Industrial Safety Engineering
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Lecture – 34
Computation of combined process parameters Laplace transform analysis

Hello everybody, we will continue combined process. Today our issue is computation of the combined process parameters and we will see the analytical solution of that computation and it is again a mathematical issue and many of you will like it and many of you will may not like it, depending on your background and interest, but nevertheless, this is an important topic and useful one. So, let us have some discussion and try to get as much as possible from this lecture.

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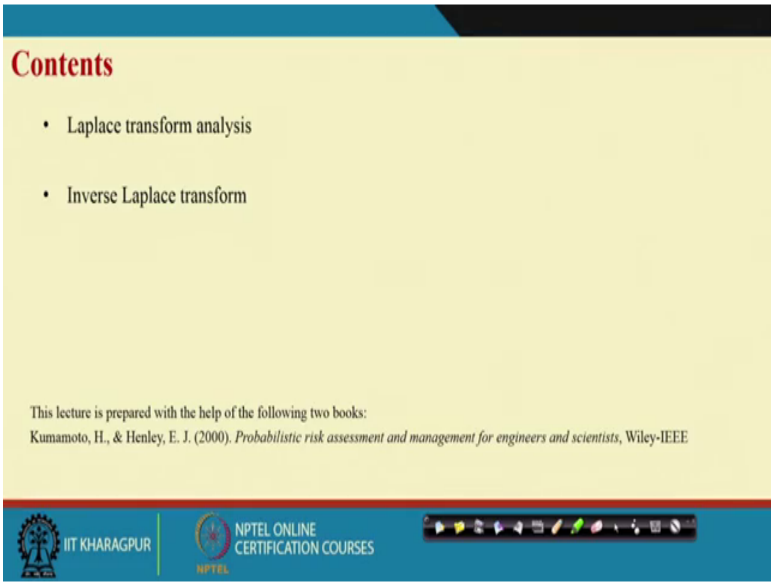

So, we will discuss Laplace transform analysis and Inverse Laplace transform with reference to this, particular topic. What is this? That the parameters of the combined process will be computed and we have again taken most of the material from our main book Probabilistic Risk Assessment and Management written by Kumamoto and Henley.

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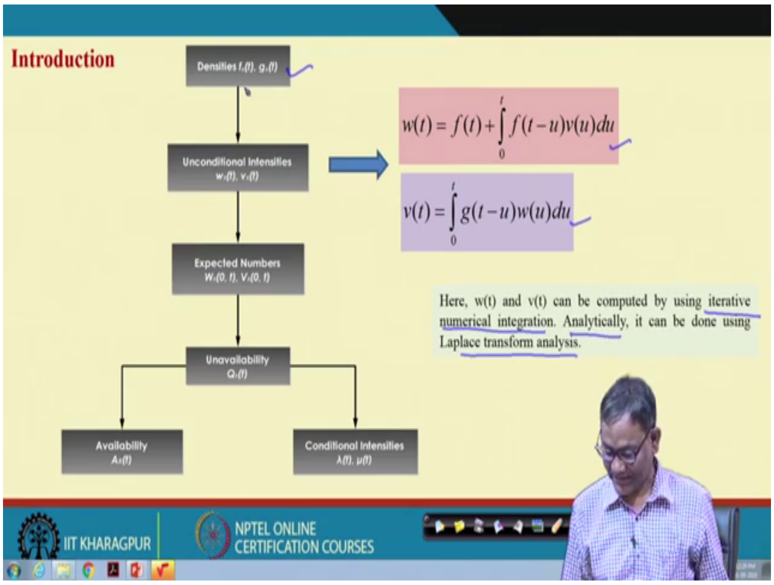

So, I repeat this diagram, as well as the fundamental equations because this is what is the crux of the matter for the combined process when we talk about the parameters and distributions of the combined process. So, let me repeat again, you know the densities. So, combined process means failure density and repair density, so, two densities known to you. So, you want to compute the, unconditional intensities w t and v t what is w t and what is v t already told to you in part in last lecture. We have elaborate derivation for this and we found out these two equations.

Now question is, if you see the equation, is it a ordinary equation something like, which is having the closed form solution or what way you will compute the w t, v t if it is not a closed form then, we most of the times, we rely on iterative numerical integration. There are many numerical methods, numerical techniques, Newton Raphson is one of them. So, what you can do? You can use the numerical integration methods and then find out the value of w t and v t, but the, if you are interested to do it analytically. So, Laplace transform analysis can be a good tool; obviously, you require to know the form of f t and g t or Laplace transform analysis. And that is the minimum requirement, whether you will go for the numerical integration or the Laplace transform analysis by 30 minutes of time.

We will see the Laplace transform analysis to compute w t and v t and where the failure intense, the failure density and repair density are following exponential distribution ok, because component useful life usually follows exponential distribution. So, that assuming an exponential distribution is also realistic. So let us see, what is Laplace transform and inverse Laplace and how it will help us in computing w t and v t.

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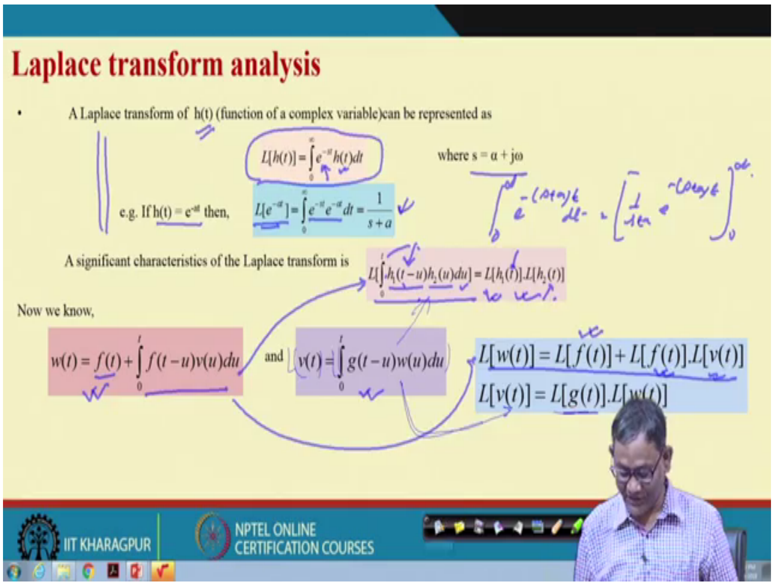

So, Laplace transform it is taught or it was taught to you in your engineering mathematics, particularly in the engineering degrees the primarily, in the second year or in the first year. And if you are student then definitely, it is and if you have gone through this, it is in your memory, but if you are an engineer in practice; you have read, maybe several years back.

So for you, it will be difficult to remember all those things. So I will give, you very g stop the Laplace transform and it is inverse here and the definition also. So that you can quickly recollect, your that time, if you if you are not studying it in now, So here interestingly, we will basically talk about a function h t and we basically are interested to find Laplace transform of h and this h t function, when we basically want this Laplace, we use this equation; that means, Laplace of h t is 0 to infinite e to the power minus s t h t d t, and this e to the power minus s t contain one term is, this s basically represent a complex variable, alpha plus j omega. Ok.

So, now question comes that, what is h t? H t can be any function of different forms. So for example, if we consider that h t is e to the power minus a t, then the Laplace of e to the power minus a t. So, you put h t here. So, 0 to infinite e, to the power minus s t and e to the power minus a t d t because of that, these are all exponential 1, so, e this will be e to the power minus s plus a and then d t integration 0 to infinite.

So, what will happen its integration will be 1 by s plus a, and e to the power minus s plus a, into t is (Refer Time: 07:30) and ultimately 0 to infinite, you put and finally, you get this equation.

So, if your function is exponential type then, it is easier to get the Laplace transform of it. So, in this is what is the definition for Laplace transformation.

We will not go into further of Laplace transformation that fundamental issues, we will use here a fundamental properties of or significant characteristics of Laplace transform which basically helps us in analytically solving v t and w t that, unconditional failure and repair intensities, that is this. That means, Laplace of int 0 to t, h t minus u and h two, t u and d u this can be written equivalently this. So, Laplace of integration of these can be written like this.

So, see here h 1 t minus u is there please mind it. So, t minus u, converted to t only and h 2 u d u that is no problem and then h two, there it is h 2 t only no problem, but the basic very important one is this, second one is here is integration. So, now, if you recall or if you just see our equation that what is w t equation, what is b t equation. You see that it has two component here, one is f t another one is integration of this.

Now, if we use this property then, this will ultimately lead to this equation. So, Laplace of w t if we say, then this will be Laplace of f t, that is the first part. Where second part is, Laplace of 0 to t means, integration of this, now we will rely this, rely on this, we will rely on this equation. So, f t minus u become, f t and v u d, v u d u that become v t. So, what it what does it gives? It give that, if we know f t and v t, then we will be able to compute w t using this Laplace transformation.

So, similarly the other equation, this equation can be converted suppose, I take the Laplace of this, then this is Laplace of this and using this formula, you can find out. This will be Laplace of g t times a Laplace of w t.

So, this simplification help us in computing our requirement means, w t and v t here and you see that how it is possible. So, our two unconditional intensities, one is unconditional failure intensity another one unconditional repair intensity, when it is transformed to Laplace domain then we got these two equations, this and this equation these two equation, we will use given f t and g t and we will show you that how the different parameters will be computed.

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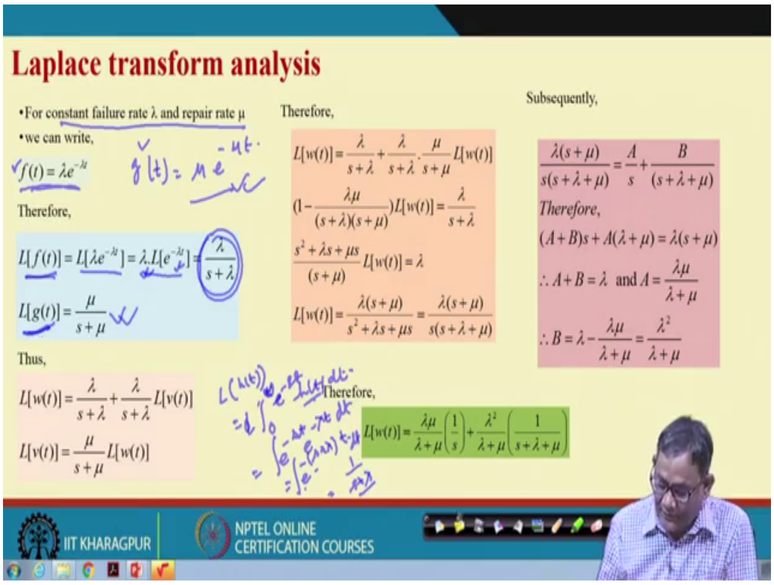

So now, our work starts, we will consider constant failure rate and repair rate. So, if the failure rate is lambda, then this is basically the exponential distribution from the failure point of view. Similarly, you can find out g t from the repair point of view, it will be mu e to the power minus mu into t ok.

So, in order to use the Laplace that transformation, what are the equation what are the different Laplace transformation required there? One is relate to f t, another one is relate to g t, because we do not know v t and w t, that is what will be computing. So, what is the Laplace of this f t is Laplace of lambda e to the power minus lambda t. So, lambda Laplace of e to the power minus lambda t equal to this, this is obvious, because you have already seen, what is Laplace of h t integration 0 to t e to the power minus h t, h t d t, integration 0 to infinite. (Refer Time: 13:03) ok.

So, e to the power now, what do you want, e to the power minus lambda t; so, e to the power minus lamb h t, h t e to the power minus lambda t. So, these two combined e to the power minus s plus lambda into t so; obviously, it will be Laplace of this will be, this. So they mean, this equal to integration of this d t, integration of this, d t e to the power d t, this will give you 1 by s plus lambda, already one lambda is there. So, it is lambda by s plus lambda so similarly, if you do this, this will give you, mu by s plus mu. So, we got Laplace of f t and Laplace of g t.

So now, we will use the formula that Laplace formula. I do not know why it is happening like this. So, this is the formula. So, L w t equal to L f t plus L f t into L v t. So, we know this, this is known. So we will put here, similarly from the second equation, this is known we will put that one. So, let us put.

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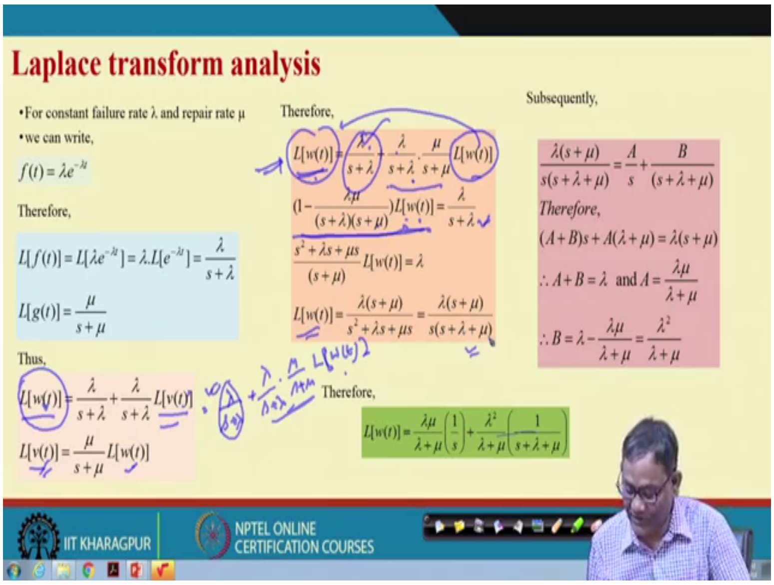

So, you will, you will be getting this one. L w t equal to lambda by s plus lambda plus lambda by s plus lambda into L v t and L v t is mu by s plus mu in L w t.

Now, we are interested to know the w t. So, in this equation, put the value of L c t into this equation. Are you not getting this one, definitely what will happen this will become lambda by s plus lambda plus lambda by s plus lambda L v t means, mu by s plus mu into L of w t. That is what is put here, L w t this portion is this, this to this, L w t.

Now, you have a equation where the only unknown is this l w t, but lambda mu, s is the that complex variable. So, you bring this to this side and do the algebraic manipulation. So, 1 minus lambda mu by s plus, lambda s plus mu into lambda t equal to this, because this, this one remains. So, after algebraic manipulation, you will be getting this equation, this is not a tough to do it. It is just, just you algebraically, you manipulate there is nothing great.

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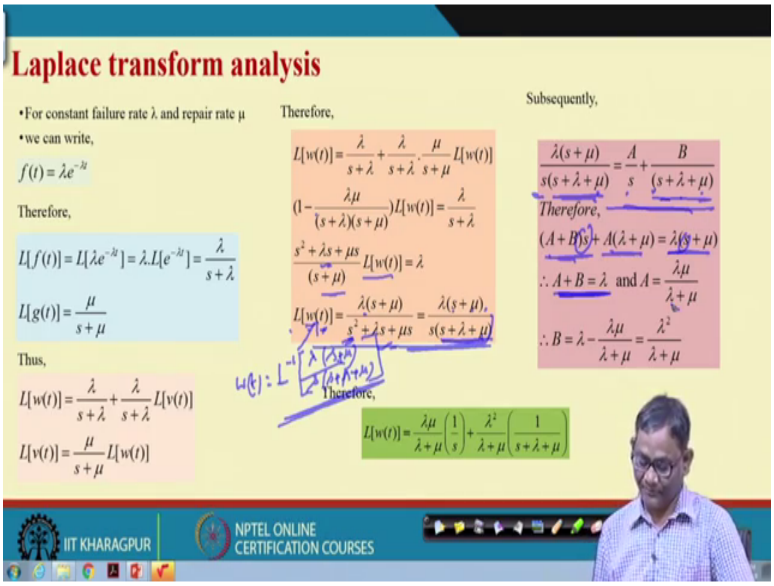

So, once this is nothing, but this. So, then lambda t equal to s lambda into this divided by this quantity. So, that can be lambda s plus mu and if I take common s then, that will be s plus, this one, so, what we require basically if now, if this is my equation then, if I want to find out the w t. So, I have to take inverse of this. So, in order to find inverse of these, so I can write the w t equal to L inverse, L inverse lambda s plus mu by s into s plus lambda plus mu ok.

Now, we have to make it, compatible to Laplace inverse, so what we do that, lambda into s plus lambda by this, this quantity we are writing like this. So, lambda into s plus lambda by this equal to A by s first part and that, the second part of the denominator, B by lambda plus mu and plus s. So now, if you solve this equation, what will happen ultimately? You will find out A plus B into h into A lambda plus mu equal to this and here, A B will be equal to lambda, because s and this s terms will be compared and the remaining will be one will be compared ok.

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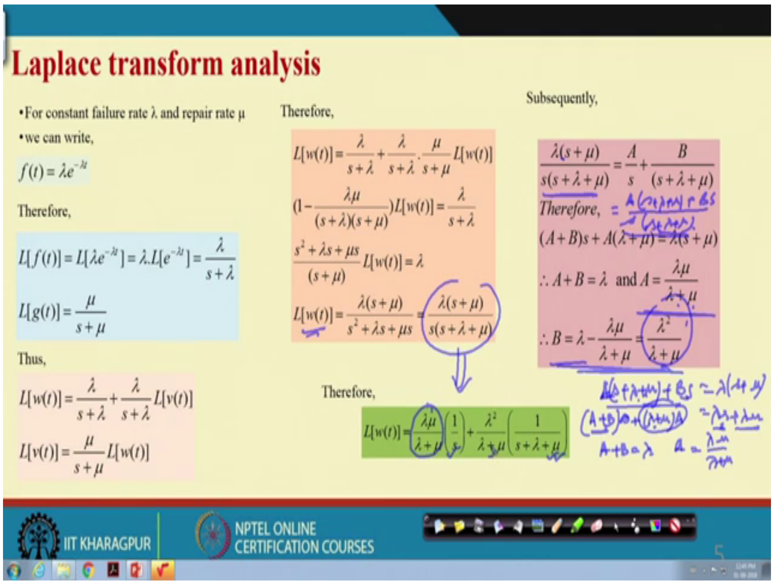

So, from here you may be wondering that, how come here, how it is coming. So, what you do basically this one, the right hand side, you can write this as, s plus lambda plus mu and the top side, what will happen? A into s plus lambda plus mu plus B s this portion is and this portion same, the numerator denominator same, now question comes with the numerator equality. So, here in the numerator side is what? A s plus lambda plus mu plus B s and this side equal to lambda into s plus mu, this will be lambda s plus lambda mu.

So, now this side also that s part, you find out, this A plus B into s A s and B s and plus lambda plus mu, the remaining portion. Now this and this s, we will from this, we can find out that A plus B equal to lambda.

And also we will find out, that is, I think A into A into this. So, we are also finding out that, A equal to lambda mu by lambda plus mu from this equal to this.

So as a result, ultimately you by solving this, you are finding out A as well and putting here, you are also finding out B. Now in this equation L w t equal to this one is converted to this, what is this? A by s plus B by this, what is A? A is lambda mu by lambda plus mu. So, lambda mu by lambda plus mu plus into 1 by s, second one, 1 by s plus lambda plus mu into B is lambda square by lambda plus mu.

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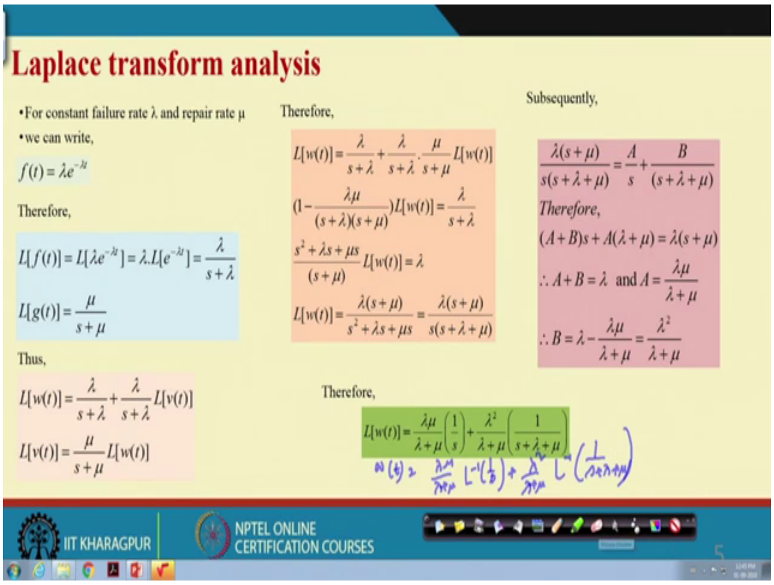

Now, if you take the transform, Laplace transform what you will get, w t equal to lambda mu by lambda plus mu, L transform 1 by s plus lambda square by lambda plus mu L transform 1 by s plus lambda plus mu. So, you all know that, what will be the equation. So, 1 by s means, it will be 1 and this one will be, e to the power minus lambda plus mu into t. So, that is what we, we got and we are writing in.

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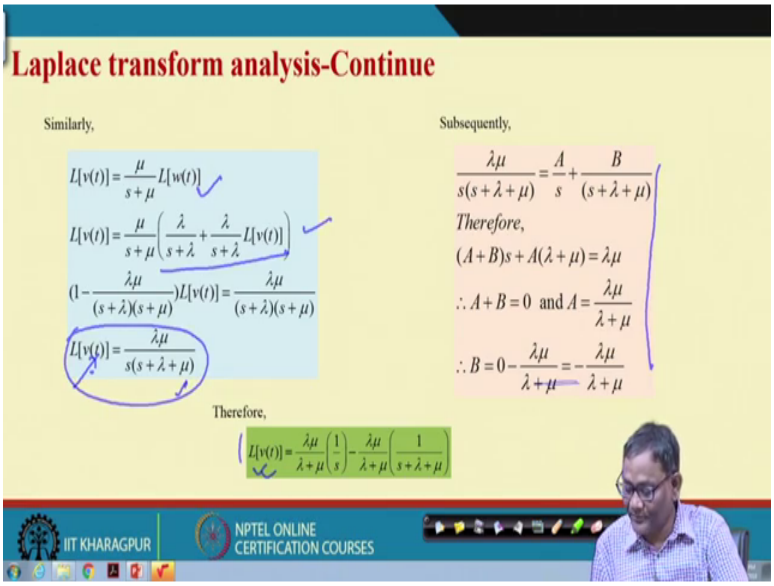

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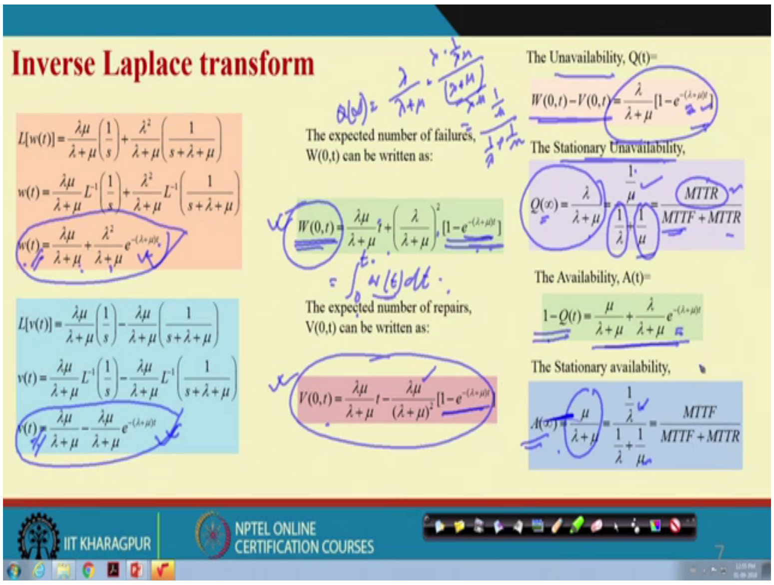

So, this is this is the part now, you are doing the inverse, you see inverse this one, we seen in the last slide. Now we take the Laplace inverse, we will be getting this, L inverse 1 by s is 1 L inverse, 1 by s plus lambda plus mu e to the power, minus lambda plus mu t. So, this is the fundamental that is the equation for us. Ok.

Similarly, for v t, so, for v t what will happen, you will, you will find out that, we have already seen that what is L v t and then put all those things and you will find out this equation then, take the inverse of it and you will find out this and finally, v t equal to this equation.

The way w t is computed using Laplace inverse, first Laplace transformation of w t followed by Laplace inverse of that quantity. And then what happened, you were getting w t and the same manner for L v t and you are getting, this equation, this is for w t and this is for v t. So, but keep in mind that, we are considering constant failure and repair rate; that means, we are basically, constant failure means, from the useful life period of the component. Ok so, to recollect again, because this is maybe new for some of you. So, for recollect, recollect this again, this one; yes I am again going back to this. So, what we have done please remember, what we have done?

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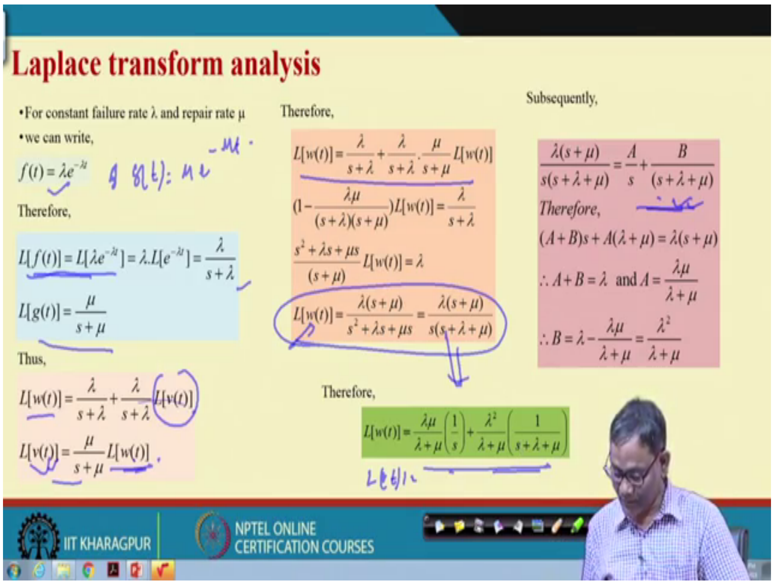

We have considered these, then also, we consider g t equal to mu e to the power minus mu t and then we have computed the Laplace of these as well as Laplace of this. And we already have the, this two equations earlier, we have seen then in this equation L v t is put from this equation. So, that we will have only equation related to L w t and that is what, you got here. Finally, L w t is of this form, but now in order to get, w to you want Laplace inverse transformation of Laplace, Laplace inverse of these. So, to make it compatible to the, our original that Laplace equation, you are basically finding out doing algebraic manipulation like this and then this equation is becoming this.

Now, if you want to do this for v t, you can do this, because w t is known, L w t is known, put here, this equation, what will happen, mu by, this into this quantity and then in the same path, you follow, you will be finding out L v t, L v t in this form, this kind of form not this equation this kind of form, that is what we have shown to you later. So, then L v t, that it is similarly, that we put here, you see L v t, L w t, L w t is put, this is basically L w t, this is put here, then 1 minus of this, this, this and finally, L v t equation is this.

Now, question is this, is we want to find out v t. So, we want to take the inverse of it. Now this one should be written, in this form, we want A and B and this, A and B are found out and then it is put here. So, this is the second equation.

So, now if you take inverse of, Laplace inverse here, you will find out, this will be 1 and this will be what again; E to the power minus lambda, plus mu into t. So then, the resultant equation, we got the equations, one is for w t, this equation. Another one is for v t. So now, what will happen, if your if you do not consider the exponential that assumption, that the failure rate or failure density and repair density is exponentially distributed with mu and this.

In that case, what other distribution whatever accordingly you have to do it ok, many a times you may not this may not be that much simple it may be complex sometimes, it cannot be done may be ok.

So, anyhow so once you know the w t and v t. If you know w t then, in order to know the expected number of failures within zero and t, what is the formula? Formula is 0 to t w t into d t, but w t equation is known to you that lambda mu by lambda plus mu plus this, if you put here and take integration. So then, the first one is lambda mu by lambda plus mu into t will be coming second one, lambda by lambda plus mu square 1 minus this will be coming ok. So, if you take the derivative integration of this from 0 to t, you will be getting this equation, this is what is the expected number of failures within time t and similarly what is the expected number of repairs? You just integrate this from 0 to t, you will be getting this expected number of repairs. So, this 1 minus, this is coming here and here also 1 minus this is coming here you derive.

Now, what is unavailability? Unavailability is expected number of failures, minus expected number of repairs. So, at what is unavailability at time t? It will be expected number of failures minus expected number of repairs. So, that mean this minus this will give you this equation, so; that means, we are able to compute w t, we are able to compute v t, we are able to compute capital W 0 t, capital V 0 t and we are able to compute unavailability, which is this, physically means this is unconditional failure density, unconditional repair density, unconditional failure intensity, unconditional repair intensity, then expected number of failures, expected number of repairs, unavailability of component.

So, now if you put t equal to infinite it is very large, that is equal to infinite then, you will get steady state your stationary unavailability. So in this equation, if you put t equal to infinite then, you will get this quantity will become 0 and then your steady state or stationary unavailability is lambda by lambda plus mu and you can, if you do little bit of algebraic manipulation here, you will get this equation so; that means, what are you doing lambda by lambda plus mu, that is my stationary unavailability. So, if I multiply this lambda by 1 by lambda mu and then the denominator also lambda plus mu by then lambda mu, then this quantity will be 1 by mu and lower one will be 1 by lambda plus 1 by mu.

So, now what is the lambda and mu lambda is the that is basically the a parameter of the exponential distribution here and we have seen earlier that is exponential distribution, the mean time to repair will be 1 by a mean time to failure will be 1 by lambda, mean time to repair will be 1 by mu and so that is why, 1 by mu, we are writing mean time to repair, 1 by lambda mean time to failure and this so; that means, the stay stationary unavailability is mean time to repair by mean time to failure plus mean time to repair.

Similarly, you can find out the availability, you can find out availability once unavailability equation is known. So, 1 minus unavailability will be the availability, equation will be like this here if you put t equal to infinite then, your availability equation will be developed and now t equal to infinite, this quantity will become 0 and you will get this value. This can be converted in this manner. So, availability is mean time to failure by mean time to failure plus mean time to repair and these two if you sum then that will be one.

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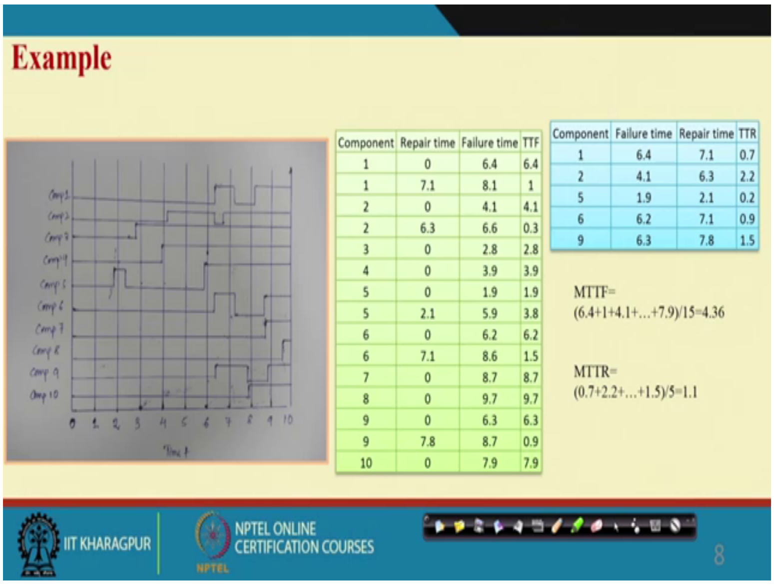

So, this example we have seen earlier same way, we found out that there are 15 different t t time to failure and also from this diagram, we found out that there are there are 1, 2, 3, 4, 5 repair time. So, time to failure is known. So, what will be the mean time failure? Mean time to failure will be all time to failure divided by the number of observations; mean time to repair is also all time to repair divided by number of observations. So, that is what you got you sum up all those the TTF values divided by 15 is 4.36.

some of TTR values T some of TTR values, divided by 5 is the MTTR. So, your from your experimental data you have all the TTF values, you have all the TTR values, you are calculating the mean time to failure and mean time to repair.

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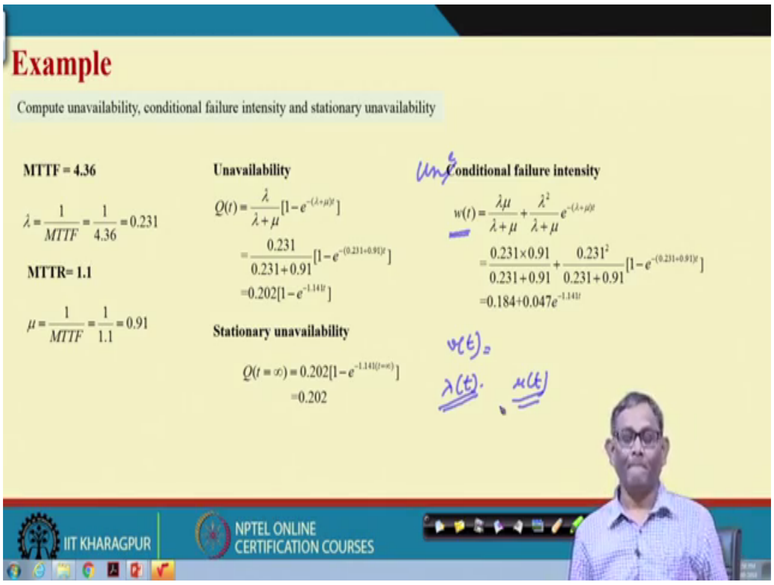

And if I know mean time to failure and mean them to repair, then with the derivations what we have obtained earlier using this you can find out lambda, you can find out mu, you can find out unavailability, you can find out stationery unavailability, you can find out conditional failure intensity, unconditional failure intensity, this is your unconditional failure intensity. This w t similarly you can find out, you can find out unconditional failure repair intensity. Ok. So, you can you can find out lambda t, which is basically conditional failure intensity, which we can find out mu t, which is your unconditional a conditional repair intensity, all those things you will be able to compute. Ok.

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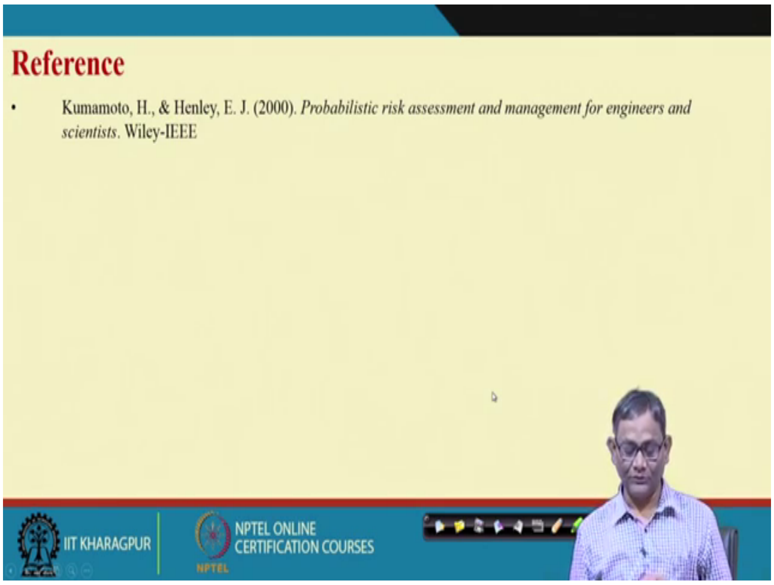

I hope you got it and if you have any problem, please put question in the discussion forum and try to learn this mathematics as much as possible.

Thank you very much.