

Industrial Safety Engineering
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Lecture – 27
Quantification of Basic Events for Non – repairable Components

Hello everybody. We will discuss today Quantification of Basic Event for Non Repairable Components. So, fundamentally it is quantification of basic events. So, let us see that what are the contents of today's presentation.

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Contents

- Introduction
- Reliability and failure distribution
- Failure density function
- Failure rate

This lecture is prepared with the help of the following two books:
Kumamoto, H., & Henley, E. J. (2000). *Probabilistic risk assessment and management for engineers and scientists*, Wiley-IEEE, and
Trivedi, K. S., & Bobbio, A. (2017). *Reliability and Availability Engineering: Modeling, Analysis, and Applications*, Cambridge University Press.

The slide also features a handwritten fault tree diagram in red ink. The top event is labeled T_{TP} . It branches into two main paths. The left path is labeled 'Basic event' and includes a list of events: 'Pressure Tank', 'Pump', 'RV', 'P1', and 'P2'. The right path includes a 'Switch' event. The diagram shows the logical relationships between these components leading to the top event.

We will introduce that what do we mean by quantification of basic event and then reliability and failure distributions will be discussed, failure density function and failure rate, hazard rate that will be discussed. So, the lecture we have prepared following two books; one is Kumamoto and Henley, Probabilistic risk assessment and management for engineers and scientist. And, Trivedi and Bobbio, Reliability and Availability Engineering that is Modeling Analysis, and Application Cambridge University Press.

So, far we have discussed many things primarily the qualitative aspects of industrial safety engineering. If you recall from the first lecture till the last lecture that mean, up to the analytic hierarchy process where we have ultimately found out the best alternatives. So, mostly we have relied upon the qualitative concepts and followed by certain algebraic or mathematical operations. In between, in both in case of fault tree analysis

some of the algorithms we have discussed, but we have not discussed that much of quantitative aspects of the industrial safety engineering.

So, today I think this is the first class where we will be going into little more mathematical issues and I hope you all will enjoy. Definitely, I will not go into that depth of mathematics that which will be difficult for you to grasp, but the fundamental basis of probability, reliability and distribution related things will be discussed here with the help of examples in a very simple manner and I hope you will enjoy it. And, when I talk about quantification of basic events, then you must know that basic events basically related to blot i or other way the fault tree analysis where, there in fault tree there is stop event and there are several basic events; ultimately, the basic events are those events which are basically alone or in combination leading to the top event to occur.

So, if you go by the system break down concept, then you have seen that the system is broken into sub systems, sub sub system and finally, component to part level and the basic events are primarily related to the component or the part related events. So, by basic event quantification, we are trying to tell you that how do quantify the failure probability of the component or essentially the parts of the component if you further break the component into parts ok. And this is obviously, a simplification by saying up to component or part level but for the sake of understanding and for most of the practical purposes this concept works well.

So, then, let us understand that one is our basic event and then we say it is basically that in any fault tree, so if you broke into different components, so you found out that this is 1, this is 2, this is 3 and this is 4. So, these are the basic event and this is the top event and in between may be here you can create event that is your intermediate event that must you know. And if I say this is totally a system, then the from system to this component level. And, if you consider the pressure tank system, pressure tank system, what we have discussed earlier pressure tank system and there you have found out lot of things like including pressure tank, then pump then your relief valve, then discharge valve, then alarm, then pressure gauge and then switch and then timer so many things.

So, when we have broken the pressure tank system in to component, we found out that relief valve is component and then pressure gauge is component switch is in other component, also pump we have consider at the comfort level valve pump has lot of other

pump it is own parts or component. So, suppose we are interested to understand that what is the probability that pressure gauge will fail when this supposed to be supposed to work or what is the probability that the relief valve will fail.

So, these are what we are saying that quantification of basic events; so, the circle quantification. So, essentially we will go by the reliable mathematics given in the reliability engineering site and you have seen that the both the reliability and safety sides are mainly common by the probability distributions. So, there are lot of commonality from mathematics point of view. So, we will be using those mathematics today.

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Introduction

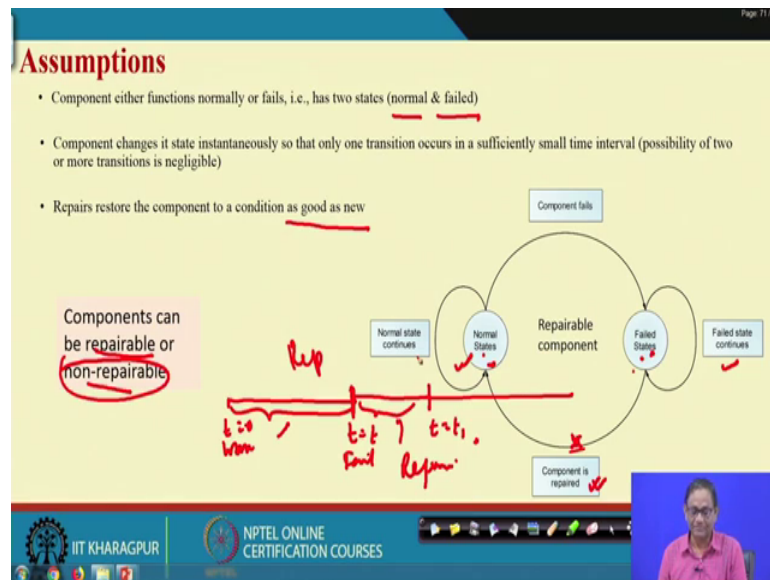
- System failure is caused by the failure of an individual component or a combination of components failure.
- Accurate description of component failures and failure modes is necessary for the identification of system failures.
- Risk assessment of a system involves identifying the possible scenarios that a system can fail and probability & consequences associated with each scenario.
- The probability of occurrence of each scenario is obtained using fault tree analysis and the probability of the basic events or root events.
- **Objective**:- To obtain the probability of the basic events using mathematical models.

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So, I just repeat this because, already I have given the idea that system failure caused by failure of individual component or combination, you all except it. Accurate description of component failure or failure modes is necessary for identification of system failures because, in fault tree or you have seen this one.

Now, risk assessment of a system involves identifying possible scenarios accident scenarios that is system can fail and probability consequences at is scenario. Now, how do get the probability unless you know the component level failure. So, the probability of occurrence of each scenario is obtained using fault tree analysis and probability of the basic events or root events. So, what is our objective, is objective is to find out the probability of basic events using mathematical models ok. It is not a conceptual model or graphical model, it is a mathematical model.

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Now, you think of the component, the component can be repairable can be non repairable. So, some kinds of valve which once failed cannot be repaired. It should be thrown and then new valve to be you know replaced by new valve, but there may be some valves which can be repaired also.

So, which is not reparable that requires one kind of mathematics and another one which is repairable that requires another kind of mathematics. So, we will discuss in today's presentation or particularly this presentation that with reference to non repairable units. The non repairable case suppose the unit start at t equal to 0, it starts working and t equal to t_1 , it fails; after that there is nothing more possible.

So, that mean this state is working. In fact, up to these it is working at this particular time, it failed. So, it fail. Now, after that there is nothing more because this unit to be completely thrown off, but if it is a repairable one, what happened, here the repair will start. Suppose, again t equal to t_1 , it is repaired completely. So, this is our repair time. So, repair this is working time, this is repair time and again then what happened after repair there will be working.

So, this repairable concept this one is given here. So, a component have two states normal and failed; normal mean working failed means either it is it will be completely replaced or it will be repaired and reuse. If it is completely replaced those units are

known: as non repairable unit and if it is repair and use they are known as repairable unit.

So, in the repairable unit there are the important thing is that once suppose the normal unit it fails, fail state this path will not be there for repair non repairable unit; for repairable unit the component will be repaired and again it will come back to the normal state. So, as a result this is most general one; that means, one the system working, it may remain in working state, it may go to the fail state and remain at the fail state or after that when you it is repaired, it will again come back to the normal state.

If it is non reparable this part either it is in the normal state remain in normal state and fail state it goes to fail state and remain in fail state. It will never come back to the normal state again means repair is not possible. So, we will discuss non reparable unit case today and the replacement will be such that it is as good as new. And in case of repairable unit also, later on we will see that the repair will be such that it is it will be as good as new. So, there will be variety may be as good as the old.

So, that variety will be there also you repairable unit. For the time being now, switch off the repairable part or repairable concept from your mind. What you think that you have a you have components which whose failures are the basic events. So, that component ones failed it is replaced completely, so that in the means it is a non repairable unit or I can say use and throw type of units.

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Non-repairable Unit: Reliability and Failure Distribution

- As the unit is non-repairable, the only event of interest is the instant at which the unit experiences failure. Let the unit A be initially in the up state at time $t=0$. The instant of time at which it will eventually fail is a continuous random variable, $X \geq 0$, where X represents the time to failure of A.
- The cumulative distribution function (CDF) $F_X(t)$ of the time to failure X is defined as

$$F_X(t) = P\{X \leq t\}$$
- Similarly given two time instants t and $t+\Delta$ (where Δ is a positive quantity), the probability that the random variable X lies within the time interval $[t, t+\Delta]$ gives the probability that the failure of A occurs in that interval. The probability is given by

$$F_X(t+\Delta) - F_X(t) = P\{t \leq X \leq t+\Delta\}$$
- The survival function of random variable X , denoted by $R_X(t)$, is defined as

$$R_X(t) = P\{X > t\} = 1 - F_X(t)$$
- $R_X(t)$ is also called reliability of A and is also defined as

$$R_X(t) = C(t) / N$$
 where, $C(t)$: number of units functioning at time t
 N : Total number of units

Handwritten notes and diagrams on the slide include:

- A graph showing the failure rate $f(t)$ as a function of time t . The curve starts at the origin and increases, labeled "failure".
- A graph showing the reliability function $R(t)$ as a function of time t . The curve starts at 1 and decreases, labeled "survival".
- A graph showing the cumulative distribution function $F(t)$ as a function of time t . The curve starts at 0 and increases, labeled "failure".
- Handwritten calculations: $100 - 90 = 10$, $10/100 = 0.1$, $0.9 + 0.1 = 1$.
- Handwritten notes: "X = time to failure", "N = total number of units", "C(t) = number of units functioning at time t".

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So, we will define important concepts here. Please stay take your time and pause the video if require and then see the what is written there and again you run the video and understand what I am and listen carefully what I will be saying and so that your concept will be clear here.

So, it should not if your concept is not clear later on you will find out lot of confusion and something what is what you feel that obvious that may not be the correct one. So, what we are discussing here as the unit is non repairable, the only event of interest is the instant at which the unit experiences failure. I said to you, I have time t equal to 0, unit is under start and operating and at t equal to t suppose it fails. So, that mean what is our interest is that when it fails that mean at t equals to t is fail that how I mean, how long you have that component work. So, this is t time this instant is this total length of work is t . Now, second is let the unit A be initially at 0, I told you ok.

So, the instant of time at which it fails, it is a continuous variable and if we represent it at by x , so that mean X greater than 0, so the X what is X represent X represent the time to failure time to failure, X is time to failure. Suppose, the component fail here then this time if it is X_1 , suppose component another component fail in this time X_2 . So, the time to failure each X which is a random variable you do not know when it will fail. So, obviously, it is time depend, the component is time dependent case.

Now, if you have identical components, suppose same similar components several in number and you put them here at time t equal to 0, put them under test you may find that some will fail with in X_1 time, some will fail X_2 time, some will fail X_3 time like this. So, at different time interval that will fail.

So, now if you if you basically interested to know that how many fails at a particular time interval and then plot against time, what will happen? Suppose, this is my time and at this time how many fails, at this time how many fails, at this time how many fails. So, you will get basically a frequency cumulative frequency. At this time suppose this much fail at this time, this plus this total number of number fails is this much this much like this you may get a curve like this where if it is cumulative frequency, then it will be hundred.

Now, if you divide this cumulative frequency by total number of units put under a test at time t equal to 0, what you will be getting, you will be getting cumulative probability;

that is what is given here $F_X(t)$. So, that mean a time t equal that mean X equal to t at this point at this point, this equation what it will give you, it will give you the probability that X means the time to failure is less than equal to t .

So, as the components we are saying the components are identical in nature and we are interested in probability. So, that mean you must have sufficiently large number of components and then within that period of time how many fails, if you divide it by the total number or put under operation then what will happen you will be getting the probability.

So, that probability what happened as all are identical components. So, we can say this is the probability that the component will fail within that is period of time. That is the cumulative probability $F_X(t)$. What does it mean? Then, that cumulative distribution function $CDF_X(t)$ of the time to failure is defined like this, what is the interpretation? Interpretation is probability that X what is X time to failure the time to failure will be less than t less than equal to t the probability value is this.

So, this axis is cumulative probability this axis is time. Suppose, you may be interested to know that what is the probability that within in between two time periods may be as very small you will create. Suppose you create this is $t + \Delta$ and this one is this one is your t .

So, then there are two probabilities with reference to t that mean probability X less than equal to t , this probability and probability X less than equal to $t + \Delta$, this probability is this, then if you create like this that mean cumulative probability here minus this, this is nothing but that what is the probability that that in that the probability X is what time to failure in between t and $t + \Delta$. In between, what is the probability that how many that will failure will take place.

So, similarly given to, I am reading again giving time instant t and $t + \Delta$ where Δ is a positive quantity the probability that, the random variable X lies within this given the probability that the failure of A occurs in the time interval, the probability is given by this; means within this time interval, the failure occurs that is the probability the difference between the two is this understood. First probability X less than t , this is t what is the probability this is the probability that the unit will fail the time to failure of the non repairable unit is less than equal to 2 is this probability; second one is that within

these interval t and t plus Δt , it will fail what is the probability this is the difference is the probability.

Now, now if you just do the another one what you will do that instead of here you are writing $F_X(t)$ that is the failure; suppose, you want that how many survives at t equal to 0, suppose n units are there and t equal to suppose t there are suppose $n-1$, then the n suppose $n-1$ units spell, then how many survive, n minus $n-1$. This many survive at t equal to t .

Suppose I want to know: what is the probability of survival. So, that mean as components are identical component with the test I can say that the probability of that a component will survive after t time t this is nothing but the reliability of the component. So, this will be defined by the $R_X(t)$, what is $R_X(t)$? Probability that X greater than t what is X , X is the time to failure. If probability X greater than t that mean it, it will be it will be it will be surviving at surviving up to t up till t X equal to t , so.

So, here I am saying this is t what is the probability that it will fail in the other diagram we want to create where we will say it is $R_X(t)$, we will say $R_X(t)$. That mean what is the probability that it will survive? So, then in the you may get a distribution over like this ok. So, that t it will survive that this is the probability. So, this X is when $R_X(t)$ things are like this. So, this is my survival function, survival function and when it is that probability of failure, then we are saying this is the case this is known as, that is known as failure distribution or failure function.

So, failure function and survival function, if you see that the that the probability value here; one is probability X less than equal to $2t$, another one is probability X greater than t if I add the 2, then it is the totality and as a result what happened that $R_X(t)$ plus $F_X(t)$ is 1 and that is reflected here. That mean if you know the cumulative probability distribution of failure, then you also know the probability distribution know survival.

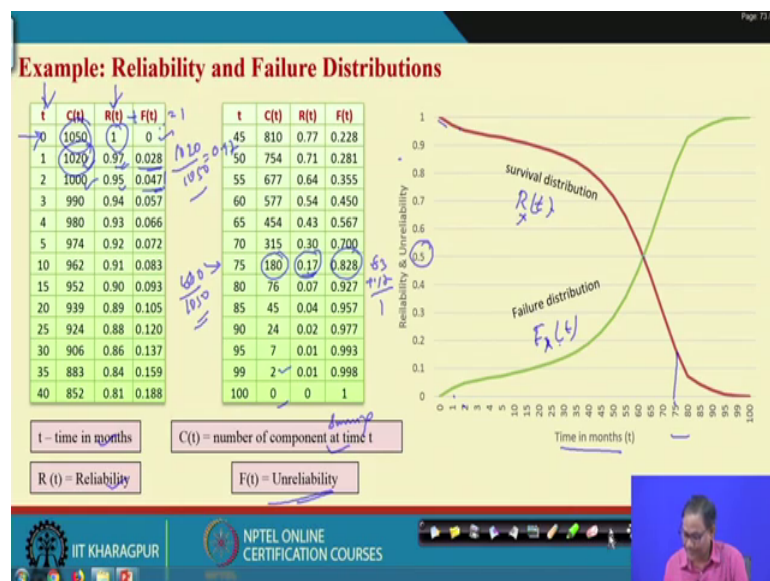
So, one is the survival function a random variable X denoted by $R_X(t)$ will be defined like this and you see that survival function what you have written $R_X(t)$ equal to $c(t)$ by n . What is $c(t)$? Number of units functioning at time t divided by total number of unit puts under operation at time t equal to 0. So, if I put hundred under test and at the end of 1 month, suppose 90 survives then my reliability at time t equal to 1 month that is 0.9.

Then, what is the failure probability $F(t)$ cumulative failure probability that will be 10 by 100, 0.1.

So, 0.9 plus 0.1 that will be always 1. So, you take any point here suppose if this is this is this is for the at time t , so this is my survival and this is my failure probability a survival is this survival probability and failure probability. So, you will be able to find out this kind of relationships. So, what we have discussed then, we have discussed essentially that a non repairable unit it will fail after certain time of operation and we want to know what is the time to failure. Time to failure is a random variable which is denoted by X and it is basically time dependent it is over time.

Then, we want to find out the what is the cumulative probability a function or cumulative that is failure or failure function then, it will be $F(t)$ a capital F X t which is probability that the time to failure will be less than equal to t means the component will fail within time t and what is the survival? Survival is reliability which is probability that the component will survive up to time t . So, it even if it fails, it will fail after time t not within time t and the probability of failure and that that cumulative probability and as well as the survival functions they are basically complemented to each other in the sense that the sum of the two will become 1 and that is what we have discussed in so far.

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Now, what we will do, we will see how this can be means plotted, the survival function and failure distribution. So, you understood it. At t equal to 0, 1050 units are under

operation. Again, I am tell you it is the identical units let if it is valve only one kind of valve it is. So, then t equal to 1 if I say t in months, the number of units survive is 1020, t equal to 2, 1000 like this t equal to 99 to and t equal to 100 it is 0.

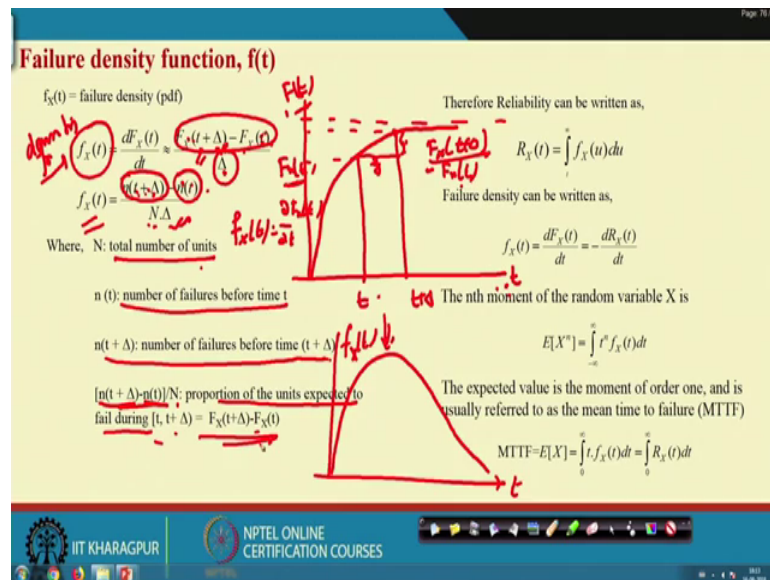
So, then what is reliability? Reliability is at time t equal to t how many survives means, what is the proportion of surviving. So, at t equal to 0, we what will be what will be reliability will be 1. Why? The reason is 10 nothing failed it is just start it. So, then t equal to 1; so, how many what is the number surviving, 1020; how many initially put 1050? What is this, this value is 0.97; if I go to the 75th 5th month, so how many surviving, 180. How many was put to under tested t equal to 0, 1050. What is the ratio 180 by 1050, this is 0.17, this is what is plot here.

So, at t equal to 0, it is 1, t equal to 1, it is 0.97, t equal to what that 0.75, it is 0.75, it is 0.17 t equal to 75, it is 0.17. So, if you plot t means this X this row column versus R t is this column, so you will be getting this curve survival function if I write like this R X t . So, now what is F t ? F t is R , R , R , t plus F t equal to 1. So, F t will be 1 minus R t . So, 1 minus 1 equal to 0, 1 minus 0.97 is this, 1 minus 0.95 is this ok. You are getting some different values that mean it should be point 0.5, but it is 0.047. The reason is here we have used 2 digit here, we have used 3 digits.

So, it is basically that when we have done it in Excel this one, but on other end ok, this plus this equal to 1. Similarly, if you come here 0.17 and this is nothing but 0.83. You see 0.17 plus 0.83 this is 1. So, at any time any time you just check. So, where it coincide, it is 0.5 because probability of surviving and probability of failing that 2 will be equal.

So, that sense so that means, your F X t is this, this is survival. So, this way you will compute, I hope that you understand. Now, other thing that t in month, the reliability number of component at time t , number of components surviving at time t surviving at time t and unreliability.

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Then, we will go to the failure density function. Let us start with this cumulative $F_X(t)$. Let us start with this one, this side t and this side $F_X(t)$. So, what is $F_X(t)$ will be maximum it will be 1. So, this is 1. Suppose, this is the curve, this is your curve. Now, you think of that t , this is t and this one is your t plus Δ . So, tell me what is this length this length is Δ , what is this value, this value is; here this value corresponding to this and this value is $F_X(t + \Delta)$ and this value is $F_X(t)$ the difference is minus $F_X(t)$. So, then if you divide this $F_X(t + \Delta) - F_X(t)$ by Δ , this minus this, what you are getting, you are getting the top portion and this divided by this is nothing but the slope.

So, slope these minus this by this nothing but the slope of this with respect to what, with respect to t . So, that mean if you create something like this $F_X(t)$ which is $\Delta F_X(t)$ into $F_X(t)$, the derivative of cumulative distribution function that is known as probability density function, density function ok. So, if you do this finally, this, you will get if this is the curve of $F_X(t)$ versus t and if you want to do the same thing t versus $F_X(t)$, then you may get a curve like this.

So, the suppose you want to get back the cumulative distribution what you require? You require to do integration and now if $F_X(t + \Delta) - F_X(t)$, we already seen that what is $F_X(t + \Delta)$; that means, number proportion surviving a t plus Δ that mean if $n(t + \Delta)$ is that is failed I am sorry that proportion failed at t plus Δ that that is that is

So, then these by this minus this is nothing but $N(t + \Delta t) - N(t)$ by $N(t)$ which is $\frac{N(t + \Delta t) - N(t)}{N(t)}$. So, that mean this quantity can be written like this and already Δt is there below. So, that in once you have data, you can create $F_X(t)$ by using this formula provide. That is number of failure at times t number of failure before time t plus Δt and then $N(t + \Delta t) - N(t)$ by $N(t)$ proportion of unit expected failing between this, this minus this by Δt . That mean, these minus this by Δt . So, this one sorry, this minus this by n this is basically proportion failing. I hope you understand, you said no yes I hope so anyhow.

Now, now come to the issue some more concept here. What happened, these descriptions I have already given to you. That means what we have mean by this. Now, if this is the case, then can we not write this that what is the reliability, then reliability is integration of F X u d u and definitely it is t 2 infinite.

Failure density function, $f_x(t)$

$f_x(t)$ = failure density (pdf)

$$f_x(t) = \frac{dF_x(t)}{dt} \approx \frac{F_x(t+\Delta) - F_x(t)}{\Delta}$$

$$f_x(t) = \frac{n(t+\Delta) - n(t)}{N\Delta}$$

Where, N: total number of units

$n(t)$: number of failures before time t

$n(t+\Delta)$: number of failures before time $(t+\Delta)$

$[n(t+\Delta) - n(t)]/N$: proportion of the units expected to fail during $[t, t+\Delta) = F_x(t+\Delta) - F_x(t)$

Therefore Reliability can be written as,

$$R_x(t) = \int_t^\infty f_x(u) du$$

Failure density can be written as,

$$f_x(t) = \frac{dF_x(t)}{dt} = -\frac{dR_x(t)}{dt}$$

The n th moment of the random variable X is

$$E[X^n] = \int_0^\infty t^n f_x(t) dt$$

The expected value is the moment of order one, and is usually referred to as the mean time to failure (MTTF)

$$MTTF = E[X] = \int_0^\infty t f_x(t) dt = \int_0^\infty R_x(t) dt$$

Suppose, I have my distribution, t can be 0 to infinite, I have distribution suppose like this. So, this is my $F_X(t)$. So, at t equal t , this one if I take the probability of this from 0 to t from 0 to t $F_X(t) dt$ what is this? This will be integration of $F_X(t)$. This is basically $F_X(t)$, now what we know that $F_X(t) + R_X(t) = 1$. So, again 0 to t already taken, but this

portion from this plus this will be 1. So, that is why $R_X(t)$ is $R_X(t)$ is integration t , it is not starting from 0 from t to infinite to infinite this. You are using you here, you can use t also no problem.

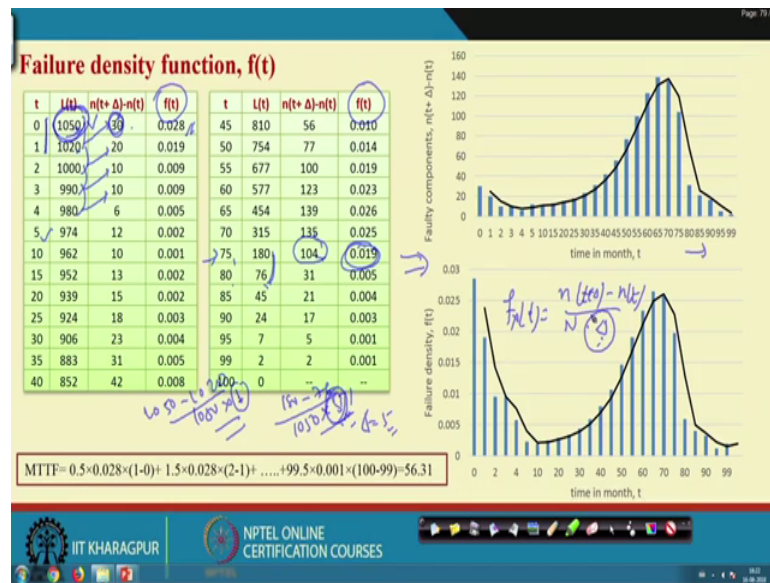
So, t is changing from t to infinite $F_X(u) du$. So, u changing from t to infinite that sense, now, so then another one. So, that mean what happened if I know the failure density $F_X(t)$ I can find out cumulative failure, you can find out the reliability function also survival functions also. And the relationship between that $F_X(t)$ and $R_X(t)$ once you capture you can write that $F_X(t)$ equal to $\frac{dF_X(t)}{dt}$ which will be nothing but minus $\frac{dR_X(t)}{dt}$ by $\frac{d}{dt}$ $R_X(t)$ by dt , why because $F_X(t)$ is $1 - R_X(t)$ then $1 - R_X(t)$.

So, this is nothing but minus $\frac{dR_X(t)}{dt}$ by dt . So, that mean if you know $F_X(t)$ capital $F_X(t)$, you can know you can find out the small $F_X(t)$. If you know capital $F_X(t)$, you can find out the survival function $R_X(t)$. If you know the $R_X(t)$ also, you can find out the $F_X(t)$. So, there are relationships between $F_X(t)$ $R_X(t)$ small $F_X(t)$. This is cumulative probability distribution, this is reliability or survival distribution, this is failure density.

So, few more things like your n th moment of random variable will be this formula and why this is required. You just Google it or get through some books that details we are not going, but only thing you remember that you will be interested in the meantime to failure. Then, what is the mean average time that we will that our component will survive.

So, then using these moment equation, the first moment will give you the give you this value. So, first moment is this $\int_0^\infty t F_X(t) dt$, this is nothing but again $\int_0^\infty R_X(t) dt$. Showing you the expected value of X what is X , X is time to failure will be found out by this. So, now what will quickly we will see that same example that all those calculation.

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You see you have seen $L(t)$ is there $n(t+\Delta)$. So, what is this 30, 30 is coming from that this minus this, 1050 minus 1020 this is 30 then, these two will give you this difference between these two is this difference between these two is this. So, that mean $n(t+\Delta) - n(t)$. So, you are getting that number of units spelled within the time. So, 0 to 1 month, within 1 month, 30 unit spell; first month to second month that second month 20 units like this. Then, what is our $F(t)$, $n(t+\Delta) - n(t)$ by N , capital N . So, this quantity is 30 by 1050.

So, in this manner, our $n(t+\Delta) - n(t)$ is 10 for the 750 unit and if you divide this by 1050, it will be like this. So, there are two 2 graphs given you; one first one is the number of units spelled within this small time interval. So, 3020 like this and the X axis is time. So, then you are getting this kind of frequency polygon and another one is that related to the failure density. So, this density values is coming like this.

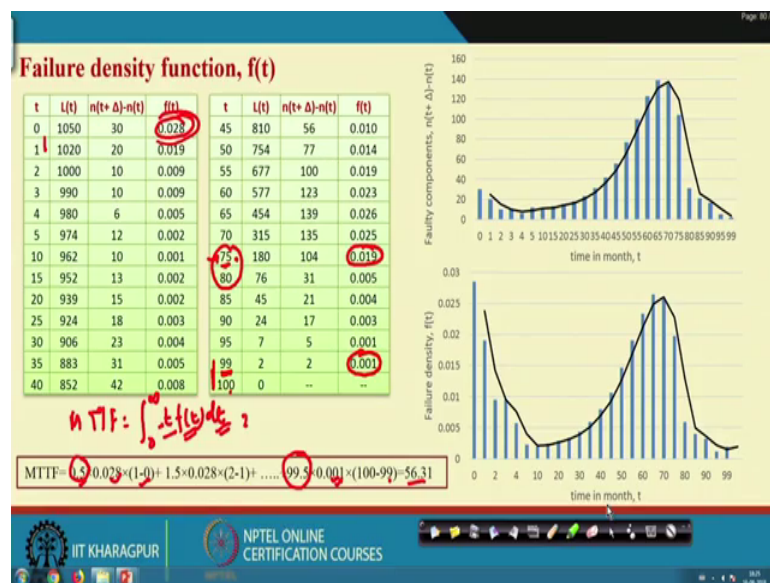
So, let me go back if we have done anything wrong. So, please remember we are saying that $n(t+\Delta) - n(t)$ by capital N into Δ . So, that Δ part I am not explained in the first package. So, this minus this is this by capital N is this. So, this minus this is this by capital N is this, but what is the Δ ? Δ is 1, that is why that is why what happened; 1050 minus 1020 by N is 1050 into Δ is 1 minus 0, that is 1 this.

But when you are calculating this, so, what is this 104, the difference between this two. So, then this is nothing but 180 minus 76 by 1050, 1050 is the even beginning one, but

delta what is the delta value 5. So, be careful when you develop this probability failure density, be careful that if the t the difference is in unit, then this will be 1. So, you can write that $n(t + \Delta) - n(t)$ by capital N . That is what at the beginning, but in order to understand this concept later on after 5 the 5 units are increased. So, this 5 will be 5 time units difference is there. So, that 5 time units, this is delta equal to 5 from then onwards.

So, this also required to be divided. So, that mean our $F(t)$ is $n(t + \Delta) - n(t)$ by capital N into delta. You may forget this part. This delta will be the difference time difference here initially from 0 to 5; only one difference is there, but at the later state the difference is 5. So, 5's will be divided keep in mind, most of the many times we may commit mistake here. Then, what is our mean time to failure. So, you just see what is your mean time to failure t the mean time to 0 to infinite $t F(t) dt$. So, that is what is shown here.

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So, mean time to failure, integration 0 to infinite $t F(t) dt$. So, we are doing it; first one is our t the in between the middle value 0 to 1, the middle value we are considering suppose 0 to one the middle value. Then what is the $F(t)$ value, $F(t)$ value is these point this. Then what is the dt value, the time interval delta here it is 1. So, that is why 0.5 in between 0 1, the time is 0.5 that is that is our t then what is the $F(t)$ value? $F(t)$ value is 0.0 to h this and what is the dt that interval, 1 minus 0.

Similarly, if you come to the last one here you see what is the middle value the t value, 99 plus 100 by 2 99.5. What is the corresponding F t value, density value 0.011. What is the delta t value the difference 100 minus 99 this, but if you come to let it be the let it be the this 17551, then what is the what is the your this one t value in between that is 77.5 what is the probability value, probability value is this and what is the interval value is 5.

So, those things you have to consider while writing this. This is basically this equation in numerical term is this. I hope you have understood this one. So, then what we have discussed so far. We have discussed interesting things from cumulative failure distribution to survival function to failure density function to mean time to failure, all related to the component which is or unit which is non-repairable and these are the plots. So, let us see some more one more concept and then we will finish our lecture today.

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Failure rate/Hazard rate [r(t)]

Failure rate/Hazard rate, $r(t)$ = Probability of failure per unit time at time t

$r_x(t) \Delta = \frac{\text{number of failures during } [t, t + \Delta]}{\text{number of survivals at time } t}$

$r_x(t) \Delta = \frac{n(t + \Delta) - n(t)}{R_x(t) \Delta}$

$r_x(t) \Delta = \frac{f_x(t) \Delta}{R_x(t)}$

$r_x(t) = \frac{f_x(t)}{R_x(t)} = \frac{1}{R_x(t)} \frac{dR_x(t)}{dt}$

- The failure rate $r_x(t)$ gives the probability that the failure will occur in the next unit of time.
- Hence from the behaviour of $r_x(t)$ the chance or the aptitude of the still working unit to fail can be observed.
- Three different behaviours can be categorized:
 - Increasing failure rate (IFR):** If the failure rate is IFR, the conditional failure probability increases with the unit's age.
 - Constant failure rate (CFR):** If the failure rate is CFR, the conditional probability does not vary with the unit's age.
 - Decreasing failure rate (DFR):** If the failure rate is DFR, the conditional failure probability decreases with the unit's age.

Handwritten notes on the slide include: $f_x(t) \Delta = \frac{n(t + \Delta) - n(t)}{\Delta}$, $r_x(t) \Delta = \frac{f_x(t) \Delta}{R_x(t)}$, and $r_x(t) = \frac{f_x(t)}{R_x(t)}$.

Another one important very very important one is hazard rate or instantaneous failure rate. The failure rate actually hazard rate instantaneous failure, what we are defining that that probability of failure per unit time at time t ; that means, that the suppose you think that time t up to this the things have survived, the component has survived then at t plus delta, delta within this small unit time.

So, we are interested to know: what is the probability of failure for this unit time when the component has already survived up to time t . So, that one can be written like this that we are creating one another component or another that parameter or another distribution

as such which is $R(t)$ and if you multiply by this, the interval that Δt small unit then this can be written like this; number of failures during t plus Δt by number of survival at t equal at time t .

So, this is known as probability instantaneous failure rate or otherwise I can say the hazard rate; hazard rate meaning that your component has survived up to time t . Now, what is the probability that it will fail in between t and $t + \Delta t$. So, that is what is given here. So, number of failures during t plus t and $t + \Delta t$ by number of survival. Number of survival up at time t it is nothing but the reliability.

So, if this one can be written like this, number of failures during t and $t + \Delta t$. So, $t + \Delta t$ the failure is $n(t + \Delta t)$ and it is $n(t)$. So, the difference is this 1. Now, then I then if you if you if you want to write that reliability part, then up to time t that what is the $R(t)$; otherwise we can write $C(t)$ no problem, you write $C(t)$ here, we have $U(t)$ that is number of components surviving up to time t . So, this by this it is nothing but the reliability number of survival. And then, we are writing that basically how many put under operation you are making the you are making the top portions, so $n(t + \Delta t)$ minus $n(t)$ by this.

So, they then what happened $n(t + \Delta t)$ by $n(t)$ into this into if you multiply Δt that will become basically $F(t)$ into Δt by $R(t)$; so, $R(t)$ into Δt because what is $F(t)$? $F(t)$ is $n(t + \Delta t) - n(t)$ by Δt into n . What is this quantity $n(t + \Delta t) - n(t)$ by N . So, this is nothing, but then this Δt will come here. So, that is what you are writing here, the top portion you are writing $F(t)$ into Δt , bottom one is $R(t)$ then $R(t)$ into Δt , Δt , Δt will be cancelled out, so $R(t)$ equal to this.

So, that mean probably failure density by the reliability is hazard rate and which is minus 1 by $R(t)$ into d/dt $R(t)$ by dt correct because $F(t)$ is minus d/dt $R(t)$. So, so, here number of failures during this by $n(t)$ right, that will be better; otherwise you will be confused. So, number of failures during t and Δt $n(t + \Delta t) - n(t)$ by $n(t)$. So, $n(t + \Delta t) - n(t)$ this is divided by N and this is the quantity.

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Failure rate/Hazard rate $r(t)$

Failure rate/Hazard rate, $r(t)$ = Probability of failure per unit time at time t

$$r_x(t) \Delta = \frac{\text{number of failures during } [t, t + \Delta)}{\text{number of survivals at time } t} \quad \text{per unit time}$$

$$r_x(t) \Delta = \frac{n(t + \Delta) - n(t) / N}{L(t) / N}$$

$$r_x(t) \Delta = \frac{f_x(t) \Delta}{R_x(t)}$$

$$r_x(t) = \frac{f_x(t)}{R_x(t)} = - \frac{1}{R_x(t)} \frac{dR_x(t)}{dt}$$

Handwritten notes:
 - $r(t) \Delta$ is the probability of failure per unit time at time t .
 - $r_x(t) \Delta$ is the probability of failure per unit time at time t for a specific unit x .
 - $f_x(t) \Delta$ is the number of failures during the time interval $[t, t + \Delta)$.
 - $R_x(t)$ is the number of survivals at time t .
 - $L(t) / N$ is the number of units in the population at time t .
 - $n(t)$ is the number of units surviving at time t .
 - $n(t + \Delta)$ is the number of units surviving at time $t + \Delta$.
 - $f_x(t) \Delta$ is the number of failures during the time interval $[t, t + \Delta)$ for a specific unit x .
 - $R_x(t)$ is the number of survivals at time t for a specific unit x .
 - $dR_x(t) / dt$ is the rate of change of the number of survivals with respect to time.

- The failure rate $r_x(t)$ gives the probability that the failure will occur in the next unit of time.
- Hence from the behaviour of $r_x(t)$ the chance or the aptitude of the still working unit to fail can be observed.
- Three different behaviours can be categorized:
 - Increasing failure rate (IFR):** If the failure rate is IFR, the conditional failure probability increases with the unit's age.
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 - Decreasing failure rate (DFR):** If the failure rate is DFR, the conditional failure probability decreases with the unit's age.

So, fine that that $R \times t$ is very important concept which is known as hazard rate and you will find out that these the if you actually really want to draw the curve that R a $R \times t$ these are this t , you will find a usually you find a curve like this which is known as bath tub curve bath tub curve. So, this there will be three regions, this is early failure, early failure, this is the constant failure, constant failure and this is worn out phase, worn out failure.

Means, you think of any component or any product or any curve everything that at the beginning, you will find out that there will be problem more problem and then slowly that it will it will basically stabilize and then after that after certain age it worn out will be more and it would it will basically that quick failure rate up to. If survive up to this, then immediately in the next instant of time or next small time interval the probability of failure will be very high. And here what happened, a suppose the initially the probability of failure is that in next in a in small interval of time, the probability will reduced.

So, this is bathtub curve, it is a very interesting one. And depending on actually that hazard rate has shown that increasing failure rate a failure rate is I for that the conditional probability increases with it is unit age. Similarly, constant means it is not decreasing or increasing, stagnant decreasing means with age it is the failure rate is decreasing like here early phase this decreasing, this is constant, this is increasing; so, IFR region ca, CFR region and DFR region.

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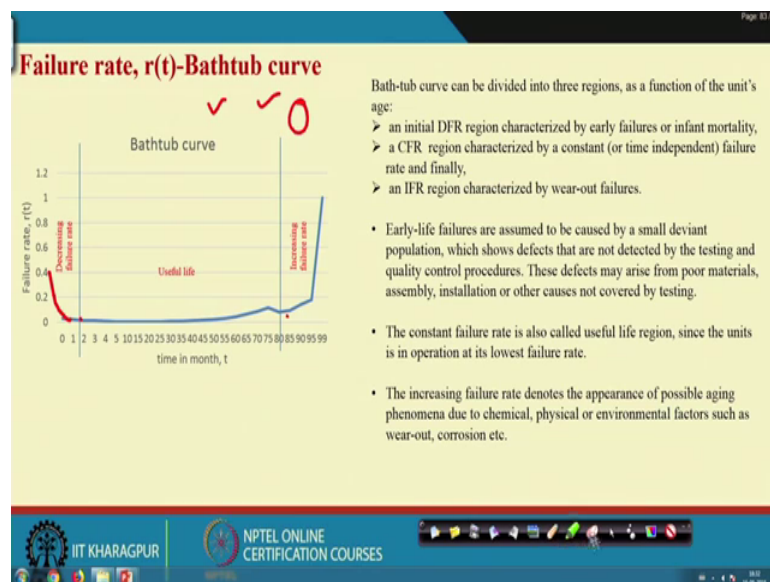
Failure rate, $r(t)$

t	L(t)	$n(t+\Delta)-n(t)$	$f(t)$	$R(t)$	$r(t) = f(t)/R(t)$
0	1050	30	0.028	1	0.028
1	1020	20	0.019	0.97	0.019
2	1000	10	0.009	0.95	0.01
3	990	10	0.009	0.94	0.010
4	980	6	0.005	0.93	0.006
5	974	12	0.002	0.92	0.002
10	962	10	0.001	0.91	0.002
15	952	13	0.002	0.90	0.002
20	939	15	0.002	0.89	0.003
25	924	18	0.003	0.88	0.003
30	906	23	0.004	0.86	0.005
35	883	31	0.005	0.84	0.007
40	852	42	0.008	0.81	0.009

t	L(t)	$n(t+\Delta)-n(t)$	$f(t)$	$R(t)$	$r(t) = f(t)/R(t)$
45	810	56	0.010	0.771	0.013
50	754	77	0.014	0.718	0.020
55	677	100	0.019	0.644	0.029
60	577	123	0.023	0.549	0.042
65	454	139	0.026	0.432	0.061
70	315	135	0.025	0.3	0.085
75	180	104	0.019	0.171	0.115
80	76	31	0.005	0.072	0.081
85	45	21	0.004	0.042	0.093
90	24	17	0.003	0.022	0.141
95	7	5	0.001	0.006	0.178
99	2	2	0.001	0.001	1
100	0	---	---	---	---

So, we have plotted this one, already we have they found out that F t R t is known. So, r small r t will be small R t this will be nothing but F t by R t . So, this way we have computed.

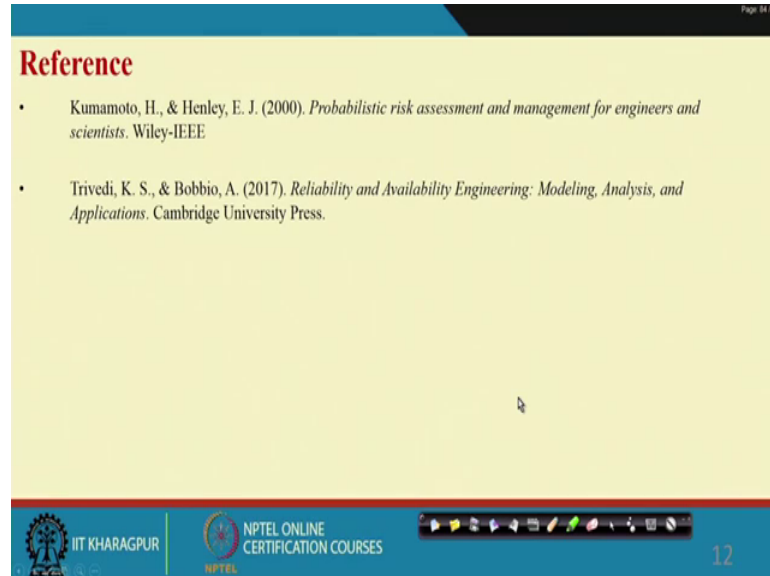
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And finally, you have got this plot for the data. So, this is our hazard rate. So, actually that are a it should be it should be may be like this, it should be like this, but the from the from the over hypothetical data we got this. So, this is this should be early decreasing failure rate, early failure constant part increasing part and that is what is the what is to be

understood from hazard rate. Hazard rate or instantaneous failure rate is a very very important concept for that component failure analysis.

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Reference

- Kumamoto, H., & Henley, E. J. (2000). *Probabilistic risk assessment and management for engineers and scientists*. Wiley-IEEE
- Trivedi, K. S., & Bobbio, A. (2017). *Reliability and Availability Engineering: Modeling, Analysis, and Applications*. Cambridge University Press.

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So, I hope that you understand and it is really a good exercise to follow through. Please practice all the numeric that data given to you, use the formulas and you yourself plot this in excel and check if there is any anomaly please report to us.

Thank you very much, thanks a lot.