Industrial Safety Engineering
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Lecture – 32
Quantification of Basic Events for Repairable Components

Hello everybody, we will start Quantification of Basic Events for Repairable Components. In fact, I have discussed little bit of this in my first earlier lectures, and primarily we have discussed the basic event quantification for non repairable system and there we have discussed about your reliability, failure distribution, failure density, then failure rate. And, then I have discussed similar parameters for repairable, repair process under repairable systems.

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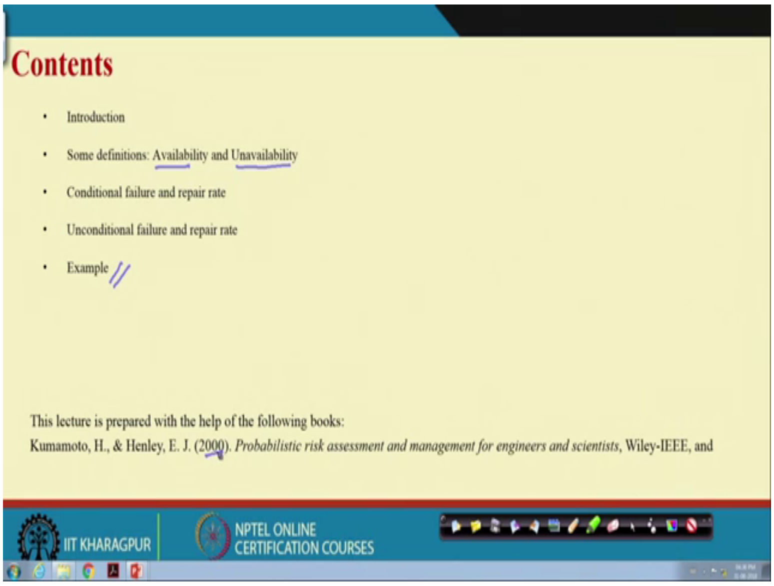

Today we will extend this and we will see some of the other important parameters, for repairable system or repairable component in this case. So, we will discuss today what is availability, what is unavailability, what is conditional failure and repair rates, and unconditional failure and repair rates. And, then we will give one example and we will from that example we will see that how those parameters can be computed. And most of the materials for today's lecture is has been taken from this book Probabilistic risk assessment and management for engineer and scientist written by Kumamoto and Henley 2000.

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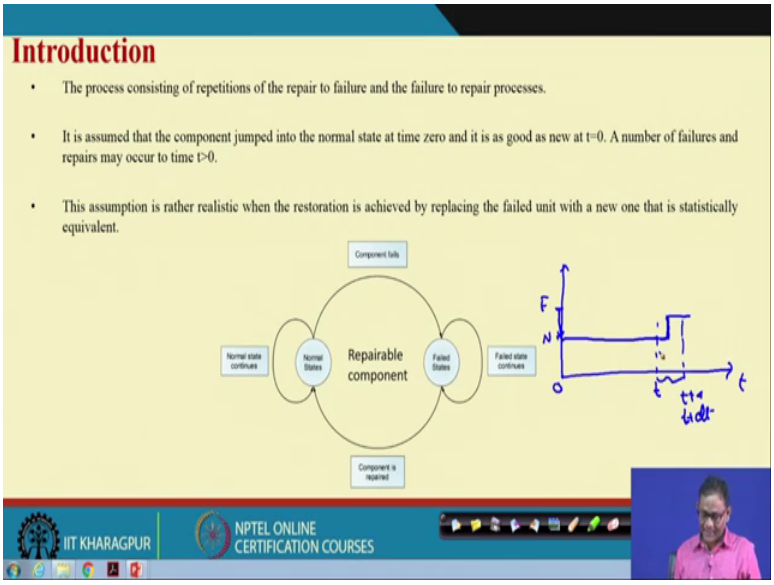

So, let me recollect what we have discussed so, far related to repairable system or a repairable component. So, you know that we have assumed 2 states; one is the either the component is working or the component is at failed state. So, if the component is working we say it is normal state, and if it is component is in failed condition we say it is failed state. So, now when a component is at normal state it may continue with the normal state or it can go to the failed state, that is going from normal state to failed state is we say that is the transition from normal to failed state. And then the failure process, repair process start and after sometime the repair is computed and again the component will come back to the failure for the normal state.

So, it is similar to like this a component overtime time and the component either at the failed state or at the normal state at that time t equal to 0 it instantly jump to normal state, and then it start working, start working and continue working. And, maybe at up to time t it is working condition and then here it and with in between t plus delta or t plus delta t d t d t time this within this time, it may it will undergo a failure then it will its, it will shift to this is the failure state and maybe failure process starts. So, that is what is the in terms of that graphical representation given here, and this is what is happening over time.

So, I have told you earlier that the failure process time we have time to failure and then time to repair. So, both the things will be combined here and that is why this is a combined process, where that failure to repair and again repair to failure both the things will be combined here.

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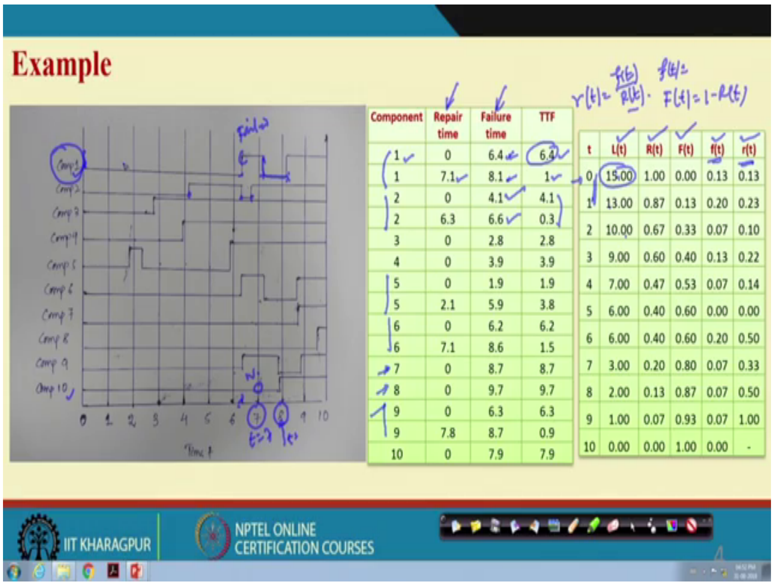

And 1 more example let us see that a better example, where suppose there are 10 identical components put under test, and we are observing overtime that what is happening with the component; whether it is under normal state or at failed state. For example, if you consider component 1 and the time 7 units minute t equal to 7 here.

So, then the component 1 is at the failed state because, here the failure states here it is failed. So, if you consider component 10, you see that component N is under normal state. So, what happened to component 1; the component 1 once it failed then repair starts and I think at around 7 it is under failed state, and it failed at around 6 point some units, and it is repaired again at 7.1 units and then it jumps to the normal state and again it continue work working at the normal state and finally, what happened? Again at unit time unit 8.1, I think 8.1 it failed and the failure that failed state continuous. So, as it is a repairable system what happened?

So, long it is working. So, it is normal and when it fails it goes to the, it is goes to the failure state, and repair process start from there. So, that is what is the concept. Now as all the components are identical in nature. So, we can use this phenomenon it is experimental data, to compute that whatever the same survival function, that is reliability function, this failure distribution, failure density and failure rate using the formula already given to you. Now, you see how you will use this information, what is shown in the graph and you ultimately come to time to repair time and failure time of each of the different components, and from there how do you find out the time to failure and also you can find to time to repair.

For example, although there are 10 components, but because of the combined process in nature failure to repair and repair to failure, what happened actually many of the components are going through failure as well as repair process and again after repair it is working as again at the normal state. So, number of observations that the failure observations is therefore, not 10 it is more than 10. So, for component 1 you see that repair component 1 we started with 0, and then failure repair at the time when it is time t equal to 0 we are assuming that it is instantly repair to normal state. So, the repaired time is 0 and then you see that it work up to this time, in this time which is equivalent to which is basically 6.4 unit time.

So, that mean the component 1 failure time to failure is 6.4, but what happened that, we have considered 10 the time units that in between that 6.4 unit time to 7.1 unit time it was under failed state, but at the 7.1 unit time, this component is again at the normal state because repair process completed. So, what that is what is written here, it failed at 6.4 unit time and again it is repair at 7.1 unit time. So, the difference actually is this one; this is the time to repair. Now, as the component 1 is again operational or at normal state at time unit 7.1; so, we are considering the repair is as good as new.

So, then again you are looking for that how long it basically works, it works from 7.1 to 8.1 unit. So, 7.1 it repair completed it started working and 8.1 unit again fail, and then remain in the failed state. So, that is that is why what happened for component 1 we got 2 time to failure data; one is 6.4 and another is 1. In the same manner or similarly we observe the failure of component 2 component 3 component 4 like this up to component 10 and these are documented here. So, component 1 within this 10 unit of time undergoes 2 fail 2 failures, one at 6.4 unit of time another one is at time failure time at 7 point failure be 8.1 unit of time.

So, 6.4 and 8.1 unit of time 2 failures; component 2 first failure at 4.1 you see that it fails here 4.1 it repair a in this time as 6.3, but again it undergoes failure at 6.6; so, 4.1 and 6.6. So, 2 times it fails it repair and again it fails. So, accordingly you got 2 time to failure data for component 2, in the same manner component 3 1 time unit component, 4 1 time to failure component, 5 2 time to failures, component 6 2 time to failures, 7 1 time failure, and 8 1 observation; similarly 9 another 2 observation for time to failure and 10 1 observation for time to failure.

Essentially if we count that how many then observations we obtain we obtain 15 observations. So, that is why when all those observations we have counted that as if there are 15 units which are put under test. This is possible because of 2 things we say that at the that time 0, it is instantly repair it instantly repair and it is at the normal and with the feature that a property that it at as good as new. And, again when we repair at in a after certain time maybe a time t equal to t naught or t equal to t 1, at that time if the repair is completed we are saying that the repair is also such that it is as good as new.

So; that means, whenever repair takes place another that the new component has been put this is the similar analogously we can tell like this. And as a result this experiment is basically analogously in experiment where 15 identical items are put on the test. So, as a result at t equal to 0, there are 15 units. So, then what will be the reliability that formula you know that there is basically all 15 surviving divided by number put in the test.

So, one and in this manner, that how many failed within 0 to 1 unit the 2 units failed. So, like these 13 surviving 10 surviving this much you have done already. As a result you are in a position to compute all R t the reliability or survival function and F t is 1 minus R t so, that is the failure function that you can compute once you know R t you can compute the F t. Similarly, small f t which is basically density function so, that you can compute for how basically that the within the time interval, what is the number fails and divided by total number put under test at t equal to 0 into divided by the time units. So that means, this minus this divided by this into the difference in time units.

So, that formula I have already given to you. So, you know this. So, you can find out the F t and at the same time R t also you can calculate what is basically R t is basically failure rate or instantaneous failure rate, which is small f t density by the survival. Means it is something like this that what is the probability that the component will fail at within t and t plus delta t giving the details survived up to time t. So, these things you have already computed and, but we have given you the example and now with reference to these, where although we have 10 component under test. But, because of the combined process; that means, it is normal then it is under failure state process then once it failed it goes under repair process, then once is repair again it will go under the failure process.

So, there in that manner what happened? Every component is undergoing through failure and repair and as a result in this case in from this experiment every component is giving us one or more repair time or failure time. So, now, we have considered only the time to failure and accordingly we have computed these parameters. So, similarly you can find out the time to repair and the repair related parameters by repair distribution, repair rate you can compute ok. And, I hope that you will be able to do it and I request all of you to practice this the failure point of view I have discussed, from the repair point of view find out the time to repair from this from this figure and at the same time you find out the failure repair distribution, repair rate, repair density all those things you please find out.

But interestingly what will interestingly what will happen under such combined process that, this reliability or the density or the failure rate they are not the only parameters that describe the state the properties of the component from failure and repair point process point of view. In fact, what if we talk about the reliability and then by definition of reliability if suppose let us talk about the component 1 ok. So, at the 8 unit of time suppose when your t equal to 8, that time if you see the components condition at time (Refer Time: 16:43) it is basically normal.

Now, if I want to know what is the reliability of this component t equal to at t equal to 8. So, if we ignore the failure process here. So, what will happen ultimately? Ultimately you will you will you will basically will get a wrong results because reliability will not talk considered in between that failure states. Reliability means what is the probability that the component of the system will work within this (Refer Time: 17:21) some time here t equal to 8 without failure basically. But what happened here the component of failed and again repaired and it is working.

So; that means, we repair some other measures, which basically will be may be better measure from the repairable units or component of systems behaviour point of view. So, those measures will be of discussion or I can say the focus or the focal points for today's lecture. So, let us see what are those parameters or the and or the distributions, which ultimately describe the combined process more meaningfully.

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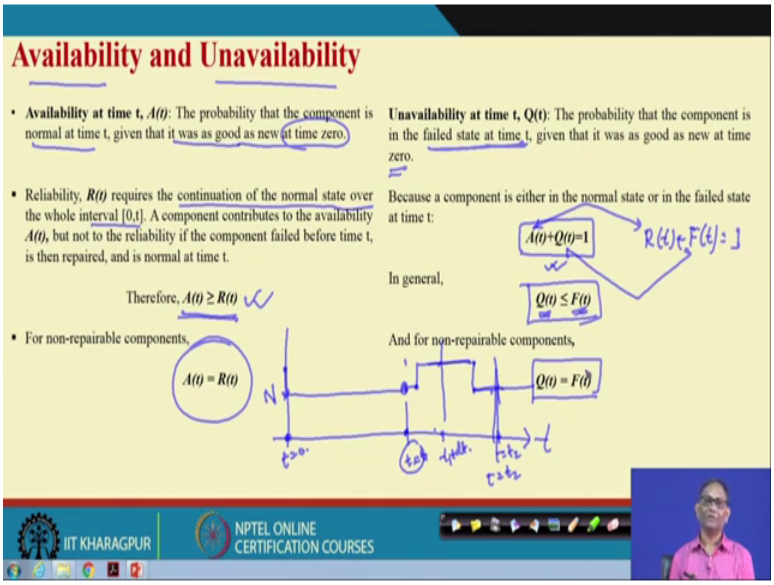

So, we will introduce now to you 2 important concept; one is availability and other one is unavailability. Now let me draw a figure here, suppose I am interested for a particular component let it be a bulb and then what happened it is when it is purchased, let t equal to 0 a new one is purchased. So, we are assuming that it is basically repair to new normal state and then it undergoes operation and at time t equal this is t. So, at this time period it is t plus delta t respect.

Now, if I ask it failed and again suppose it is in failed condition then what happened at this point in time it is repair and then it is again under normal condition let it be like this. So, what happened suppose at any point in on time for example, at this point. So, is the component available you will say yes if I say whether it is available here suppose t plus delta t exactly, you will see know again it is available at suppose this is the t equal to let it be t 1, t 1 plus delta t and if t equal to t 2 is it available? Yes. Because of the repairable nature the component will be available or unavailable at certain point in time that is that is very important.

And you must know that of a repairable system whether the whether component for repairable component whether the component for repairable system whether the system is available or unavailable at a particular point in time, and that definition is also very very important for safety strategy. So, that is why now we formally defined what is availability. Availability is the probability that the component is normal at time t given that it was as good as new at time 0. So, at time t equal to 0 it was new and now you are considering it a time instant (Refer Time: 21:05) t equal to t here, whether it is what is the probability that it will be available, it is available here? If I say no t equal to this what is the probability that it is available here.

So, you have to keep in mind this one. Availability means at a particular point on time we are interested to know whether the component of the system is in normal state or not. Then unavailability means just the opposite that at a particular point in time, whether the component is failed state or not in component is in failed state or not, but for both the cases please remember that the component was as good as new when it was first manufacturer or first started operation.

That means, at the beginning when t equal to 0, they as good as new that is what is given here. Probability that the component is normal at time t, given that is it was as good as new at time t equal to 0. What is unavailability you see? The probability that, the component is in the failed state at time t, given that it was as good as new at time 0 ok. So, that is why what happened a component will be at a particular point on time, it will be either available or not available 2 states. As a result the probability of availability or unavailability so; that means, if the sum of probability of availability and sum of probability of unavailability will be 1 that is written here. So, if you recall my previous lectures. So, in one lecture I say that reliability plus failure that is cumulative distribution that also one we should.

Now, we are saying availability and unavailability the probability is also one, but there is fundamental difference between because reliability and availability as well as unavailability and failure probability fundamental difference is there. So, I told you earlier, I started describing what is the difference when you talk about reliability, reliability requires the continuation of the normal state over the interval 0 to t ok. So, if I say I want to know the reliability here at t equal to t 2, see what happened for this component this component has failed here. So, it violates the assumption that it is continue or working within this so, but for availability it is not. So, a component contributes to the availability, but do not contribute to the reliability if the components failed within the unit of time.

So, as a result a component availability probability of a components availability will be more, then the probability of the of the components available reliability or other sense I can say the probability of component having at normal state and probability of component fails with the less than that time, they talks about 2 things one is availability the first one, and second one is the reliability and because of the requirement of continual that normal state condition, these relationship holds for any component or system also. That means, availability is more than or equal to reliability. It is obvious because that failed once it failed it will not contribute to reliability, but once it failed it is repair and then again it is under normal condition, that that contribution goes to availability so, fundamentally this is correct.

But if the component is not repairable means it is a non repairable component, then the failure then the again you will not get it under normal state. So, at this situation both will be same availability and reliability are same I hope you understand this. In the same manner analogously we can say this that unavailability is less than that cumulative failure probability. And this one will be equal when component is non-repairable ok. I hope that you understand the 2 concept one is availability another one is, unavailability the relation between availability and unavailability, the relation or the inequality from availability and reliability point of view and unavailability and failure probability point of view that relation that you understand.

And for non repairable system availability equals to reliability and unavailability equals to failure probability. So, that is why the reliability and failure probability are sufficient enough to explain availability or unavailability. So, you do not require availability unavailability separately there. But when it is a combined process reliability and failure probability will not give you the complete picture, only as availability and unavailability is also taken into consideration, this is our first concept.

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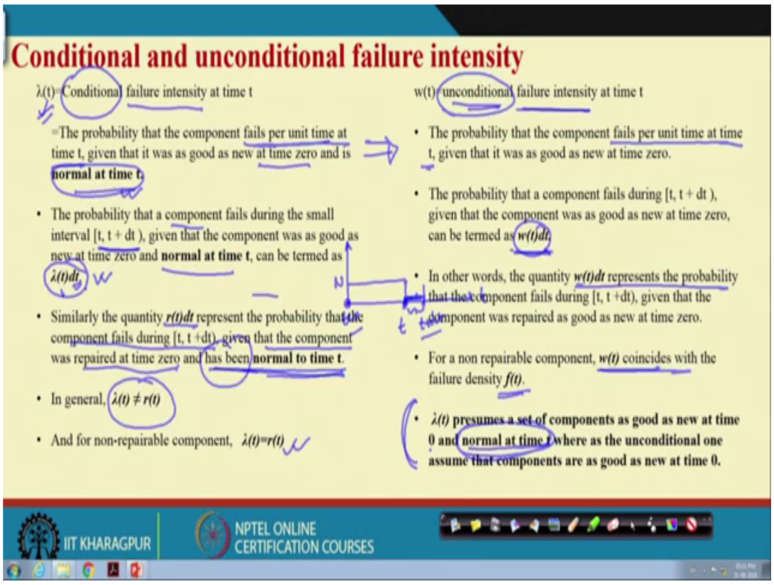

Then the second another important one is that very important one, that failure intensity. Now failure intensity can be conditional can be unconditional ok; so, both case we are talking about failure intensity. So, what is the failure intensity? So, failure intensity is also the probability that a component fails per unit time; fails per unit time at time t; here you see the component would fail component fails per unit time at time t.

Now, now go to the availability again let us go back to availability probability that component is normal at time t, but where what we are saying under conditional? That we are talking that fails per unit time per unit failure, but when? Given that it was as good as new when at time t equal to 0 and normal at time t ok. So, if you draw suppose you draw the figure again, well suppose this is the figure.

So, this is t what we are saying? Suppose t times elapse we are we are having basically a unit time let us consider t plus small unit time. So, usually we should t plus delta t. So, we are interested to know that what is the probability that the component will fail per unit time here within small if we if we make the time is in very small, but given that it was as good as new when t equal to 0 plus it is failed at time t equal to it is. So, I just taken a 1 minute please yes.

So, time t equal to 0 and normal at time t equal to t so; that means, suppose this normal time t equal to t. So, similarly we can do same thing for the repairable case repair process also, that mean the conditional repair intensity unconditional repair intensity. So, first we are describing the conditional failure intensity and unconditional failure intensity. So, what are the condition here? Condition imposed is that the component is normal at time t what is the probability that the component fails per unit time? At time t given that at time t is equal to 0 it is as good as new and it is normal it is normal at time t.

Now, if I go to the unconditional part here, what is the difference? Probability that the component fails per unit time at time t, given that it is as good as new at time t equal to 0; no condition imposed on the component at time t that is why the unconditional word is used. In fact, for all the component cases at time t equal to 0, we are considering that it is a new component. Now, this is denoted by lambda t. then what is lambda t d t? Lambda t d t talks about that a probability that a component fails during this small interval within this t and t plus delta t, the component will fail what is the probability that component will fail within this small unit of time?

When we are talk and when you make it this small unit at unit time, then it is basically lambda t; that means, when I make this delta t equal to 1 it is nothing, but lambda t. So, what are the condition imposed? Condition imposed is when at time t equal to 0 it is new, another condition imposed is it is normal at time t equal to t. Now, now come to the another important quantity that is r t and d t. So, you know what is r t? I explained earlier r t is failure rate then what is r t d t? Failure rate times multiplying the small time unit. It is represent that the component fails during t plus delta t that component was repaired at time t is equal to 0 and has been normal to time t.

Its time t equal to 0 it is repaired to in such a may not that as good as new, but it has been normal to time t. But in case of lambda t d t we your not saying it has been; when once it repair after repaired in the next that time we are considering that is normal it is not like this. In the first case at that particular time in stand it is normal that is why we are writing it is normal is normal at time t, but here your writing has been normal to time t.

So, as a result both are talking about probability that the component will fail within the small interval t plus delta t, but in the first case lambda t delta t we are talking about that this probability, that the component is as good as new at time equal to 0 that is 2 for the both the cases, but it is normal at time t at normal state, but in the second case it has been normal. So, that in between there is no failure.

So, as a result what happened these 2 lambda t and r t they are not equal and if you consider that it is a non repairable system then it is equal. General case repairable case it is not true. So, analogously you can create this w t d t. So, what happened? Only the condition at time t that is that is relax here in the unconditional case. So, as a result what is w t d t? W t d t is the component fails in the small interval t and t plus delta t given that the component was nu or as good as nu at time t equal to 0.

And the quantity w t d t represents the probability that is what we already told you and now what will happen for non-repairable component w t and F t failure density and unconditional failure intensity they are they basically equal the crux of the matter. The difference here apart from the definitions given here the difference here is that lambda t which is basically conditional failure intensity, it presume a set of component as good as new at time 0 and normal at time t. Whereas unconditional case this assumption is relaxed please keep in mind. So, what you have learnt so, far then you have learnt many things quickly for today's presentation point of view.

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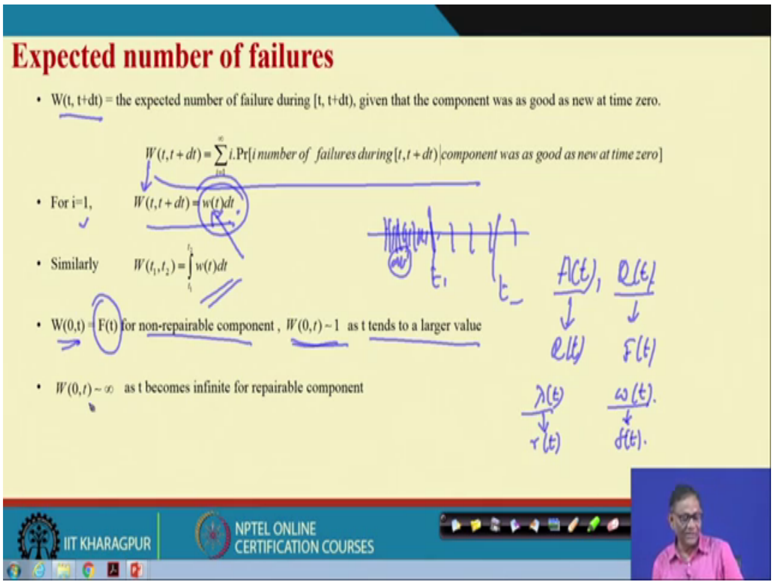

Let me tell you that you know availability, you know unavailability, you also know the their availability relation with reliability unavailability with F t and then what we have discuss? We have discuss lambda t conditional failure intensity w t unconditional failure intensity, then lambda t its relation with failure rate and unconditional intensity with the failure density.

Apart from this with example, we have seen that how the computation is possible, but that availability and unavailability, this intensity and unconditional intensity that computation I will give you after 1 or 2 minutes. But before that one more important concept you must know that we are talking about, that if once a component fails, it will be repaired and again it will work and again it will it will be repaired and repair process will bring the component to as good as new condition.

So, that is why we are interested to know another quantity which is expected number of failures. So, what is the expected number of failures in between t and t plus delta t a small time, given that the component is as good as nu at time t equal to 0, then this is the formula is very simple one. So, it is basically sum of i times probability that i number of failures during this small interval of delta t time, and given at the component is as good as nu at time t equal to 0. Now if we if the delta t interval is so, small that only one failure can take place then this lead to this quantity.

Small you choose the your time interval like your basically choosing the time interval; is make the time interval very small delta t very small, in such a manner you make that within is very small time only if failure occurs only one failure possible. Then i will be 1 and in that case this is W t expected that is number of failures will be w t's into d t where w t is unconditional failure intensity and d t is the small time interval.

Now if you want to know within a within 2 time interval like t 1 and t 2. So, in then you do integration ok. Just what will happen if it is a non-repairable component? This what is the expected number of failures in between 0 to t it will basically the failure probability. And, it will become 1 if you consider longer time sufficiently long time and the number of; that means, expected number of failures will be infinite, if t is infinite for repairable system.

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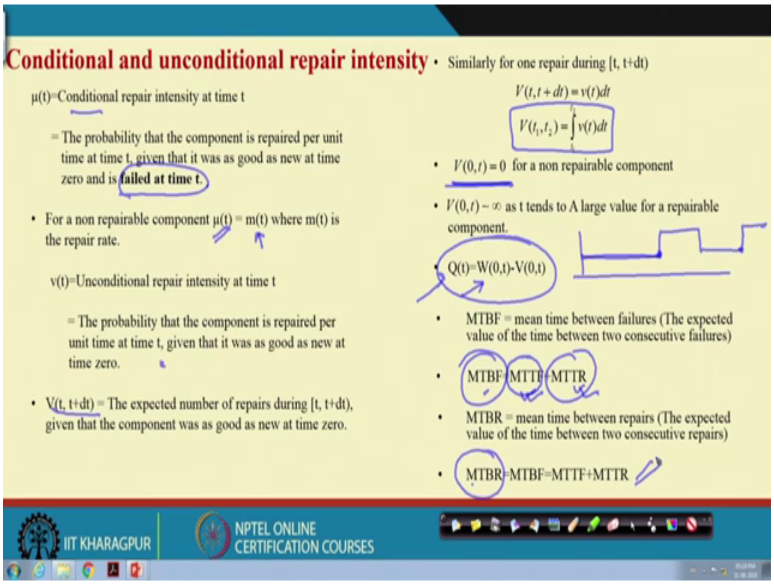

So, I will not discussed this conditional or a conditional repair intensity. The discussion what we have made so, far with reference to conditional and unconditional failure intensities that is sufficient for you to understand this slide. I have given sufficient material in the slide so, that even if you do not understand many a times. So, you can go through and recollect what you have learnt earlier at the same time, you pause the video again listen and understand

So, here only thing I want to tell you when you are talking about conditional, when you are talking about conditional part, you see that failed at time t equal to 0 when you talk about unconditional part such situations such condition is relaxed ok. Otherwise what happened this mu t? Mu t is the conditional repair intensity and m t is the repair rate.

So they will be, they will be same for non-repairable system and like expected number of failures you also required to know expected number of repairs. So, that is what is V and V t d t and t plus delta t. The expected number of repairs means within t small time interval how many component will be repair, when we consider that they were good and they were nu at time t equal to 0 ok. Analogously you will within a time interval t 1 and t 2 this is the number repaired. And, similarly this will be 0 that repaired that number expected number of repair will be 0 for non-repairable system.

And expected number of repair will be infinite for repairable system is t is very large, t tends to infinite and another interesting one is this. What is this Q t? Unavailability, probability that the component is unavailable at time t equal to 0 when it is good unavailable at time t equal to t when it is as good as new at time t equal to 0. This is the different between number expected number of failures and expected number of repairs ok. And, in combined system in case of non repairable case you have seen that meantime to failure. And, when we talk about the combined system and only repairable, but we have considered we have seen that meantime to repair.

Now the failure and repair both processes combined. So, you are getting meantime between failures or meantime between repair. So, normal fail again normal again fail again normal like this. So, now, this is your normal time this is fail time; so, now meantime between failures. So, this is my first failure and then it is repaired and then again it is second failure. So, what is the meantime between failures? So, that is why it contains both meantime to repair failure and meantime to repair. The similarly meantime between repairs also will be the same, meantime to failure plus meantime to repair. So, this is what is the final one. So, I just ask you to go through this book Kumamoto book and you please see this example, similar example there.

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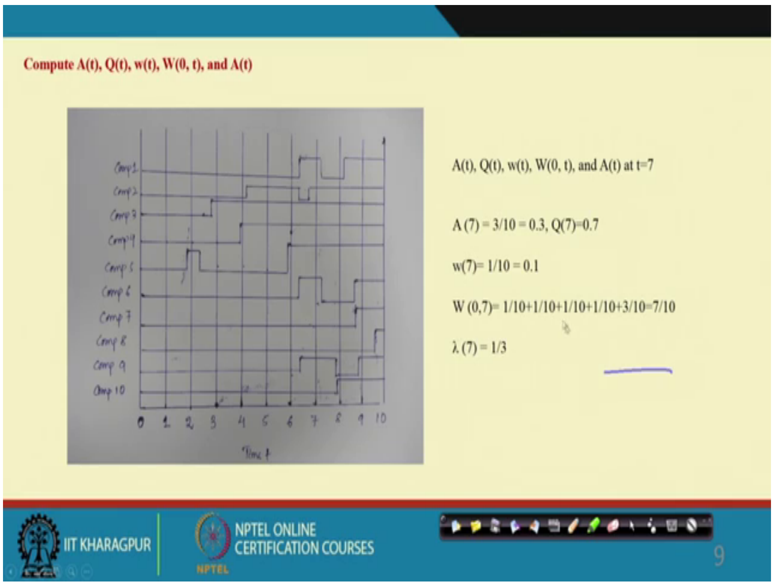

And the computation part, whatever definition I have given to you here that is sufficient, and you will be able to compute availability, unavailability, unconditional failure intensities, expected number of failures all those things. So, with reference to this, this is given, go through practice it, write on the discussion forum and the books like this.

Thank you very much.