Industrial Safety Engineering
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Lecture – 28
Quantification of Basic Events – Hazard Rate

Let us discuss Quantification of Basic Events. Today, we will be considering the Hazard Rate or Hazard Function.

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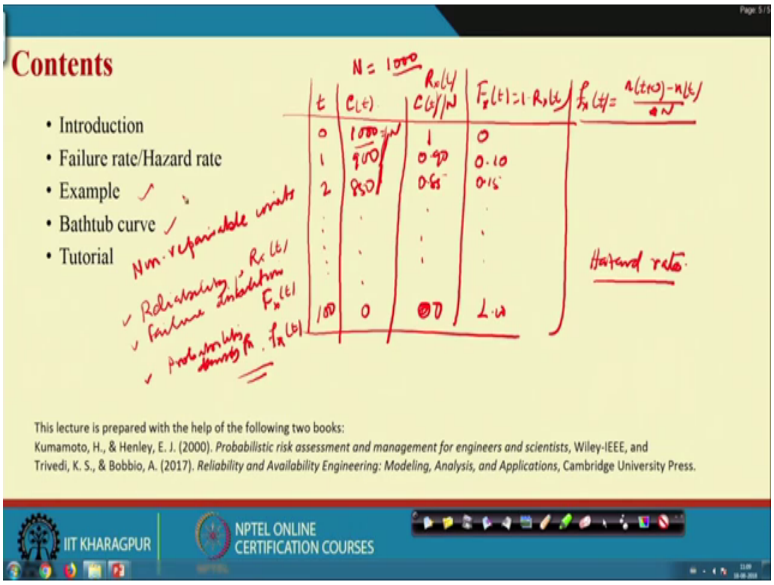

The content of today's presentation is introduction, failure rate and or hazard rate, example, Bathtub curve and 1 tutorial. So, you just let us understand what we have discussed in last class because, this class has good relation with my last class. In last class, I have shown you the concept of reliability, concept of reliability then failure distribution, failure distribution.

So, we defined reliability as R x t failure distribution as F x t. And, then we have gone to probability density function, which we defined f x small f x t. Where is t is basically the time to failure, we consider non repairable units repairable units. And, and with different formula starting from the scratch we have defined what is R a R t F t and small f t.

So, for the time being suppose let t equal to 0 and then suppose you are going to equal to 1 2 something like this, let it be may be 100 units of time. Then, let you are conducting an experiment, where you are putting suppose N number of identical units into operation. And, then you are finding out suppose a let L equal to may be N equal to may be 1000. And, then what you are doing you are finding out that, what is the number of items surviving after certain time interval.

For example let t equal to 0 the, if I say C t is basically the survival units then here it will be N which is 1000. And, in this manner suppose in the next unit is 900 then 850. So, like this you found out there are the 110 that is 0. So, then number of units survived t by d C t by C t by N that is your R x t or reliability.

So, it will start 1000 by 1000 1 then this is may be 0.90 0.85 in this minus finally, it will be it will be 0. So, then what we have seen, we have seen F x t, which is basically 1 minus R x t. So, 1 minus a this will be 0 this is 1 minus 0.9 9.10 1 minus 0.85 dot 0 0.1 5 like this and finally, it is 1.0. Then, you have seen that f x t, this one is nothing, but what you have done that that number of unit spell in n t plus delta minus n t by N.

So, if I consider this 2, there may 100 unit spell and then out of 1000 units and that manner you have computed the f x t. So, even you go for the second time interval you will consider the difference between these 2 third time interval like this f x t also we have computed. Today, we will be discussing that another important concept called hazard rate. This hazard rate and with example will be receiving finally, the our that bathtub curve will be discussed and then 1 tutorial you will see.

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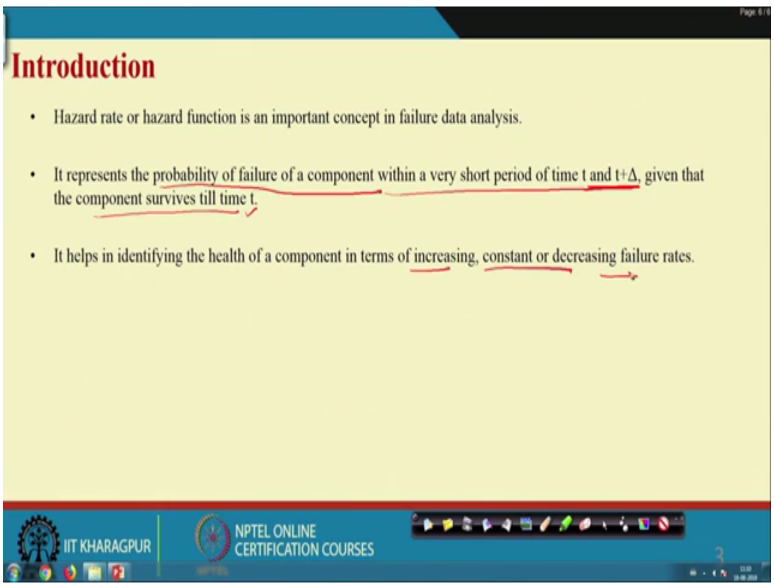

So, what is hazard rate? Hazard rate is or hazard function it is basically the probability of failure of a component within very short period of time let t and t plus delta, given that the component survive till t, till time unit t. It helps in identifying the health of the component in terms of increasing the constant and decreasing failure rate that also we will see later on.

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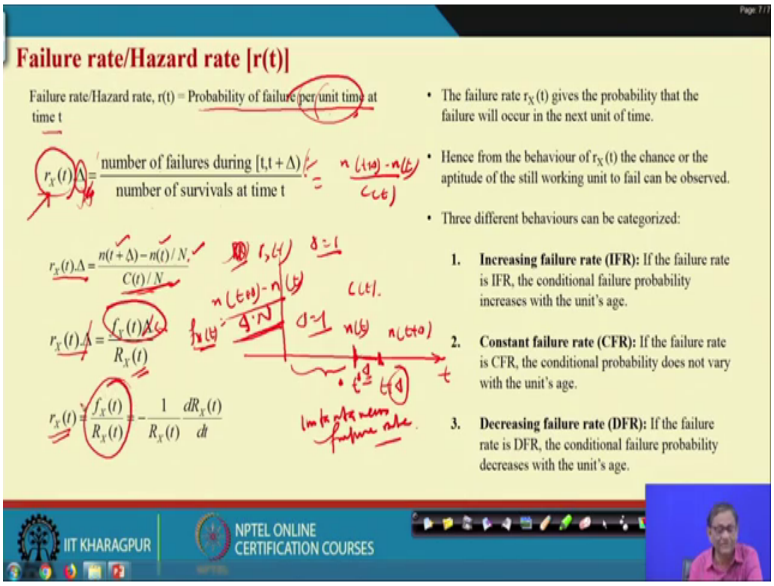

See what is hazard rate here? Be careful, we are saying that hazard rate is probability of failure per unit time at time t; that means, we are we have some timeline that is t. And, suppose this is our hazard rate h t. What we are basically interested to find out suppose the unit survive till time t. Then, we are creating another time interval that is delta. So, this point is t plus delta.

Now, this is basically that that probability that the unit survive of 2 time t, it will basically fail within this small interval delta t. That is what is the hazard rate that is why this is all this is also known as failure rate or more precisely instantaneous failure rate; instantaneous failure rate.

So, this delta is very small amount of time very small time, then by this definition if we denote that r x t is the hazard rate. Let, it be r x t; r x t is the hazard rate. Then, it is number of failures during this period of time and we are saying that per unit time. So, that mean divided by delta if you put here we can multiply delta in this side no problem. So, number of unit failures during t plus delta t by number of survival at this time period.

So, that mean r x t into delta the time period is this, because we are interested in per unit time. In delta time, how many what is the unit spell? That number of survive here and number survive here the difference will be number of units spell in delta time. So, then per unit time it will be by data. So, instead of writing here by delta you are writing here multiplying this sets so, no problem. So, then what is the number of failings at this point in time that is n t. What is the number failing that is n t plus delta.

So, number fail of failures during these is n t plus delta minus n t. Then, number of surviving at this place is C t that is what we have seen earlier. So, then this one can be written. So, this can be written like this n t plus delta minus n t by C t.

So, this can be written like this again dividing a in capital N, which is the number of units that put under test at the time t equal to 0. So, if you divide this then you see what you are getting r x t delta equal to this f x t into delta, where from this delta is coming. Suppose if you consider delta is equal to 1 what I have given in few minutes back I said that n t plus delta minus n t by this.

This will be your f x t provided delta equal to 1, but we are we do not know whether the delta 1 or delta less than 1, but it is a positive quantity.

So, then if it is delta time then it delta will be multiplied here. So, that is what n t plus delta minus n t by N will be f x t into delta, that is what is written here. And, this one is R x t the reliability part we have discussed earlier. So, this is R x t. Now, this delta and delta will be cancelled out. So, what is R x t? R x t is f x t by this clear.

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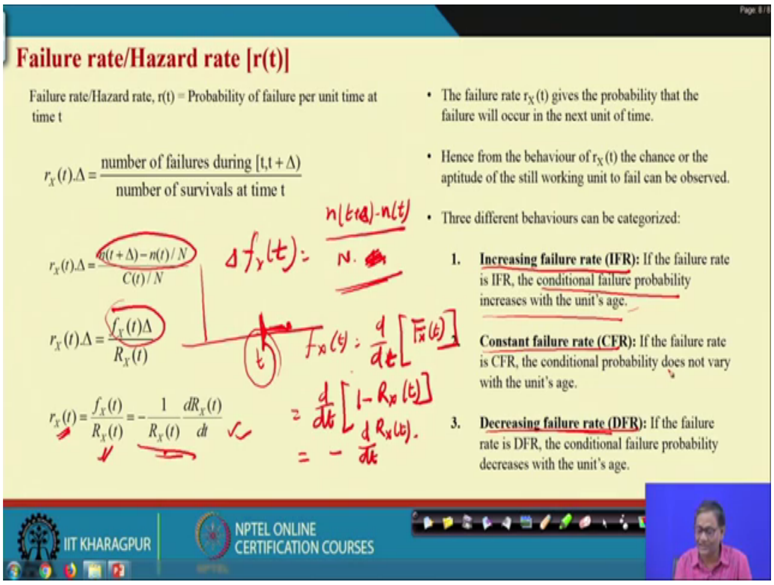

If, this not clear let me explain again, that our f x t will be the density that n t plus delta minus n t by N into delta, because this is delta.

So, now so, you have the quantity top side that n t plus delta minus n t by N. So, if I remove this I have to write like this, this is what is written here. So, then this much is. Next is that, where how you are getting this? Because, see ultimately what is f x t? F x t is derivative of d by d x d i d a d t d by d t into it is cumulative function f x t. What is the that f x t, that is 1 minus R x t, because F x t plus R x t equal to 1.

So, then if you take the derivative this will be 0 and this will be minus d by d t in R x t. So, if you write f x t in this form then, r x t will be 1 by capital R x t into d capital R x t by d t. So, that mean the hazard rate in terms of reliability. So, that is the definition and the derivation of hazard rate. Interestingly, it is basically gives 3 important explanation, 1 is increasing failure rate, constant failure rate, and decreasing failure rate.

So, depending on the this R x t, if it is increasing then the conditional failure probability increases with age. We said that this survive of 2 time t and then what is the failure probability that it will fail in the next unit of time within next unit of time.

So, conditional to this survival this will the failure rate will increasing; that means, failure rate increases, probability of failure increases with increase in age. If, it is decreasing failure rate, probability of failure decreases with units age. If, it is constant failure rate; that means, there is no change in the probability of failure with the, with age at least for a particular period of time. So, that is what is the explanation of hazard rate.

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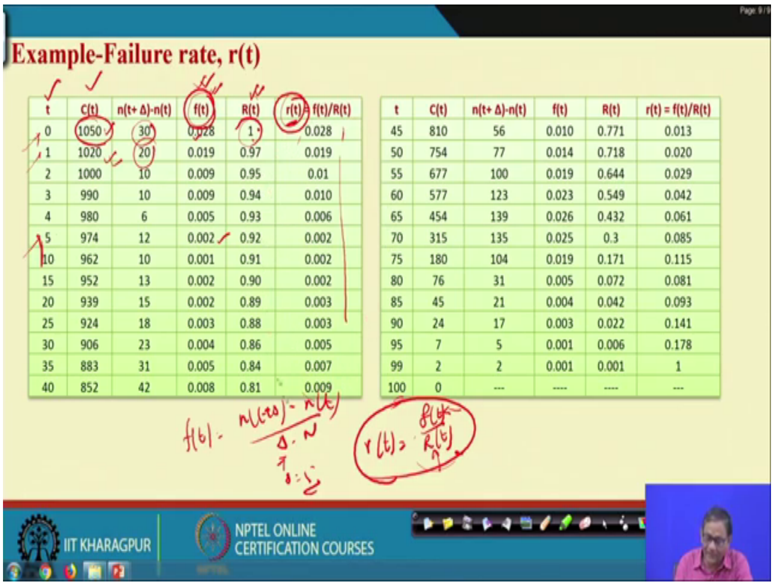

But, hazard rate is better understood by hazard by Bathtub curve. So, that we will describe, but before that you please see 1 tutorial here. We have considered t equal to t 0 to 100. So, 100 different units please see that first 0 followed by 1 like this up to 5 the change is 1 unit and after that the change is 5 unit.

So, then at the at the beginning the number of units put under operation was 1050 and then slowly as that age increases or time increases, the number of units starts failing. And, at the at 1 unit of time; that means, that number surviving is 1020 and this we have discussed earlier in this manner. So, then the number of units failed during in between t equal to 0 to t equal to 1 is 30, number of units failed in between 2 and 1 is 20 like this.

Now, what is our f t here you will have confusion, if you consider delta equal to 1 all the time. Then, what you will do n t plus delta minus n t by original that number put under operation that you will divide. This will work up to this.

After, that what happened here? Here, what happened this minus this is 5 unit difference so, delta to be (Refer Time: 15:12). So, that mean f t here is n t plus delta minus n t by delta into N. This delta will be 1 for the first 5 after that it will be delta will be 5.

So, in this manner we have calculated. So, once f t is calculated and also you know C t, you know the R t also. So, what is R t? R t is basically C t by N. So, this now, if I know R t and if I know small f t you know hazard rate, because hazard rate r t is that density by reliability. Because, this is the probability of failure or otherwise it is basically it is failure density. Now, within certain period how what is the how many fails? And, then R t is the survival function this is the conditional part.

So, you have f t you have R t. So, small r t is f t by capital R t, in this manner you have computed. And, I am sure that you all will be able to compute this, you practice huge excel sheet and take the same data. So, what data you require you require only this t and C t data rest of the data will be derived from or computed from this C and t. So, you got now the hazard rate or hazard function or instantaneous your rate. And, sometimes we say it also failure rate in many books unit is written failure rate.

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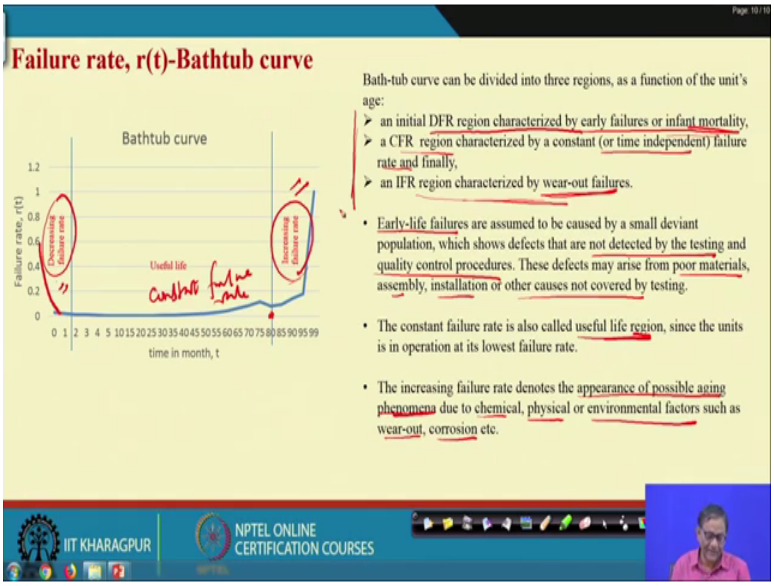

So, then if you plot this, you we have plotted this hazard rate and we got this kind of curve. So, if you see this curve here what happened from this point this axis little actually no this is the reason.

So, tradition when in general when you talk about the life cycle of the unit in (Refer Time: 17:34) it will be like this; it will be like this. But from our data we got this blue colored that curve, but if we consider the life cycle of the unit, that we then you will find out the hazard rate will be like this.

If, hazard rate is like this, then there are 3 regions; 1 region this 1 is the decreasing failure rate, this region is known and that is basically which a early failure zone. And, then after that what happened why this early failure actually it takes place, this is basically the DFR capitalized by early failures or infant mortality. And, then followed by there will be a long period of time, when the hazard rate will be constant. So, this is decreasing failure rate, here constant failure rate and this is increasing failure rate.

So, this constant period rate that period will be quite large, compared to early failure rate and that is the burn out page. So, then last page will be the increasing failure rate, this increasing failure rate is basically wear out or burn out failures, because that is the end of the life of the unit under consideration.

So, that is the beauty of hazard rate, because it gives you the bath tub curve. And, then you initially when the product under operation, what will happen because of basically that infant related means some kind of I can tell you that, some kind of taste problems, some kind of quality check problem, the product will face more failure initially.

And, then slowly those quality problem will be checked and finally, it will be rectified, and then the failure rate decreases very quickly, and reaches to the constant failure rate, which is the useful life. And, then the this will follow and finally, here because of age effect, because of aging effect, the component or the unit will very quickly undergo to death.

So, that the 3 different region you will get in bath tub curve. You may ask us that, in the data what we have plotted this portion is not there it is although data is hypothetical, but if you in really in practice you got this kind of plot then; that means, the component early failure time, that is not taken into consideration in the analysis.

Because, the unit is very well under useful period of time most of the units. And, some units are under the wear out face. So, that is what is the meaning. So, what is let me read out little more. As I say that DFR is characterized by early failure or infant mortality. CFR is characterized by a constant this is time independent failure rate and IFR is wear out failure rates.

Now, what are the reason for early life failures as I told you there basically you say, they are not detected during testing or quality control procedure. And, the defect may arise from poor material, assembly, installation, and other causes not covered during testing. That is reason you get initially little more failure then; obviously, reasons are found out and rectified and failure rate decreases.

So, the constant failure rate is called use full life region. Very important and this is time, when the product or the unit is under full operation, under full utilization. And, the increasing failure rate here the appearance of possibly aging phenomena. So, why? Because of definitely age means wear out worn out phase. Chemical, physical, environmental factors are responsible for such wear out, also corrosion and other things will be will be responsible.

So, what we have discussed we have discussed hazard rate and we relate it to the your failure density, as well as reliability. And, then we have shown you through a tutorial that how the hazard rate is computed. And, then we have also seen here that hazard rate is typically a bathtub curve.

And, it is you will get this bathtub curve, if you have data for the system life cycle. And, there are important issue is that the early failure rate will be quickly decreasing. And, this is because of the testing and quality problem. And, the useful period of life or useful constant failure rate period will be very lengthy.

Because this is the period when the component or unit or the system is under full utilization. And, because of the age and may be over use rays, the environmental, chemical, all other factors region, what will happen the component unit or system may undergo wear out phase. And, then what will happen? It, will it is basically reaching towards the death you have to dispose of the unit from the operation. So that is what is the meaning of hazard rate or hazard function.

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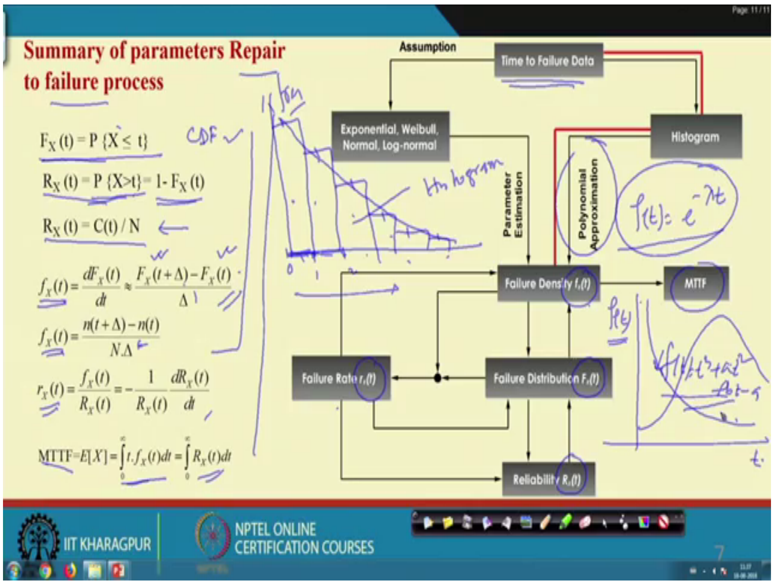

So, then let me summarize this what you have learnt so far in quantification. So, in summary you have learnt that X that what is the C D F Cumulative Distribution Function? That mean, the what will happen the if X is random variable, which denotes that the component time to failure. Then, probability that X spell within time t, that is what is F x t, which is CDF Commutative Distribution Function. Then, you have understood R x, which is reliability of the component unit or system, reliability means the probability that the unit, under unit we will operate, for the intended period of time, under given environmental condition, because for which it is the intended to work.

So, that is what that is a probability that, the random variable X, which is basically time to period it exceeds a the time or given time t, then this is this is nothing, but 1 minus the Cumulative Distribution Function. Because, the ultimately for our component or unit is 2 state system, either it is surviving or under it is spell. So, the F x t and R x t combination the sum of this two will be 1.

Then, what happened? We have shown that you have done a random experiment. And, in these experiment N units, N identical units are put under test. And, then you observe that how many surviving after certain time intervals. And, that time intervals is C t, which is basically C t may be 0 may be 1 or like this your t 0 t 1 t 2 to like this. The, time and then C t is basically the number of unit survive and t equal to 0, t equal to 1, t equal to 2, something like this. And, N is the number of units under test at t equal to 0.

So, as a result at t equal to 0 R x t will be 1 and slowly it will it will decrease and the at the end of life cycle it will be basically 0, but theoretically it is 0. And, then you have learned also f x t, which is nothing, but the derivative of the cumulative distribution function. And, then what happened through using this formula F x t plus delta this is nothing, but CDF with reference to time t plus delta and this is with reference to t divided by delta.

So, if you, as you have you are conducting experiments, you will be having the data available with you. So, a that is n t plus delta number of units spelled at time t plus delta. And, minus number of unit spell at time t divided by total number put under test and the interval of time for which the delta that t, interval of time you are considering or unit of time you are considering.

So, this also we have you know now and you with examples you know how to compute it, plus r x t which basically today we have we have discussed. So, last class we have discussed up to this. Today, we primarily concentrate on the hazard rate function and, in last class also I have discussed that what will be the mean time to failure. This is nothing, but this or this equation.

Now, this is what is already done. So, in the right hand side there is a figure, you just see the figure this figure is basically talking about the parameters. Parameters for here we are basically considering repair to failure process. It is basically repair to floor process is equivalent to non this is a system here non repair, only 1 repair to failure means it is a non-repairable system, it is basically once it fails you have to discard it.

So, the under parameter estimation, what are the things given? Failure density; it is discussed, we have discussed cumulative density function, we have discussed reliability, we have discussed hazard rate or failure rate, we have discussed mean time to failure.

So, you have done an experiment. So, and from the experiment you will get time to failure data then from time to failure data you can develop histogram and then you can approximate the histogram through some polynomial expression. So, that we have not discussed yet, we have basically given a polynomial approximation we have not discussed it.

So, ultimately in the right hand side we are saying that the theory behind the or the computational details behind all those parameters given here, but many time what happened you will just take the time to failure data. And, then you will derive all those things, not the manner we have discussed from the experiment basic from the basically fundamental or from the 0 till all kind of things of this distribution.

So, here what happens once you have time to failure data, you can plot the histogram. For example, I have TTF data I can start from 0 1 2 like this. And, this side there can be frequency means within 1 unit how many fails may be let this, 1 to 2 units let this unit fail, 2 to 3 units let this fail, this fail, this fail like this.

Then, this is nothing, but the histogram. So, we what is histogram? Because, your what is the variable x here or t time to failure. So, time to failure you are arranging them in ascending order, then creating interval here we have create interval of 1 unit of time 0 to 1 1 to 2 like this. Then, you have found out within 1 unit 1 unit of time how many fails? This is the frequency. And, then within 1 to 2 how many fails like this when you plot these, you will get this that attached bars that bar 1 2 3 like this bar will basically gives you the histogram.

Now, if you draw the midpoint of all those bars, you will you will be getting a getting a curve. Now, here this is almost a straight line, but it may so, happen that your histogram will give you a curve like this or it may give you a curve like this. When the midpoint are joined, then by polynomial approximation, we are saying what is the equation of this one, sub if this is t and this one if I say it is f t, then f t can be approximated with in terms of t through some polynomial approximation.

For example, if I say that this is the exponential part 1, then you can write that f t equal to may be e to the power minus lambda t. So, that can be an equation. So, it is just a I am not saying that this may be the equation of these I am saying this is one kind of, or you may say that it is basically t q plus a t square plus b t t minus 9 that is my f t. So, this is another kind of polynomial expression, polynomial approximation.

So, this is what is the starting of the quantification of basic event, that the how do I know the probability of failure. The hazard function the reliability and other things and please go through the book, the 2 books already given here. I hope that you will be able to understand it and you will be able to solve the assignments as well as the examination problems. If you have any query please put in the discussion forum.

Thank you.