Industrial Safety Engineering

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Lecture – 31

Quantification of Basic Events Failure to Repair Process

Hello everybody, we are continuing Quantification of Basic Events. Today, we will see the repairable unit. For a repairable component case, there will be two processes, one is failure process; that is repaired to failure process, another one is, once the item is failed or component is failed it is to be repaired that is failure to repair process. So, the in a repairable unit, there will be two process; one is the, suppose the component starts working after certain amount of time spent, it fails then it will be repaired. So; that means, one is the failure process, another one is the repair process and we are writing these are that failure repair to failure and failure to repair process.

So, we have already seen the first part that is repair to failure process in terms of non repairable unit and we have defined the distributions different parameters there distributions and then how to estimate the parameters with the assuming certain probability models or probability distributions. Now, today particularly in this lecture we will see that the failure to repair process; that means, what is the repair process, it is similar to repair to failure process. So, the analogy or so we will see at the same time we will define some of the parameters in this lecture, which is particularly for repair, not for the combined failure to failure and repair, it is the particularly for repair process.

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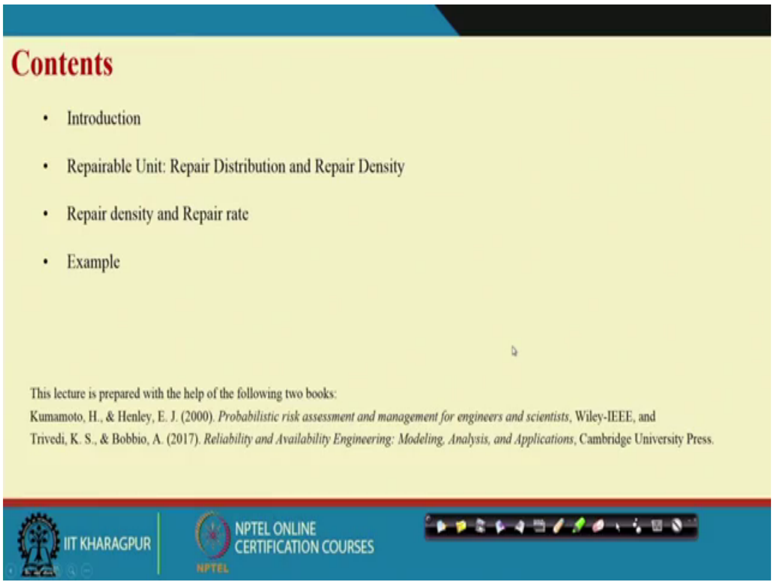

So, the content is one is the repairable unit we will revise it again and then the repair distribution, repair density, repair rate all those may be analogous to failure distribution, failure density, and failure rate. And, then we will discuss a hypothetical example how to compute all those all those parameters and also we will discuss mean time to repair. And, we are primarily following the book written by Kumamoto, H and Henley, E. J. Probabilistic risk assessment and management for engineer and scientists.

So, if you do not have this book, please have this book and go through the relevant pages and I am sure you will be able to understand it even if you do not fully understand in this class, but I will try my best to make you understand.

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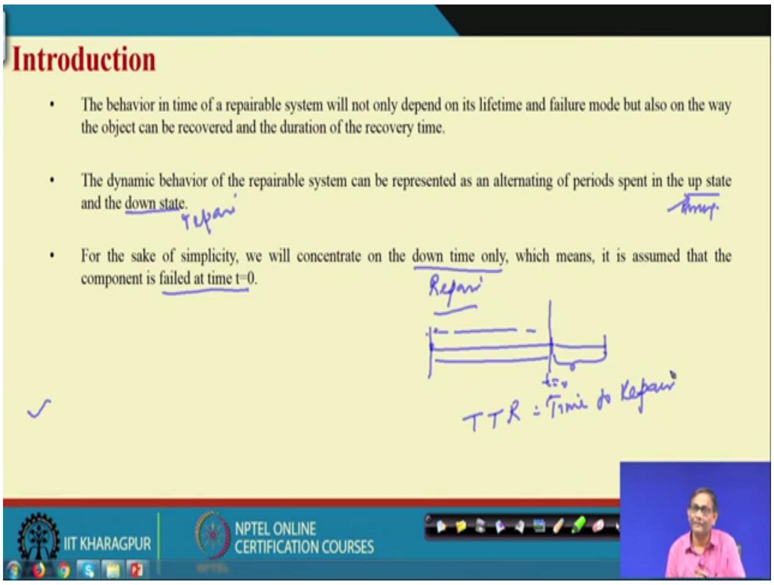

So, as I told you the behavior of time a in of a repairable system will not only depend its lifetime and failure mode, but also on the way the subject can be recovered and the duration of the recovery time. That mean here the object can be recovered, mean the component can be repaired and what is the that repair time, that also important here. And the dynamic behavior of the repairable system can be represented as an alternating of periods spent in the ups and down state, what is up? Up means working and what is down? Down means it is under repair. So, there are, there will be, there are two state, up state and downstate. Up state means it is the working state, down mean repair state.

So, for the sake of simplicity we will concentrate on down time only as I told you that in this lecture we will be interested only in repair process. So, suppose the thing is like that that suppose the component start working here and after time t it fails then this is the repair time. So, we will be interested in this, not this working part and what happened that is why we will assume that the component failed at time t equal to 0, if it is and it may not be. So, what we will do basically we will shift the origin to here, we start here t equal to 0 that is the assumption.

So, as if we are concentrating only on a failed only on a failed item or failed component we are doing repair and we are basically interested to know the several characteristics, several parameters of that process that mean one start of repair to end of repair the completion. Now we will generate random variable which will be the time between start of repair to end of repair which will be time to repair, time to repair this is known to you. I have discussed earlier.

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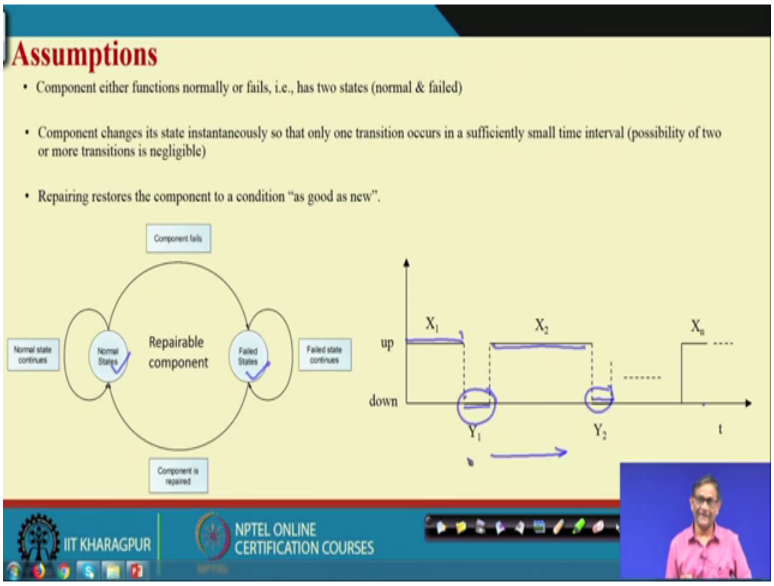

We said that the component will be either at the normal state or failure states. So, if it is in normal state it will continue at the normal state or it basically there will be transition from normal state to failed state. When it is in failed state then it continue to in to be in failed state and once it is repaired it will go to normal state ok. So, then two state system. So, this is over in on the time axis given like this, component is up means it is basically normal state. So, this is the time when it fails and then it is the repair time or failure time. So, along it is under failed state. So, again it is up so, normal time. Again it is failed and this is the repair time and in this process this is the combined process going on.

So, in the non repairable unit or only that only the failure process case we have denoted X is the random variable which basically talks about the apology that the component will that component time to failure and then here we are defining another random variable Y which is time to repair. So, what is the content of today's lecture? We are interested in this, this, similarly like this we are interested in Y and we want to find out some several parameters related to Y. So, distributions all those things and Y is basically time to repair it is a random variable.

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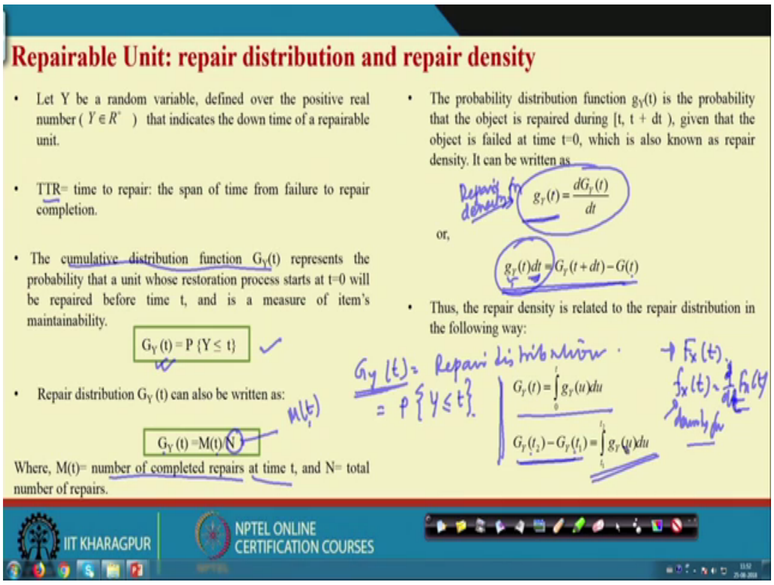

So, now, let us define formally some of the parameters. So, as I told you Y is a random variable defined over the positive real number indicates the downtime of a repairable unit. Downtime means the total repairable time and that is nothing, but the time to repair the span of time from failure to repair completion. Its fails and then start repairing and when repair is completed it will go back to normal state that time is TTR.

So, now what we will define here, first we will define the cumulative distribution function G Y t that is cumulative distribution function of repair. So, what is our G Y t that is basically repair distribution. What it is? It basically represent the probability that a you need whose restoration process starts at t equal to 0 will be repaired before time t ok. So, that mean this is probability Y the time to repair less than equal to t. So, if you recall the failure distribution there we have used failure distribution F x t and we say this is cumulative failure distribution or failure distribution we say that probability that the component will fail or x less than equal to t with in time t. Here we are concentrating on the repair process, so what is the probability that the component will be repair within time t is G y t capital G y t that is what is written here. So, our first concept is repair distribution.

Now, suppose what happened at time t equal to 0 there are N number of capital N, N number of identical component under failed state. And, you started repairing them from t equal to 0 you start and then maybe after time t you found out that a M t, M t this is the number of component completed repairs within time t. So, this M t is nothing, but the number of component repaired within time t. Then what will be the repaired distribution? Repaired distribution is number of component completed repaired by total number of component under repair at time t equal to 0. So, that G y t this is the from probabilities concept this is also probability, but when you have the data you are able to find out the values. So, this is the way we will basically compute the repair distribution or repair probability.

Now, this cumulative distribution function when you take derivative of it then you will get density function that is what you have seen earlier in failure distribution we have seen earlier F x t and then we say that there is small f x t which is d by d x F x t capital F x t. So, the derivative of if this is the cumulative distribution this is your density function. You can go from the basic fundamental of density function I mean what is density that from the histogram also, but those things we have discussed earlier. So, also we have discussed that if you know the cumulative distribution function then you will be able to find out the density function by taking the derivative of it what time.

So, that is why here from the repair distribution to repair density function. So, what is the repair density function? Repair density function, repair density function small g y t equal to d by d t d by d t G y t by d t. This is sorry this is t also. So, this is our formula. Now, this can be written in this manner. Then what is this value g y t into d t? This is basically talking about that probability that the component will be repair in between t and t plus d t within this small d t time the probability that component will be repaired.

So, this is density, this is the time interval multi density multiplied by time interval will give you the probability and it basically talks about within a small small interval in time what is the probability that a component will be repaired. So, now this one can be written in this form also, if I know the density I also know the distribution; that means, what is the cumulative distribution if you know the density, it is the integration of density. So, that is what you have written here. So, G Y capital G Y t equal to 0 to t small g y u d u, so and if you take integration here it will be G t right hand side will be G t.

Now, again that this one if I have two different times the t 1 and t 2 in between what is the probability if that the component will be repaired between t 1 and t 2. Then you also can find out from this, from this equation you integrate in between t 1 and t 2 g y d y.

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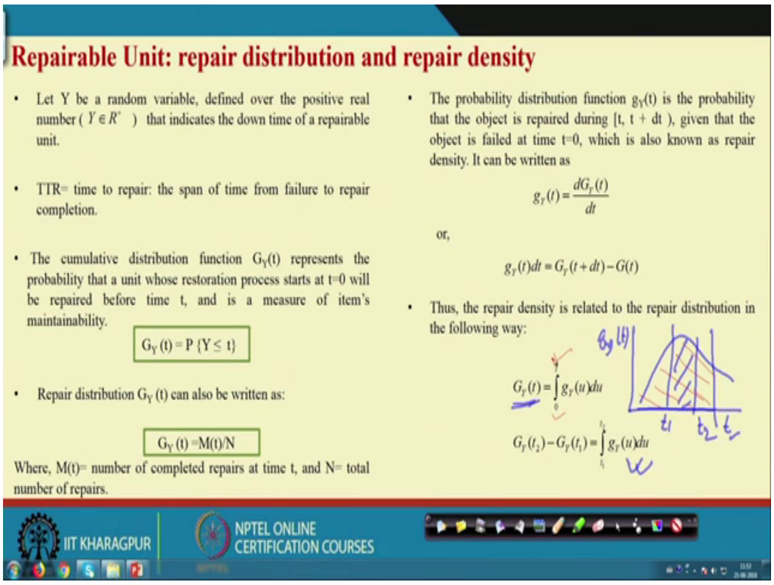

So, suppose if I say my repair distribution is this that is g y t I take two values that is t 1 and this one is t 2. So, what is the probability that component will be repaired within this t 1 and t 2? It is the area under this curve in between t 1 and t 2 that is what you are writing. Suppose if I consider that t this my t value is, t value is this then I want to know what is the probability the component will be repaired within time t. Then that is nothing, but G y t and this is the integration of under this equation the area under the curve from 0 to t.

So, that is what we have seen the same thing in failure distribution case. Now in repair distribution here we are seeing the same similar thing. So, there same only the, but difference each one talking about time to failure, another one today is one talking about time to repair. I hope that you got it. So, what do you learn now repair distribution, repair density function and their relation and we are considering only the repair process and with certain assumptions already given to you.

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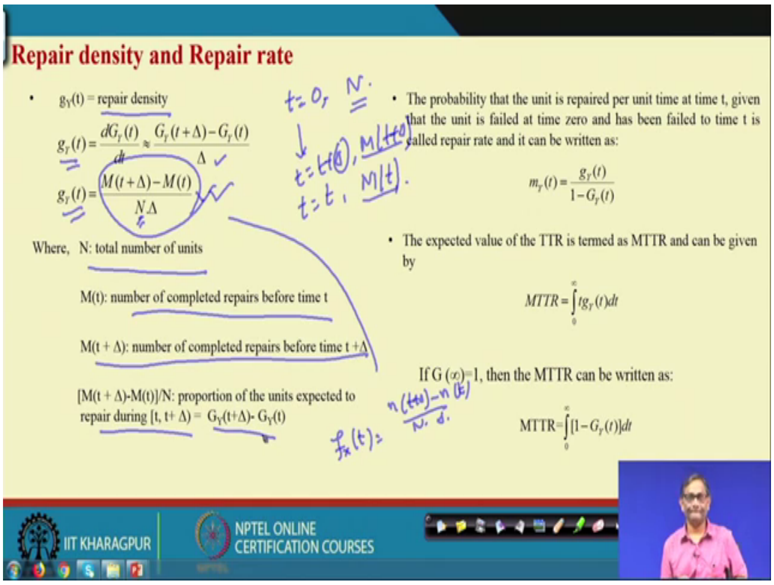

We will now go to repair rate from repair density to repair rate. So, repair density you have already seen that d by d t into G y t this can be written like this G y t plus delta minus G y t by delta. So, now G y t plus delta minus G y t if I write in terms of M, what is M number of units repaired within t plus delta then G t plus delta will be M t plus delta by capital N because capital N is the number of units under repair at time t equal to 0. Similarly G y t will be M t by N then the resultant quantity will be like this. So, if you at t equal to 0 there are N component fail component under repair. And, at t equal to t plus delta M t plus delta is repaired and at t equal to t M t components are repaired then the difference M t plus delta minus M t this is nothing, but the N number of components repaired within delta amount of time.

So, and divided by if you put divided by N ensure that that will be the proportion of component which is repaired by time delta and if I want it any time in unit time then you have to divide it by delta and that is what is the density. So, repair density in terms of experimental data. It is again if you can you have found out this one in failure density also. So, if you recall a failure density what we have written there, we have written fx t equal to we have written n t plus delta minus n t divided by capital N into delta, this you have seen earlier list.

So, all of you know N is total number of units, M t is number completed repairs before time t before time t plus delta and the difference by this the proportion of unit expected to repair within this time which is this. In other way when we actually go in terms of probability we say that probability that the one component failed component will be repaired with the next time t and when you have experimental data we have computing in this manner. So, this is basically repair density explained. Now, we go to repair rate. What is the repair rate? Repair rate is that probability, repair rate the probability that the unit is repaired per unit time

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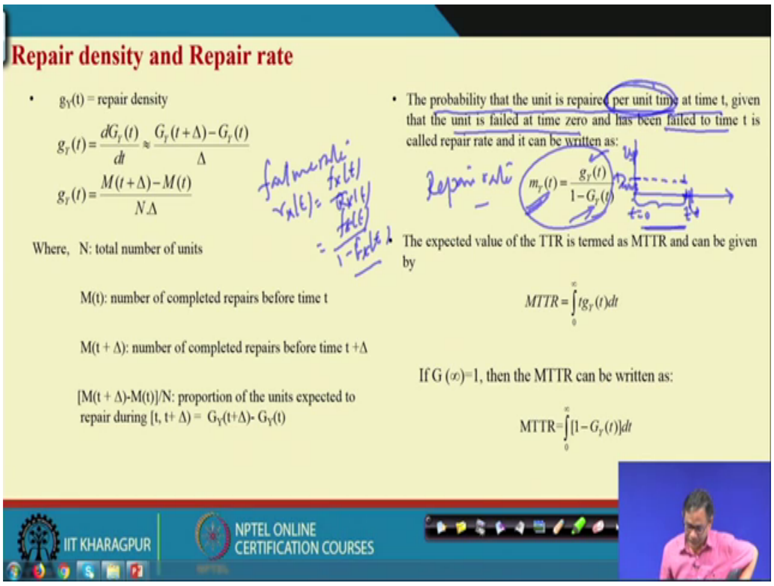

Please remember, unit is repaired per unit time at time t given that the unit is failed up to time at time zero and has been failed up to time t; means suppose this is my repair at the time t equal to 0 the component fails. And, this is time t equal to t till now the component suppose this is my downtime and this is my uptime, till now the component is here.

Now, what do you want basically, what is that it will be repaired, it will be repaired in immediately instantly after time t. So, that ratio each talks about main mean there are this is a conditional one that mean up to time t it is under failed not repaired, it is in the failed state. And, at same time it will be repaired what is the, what is that quantity this quantity is talked I am is denoted by our repair rate.

So, it is similar to failure rate, in failure rate what we have considered that the component is working at time 0 and it is working time till time t equal to 0, but what is the probability that it will be failed immediately after that; that means, instantaneous failure and there what happened we considered that the failure conditional to survival up to time t. Here also repair conditional to that the system is at failed up to time t. So, the same way it is written here. If now, let me go back again for that failure rate because we have we have seen this one earlier failure rate r x t, there what we say that we say f x t by r x t, what r x t r x t 1 minus F x t.

Here the equivalence is capital F x t is equivalence is G y t and small f x t is equivalence is small g y t, I hope that you understand now. Then what is the that within the within very small amount of time let it be delta time, it will be or otherwise in that per unit time it will be per unit time it will be repaired.

So, this is an important concept for all of us. Now, we will see that as we have the repair process our sole purpose here is to repair as quickly as possible and the parameter of the repair process which basically talks about what is the performance of the repair process is MTTR: Mean Time to Repair.

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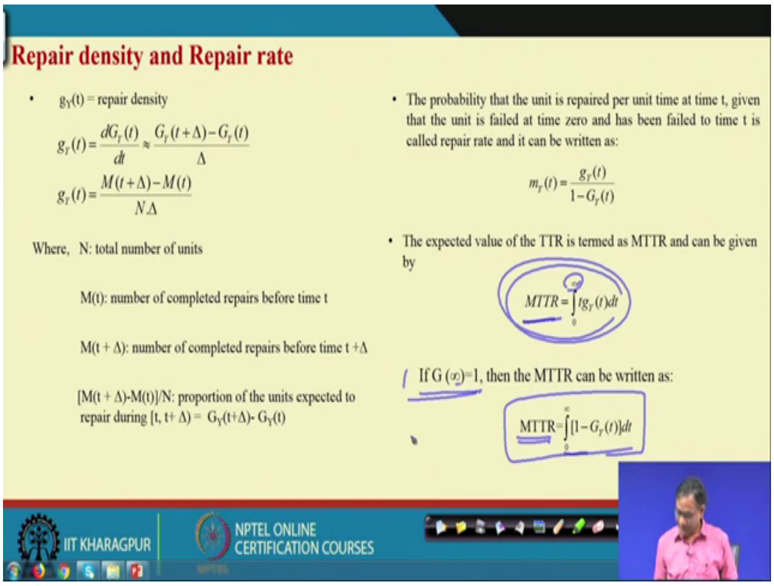

So, now mean time to repair will be computed using this formula that mean t g y t d t 0 to infinity changing from 0 to infinity. Now, when this time is infinite suppose I given a failed component you are given infinite very large extremely large amount of time, what is the probability that it will be repaired? It will be repaired with one probability. So, then if this is the condition I mean when t equal to infinite then your mean time to repair is will be with this formula 1 minus G y t d t fine.

So, I hope that you got the idea of repair process, repair process means it component failed and you start repairing it. Now, time to repair is the random variable which is the positive quantity and that follow certain distribution definitely and we had we have tried to find out all those distribution. First the repair distribution, that mean the component will be repaired within time t. Then we have seen that repair density and then we have gone for repair rate.

And finally, we have seen that: what is the performance measure in terms of mean time to mean time to repair. I hope that you have understood all those things because it is similar to failure distribution. So, let us now see one small example that how we will come make all those distributions.

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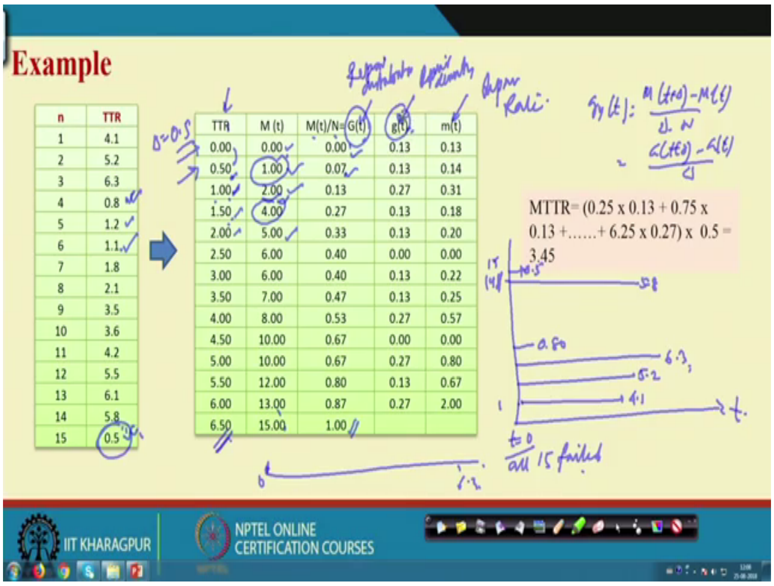

Let us see, suppose we have 15 identical components create a time line t here and this side time t equal to let it be 0 all 15 failed. Now, your repair starts considered that the repair process is similar for all the identical components.

Now, the first one will be repaired, first one let that this is the first one it will be repaired may be by 4.1 time unit, second one 5.2, third one 6.3, fourth one 0.8. So, like this if I go for the fourteenth one 5.8, fourteenth one and fifteenth one is 0.05, 0.5 not 0.05 fifteen one. So, this is what is the suppose you have done this experiment 15 identical component you have considered at time t equal to 0 everything has gone to every component gone to identical repair process and then you found out that this is the distribution.

So, now we want to with this data we want to find out what will be the G t value that the repair distribution, repair density and repair rate. So, repair distribution a repair density repair rate. So, as the TTR values are known, so we added here 0 also because at time 0 anyhow let us consider from here we just found out this TTR values we arranged in ascending order. So, first 0 then followed by we have written 0.5 because this is the one component which is repaired within this time then the next smallest unit is next smallest unit is what we have written 1.1 ok. So, number of units repaired 1 and 1 it is I have written 2.

So, is there any other value 0.8 yes 0.8 is there. So, if I arrange them in ascending order that mean from 0 to the maximum one will be I think some value 6.3 6.3. So, you will get 6.3. Now, what happened we have created TTR this is also time to repair for individual component we are created a generally general TTR time to repair where we started with 0 and then we have given 0.5 increment.

So, 0 to 0.5 1 1.5 to like this; so, I can say let the delta is 0.5 that way you have created. Then what we are finding out that how many component repaired at each of the. So, as each of the time now if I want to know what is the amount repaired at time t equal to 0 it is 0, but number of component repaired at time 0.5 we found that one component is repair. So, we have written 1. So, number of component repaired by 1 unit of time that we found that 0.5 and 0.8 is 2, similarly from 1.5 so, that when this 2 plus 1.1 and 1.2 there less than 1.5. So, 4 units like within to 5 units, similarly within 6.5 units of time all 50 units are repaired. So, this is basically M t means cumulative number of repair.

So, at with in time 6.5 we have need all 15 components have been repaired here. We have not considered that in a something not repaired, we will consider all put under test repair, put under repair is repaired. Then your repair distribution is M t by N this one and this is basically 0 N is what, N is 15 then 0.7 like this is this is 1. What will be the g t values, g t values will be that this minus this G small g y t what we found out M t plus delta minus M t by delta into N or we have found out G t plus delta minus G t by delta. So, M t plus delta by N that is g delta equal to s.

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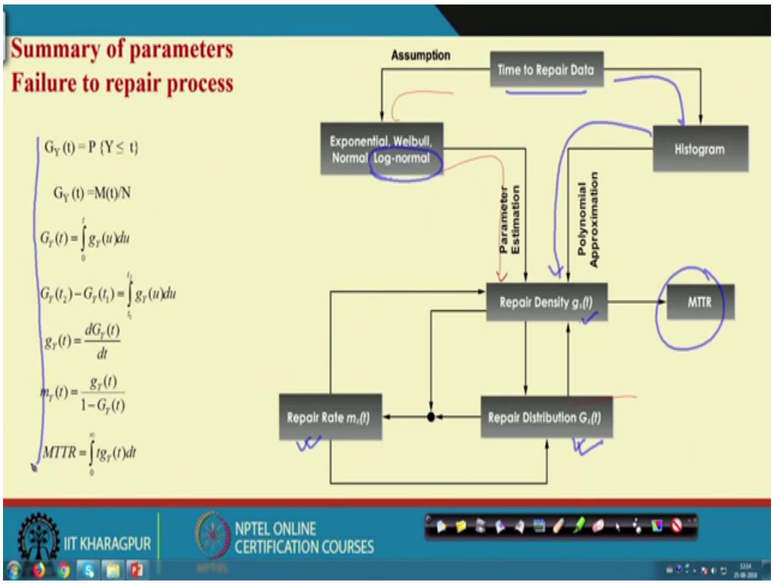

So, now the using the formula or if you want more conventional way of learning so, you just see I have already given to you sorry what is happening you see M t plus delta minus M t by N into delta. So, this formula you use here and you will be getting G t values and then M t values M t values will be G t by 1 minus this.

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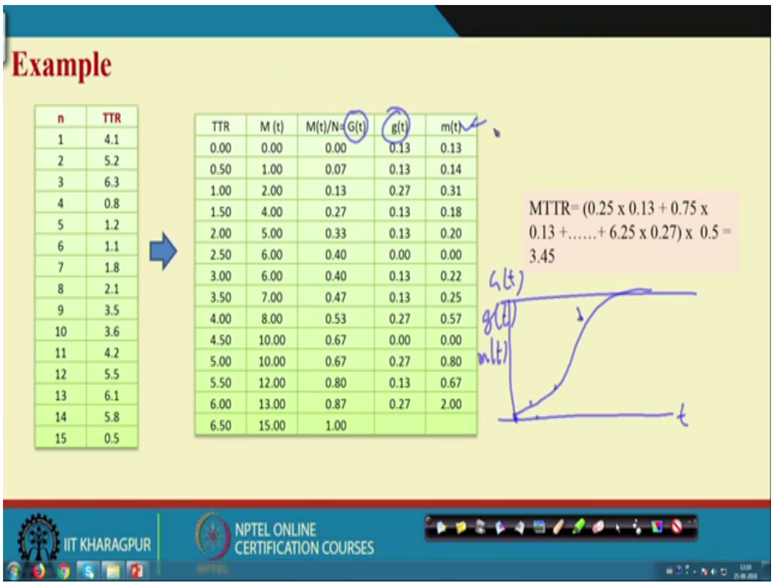

So, this will give you these values. So, now, you can plot, if you plot what happened this side t and this side your G t this side maybe small g t, this is small m t. You will get different plots now this value G t 0.7 0.13 like this it is up to 1. So, first when t equal to

0, it is 0 when t equal to 0.5 it is basically 0.07 and when then point a 0.13 or some curve like this we will be getting that is g t and similarly small this one also you plot you will be getting some other type of curve and hazard rate here it is basically repair rate. You will get repair rate equivalence failure rate the similar curve we will be you will be getting, is it constant no here it is changing.

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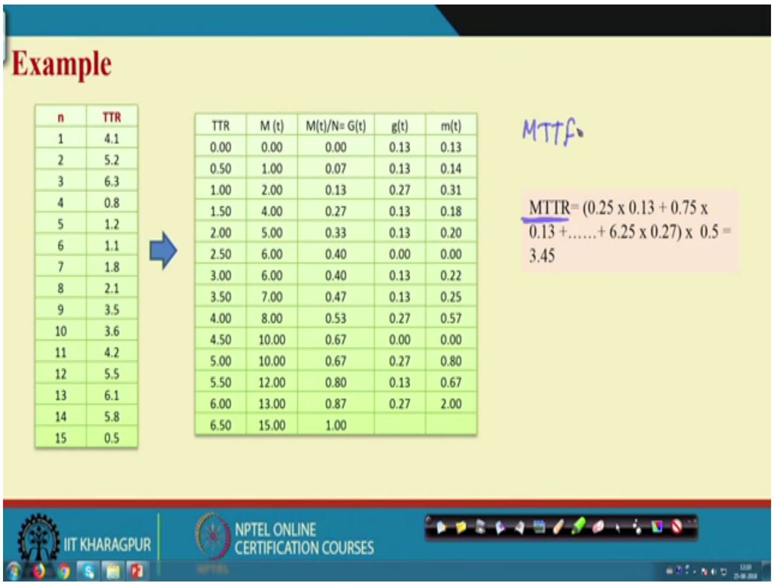

So, you definitely have understood this and then another issue that how to compute the mean time to repair from this data. How do you computed mean time to failure from this data, same manner you compute what is there, what is the formula for MTTR integration t g t d t 0 to infinite.

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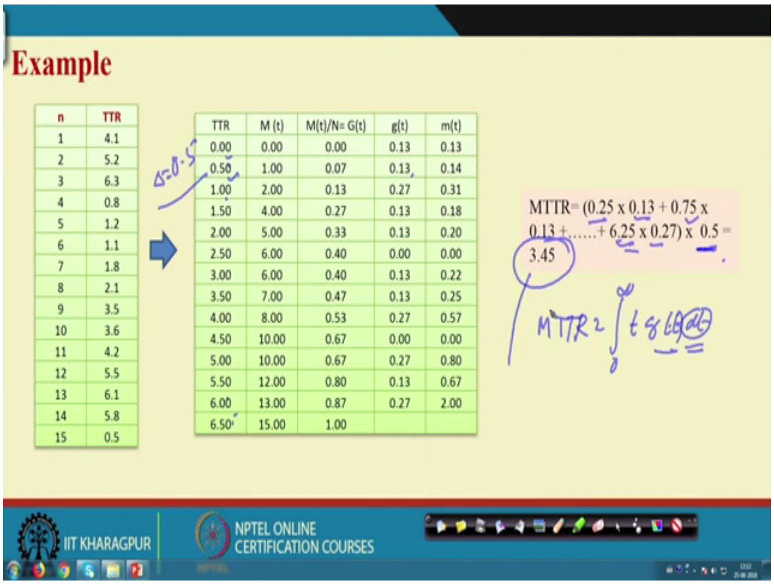

Now, you see what we have done here what is the d t value everywhere I said that the difference is 0.5. So, that is your d t or delta values. Then what will be the t values, t values will be the in between; so, here 0.25 first one 0.25 1.25 sorry 1 and 0.5 that is 0.75. So, similarly last one 6 and 6 plus 6.5 by 2 6.25 and what is the g t values g t values are there. For first one it is 0.13, second one 0.13 and the last one 0.27 and what is the d t value, d t value or delta value equal for all the cases t values. So, that is why we have multiplied finally, this one and you are getting this values this is what mean time to repair.

On an average if a component is failed then it, on an average a failed component will be repaired by this unit of time, this is average time. Now, summary you are seen the summary related to time to failure data now time to repair data. So, once you have data, you develop histogram, then through polynomial approximation you can find out G x t. And once you know G x t, the other distribution you will be able to find out, you will be able to repair distribution, you will be able to find out repair rate, you will be able to find out mean time to repair.

And we have given you in reference to time to failure data, how the exponential and Weibull distribution is used. So, here also you have to find out what is the appropriate distribution. For repair process many a times you we use log normal distribution. So, that mean time to data if follows log normal distribution; that is another path you find the assumptions, estimate the parameter then you get this. Again you find out all other things and the most fundamental formulas are given here. These are the most fundamental formulas and these formula formulas, if you compare with the failure process formulas. only failure process, then you will find out 1 to 1 analogy here. So, it is very easy to remember; that is why.

So, now what happened, we will move to the combined one; that means, 1 failure process,1 repair process, if we combine the two then that is the most general one that is the repairable unit case. A repairable unit once it fails it will go through repair process then it will work till failure that is a failure process. So, these two process will be combined and we will be telling that is combined process, which is most aptly fit for the repairable component or repairable system failure analysis.

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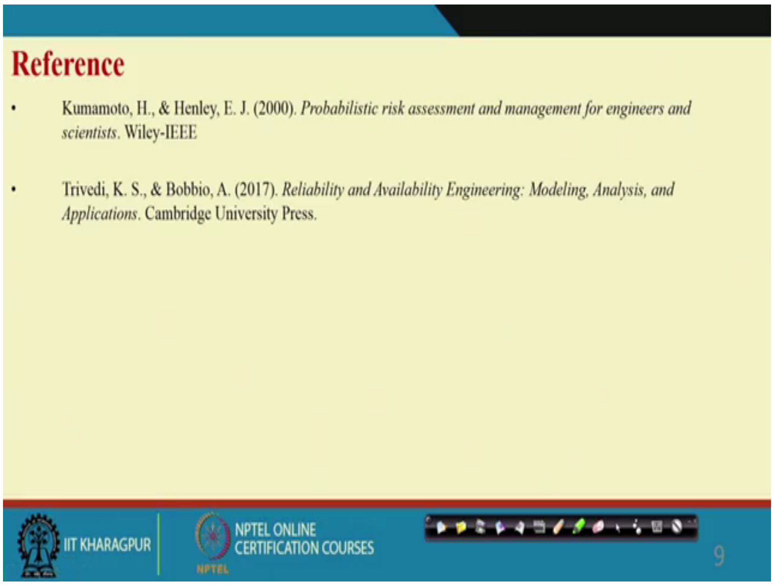

References are already told to you.

Thank you, I hope you have understood it.