Industrial Safety Engineering
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Lecture – 37
Systems Safety Quantification: Truth Table Approach (Contd.)

Hello everybody. Let us start Truth Table Approach. Today's topic is System Safety Quantification Truth Table Approach.

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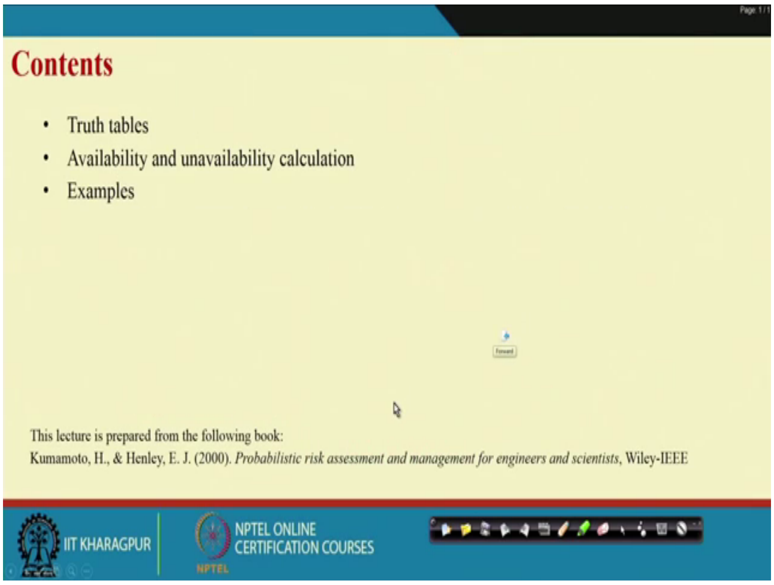

The contents of today's presentation, we first described truth tables with different gates and gate or gate and mixture of and and or gate and as well as, we will show you that when the traditional approach is not working. And under such situation, how truth table will help us to compute the availability and unavailability of a system and we will show also some of the examples.

And this lecture is prepared from the book Probabilistic risk assessment and management for engineer and scientist, written by Kumamoto and Henley 2000 published by Wiley.

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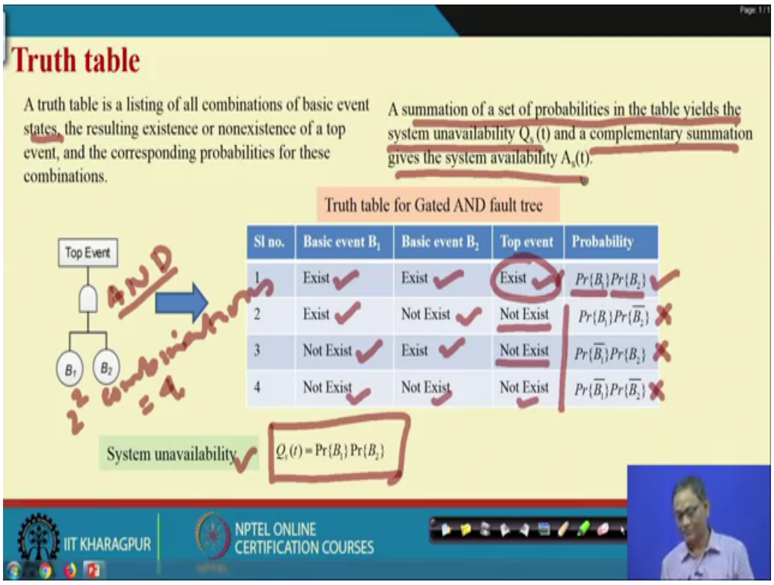

Let us first understand; what is truth table? Truth table actually list all combinations, all combinations of basic events states. Then we will find out for which combination the top event exists and which combination top event does not exist. For example, if we consider a fault tree, having two inputs that mean two basic events and top event is linked with an and gate with an and gate then, what are how many states or combinations possibles here.

So, from the components point of view, either the component failure exist or not exist, so as a result because of two components, so we will be having 2 to the power 2 combinations, which is 4 combinations. So, the 4 combinations, first one is basic event B 1 exist, B 2 exist, so if both B 1 and B 2 occur then, top event occur, so top even exist. Then what is the probability, probability of basic event B 1 exists is probability B 1 and B 2 exist is probability B 2. So, as both the events are independent and linked with and gate top event exists probability will be probability B 1 into probability B 2.

So, this is a combination or a system state, where the top event exists means top event system is unavailable. Then, second one, the suppose B 1 exists, B 2 not exist because of and gate top event does not occur, so system is available. So, this is this will not contribute to unavailability calculation. Third one that first B 1 not exist means it is working, B 2 not exist mean not working then also top event will be no exist. So that mean top system is not unsuccessful. In the fourth one both B 1 and B 2 are working. So, system also will work, so this will also not contribute to unavailability calculation.

So, as a result what is the unavailability here, the probability of top event exists, this is one situation, there is a first combination when both B 1 and B 2 exist. And then, the resultant probability is probability B 1 into probability B 2 as B 1 and B 2 are independent events. So, as a result the system unavailability Q s t is known by this equation Q s t is probability B 1 and probability B 2.

In fact, this thing you have done earlier also when we have given and gate fault tree then, we have we have seen that the top event is the intersection of the bottom events and the probability of top event is basically, that that multiplication of the bottom event probabilities when the basic events or bottom events are independent in nature.

So, then, this is the equivalent representation of and gate in terms of fault tree ok. So, then what is system unavailability, system unavailability is the probabilities that will a summation of set of probabilities in the table yields the system unavailability Q s t. So, where top event exists, that probability you have to add. Now, here only one situation so that is why this probability is considered, if we want to find out the unavailability which is 1 minus Q s t or the other situations probabilities sum of these 3.

So, that is why a complimentary summation gives the system unavailability. This is what is truth table. So, in truth table you find out all possible combinations and then you find out that for which of the combination, the top event exists means the system failure takes place, for which of the combination the top event does not exist mean the system failure will not takes place or basically, you can find out the combination with the system states; whether it is failing or not failing.

So, the system state when or the combination when the system fails that leads the probability of that combination lead to the unavailability and the systems and the combination for be the system state is successful that combination probability will lead to the system availability and some of the two will be obviously, 1.

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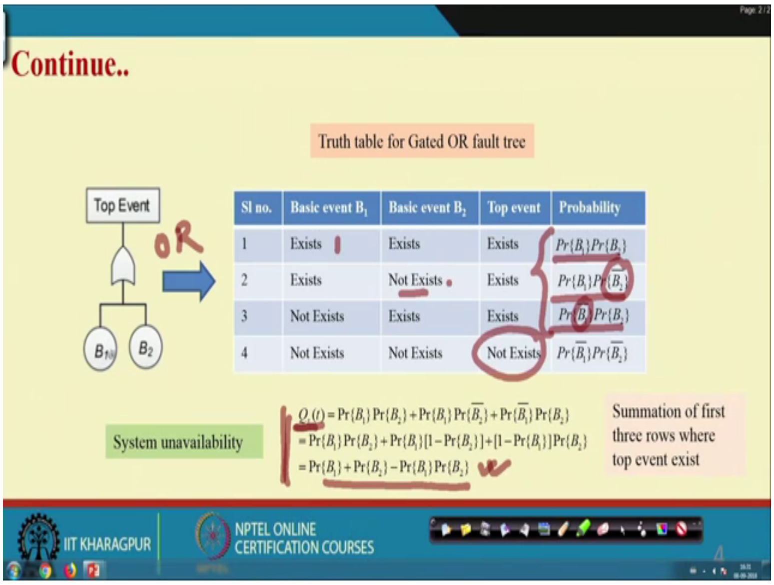

So, what will happen if we just see the equivalent truth table for or gate. When you use or gate, it basically talks about if B 1 occurs, top event occur B 2 occur top event occur B 1 B 2 both occur top event occur. So, that mean there will be 3 street combinations, when the system will be unavailable. Only one combination with both B 1 and B 2 does not occur, that time only system will be available, that is what is given here system not top event not exist means system is available.

So, then probability of first 3 where system top event exists; that means, system is unavailable, so you summed up the probability this one, this one and this one this is what is done here. So, if B 1 exists the probability is called of B 1, a B 2 exist probability of B 2, if then if not exists B 1 B 2 bar so that is why B 2 bar is here B 1 bar is there and then B 1 bar is nothing, but one minus B 1, so that combination ultimately lead to this probability.

Now, that already seen by you, means this equation you have already seen, if there is an or gate with 2 inputs, what is the probability of E probability of B 1 plus probability of B 2 minus probability of B 1 into probability B 2 that you have seen. So, truth table is also giving the same thing because, it is equivalent representation ok. So, this is or gate.

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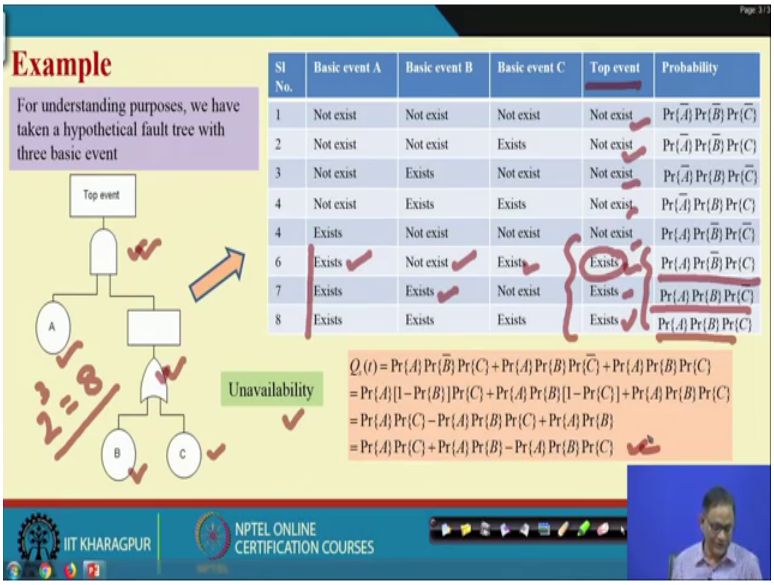

Now, using this principle we have given little complicated one, it is not exactly only and or, but combination of and and or gate. And here, how many basic events are there, there are 3 basic events we have written, A B and C otherwise, you can write B 1 B 2 and B 3. So, there are 3 basic events, so each having 2 states. So, there will be 8 2 to the power 3 equal to 8 combinations.

Now, because you know the system configuration and also and and or gate combinations, so finally, what will happen, you will find out some of the combinations when top event exist, some of the combination top event will not exist. For example, this is and gate, if this and gate then A must occur plus because, of this is or gate either B or C or both B and C must occur, so that, the top event will occur. So, that is what is a seen here, for example suppose, when all the bottom event does not do not exist, top event also do not exist.

Suppose the first one C exists other two not exist not exist. So, similarly, not exist not exist not exist. The combination when A exist means, A is failed, B does not fail, but C is failed then, definitely top event will occur. So, you see that 3 condition when A exist, this is a must because of and gate. Now either B or C or both must occur, here C exists that is a top event exists A and B exist that is why top event exist here both a and B and C exist that is why top event exist so, this is the combination.

Now, you know because of the two state components, so you know the probability. So, as they are independent, you are just multiplying those independent event probabilities and then, system is unavailable when you sum up all those probabilities ok, so this is what is truth table.

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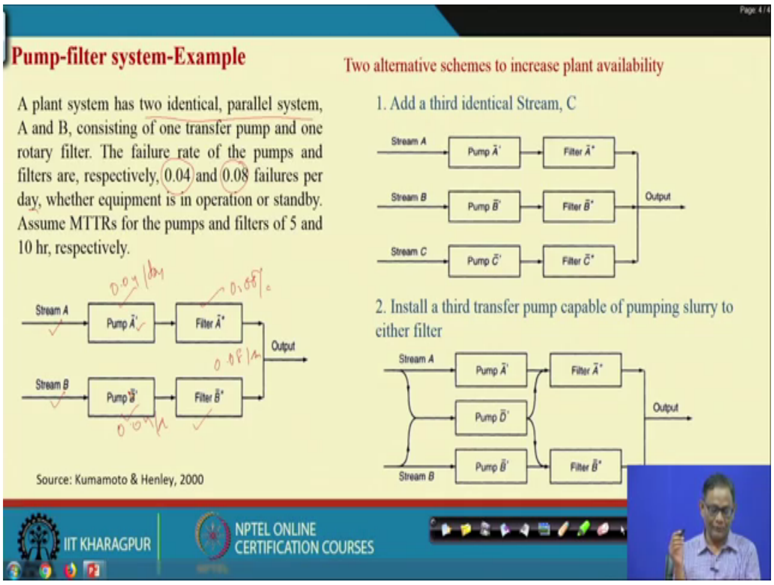

Now, we will see one interesting example here, this example we have taken from the book Kumamoto and Henley. Here the issue is, I am just reading out the description of the plant, plant has 2 identical parallel systems, so known as stream A and stream B. So, stream A and stream B they are identical, but parallel system. So, both the stream consists of 1 pump and 1 filter in series and two stream are in parallel ok.

So, now the failure rate of pumps, both the pumps pump A and pump B both the pumps are failure rate for both the pumps is 0.04 and both the filter is 0.08 per day. Failure rate here this one 0.04 per day and this is also 0.04 per day, 0.08 per day and 0.08 per day ok.

And you see that, another issue is that the if the pumps or the filters fail then, there they can be repaired and the repair time is also given that this is in terms of mean time to repair for pumps and filters 5 and 10 hours respectively. So, that MTTR for pumps is 5 hour and MTTR for filter is 10 hour ok. So, this is a system it is basically, combined system that failure and repair both the processes are involved.

Now, as plant engineer so your full output is possible, only when both the both the stream work. So, that mean the full output combination is both stream A and stream B that mean 2 parallel stream must work. Otherwise, if one of them work then that gives you half output and if none of them work gives you 0 output. Suppose you want to improve the system because, in order to get full output and where full output requires both the streams to work and then, it is quite likely that there the one of them may fail and under such situation there will be huge loss.

So, as a result the management wants to improve the system and they have 3 options now, one is keep the status quo means, the two parallel stream and proceed or add or C to alternate approaches, one is add a third identical stream so that means 1, 1 stream will be will be redundant or standby or you add a pump which is capable of pumping slurry to both the filters.

So, as a result what happened, if we include these 2 alternatives with the existing one then, we have three different choices. One is maintain the status quo, second one is add a third identical stream C and third one is add a third transfer pump which is capable of pumping slurry to both the filters. So, we want to see the availability and unavailability of each of the systems and then we want to know which one is based or which means which one we should continue so, that if the on some more cost related data are available with us.

So, with this view I am just explaining that, how the ultimately the traditional series and parallel system, as well as that minimum and and or gate system and then how the truth table approach will help us in finding out the unavailability availability, as well as helping you to compare three different choices. So, this is a example, but it can be incorporated or augmented for more number of choices also and that is what is the basis of this presentation with reference to pump filter system. I have taken this example and the resulting calculation from the book written by Kumamoto and Henley probabilistic risk assessment and management for engineers.

So, you have 3 systems now, this system this you have to.

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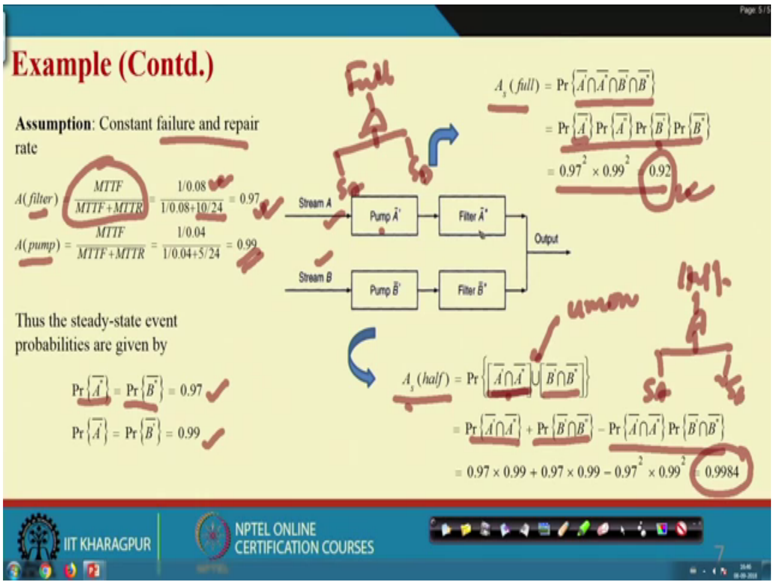

So, the first one, what is the assumption here that we are assuming that constant and failure and repair rate for both pump and filters. So, if this is the case then, you have seen earlier when we have discussed quantification of basic events that time we have shown you that availability is A can be measured by mean time to failure divided by mean time to failure and mean time to repair. This means with this equation availability is measure.

Now, what is the mean time to failure for filter. So, filter failure per day 0.08 that is the rate so then 1 by that rate will give you the mean time to failure and what about the mean time to repair. Repair is basically, 10 hours you have 24 hours a day, so that is why 10 by 24 and 0.08 is also that much that is the rate per day. So, we have converted everything in terms of day and then, you got the availability for filter each 0.97, please remember that both the filters are identical, so its availability also identical means, 0.97. In the same manner availability of pump is 0.99, each of the pump 0.99 ok.

So, probability of filters that since A that is a success is 0.97, probability of pump 0.99 that is what is computed. Now, what we want in our interested to know, when we will be getting the full production. So, we want to know availability for full. So, availability of full means both stream A and stream B should be successful. So, what is the probability of stream a successful when both pump and filter work, similarly for stream B both pump B and filter B should work.

So, here it is in series, so this and this pump A and filter A work, simultaneously that probability will give basically from the intersection point to be it will come. So, let me take a situation of like this that, A is dot bar interaction A double dot double dot bar intersection of this B and B double dot that is it in here. So, A is fully probability of this, so now, you know all those events are independent, so you are multiplying all the probabilities and you are getting this.

Now, what is the probability that of A dash bar this is nothing, but the availability of pump which is availability of pump is 0.99 and availability of filter is 0.97. So, filter is this one point A star pump 0.99 and B star, that is also pump 0.99, so that is this one ok. So 0.99 square and 0.77 A 0.97 square that when you multiply the 2 you are getting 0.92. So, what is the inference from this example, inference is that, that for the probability that the system will be available for full production is 0.92.

Now, what will be the case for half production, if one of the two streams work then half production. So, that have been either, then now one of the two stream work means, the either the stream A works or stream B works. So, stream A work is A dash bar intersection A double dash bar, stream B works means B dash bar and B double dash bar intersection. Now one of them work that is why, the union is given here union. So, it is some kind of or gate. So, for full production it is an and gate that full then, and gate, then stream A stream B and here half or gate stream A and stream B ok.

So, that is why what happened now, this is stream A and this one is stream B, so probability stream A plus probability stream B minus the intersection of the 2 So, as a result, the that availability half for the system the probability is 0.9984 ok, so that means, that if your system is like the swain, you have some component in series and then, then the resulting component or similar another component in parallel, so you can very easily with and or gate combination you can find out the availability or as such an availability of the system.

So, we now for the first alternative or first choice when we have only 2 stream and then A s full and A s half the probability, other way I can say the availability is calculated and the first one is 0.92 that is A s full, availability half is 0.9984. We will see the same thing how to calculate the A s full and A s half for the another for the second alternative or second choice when we have three identical stream A B and C.

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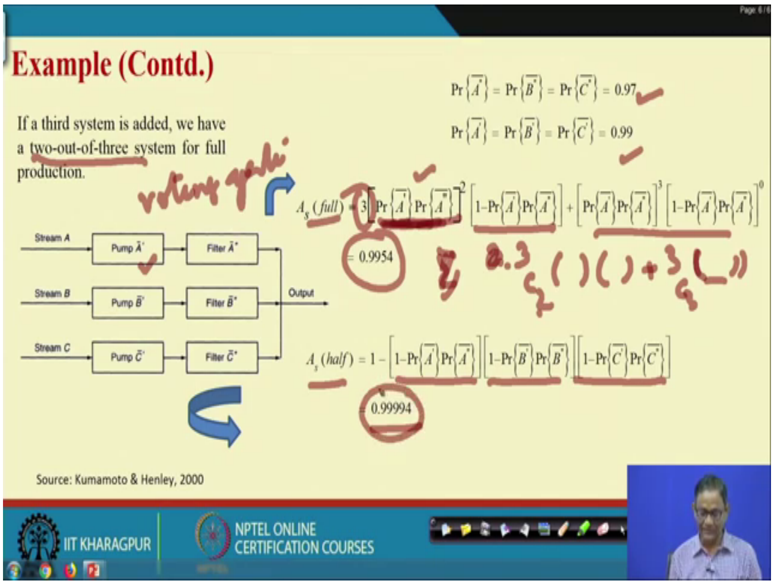

For 3 identical stream A B C, so again the as identical that mean the pump A B and C they are availability or probability of successfully working is 0.97 and then the ok, A B C 0.99 and 0.97. So, A star, B star, C star that is basically the filter and A dot A dot B dot C dot or A dot be a double dot B double dot C dot filter, this is simple you know what is already written here ok.

Now, what happened, you have three identical streams, but for full production you require 2 of the 3 should work, so it is a voting gate combination. So, it is voting gate, voting gate in terms of A s full as well as A s half. For A s full 2 out of 3 stream must work that mean either 2 or 3 stream must work. For half production 1 out of 3 means 1 or 2 or 3 that must work.

So, now we use the binomial theorem that, you have seen in the last class. So, here what happened the stream A availability, stream B availability and stream C availability they are same, so their unavailability is also same. So, because of this assumption that mean, we have 3 different ways to 3 different streams, each of having each stream is having a equal probability of availability or unavailability, so you can use the binomial theorem that is what we have discussed in last class. Now using these what are the combination for A s full. So, it is basically, sum of or other way, let me make it so, 2 out of 3 so 3 choose 2 and some combination will be probability will be there plus 3 choice 3 and the probability will be there.

So, when we are saying the 3 suppose the stream A stream B stream C they are availability is what probability of A s and probability of A dot and probability of A double dot bar success. So, similarly second one, similarly third one and as because each of the streams and A dot B dot C dot that bars they are basically, what do you said that they are equal. So, we can write only in terms of a no problem there.

So, this is the first for success part it is square. So, this square, so, 3 C 2 will give you 3, now you have 2 multiplication 1 of these probability times square and what is the complement, complement will be 1 minus these to the power 3 minus 2 1. Then, plus 3 C 3, 3 C 3 mean 1 and then these the first part this cube and the second part to the power 0, so that binomial theorem we have used earlier. So, this is your probability that, your system will give you full production 0.9954.

Now, then for A s half what you are doing here, A s half means, 1 minus unavailability. So, that sense you have complete system means no production. So, if one of the stream work half production will be there. So, there are three identical streams so, none of the stream works that is no products, so 1 minus no production will be as half. So, here this is the this is the probability for, this is not working, this is not working, this is not working as 3 identical streams up there, they are multiplied and you got this probability value ok.

So, I think now, you understand application of binomial theorem here.

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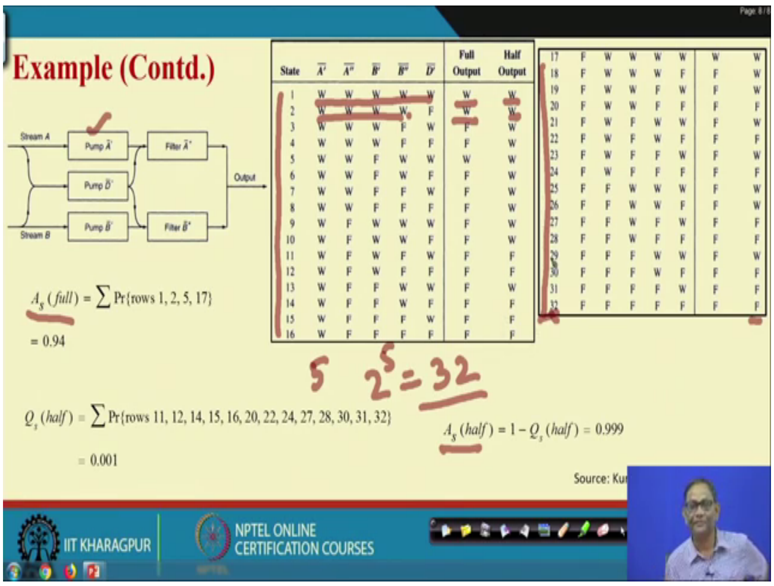

Then come to the third one. Third one you it is a different one. Here it is not a series system or parallel system or combination of series and parallel pure series and parallel system, it is a bridge network.

So, like in the earlier that n and or combination approach whatever, we have seen earlier that that cannot be applied easily here, unless you can make its equivalent representation, but under such bridge network it is difficult. So, here we are not using those traditional way of doing not traditional, the general that series parallel or n and or combinations here we are using truth table.

So now, how we are using truth table uses see, how many possibilities are there ok. So, possibilities, now question is that pump how many pumps 3 pumps and 2 filters. So, how many component, 5 component. So, each component can have how many state, 2 states. So, that is why the combination will be 2 to the power 5 equal to 32, you see the 32 combinations are written here.

So, first combination W means working, everything working. What will happen, full output is as possible or not working half; obviously, full is there half is there. Second one, as per requirement at least 2 of the stream 2 stream work, so that is why second one first 4 work, full possible half also possible, in this manner like last one when all fail. So, you have 32 combinations, now you see what do you want to do, you want to find out the A s full or A s half.

So, you find out under which of the combination as full is W means working, which of the combination as half is working.

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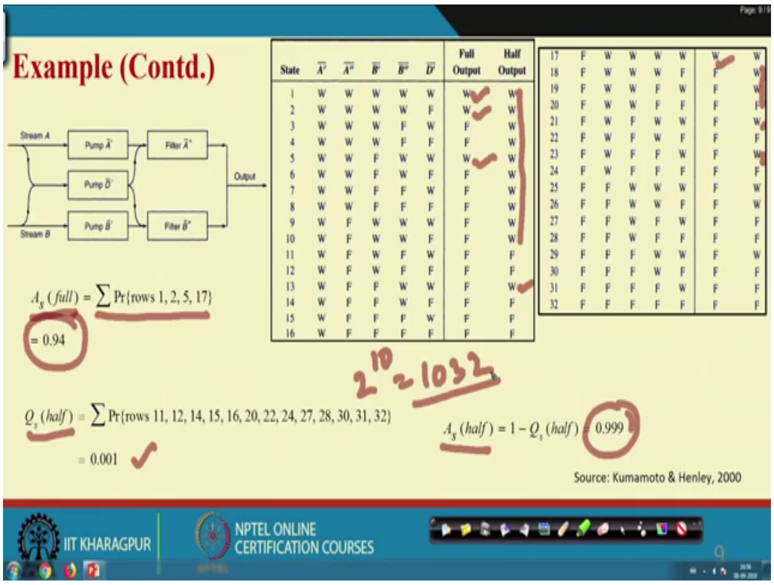

So, if you see carefully if the table you see that A s full, 1st combination, 2nd combination, 5th combination and 17th combination, all other cases it is not possible. As a result, we are writing A s full is summation of probability of rows 1 2 5 and 17. And you know that, what is the working means; what is the probability that values are known to you.

Now, if you if you find out the probability of the 1st row, 2nd row, 5th row and 17th row and you add them, you will be getting A s full is this. Now similarly, you will find out the, that means A s half, now you see that how many rows having W, a plenty of rows is having W, so W. So, you can find out each of those rows with corresponding probabilities then, row probabilities then, sum up it is possible, otherwise what happen as there are less number of F, means failed case.

So, first you may find out the Q S failure half over half production first find out and then A s half will be 1 minus that failure, so it is 0.999 so, this is the use of truth table. So, when you are not able to develop series parallel or combination, so there are the simple formula not you are not able to use it, mathematically it a little cumbersome. So, that is better go for truth table and truth table means find out all combination, but if you plenty of component then, it is again very difficult because, you know you have seen, if there are 10 components in the system then, 2 to the power 10 is 1032, this much combination will be there.

So, again it is basic this enumeration will take lot of time, it is a very difficult one again ok. But nevertheless, you can you can develops up to a program and generate it, but this is basically, good approach because, you will not miss anything there and you understand actually inside what is happening. So, now, we know the availability, unavailability for 3 different systems or alternatives or choices.

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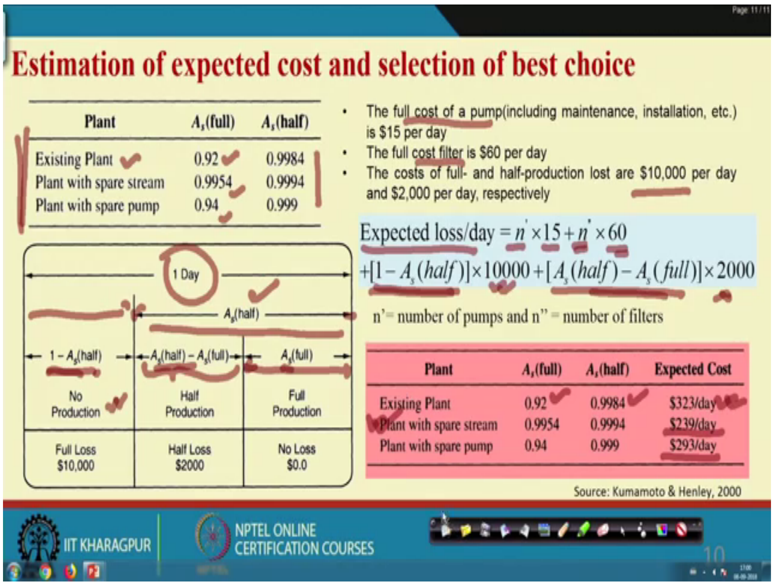

With some more data, you can find out which one is the best. So, what we have found out so far, for existing plant availability full probability is 0.92 with identical spare stream 0.9954, with spare pump 0.94 similarly, for half availability of half ok.

Now, if we know some of the data like what is the full cost of a pump including maintenance and installation, if we know the full cost of filter including means installation maintenance etcetera and if we know that if the loss for not having full production or having half production then, using this formula expected loss per day, you will be able to find out what is the value of expected loss per day for 3 different alternatives.

So, the formula is first is that, what is the that number of pump into cost of pump plus number of filter, cost of filter plus that no production into probability of no production into cost of no production and probability of half production into cost of half production. So, with this formula you will be able to find out what are the expected loss per day.

Now, how do you find out those these three things, so the loss calculation the situations it is given because, we are considering 1 day. Now if suppose if we consider 1 day, there is no production or half production so then, if we say that, this is your half production time then, this is definitely the no production type. So, 1 minus half production will give you no production. But, when you as you have seen in the truth table, half production also include full production, so that mean this can be divided again two parts; one is full production then, remaining one will be only half production ok.

So, now if I summed up full production and these minus this plus this is nothing, but A s half, again if we sum up these plus these plus these is nothing, but 1. So, these are the situation that is what is written here you see for full, for full production loss 1 minus A s half into loss. For half A s half minus A s full, that is written here.

Using this formula and the probability availability are known, so now, we using this and number of pumps its cost you found out that if you continue with existing plant, the cost is dollar 323 per day, if you go for the second alternative that is plant with spare stream and then it is basically, rupees 239 per day and if you go for the 3rd alternative plant with spare pump it is rupees 293 per day, not rupees we have we have used dollar 293 per day ok.

So, considering the expected cost, the second alternative gives you the better result.

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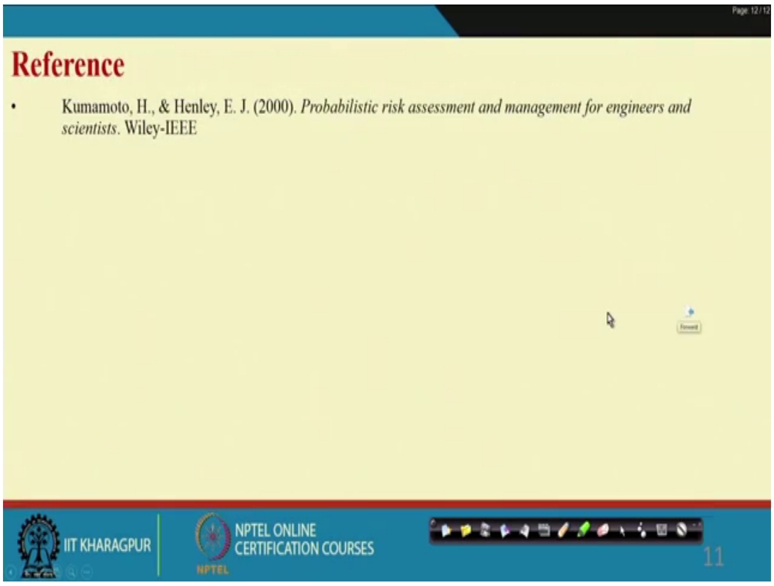

So, what you have learned here today, today you have learnt that truth table approach and its equivalence with and and or gate, with combination of and and or situation where equivalent and and or each difficult to provide. And truth table is basically complete enumeration and after that using truth table we have shown to you that, what are the ways you can calculate availability, unavailability and finally, we have seen that how different alternatives can be evaluated using the concept called expected cost. So, hope you have understood it.

Thank you very much.