Industrial Safety Engineering
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Lecture - 33
Quantification of Basic Events Failure & Repair Intensities

Hello everybody. Good morning. So, we discuss failure and repair intensities today.

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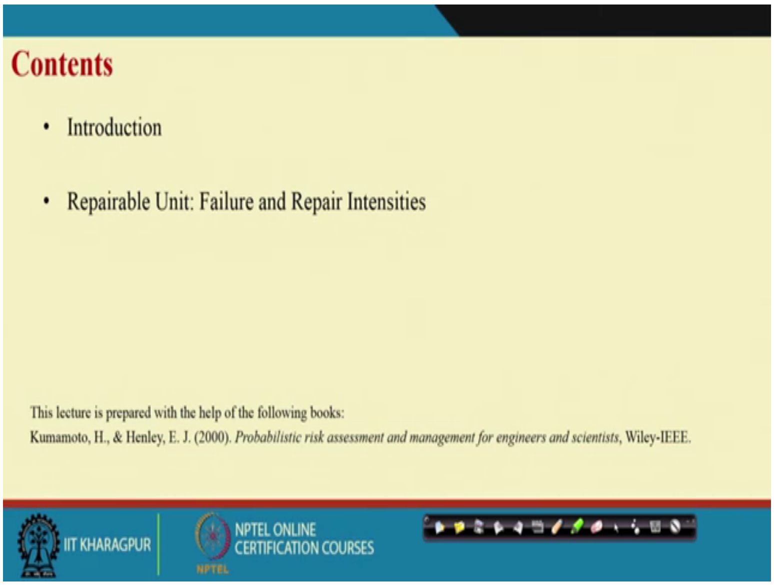

In last class you have seen that, we introduced some new parameters for combined process which is basically the repairable unit. In a repairable unit or repairable system, you know the basic characteristics are once the component fails or the system fails, it will be repair and then it will go to it is normal state means, it is in the working condition. And again under use after certain time it may fail, then it undergoes through the repair process.

And that failure process and repair process to are combine and we said that that is the combined process. And apart from reliability, apart from failure distribution, apart from failure density, from failure rate we will introduced availability, we introduced unavailability, we have introduced a different in failure intensities. Like conditional failure intensities, unconditional failure intensities similarly conditional repair intensities and unconditional repair intensities. In today's class, we will discuss the theoretical basis of failure and repair intensities. So, we are relying on the same book written by

Kumamoto and Henley on probabilistic risk assessment and management for engineer and scientist.

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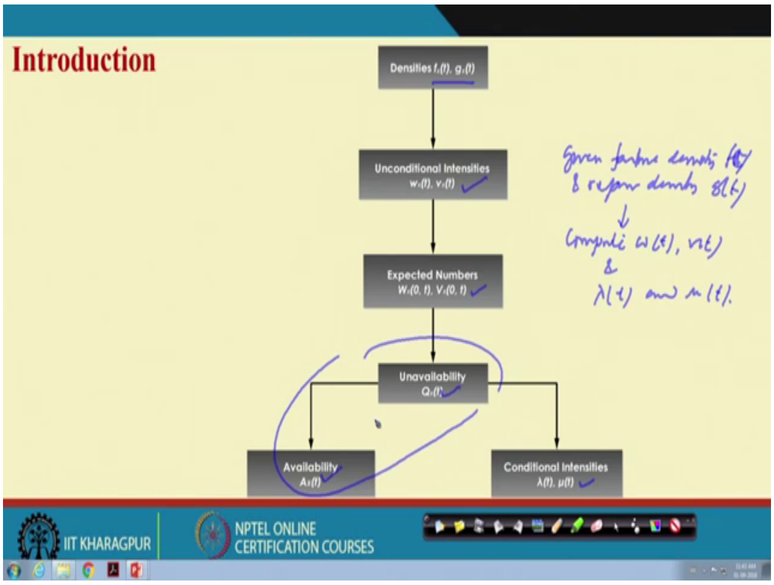

And most of the material taken from this book. So, if you recall in last class, we started with an example and then we have discussed different parameters and those parameters are listed here. For example, availability for example, unavailability for example, conditional intensities for example, unconditional intensities for example, expected number of failures.

And then we have defined all those things and finally, we have shown you how to compute the availability, unavailability and the different intensities. But we could not explain in details from the example given to you. So, today what I thought of that, let me start with that example and then I will come to the theoretical basis of that computation of failure intensities; given f t and g t where of the failure densities. So, what is the objective of today's presentation given failure density that is f t and repair density that is g t? So, we want to compute unconditional failure and repair intensities w t and b t as well as conditional failure and repair intensities lambda t and mu t.

So, we start with the example and will explain this things so, that you can relate with the previous lecture. I hope that you are going through the book as well as the video lectures regularly. So, then if there is any disconnect your revisiting or reviewing the video again, and then you are getting it understood.

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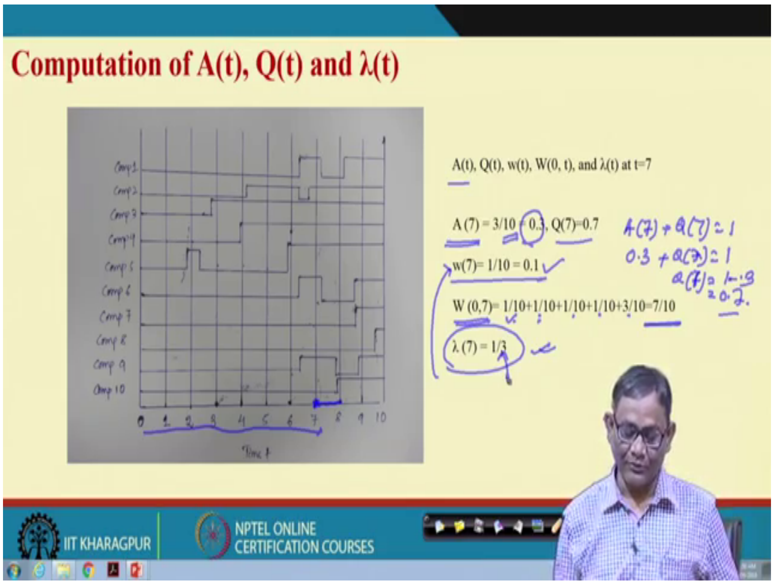

So, let us go to this example. I request all of you to listen carefully. If you listen carefully definitely you will understand every bit of the sentences, the words we are using the figures, we are showing and the computation steps we had deliberating.

So, this is the example given in the last class, I will not spend much time here. We have shown to you that there are 10 identical components, and it has it is failure repair process. And accordingly what happened we found that there is some component which has gone through failure repair failure, some component gone through failure and repair some component gone through normal state and then failed, but repair process is not completed. So, as a result what happened the meaning of availability comes handy, and you know what is the availability that the probability that component is available or normal at time t, given that and it was as good as new at time t is equal to 0. So, with this example let us try to compute what is the availability at time t is equal to 7. So, that means you are here.

So now, you see that how many components under test 10 components. So, the denominator is 10. So, how many components are normal or working at t equal to 7 that point on time, you will find out that component 10 is working, component 8 is working, component 7 is working and rest of the components are under failed state. So, that mean 3 components working at t equal to 7 unit, given that at time t equal to 0 there are 10 units working. So, that is why the availability is 0.3, 3 by 10. So, as you know that availability plus unavailability will be 1.

So, availability is 0.3 so, then your unavailability will be 1 minus 0.3 that is unavailability will be 0.7. Or other way you can find out that how many at time t equal to 7, how many component are under fail state, out of 10 component 3 are at working state so, 7 are at fail state. So, that is why Q t equal to 7 by 10 which is 0.7. So, then there is another issue that that is unconditional failure intensity. Unconditional failure intensity basically talks about a situation where we do not bother about; do not bother about what is happening at time t, at time t equal to t whether the fail component are normal or not normal. We will bother only at time t equal to 0.

So now what is the unconditional failure intensity is at t equal to 7? So, you have to find out that what in between 7 and 8 how many component fails. So, you see that in between 7 and 8, only one component fail. And although at 7 there are many failure, but in between 7 and 8 this unit of time, only one fail and a and at time t equal to 0, there have been 10 components working.

So, unconditional failure intensities there that is therefore, the number of component fails in between 7 and 8 units of time divided by the total unit of total number of components at time t equal to 0, so that is 0.1. Another one is the expected number of failures. So, we are considering here expected number of failures from 0 to 7, then you find out that from 0 to 1 to 0 1 to 2, 2 to 3 like this up to say up to 7 so, 0 to 7 this one. So, you will find out that how many 1 2 3 4 5, because from 0 to 1 there is no failure. So, as a result ultimately and from 1 to 2, there is one failure, from 2 to 3, there is again one failure, 2 to 3 this failed. And 3 to 4 this is normal, this one failed. So, again one failure like this.

So, 1 by 10 1 by 10 all will is summed up, and you got that that expected number of failure, number failure is 0.7. And then comes the conditional part, conditional part means it is a different population. When we are talking about unconditional failure intensities, we start with 10 components here with this examples of 10 numbers of the population size is 10. And we found out that in between 7 and 8, how many fails. That it fails 1. So, that is why the unconditional failure intensities 1 by 10 equal to 0.1. But in case of conditional part, we are adding another condition that the component is normal at time t equal to 7. So, at time t equal to 7 how many component are working condition or at normal state that is 3 only.

So then within 7 and 8 how many fail? 1 so, that is why the probability or conditional failure intensity is 1 by 3. Probability that component will fail per unit time given that they were as good as new at time t equal to 0 and normal at time t equal to t. I hope that you will be able to compute by your own. So now, we will see the theoretical basis of these unconditional failure probabilities or unconditional failure intensities and unconditional, conditional failure intensities unconditional repair intensities and conditional repair intensities.

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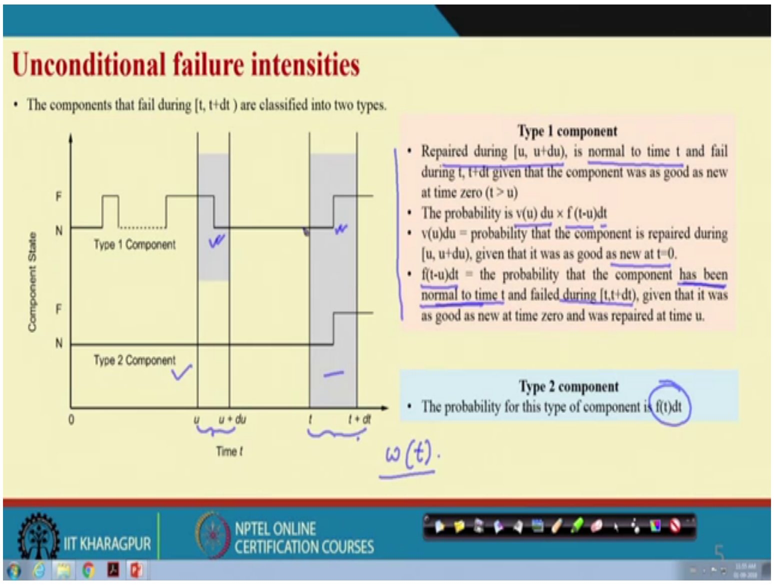

So, first we start with unconditional failure intensities. So, we are interested to know for what? We are interested to know what is w t, unconditional failure intensity. So, if you if you think that your time interval is t and t plus delta t. So, we want to know that the code is unconditional failure intensity the probability that the component will fail per unit time at t; given that the component is as good as new at time t equal to 0 and normal at time t.

So, compound that is what is basically unconditional failure intensity. So, that 2 type situations you will occur one is that component is normal at time t and it has been normal from time 0, that is this call second one. Another one is that component fails in between maybe a between and let it is repaired in between u and u plus d u and then again failed in between t and t plus delta t. So, as a result what happened? We have 2 types of situations 2 outcomes and we are saying that 2 type of the component 2 types of component also we can tell.

So, issue is that the in the first type of component so, as it fails in between this one time and it is basically then repair; repair in between u and u plus del u, so that is one condition. So, we are writing repair during u and u plus d u and then it is normal up to time t. And after that it is normal to time t, and then again fail in between t and t plus delta t. So, as a result what is happening, what is the probability? Probability here the 2 issues one is it is repaired here, and then it has been normal up to time t and then again it is it failed.

So, these 2 will be multiplied here. The probability will be v u d u which is basically it is repaired in between this time and again it is basically fail within t and t plus delta t. And as it is repaired here already you amount of time is elapse. So, f t minus u into dt. Then what is our v u d u? Probability that the component is repair during this given that it was as good as good as new at time t equal to 0. And then what is f t minus u d t? The probability that the component has been normal to time t, please read this has been normal to time t and failed during t and t plus delta t and; obviously, the other condition is then it was as good as new at time t equal to 0 and repair at time t equal to u.

So, and in this in the second case what will happen? There is no failure repair issues it is failing within these t and t plus delta t. So, as a result what happened? At the probability here that it fail you will be f t into dt. So, this kind of explanation also we have discuss earlier. So, that means, we have to consider not only these also this, because this 2 outcomes, the 2 situation can arise.

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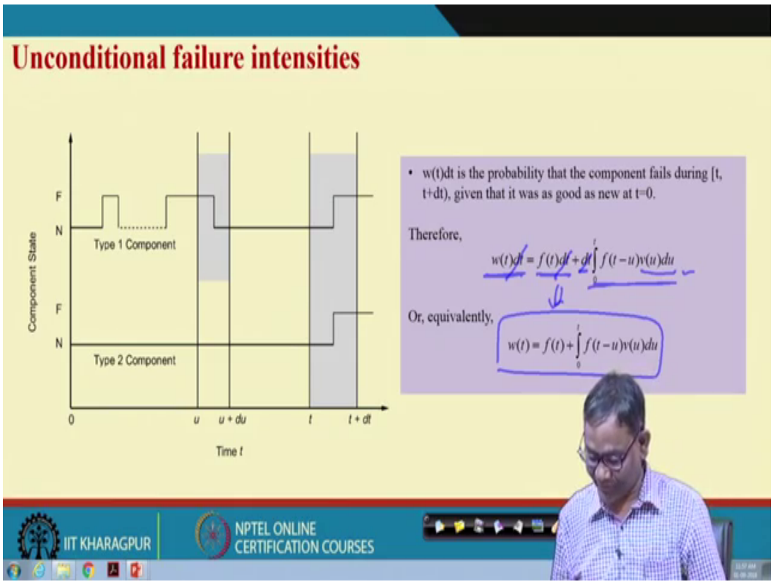

As a result as a result what happened? What will be the our intensity part unconditional intensity. So, first we will we are getting this w t into dt. What is w t into dt? This is the probability that the component fail in between t and t plus delta t; given that it was as good as new when t equal to 0.

And we found out 2 situation, one is that it fails from the component 2 that we found to the probability this probability f t dt that it fail in between this. And second one is that it is repaired and then normal from then up to that (Refer Time: 17:11) here t. And when it is repaired we assume that it is again as good as new, and as a result the dt times this probability will be used. So, dt 0 to t f t minus u v u d u. This v u d u coming from the repair side and f t minus u that it has been normal after repair up to time t so, that you will amount of time already elapsed, but when you repair it is condition is as good as new. So, this delta t and delta t will be cancelled out from this equation we will find out this equation. This is a one of the fundamental equation here. That means what is w t? W t is unconditional failure intensity which is equal to f t plus integration 0 to t f t minus u v u d u.

Now, there may be situation, suppose what will happen if it is non repairable? If non repairable case then the second component does not exist so, accordingly the equation will be w t equal to f t.

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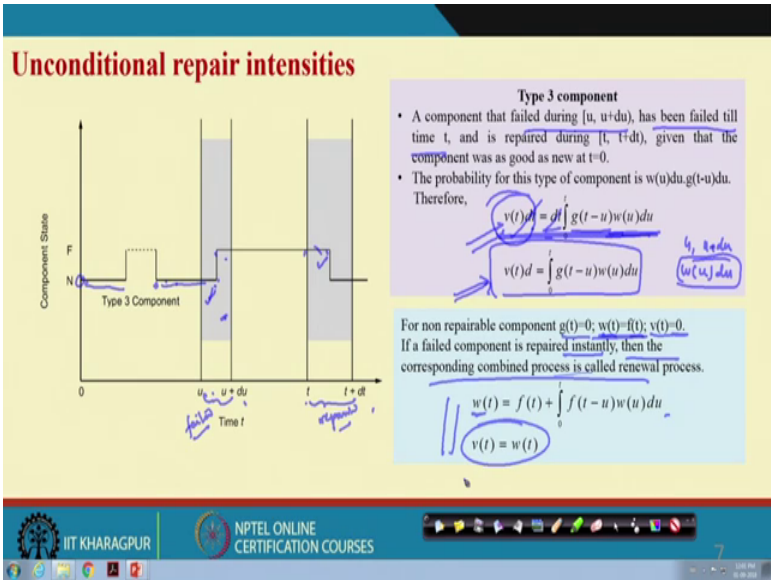

So, similarly you can go for repair intensities. What is the difference here? You see the component this is the normal state, normal, then it failed again repair at this point repair. And then it is under again normal, then again it failed here and it is in the failure state. It is in the failure state and then it is repaired in between t plus this. So, this is repaired between this. And what happened here it was normal to fail so, failed in between failed here in between this. So, that in this is a fail failed in between this we will start from the failure process. And here repaired in between this repair part.

So, 2 things will be considered because we are interested to know the repaired intensities only. So, o that mean what is the condition that, failed during u and u plus d u, and another one in the has been failed till time t, it is in the failed state up to time t and then we are interested to know what is the probability that it will be repair with this t and t plus delta t. So, then this is the one such outcome will be there, there will be not other. So, that mean now this similar situation in the failure intensities case we said that b u d u into f t minus u into f t minus u into dt. Now here our density change it is not failure density, it will be repair densities in this case and in this case it will be failure intensities.

So, that is why what is the probability that it will be failed in between u and u plus d u, failed in between u and u plus d u. That is definitely w u d u because w u is the failure unconditional failure intensity. So, this put will be there, and what is happening after that? It is basically it will be it should be has been normal. So, g t minus u and d t so, this is other part from here. It fail, that g t minus u into d t, but question is that it has been in the failed state from after that repair t after repair, sorry after failure it will be in failed state, and then it will be repaired here. So, that is why the integration is required it has been in the 0 to t, and when it is failed completely failed.

So, your equation is this. So, unconditional failure repair density is dt and b t into dt it basically talks about that that the component will be repair, in between probability that the component will be repair in between t and t plus delta t this is the case. And then this equation, now this t and this t will be cancelled out and as a result you will be having this equation. So, for non-repairable component what will happen? That w g t 0 w t f t equal to f t that I already told you and b t will be 0. If a component failed, component in is repaired instantly, then the combining process will be like this. W t will be f t 0 to t tuner d w and b t will be w t.

So, anyhow so, please remember this is little bit complicated definitely from concept point of view, but you required to know all those things. And to me you required to read and as well as listen the video maybe 2 3 times so, that you will get the concepts clear. And these concepts are essential although for practical people, it may be difficult, practical means practitioners who are basically working in the failed. for you it may be difficult to understand sometimes. But you please understand the message of the physical meaning of all those that parameters; that is very very important. And essentially then what we are looking for?

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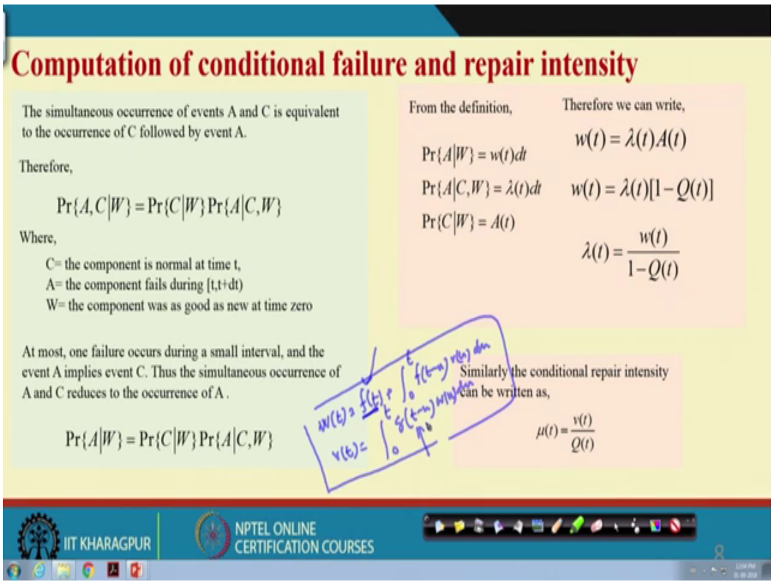

We are looking for that given the; we are looking for that given the given the f t and that is many failure density and repair density then what will be the unconditional, what will be the unconditional failure intensity? This is one formula, and another one is this one, 2 important equations.

So, before going to now computing the conditional failure and repair intensity so, you must know that the 2 important formula we have discussed, one is w t, that we say f t plus integration 0 to t f t minus u d u du. Another one we discussed v t, that we say 0 to t then this one g t minus u w u d u. These 2 formula talking about unconditional failure intensity and unconditional repair intensity. And you see that what you required to know, you require to know this failure density and repair density.

So now we will discuss another important concept call conditional failure and repair intensity; that definition already given to you. What is the additional condition in case of conditional failure and repair intensity with reference to the unconditional failure and repair intensity is that, in case of conditional failure intensity that the component will be normal at time t equal to 0.

Other condition remain same, that that are equally valid similarly, for repair intensity. So now, here we will discuss that how does conditional failure intensity will be derived and the similarly analogously you can derive the repair intensity. So, all of you know the conditional probability. If you do not know, please go through conditional probability, and we will not discuss what is conditional probability here, it is a prerequisite for you. So, for the time being you just understand few things.

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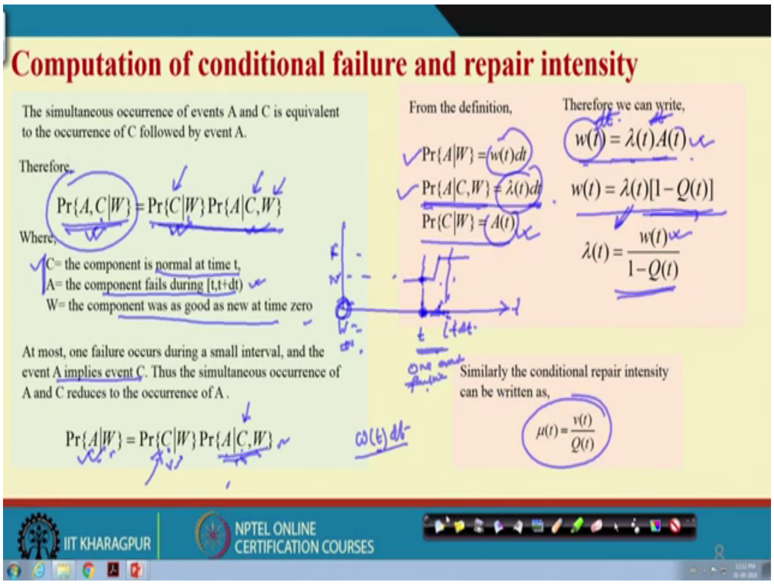

Component is normal at time t that is one condition. Then component fails during this, this is what is our subject to interest. Then the component was as good as new at time t equal to 0, that is also another conditions. Now then, if we want to know, what is the joint occurrence of A and C. What is joint occurrence of A and C? A means that component suppose if I go along time length t t equal to t the component is normal. So, component is normal at time t equal to t it is normal. And then another one what is this? The component fails during t and t plus delta t t and this is my t plus dt.

So, what will happen? It will basically from normal to it will go to abnormal state, means it will fail. So, the joint occurrence that mean it is the compound is normal till time t and then it fails, but in both the cases the condition is w; that means, it is as good as new at time t equal to 0 so, then that joint probability can be written like this probability A C given w. So, these can be if we you know the probability that conditional probability that can be written like this. What is this? The probability that the first is a component is normal at time t given that it was as good as new at time t equal to 0 this is given by this probability C given W. And second one is that the component fails given 2 conditions. One it is normal at time t equal to 0 and as good as new at time t equal to 0, and it is normal at time t equal to t. So, that C is talking about component is normal at time t equal to t, W talking about component is as good as new at time equal to 0.

And A denotes that component will fail in between t and delta t. So, these probability joint probability is the multiplication of these 2 probabilities. But here one interesting thing is there; that interesting thing is that we basically choose the interval in such a small value that only one failure so can happen, or one event one event can happen that is a failure event here because it is in the normal state, it can go to the that abnormal a failed states. So, one failure event can take place within this small amount of time. So, then if I consider the conditions, then what will happen ultimately? This A and C, what is A and C? That a means it will fail C it is normal. So, as within this small time only one event can occur that it can failed.

So, ultimately that A this portion will be reduced to this. The probability A C given W is nothing but probability a given W. So, why, because if I say probability A mean it fails within this, and it is a small interval only one failure event can occurred; so, that means, it was normal at time t. So, that mean the event a that implies that implies event C also the conditions C. So, this one can be written like this and this portion as it is. Now with these now relate to the definitions what we have already discussed. What this w t? What is w t dt? This is the probability that the component fail in between this. Then what is probability A given W? Nothing but this, so your writing like this.

Now what is probability of a given C W? This is conditional because C condition is imposed. So, this is not unconditional failure intensities, this is conditional failure intensities that is a given C once you multiply by dt that you will get the probability. So, that means, these relates to lambda t into dt. And the first one, first one what? Component is normal at time equal to t, given that it was as good as new at time equal to 0; this is nothing but the availability.

So, availability is included here. So, that mean these relation talks about relation among the first part, this part and this part, and if you put all those things into this equation, what you will get? You will get this equation. The reason is that mean probability A given W is probability A given W is w t dt then probability C given w is a t. And probability a given C w is lambda t into dt. So, this dt d t will cancel out. So, that mean w t equal to lambda t into A t and as you know that availability is 1 minus unavailabilities.

So, we can write like this and from this equation also from this equation you can write lambda t equal to unconditional failure intensity divided by availability.

So now you understand that if you know the failure density and repair density, you can calculate the unconditional failure intensity unconditional repair intensity. And once you know the unconditional failure repair intensity and availability of the parameters, using the conditional failure probability theorem you can find out the conditional failure intensity and conditional repair intensity. We have discussed about conditional failure intensity only here using this probability, conditional probability theorem. What you required to do? You also so required to derive this part.

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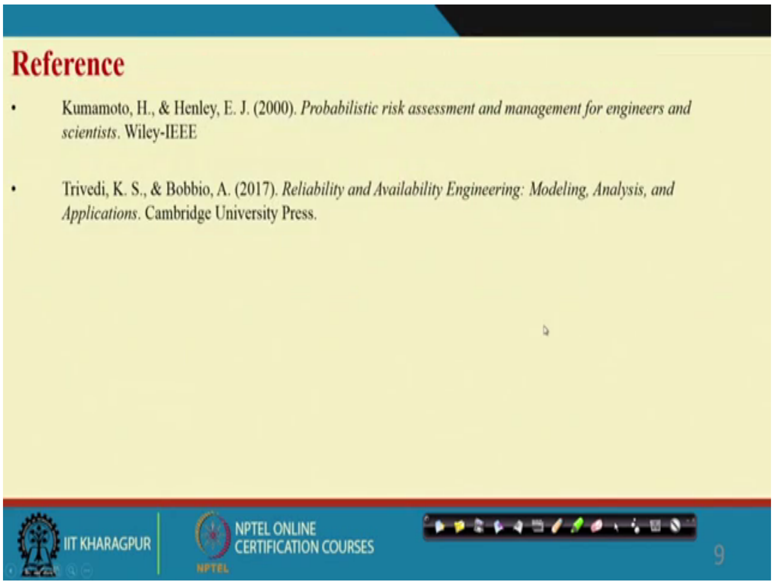

So, these are the references and I hope that you have understood it. We will see some more mathematical issues also in next class. But please remember that this week is more of mathematics, and then may be and subsequent week again we will go for more of concepts and practice kind of things or application type of things.

So, thank you very much.