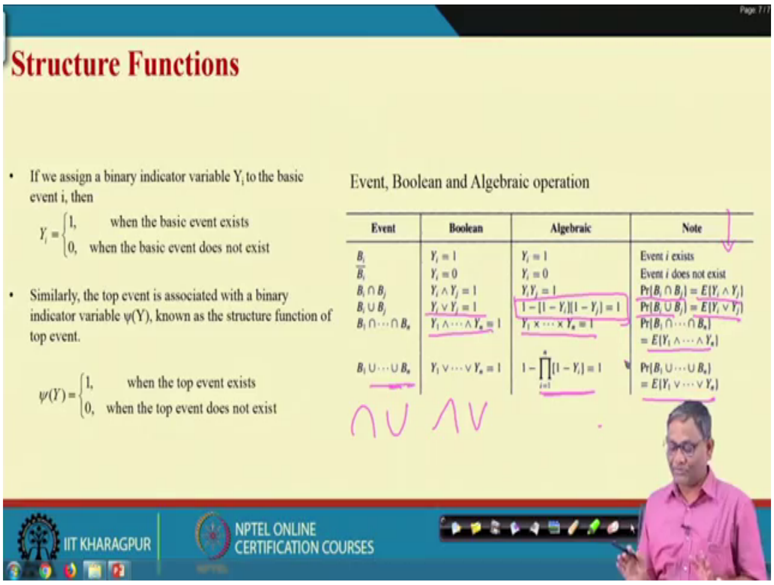

So, please understand this is my Yi union Y j equal to 1 in Boolean, but in algebraic form is this, so 1 minus 1 minus Y i and 1 minus Y j ok. So, when you are writing again interaction, you see that you are just multiplying. Then when there are n number of items, you see that here 1 minus Y i and 1 minus Y j multiplied, so here n times this will be multiplied 1 minus this. And how it is happening that you also know, because earlier we have discussed similar things.

Then what is the actual physical meaning is that, so from probability point of view, when we use indicator variable, then the probability value suppose probability B i intersection B j is nothing but expected value of Y i intersection Y j. Similarly, for union case expected value of these, similarly when there are n number of item interacting these or union of this. So, ok. So, this is what is basically you have to understand that when you are writing events, then you are writing intersection in this manner union in this symbol. You when you are writing in terms of Boolean, intersection this union this.

And when you are writing in terms of algebraic form, then you are using for intersection all indicators variable are multiplied. But, when there is union, then you are right the first finding out that complementary of this 1 minus and that 1 minus Y I, and then multiplying all those things and then finally, all that multiplied terms complete term is subtracted from 1. So, this is the way of representation further at different kind of situations. So, please keep in mind.

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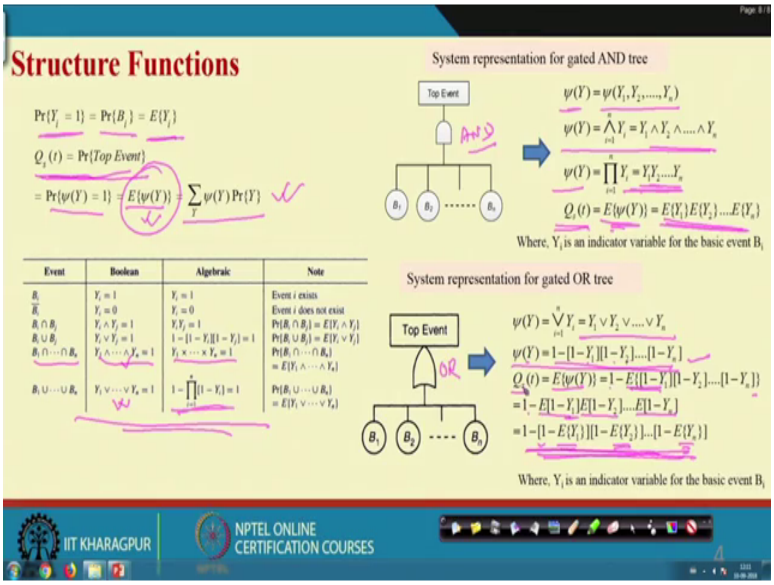

Now, with this simple the representation, then we will go to the next slide, where our aim is basically that we will we will show you that in case of AND gate and OR gate how the structure function works. I already told you that probability of Y i equal to 1 that is nothing but probability that a B i event B i b B i exist, this is nothing but expected value of Y i, where Y i is a indicator variable.

Now, then what will be the probability of top event, which is basically system unavailability, then it will be probability that psi I equal to 1, which is the top event indicator variable, and that is nothing but expected value of psi I   psi Y, and this is nothing but sum of psi Y and probability Y. So, this is this is another important expression for you.

So, what you have done here, you have created Y i indicator variable, so you are considering all Y i. And then you have also found and created indicator variable for the top event, so then ultimately what happened, you are basically finding out that multiplication of psi Y and probability of that Y, and then summing that all for all the all the events I all Y that is Y..

And this is what is our formulation will infirm and algebraic formulation we have given you. So, with these two, first we will apply this table, and see what happened to what happened to how do you compute psi Y. And then using this expected value, we will show you, how you will compute the unavailability.

So, let me tell you or you all know that as it is AND gate, so psi Y is a function of psi Y 1 to Y n, it is basically this top event is a function of all the basic events. Now, this one because of AND gate, we can you if you use Boolean, you can write like this. You see because of AND gate, AND gate is this 1. So, what you are writing, this you are writing, so this is nothing but intersection i equal to 1 to n Y i which is this, so we are writing like this. Then this one, if you write in terms of algebraic form, so Y 1 to Y n you see the same way we were writing.

Now, then what is the unavailability, unavailability Q s t is expected value of psi Y, so you are writing expected value of psi Y here. Now, expected value of psi Y means expected value of Y 1 into Y 2 in up to Y n. As Y 1, Y 2, Y n are the basic event indicator variable, so they are and they are independent, so they are their expected value of joint will be, but I actually expected value will be individual expected values multiplied. So, expected value of the multiplication of all the indicator variables will be equal to that expected values of each of the variables, and then multiply the same.

Now, if your gate is OR gate, this one was AND gate, gate is OR gate, so what will happen, you see this is the Boolean form, so you are first writing in the Boolean form. And then if you want to write algebraic form, so algebraic form is this. So, 1 minus i equal to 1 to n 1 minus Y i that is what we are writ[e] written 1 minus 1 minus Y 1 1 minus Y 2 like 1 minus Y n.

Then Q s t will be expected value of these, means expected value of the entire term. And then expected value of 1 is 1, now expected value of all those thing as Y 1 to Y n are independent, then 1 minus Y 1 to and 1 minus Y 2 they are also independent. So, as a result, you are what you are doing expected value, you are multiplying expected each of the expected values. Now, again you can enter this into the expected values into the bracket, so expected value of 1 is 1, and then minus expected value of Y 1, so this is the form ok.

So, wherever you get AND gate, expected value you will basically multiplied the terms and expected value will be expected value of multiplication of all the indicator variables. And whenever you are having OR gate, this is the algebraic expression for OR gate, this one. And then this is the expected value of Q s t, so ok. So, this is basically just representation in terms of in terms of basic event and the top event given in different a logic gates. And we may go for the complicated little complicated tree now, and then let us see what will happen there.

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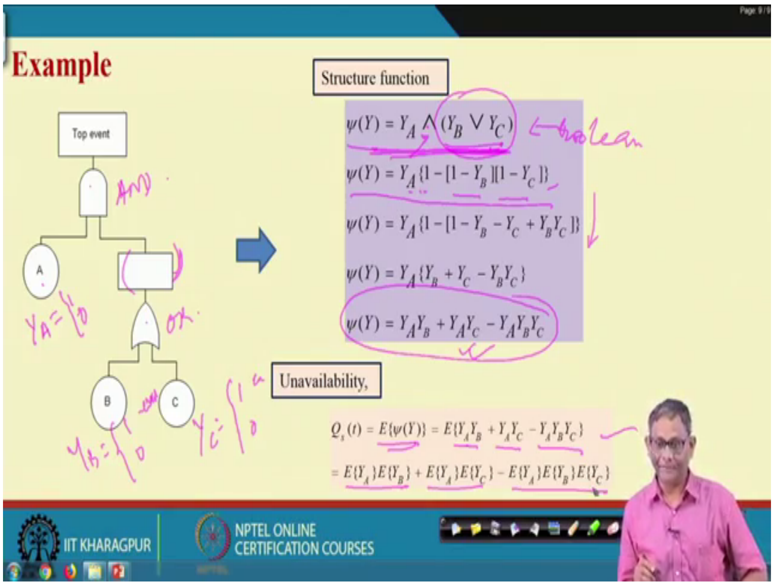

So, now instead of just one OR or AND gate, we have here little complex, simple in the sense complex in terms of the earlier trees what I have shown you here. So, now what happened here, we have one AND gate and one OR gate, and there are three basic event A, B, C. So, what will be the structure function psi Y? So, these AND gate is there, so intersection we are using. Now, this quantity you require to find out, so this quantity is nothing but because of OR gate, so Y B and Y C. So, what this Y A, Y B, Y C. Y A is indicator variable for this one; if A exist, otherwise 0. Y B is indicator variable 1 0. And Y C is again indicator variable 1 0; if it exists if it exists, it is 1, otherwise not.

So, then if I want to find out with this OR gate you earlier seen, the inputs will be union. And for AND gate, it is intersection. So, your structure function in terms of Boolean, you are writing like this. Now, you can write in terms of algebraic form, so first you write algebraic form of this, this intersection this 1 minus 1 minus Y B and 1 minus Y C. And this is for this now intersection is there. So, Y A multiplied by this. So, now if you just simplify this one, you will you are doing some more algebra. So, you will be finally, getting this equation, so Y A Y B plus Y A Y C plus Y A Y B Y C ok, so that means, this is my structure function for this kind of fault tree.

And then unavailability will be expected value of so, Q s t is expected value of this. So, expected value of Y A Y B, expected value of y Y A Y C, expected value of Y A Y B Y C this three, this will be summed up, and this will be added, and this will be subtracted. So, now Y A Y B again because of independent nature that will be multiplied and this. So, this is your structure function for this fault tree.

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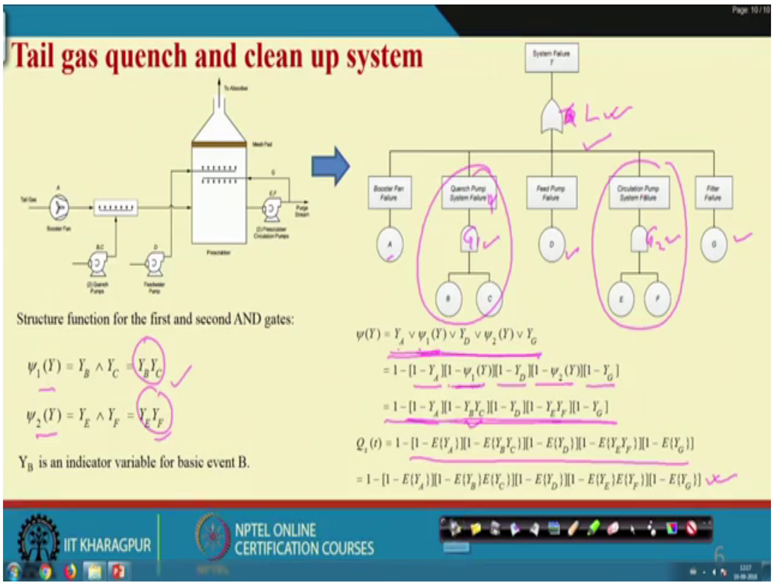

Now, we will see that another real example, which I have already given you in last class, there we have say this is basically a Tail gas quench and cleans is clean up system, there are several component. And there are three objectives, first one is that your a reduction of temperature, and then your basically quench you that through quenching, then your basically moisturization and finally, particles to be to be removed. So, all those things combined together.

We have seen the fault tree earlier; this is our fault tree ok. So, what do you require, you require to find out the structure function of this. So, given this fault tree or given this system, you have to develop the fault tree. And then from the fault tree, you find out the structure function. And once we have the structure function, finds find out the unavailability part.

So, let us see yes this one. So, this fault tree, you have seen in my last class. Now, what is happening here, this is an OR gate. So, Y A this suppose if I say this is my G 1 gate ok, this is G gate, this is G 1, and this is G 2 gate, so that means, enter this union G 1 and union this, union this, and union this ok. This is G, so you will get this one is another gate, suppose L gate let it be forget about this.

So, then what happened, here we have written 1 structure function for this small one. Another structure small one here, which we have defined here, so that means, in this tree, what happened apart from this OR gate, there are this is basically the tree has two AND gate this and this, first we have found out the structure function from these two and gates. So, psi 1 and psi two these are this one. So, if I know psi 1 and psi 2, then AB A, D and G another basic events are known. Using this OR gate, you can find out the Boolean part like this, so this union, this union, this all union.

Now, when union is there, so this will be 1 minus all union 1 minus this, 1 minus psi 1 Y 1 minus D 1 minus psi 2 Y and 1 minus Y G ok. So, this is the formula you have seen earlier. Now, then 1 minus 1 minus Y A, now this one 1 minus psi 1 Y, this structure function its value is Y B Y C, we will put here. Similarly, other one is Y E Y F, we put here. So, this is my structure function psi Y equal to this. Then for Q s t what you will do, Q s t is a expected value of this. So, expected value of this once you put, finally you will end up into this equation. So, this example we have shown that how the structure function will be use in some real cases ok.

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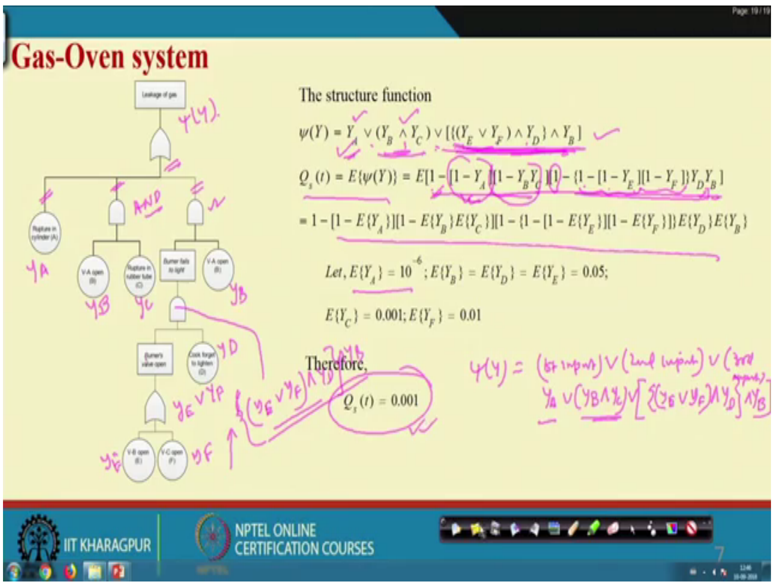

Now, let us see another example Gas-Oven system. Here you see there are many basic events. So, what we will do, we let us let us write let us write. Suppose, this is y A indicator variable, for this y B, for this y C, for this again y B, because this is this is basically valve A open, this is valve A open. Now, this one y E, and this is y F, so that means, y A equal to 1 means rupture takes place, 0 means no rupture. Similarly, y F equal to 1 means valve C open, otherwise 0.

Now, we want to find out the structure function for this. So, so if you start from this top event that I want you want to find out the psi y here. And because of OR gate, and you have to first find out that how many inputs are there; first inputs, second inputs, and third inputs. So, if that means, what you will do, your psi y here equal to that first input first input this because of OR gate union, second input, again union third input ok. So, very easily you see the first input is y i y A, so you write this one, then union you are writing.

What is the second input, second input is a AND gate with two event y B and y C two indicator variables of B and C events. Because of AND gate, you use intersection, so that means, this is y B intersection y C, then again your union is there. But, for the third inputs, you see so many things are there. So, what do you do for the third input, you start from the bottom. Then here OR gate, so y E and y F. So, with OR gate, we can write y E union y F.

So, this is now here what happened, it is AND gate. So, with inputs this and this, so this is our y D. So, we can write for this y E union y F because of AND gate intersection y D. Further again here is another AND gate, so that entirety therefore, the third input will be again intersection y B. So, what you are writing that is why, you are writing y E union Y F intersection y D, then again intersection y B this one. So, then y A you see Y A union Y B Y C intersection then union this portion Y E union Y F intersection Y D intersection Y B ok. So, this is our structure function.

So, once you know the structure function, then you know how the expected value to be computed. So, what will be the case, expected value of this suppose if I go for the algebraic form, I am not written the algebraic form here, and then one for expected value. So, algebraic form is this one in between what is given here. So, because of union, so what you were writing 1 minus, then what is the first term, Y A, so 1 minus Y A product second one; so, 1 minus Y B Y C because of this intersection formula. So, this part is over. So, this one taking care of this, this one taking care of this.

Now, third intersection the here in between again union intersection is there for third union should be for this one 1, we are writing minus we are taking care of inside. It is basically for the first one is this 1 minus 1 minus Y E into 1 minus Y F. Then this intersection this, so multiplied by this, again intersection if this multiplied by this, so 1 minus this whole that is the algebraic form. Now, you when you are finding out Q s t, you are taking expected value of this algebraic form. So, you are writing like this ok. And then as the expected values are nothing but the probability values. So, when you put all those things, finally you are getting Q s t is 0.001.

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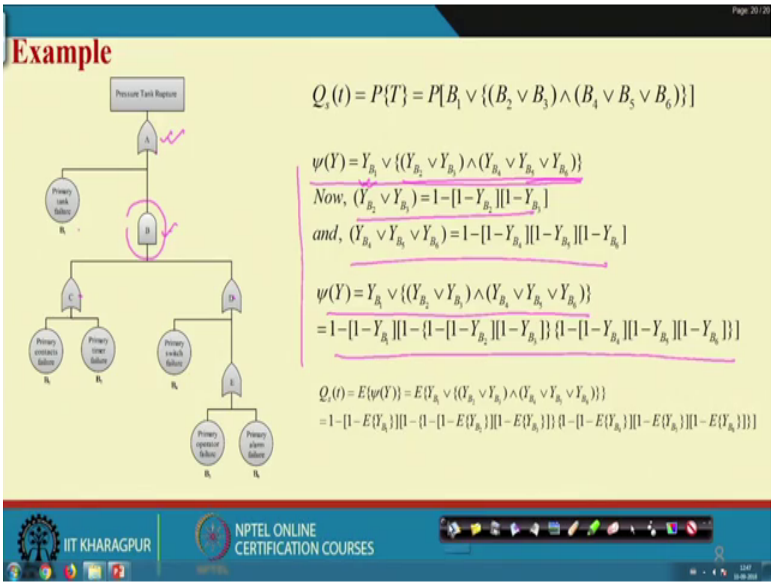

So, another example here pressure tank rupture. This fault tree you have seen earlier. And in the same manner, we have basically come found out the first you find out the structure function structure function is this one. So, how many events, six different events are there. So, put the structure function values. So, or this is basically OR gate with two inputs, so that is Y B 1 and entire this one as inputs. So, now this one is an AND gate with two or inputs, so that mean that in two or B inputs we have written like this.

And finally, we have we have basically integrated all those things, and we got this structure function ok. So, I hope you will be able to develop this structure function, because the similar way earlier I have shown you. Then Q s t you have to take the expected values ok. Now, let us see that if we put the basic event probability values, then using and use gate by gate method, find out the top event probability. And using structure function, find the top event probability is it matching or not.

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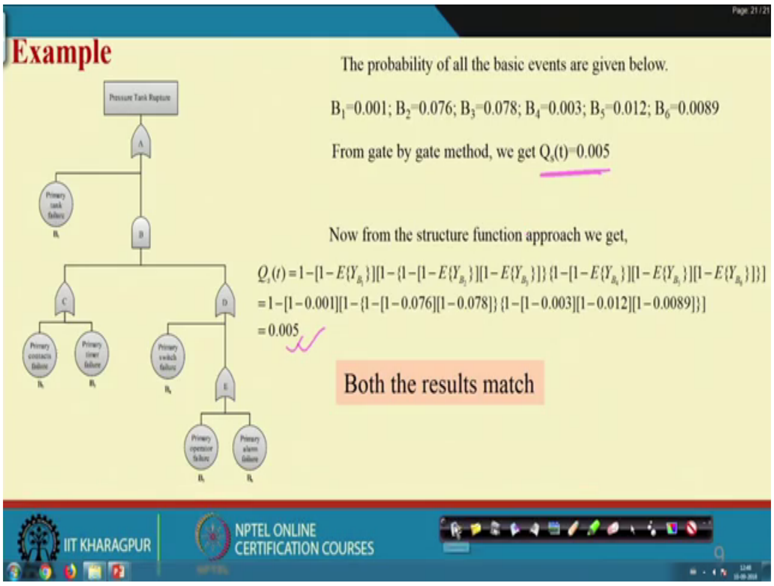

You see using gate by gate method, when we computed we found about this. Now, using structure function expected value is the probability value, we are also finding on the same probability. So, it will match, when it is a small fault tree. We seen about similar kind of fault tree, you can develop and compare the things. But, for large fault tree, it will be one actually we should we should go for programs.

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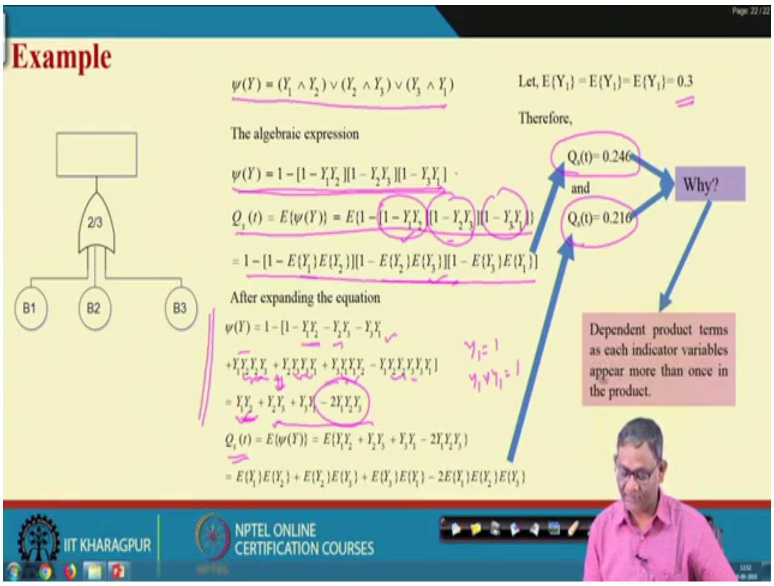

Then I will show you another interesting one, here little difference you will get. The difference not in terms of that example point of view, but use of a expected value point of view that when you should use the expected value. Should you use in between without decomposing the structure function into the lowest possible level or in between if you use expected value, what will have, what are the problem you will encounter.

So, here you see it is a boating gate case. And for this boating gate case, you can very easily define the structure function. And now, the structure once you know once you know the algebraic form, the structure function is known. Then Q s t you can use the expected value, and in this manner al[so] if you use the expected value of this, then put expected value inside and then finally, calculate the Q s t, then you are getting Q s t value point 0.246.

So, here what happened, actually the problem we will see later, but here you please understand that the product there are the product terms 1 minus Y 1 Y 2 1 minus Y 2 Y 3 and 1 minus Y 3 Y 1. So, there are three product terms. So, Y 1 this one, this one, and this one, where at least in two of the terms each of the indicator variables appearing. So, even though Y 1, Y 2, Y 3 or the basic events are independent here, but the product term 1 minus these and 1 minus these, they are not independent, because they have the common variables indicator variables.

So, now if you use Q s t expected value here inside, then you are basically committing a mistake, and that mistake is explained here. What is this? Instead of putting expected value earlier, if you further decompose this, and you will find out interesting thing is that in the first term Y 2 Y 2, in the second term Y 3 Y 3, third term Y 1 Y 1, and the fourth one Y 2 Y 2 Y 1 Y 1Y 3 Y 3. As Y 1 is 1, when the event exist; then Y 1 multiplied Y 1, it is actually 1.

So, as a result in Boolean case, so you have to write this Y 1 Y 2 Y 3 and Y 1 Y 2 like this, so you have to write like this ok. So, 1 minus 1 cancel out, Y 1 Y 2, Y 1 Y 2, Y 2 Y 3, Y 2 Y 3, Y 3 Y 3 Y 1. And this one, when you when you use this concept, and then you find out that this is being basically reduced to minus 2 Y 1 Y 2 Y 3.

So, now if you put expected value here that mean, you are basically using 0.3 and multiplying there. Here two times you are multiplying 0.3, so it is not 1 into 1 is 1, it is 0.3 into 0.3 is 0.09. So, it is making the problem creating the problem, so that is why what happened when the product terms they are not independent argumently, if you use expected value formula earlier without decomposing the structure function to the lowest possible level, you will commit mistake.

And now, what happened after decomposing to this level, when we are calculating Q s t that is a unavailability using the expected value, we are getting this value 0.216. So, we have written why this is happening, because dependent product terms as each indicator variables appear more than once in the product, I have already told you. Please be careful for this kind of fault tree, when you are using structure function and then the expected values you are using, so you should use the expected value at the lowest possible level not in between.

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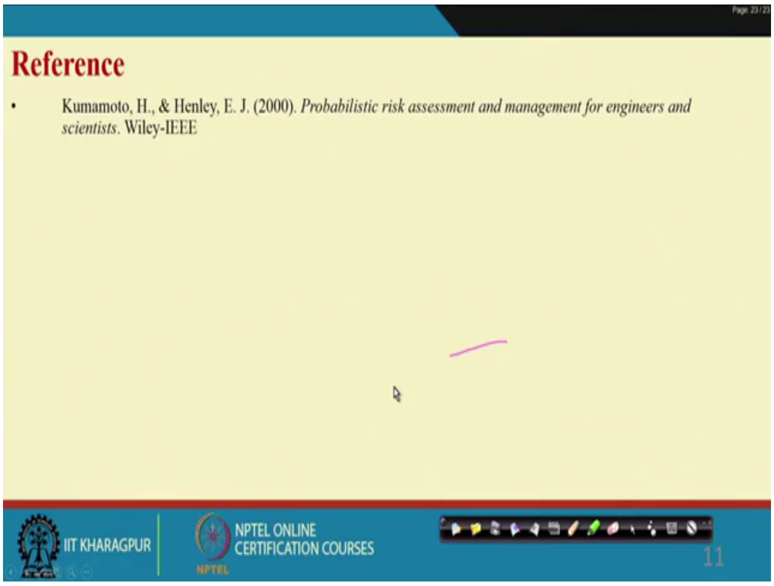

So, thank you very much, I hope that you have understood, good day.

Thank you.