Industrial Safety Engineering
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Lecture - 26
Ranking of Design Solutions: AHP Approach

Welcome, so, we are going to start another topic Ranking of Design Solutions we will be discussing the AHP Approach AHP is analytic hierarchy process.

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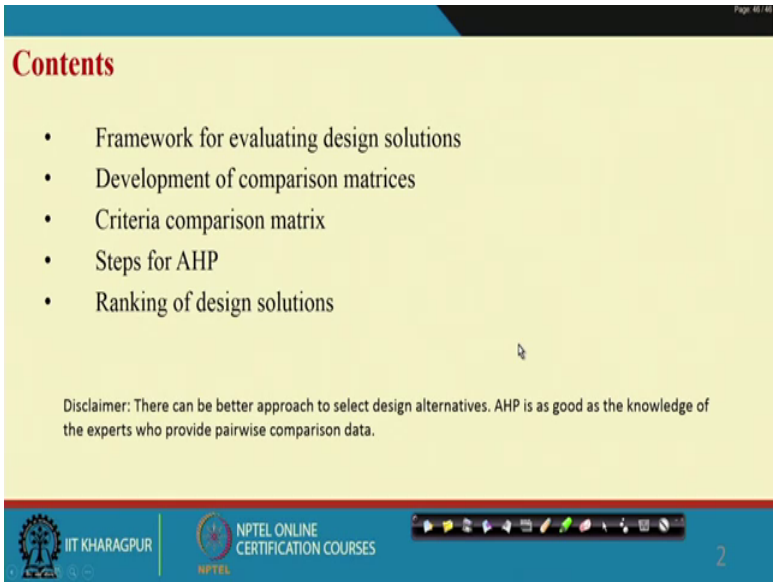

So, today contents framework for evaluating design solutions, then development of comparison matrices, criteria comparison matrix then step for AHP and ranking of design solutions. So, I hope that we will be able to complete it by 35 minutes of time and let us start with the first one that framework for evaluating design solutions.

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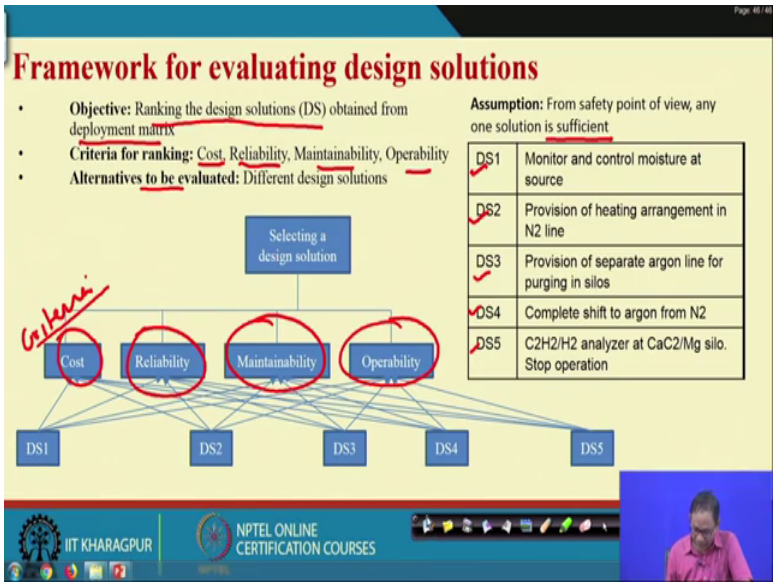

So, now I will relate this to the previous lecture, in previous lecture we have discussed safety function deployment and the sole purpose of that safety function deployment was to find out design solutions that will help in reducing the risk to the label acceptable provided the solutions are implemented.

We have given you the total approach that how you will ultimately come into getting some of the design solutions. And last lecture with reference to desulphurization plant we have identified several design solutions and for particularly hazard 1 and hazard 6 though these two hazards require similarities and solutions because they are similar in nature. So, five design solutions we have identified and; obviously, you may argue with the design with the ability of the design solutions there could be better design solutions, but that we have discussed in last class, that there can be alternative approaches, that can be better design.

But, nevertheless the approach was to help you to find out some of the design solutions. Now, we are assuming that that may be for a particular hazard to be addressed, there could be different design solutions that will be more than one design solutions or other way I can say that there could be more than one solutions that may ultimately prevent the accident scenario to happen. So, if this is the case then you required to find out the best design solution and that is what is known as ranking of design solutions. So, we have

some important assumptions here, one is that from safety point of view any one solution is sufficient.

Suppose you have identified five design solutions and we are saying that with reference to that particular hazards to come back any one of them are sufficient. It is very very conservative and restrictive one because, the design solutions are different and they could have the different potential, but never the less I am saying that any one of them are sufficient. So, if you find that no there are some better solution fantastic, but this is our assumption for comparison point of view.

The reason is if any one of them are not sufficient to fight for the safety then, that is not a good solution which you have found out using accept D so, that is not to be considered. So, as a result what happens once you have several design solutions you required to choose the best one and you require some of the criteria to have for that comparison. For example, if I considered that I was 5 design solutions and I want to select a best one, then there is set of criteria which help you in getting the best one.

So, what are the criteria we have considered with reference to design solutions which are basically interventions, it may be your hardware intervention, may be software intervention or maybe human ware interventions. But every intervention that will ultimately lead to improvement in safety there is a cost involved for implementing the intervention.

Now, second is that the intervention should be reliable on one, because it should perform the intended function for which it is developed. So, it should work within the mission time then, the intervention or the design solution that must be easy to maintain and that must be easy to operate also.

So, safety we are another criteria that how much risk reduction will be done that we have assume that it is a any one of them are sufficient. So, that is why may be the safety criticality there other criteria we have not added, but if you think that no critical through system to be added you can add another criteria also.

So, then you will choose one of the five design solution which will be ultimately weighted with the criteria satisfaction, that mean the design solutions should satisfy the criteria and at the same time there will be one which is best. So, what we are doing then

our objective is ranking of design solution obtained from s f d what are the criteria we are considering cost, reliability, maintainability and operability and I understand that you know what is reliability, what is maintainability and what is operability. So, we have discussed sometimes otherwise we will again discuss. And then alternatives to be evaluated is nothing, but the different design solutions. So, what is the framework?

If you want to use AHP so, there will be set of design solutions or set of which are basically set of alternatives. Then, there will be set of criteria based on which the alternatives will be evaluated and your ultimate goal is select the best one for best one best one best alternatives so, that is what is our framework. Now, let us see that how you will achieve in selecting the best alternatives given a several alternatives.

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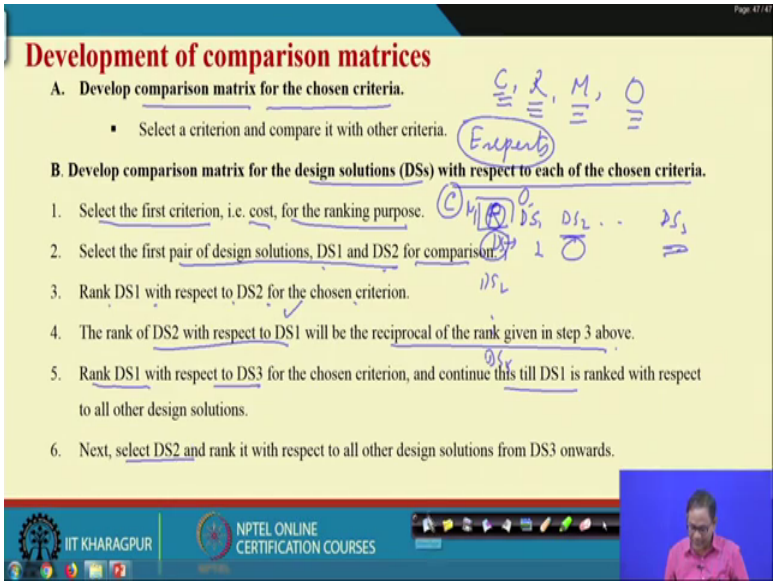

So, this is these are the steps, what are the, what is this first development of comparison matrices, what is comparison matrices? If you see the framework at the at the bottom you found there are alternatives and immediately above the alternatives there is there are criteria.

So, you have to develop comparison matrix for the chosen criteria and at the same time you also required to develop comparison matrix for the design solution, each of the design solution with respect to the chosen criteria. What do you mean by this? We have cost let it be C, then we have reliability 2nd criteria, we have maintainability 3rd criteria and we have operability 4th criteria.

Now when we choose design solution irrespective of which solution which design solution or which alternative it is but, there must be comparisons some weightage with reference to cost versus, the reliability, cost versus maintainability, cost versus operability, risk versus reliability versus maintainability, reliability versus operability, then maintainability versus operability or vice versa.

This is what is the comparison matrix and which where you are putting more importance, which is more important reliability of the design solution or cost of the design solution, maintainability of the design solution or reliability of the solution or they are equal. So, that kind of data you have together and this data are given by experts because, these are very specialized job, you cannot I can arbitrarily put some value and do it.

So, this comparison will matrix will come from experts. So, usually we consider 5 experts 3 or 5 or 7 x number experts and take the and the then come maybe if you take the average and get the comparison values, but it is not 1 expert it should be more than 1 exports preferably 5 experts.

So, another one is that suppose now again you have set of alternatives like D S 1 D S 2 to D S 5, D S 1 D S 2 to D S 5 with reference to let it be the first criteria cost how do you rate this alternatives. So, is D S 1 with reference to D S 1 the comparison will be definitely 1, but with reference to D S 1 and D S 2 if I consider cost is an important than what is a comparison is D S 1 is or D S 1 is 3 times 3 times better than more than D S 2 in terms of cost, if not it is the other way round. So, that way you have to develop the comparison matrix so, after you develop comparison matrix for cost then you go for reliability.

Then once that is done then you go for maintainability and finally, go for operability means all the criteria; for all the criteria the design solutions will be compared. So, when you start take any one criteria the approach is this, select first criterion for example, cost for the ranking purpose. Select the first pair of design solutions DS 1 and DS 2; DS 1 and DS 2 rank DS1 with respect to D S 2 for the chosen criterion that mean you will be having some scale and we will discuss the specialty scale we will use to get some values.

So, then rank DS 2 with respect to DS 1 and the reciprocal will be the rank given in step 3 similarly rank DS 1 to DS 3 still so long the DS 1 with all other; with respect to all other design solutions 5 design solutions are there. So, DS 1 to DS 2, DS 1 DS 3, D S 1

DS 4, DS 1 DS 5 like this. Then next you select DS 2 continue the process so, what I will do now, I will with example with some scale data we will see what is this.

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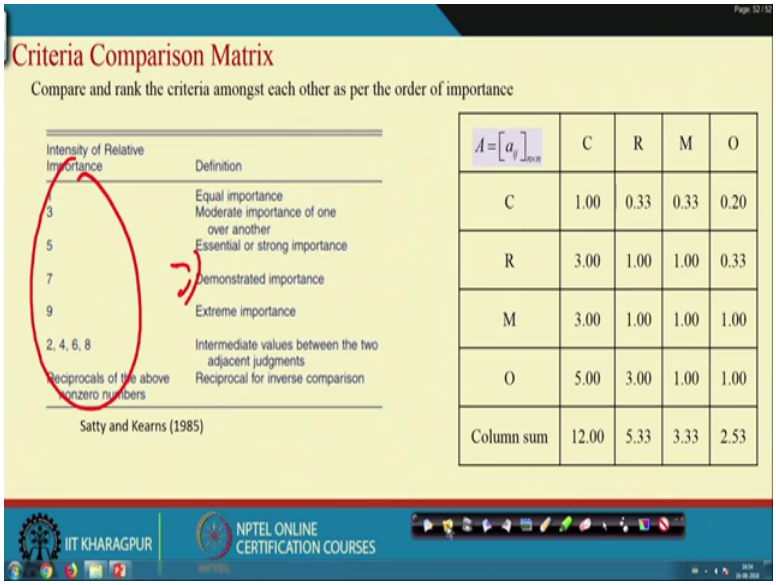

So, suppose you are interested to get the comparison matrix for the criterion, we have 4 criteria this side 4.

Now, the Satty and Kearns in 1985 from this source we found the, what are the scale, what are the values relative comparison values you will put. 1 means they are equal importance, 3 means moderate importance of one over another, 5 is essential or strong importance, 7 means demonstrated importance, 9 is extreme importance and 2468 intermediate values between the two adjacent judgements like in between 1 3 in between 3 5 like this. The reciprocal of the above non zero numbers that is the reciprocal of the inverse comparison for example, cost versus cost the same thing it will be 1. Now cost versus reliability it is 0.33 what is this actually; that means, the reliability is 3 time more important than cost. So, that is the reciprocal is point by 3, 0.33.

Similarly, maintainability is 3 times more important than cost reciprocal is 0.33, similarly operability is 5 times more important than cost then, reciprocal is 0.20. So, in this manner cost reliability, cost maintainability, cost operability, then reliability maintainability, reliability operability, like maintainability operability this way you have to you input the values.

So, let our values are like this, what does it mean? If I take this value what is the meaning; that means, maintainability and reliability are of equal importance. If I take this what is the meaning that mean operability is 5 time more times more important than cost, this 5 strong. So, this manner you will get the values, who will give the values? The expert will give the values. Then 5 expert then you just take the average, diagonal elements will always be 1 because, they are comparison with the same criteria like cost versus cost.

So, this is what is comparison matrix, later on I will come to the column sum and other issues but, comparison matrix will be created in this manner. Now with reference to a particular design solution so, what is the comparison matrix you see.

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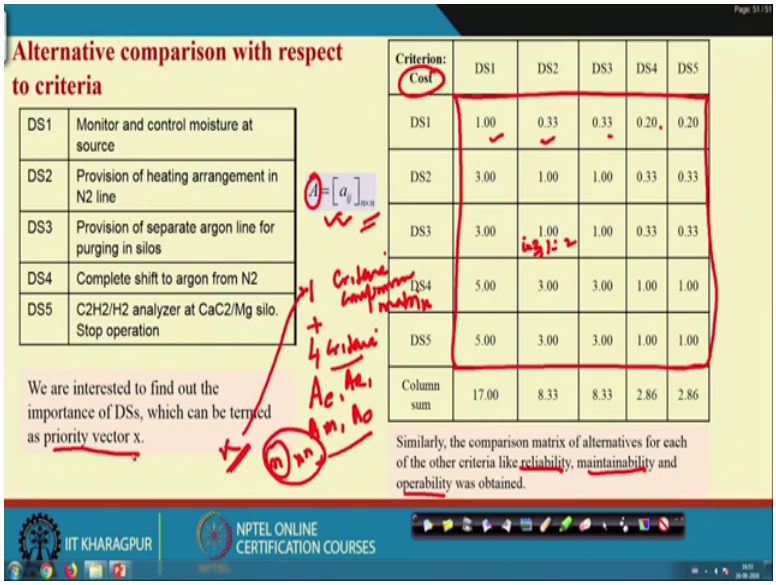

So, how many solutions we have? We have 5 solutions so, DS 1 to DS 5, DS 1 to DS 5. So, here the comparative alternatives will be compared with respect to each of the criterion we have considered cost. Now, what is the meaning of suppose, this that mean DS 4 cost is 3 times more than DS 2 because it is cost comparison that cost comparison of DS 4 is 3 times more than DS 2.

So, as a result what will happen that mean DS 2 will be, DS 4 is 3 times then DS 2 will be 1 by 3 0.33 reciprocal. Similarly, if I consider suppose DS 2 here so, DS 2 the cost of DS 2 is 3 times more than DS 1. So, then the reciprocal will be the DS 1 DS 2 0.33

again. So, like this DS 5 that is 0.25 DS 1 is 0.2 times DS 5 so, then DS 5 will be the 5 times same manner given by the experts.

So, then what we are saying that we are interested to find out the importance of DS which can be termed as priority vector x, suppose, now what is what will be our job with reference to cost, what is the priority vector for the 5 design solutions here their alternatives. So, that it that is basically our job, we want to find out x which basically talks about say which is a priority vector, which talks about how in terms of cost DS 1 to DS 5 is prioritized some scores you will get.

So, here we have used this notation that is A it is this is nothing, but the comparison matrix. So, this is A this one is our A with respect to that 5 alternatives compared in terms of cost. So, what is a i j each of the elements so, ith column and a row and jth column value is a i j so; that means, if I consider 1 2 3 so, if i here i equal to 3 j equal to 2. So, i j, what is n? N is the here say number of alternatives or number of design solutions. So, as you get the comparison matrix A for cost similarly you can get comparison matrix for reliability for maintainability and for operability, how many criteria you have chosen? You have chosen here 4 criteria.

So, you will be getting 4 comparison matrix so, A cost, A reliability, A maintainability and A our operability, in addition the cost itself will be sorry the criteria itself will be compared amongst each other so one more will be there which is that is criteria comparison matrix, criteria comparison matrix. So, if there are suppose k criteria so, you will be having one comparison matrix for this k criteria.

So, another is another is; for suppose you have n design alternatives so, that mean n cross n that comparison matrix for each of the k criteria. So, ultimately k plus 1 comparison matrix will be there. So, once you have the comparison matrix, then how you will find out the priority vector x that we will discuss next. I hope you understand so, there are set of alternatives in our example design solutions, there are set of criteria based on which you evaluate the alternatives, here the design solution and find out the best alternatives means best design solution.

So, what data you require? You require comparison pair wise comparison data. So, what are those pair wise comparison data? One is criteria to criteria comparison, another one is

the with respect to each of the criteria all the alternatives pair wise comparison will be made.

So, if there are k criteria 1 matrix comparison matrix with amongst the k criteria and k comparison matrix for amongst the design alternatives or among the alternatives here, the design solution. In our example there are 4 criteria and we have 5 design solutions. So, in total there are 5 comparison matrix, 4 with respect to each of the criteria for the alternatives and 1 with respect to the criteria, 4 criteria you get that that one. Who will give you the data? Data will be given by expert. How many experts you will choose? It is better to choose more than 1 expert 3 or 5. So, what scale you will use the Satty scale what I have given to you in this slide.

So, you Satty scale what are the meaning it is given there. So, once you complete this one you have data is available with you for doing AHP and getting the best solution. Now, we will see the how the best solution is achieved.

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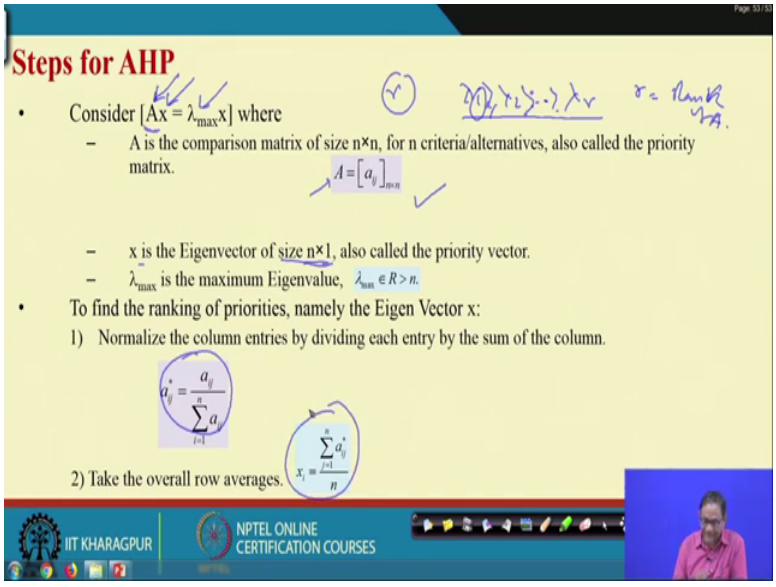

So, the mathematics behind AHP will not go into the deep of the mathematics, but please understand what is A? A is the comparison matrix. What is Ax? The priority vector then, what is lambda max? Lambda max is basically the eigenvalue of the comparison matrix A. So, while lambda max mean the you will be able you will get set of eigenvalues, how many eigenvalues depending on the size of the matrix, actually depending on the rank of the matrix.

If the rank of the matrix is r, then you will you will have r eigenvalues, r number of eigenvalues like lambda 1 lambda 2 to lambda r, where r is rank of A. So, out of these the maximum one usually the way we are in eigenvalue decomposition we find out the 1st one will be the maximum followed by the 2nd one and that is basically the approach. So, lambda max with a maximum one that is what you want to find out, you will not considered other eigenvalues only lambda max you will consider. So, A x equal to lambda max x this is the equation or which is basically characteristics equation in matrix.

So, you have to find out this equation, you have to first find out the lambda max, once you have lambda max you will be able to get the x value. So, this part is over, we have already seen this is the comparison matrix, x is the eigenvector of size n cross 1, because how what is the either in the criteria alternative if there are n criteria it will be n cross 1.

If there are n alternative it also will be n cross 1 that is the eigenvectors, this is the priority vector and lambda max is the maximum eigen value that I already told you. So, now, you may go by eigenvector, eigenvalue eigenvector decomposition so, that approach we are not using here is a simple approach we are using which will give you the equivalent result, what you do because you have A matrix.

So, you normalize column entities by dividing each entry by the sum of the column this one and then take the overall averages. So, what is it? You just see for example, this is our matrix A so, what is the sum? Sum here is 17 here 8.33 8.33 2.86 2.86 sum of the weights what is given here. Now, if you divide each of the entities in this column by this each of the entities in 2nd column by column sum 3rd column by 3rd column sum, 4th column by 4rth column sum, 5th 1by 5th column sum you will be getting another matrix. What is this? That is what is a normalized matrix this.

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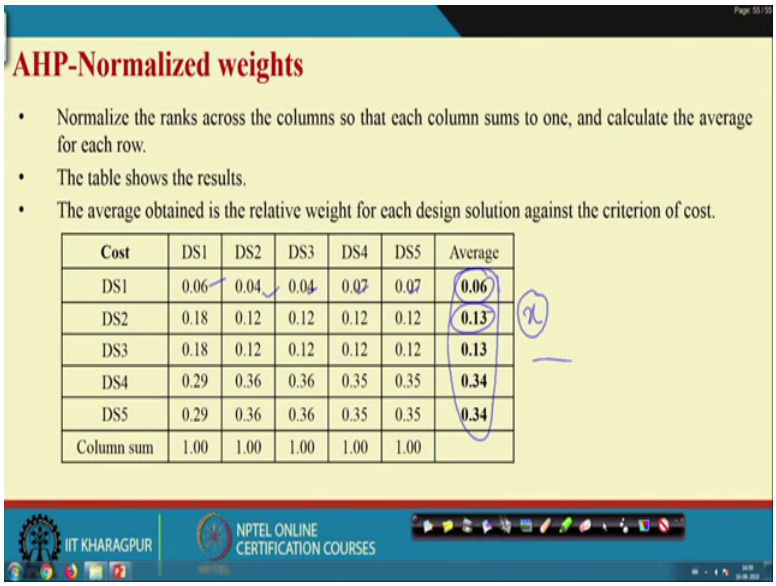

Cost	DS1	DS2	DS3	DS4	DS5	Average
DS1	0.06	0.04	0.04	0.07	0.07	**0.06**
DS2	0.18	0.12	0.12	0.12	0.12	**0.13**
DS3	0.18	0.12	0.12	0.12	0.12	**0.13**
DS4	0.29	0.36	0.36	0.35	0.35	**0.34**
DS5	0.29	0.36	0.36	0.35	0.35	**0.34**
Column sum	1.00	1.00	1.00	1.00	1.00	

So, that ultimately the column sum over 17 and 1 by that which one is this with reference to cost. So, if you will get this matrix each of the column entries will be divided by column sum, then what you do then you find out the overall row average, this row average how many rows will be there it depends on how many alternatives are there row average.

So, take the average so, this plus this plus this plus this plus this by so, this plus this plus this plus this plus this plus by pi this is the row average, 2nd row average 3rd row average so like this. So; that means, this is our x priority vector of the design alternatives with reference to cost.

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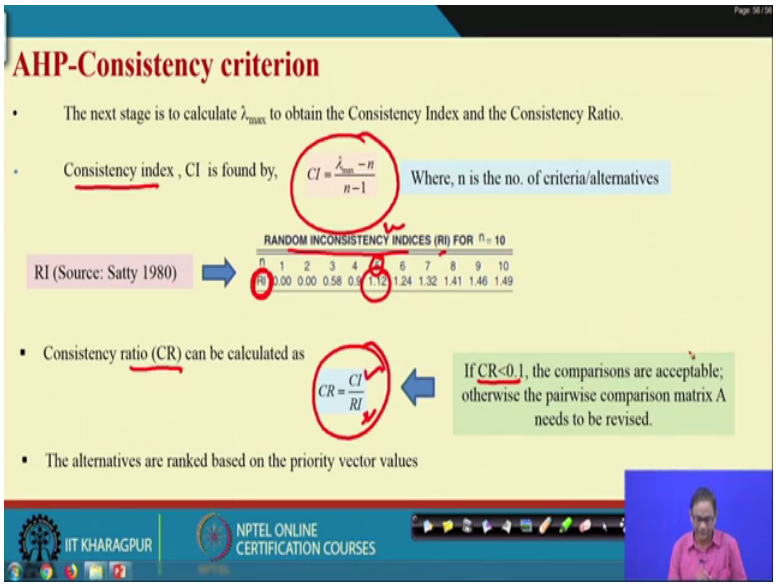

Now, what happened that you have to do some test here, so, you whether the comparison given by the experts they are consistent or not? So, you ultimately what you do you have got this matrix and finally, you have found out these averages fantastic. This means x that priority vector is formed, but this is based on the comparison pair wise comparison given by the experts so, whether experts are consistent or not.

So, this can be judged by this mathematics, what is the mathematics? First you find out the consistency index. What is consistency index? Is this lambda max minus n by n minus 1. So, n you know, lambda max already computed so, then what is then what you will find out that there is another index called random in consistency index RI. So, Satty has given the random inconsistence index for 10 number, for n equal to 10 means 10 alternatives or 10 criteria to be compared. So, suppose in our case so n is 5 then the random inconsistency index is 1.12 so, you have found lambda max and n is also known.

Now, then the consistency ratio CR will be CI by RI these consistency ratio this should be less than 0.1. So, 0.1 if then it is acceptable otherwise not so, let us see whether things are what is the status now.

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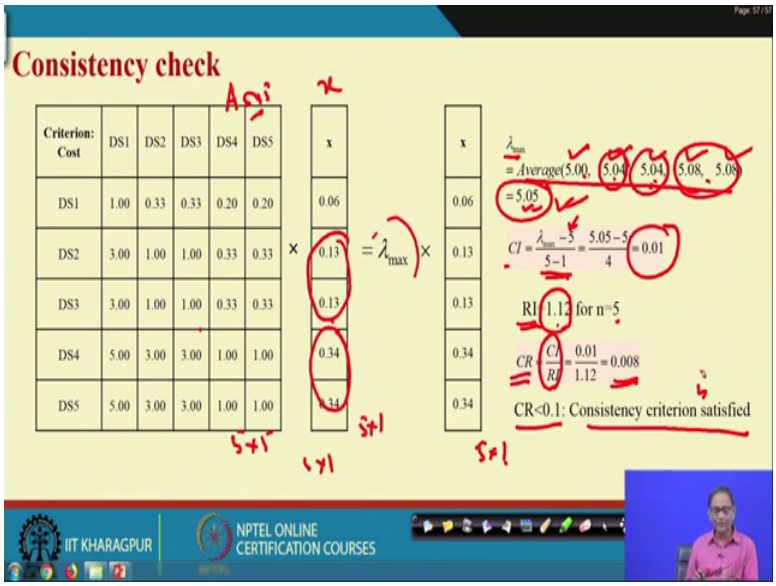

So, with reference to cost this is our matrix comparison matrix, this is the vector priority vector you have obtained now, these A this is our A and this is x equal to lambda x lambda x. Now, you are finding out lambda max, lambda max average of these this is 5. So, you know this is there the down lambda max is you can calculate how many equations will be there.

So, there this is basically 5 cross 5 this is my 5 cross 1. So, resultant will be 5 cross 1 and this is also 5 cross 1. So, you will get 5 equations so, lambda max 5 values you will find out lambda values so, that take the average of all those 5 values will be getting this. So, that mean from this equation this matrix 5 values you obtain and first value is 5 2nd 5.4 5.04 5.08 like this because, these two 13 you see 0.13 that is why this 5.04 5.04 these two are same; these two are same. So I hope you understand that mean you will be having this 5 cross 5 equations, but you want only 1 lambda value. So, you take the average you will get 5 lambda values you take the average that will be that average will be our that lambda max values. So, now, lambda max values is 5.05 and our n number of how many what is the n size of the this matrix 5cross 5.

So, 5 n equal to 5 divided by 5 minus 1 this is 0.01, then what is RI for 5 from the table I have shown you earlier it is random inconsistent in this 1.05, what is then consistency ratio CI by RI this is 0.008 it is less than 0.1. So, consistency criterion is satisfied. So; that means, the experts so, what about the data given by the pair wise comparison data is

consistent. So, you have to do it for other comparison matrix related to reliability, maintainability, operability, as well as for the criteria comparison matrix.

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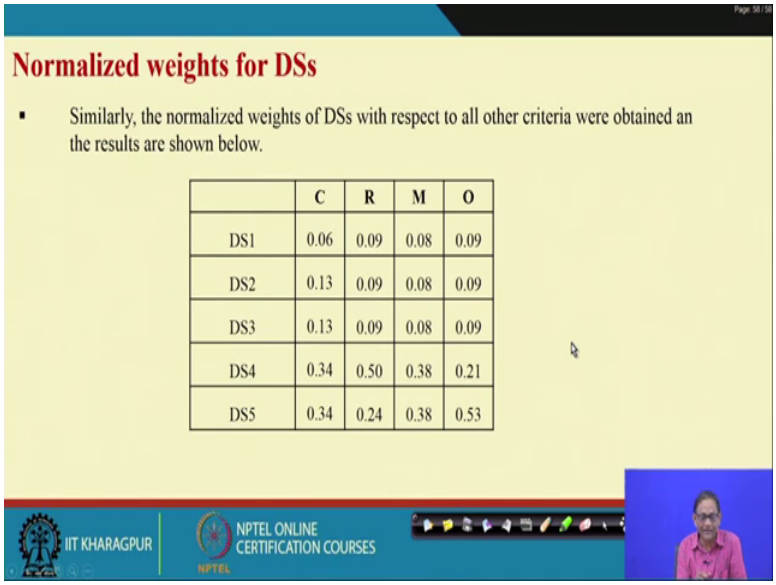

So, similarly for the criteria also you will get the normalized this values because, the column sum will be divided by each entries of a column will be divided by the column sum, the way we have done for design solution with reference to cost.

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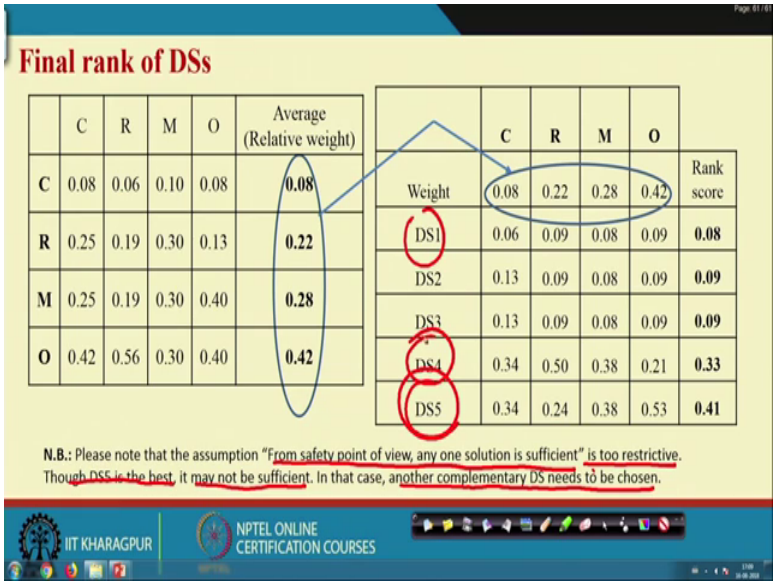

After that what will happen? So, there are two issues in a one is you have priority factor for the criteria same manner you have computed and you have the priority vector for the

design solutions with reference to C R M and O what is this? Let me go back 0.06 0.13 0.13 0.34 0.34 these are basically priority vector for 5 design solution with reference to cost.

Now, come [FL] so, what is with reference to your that DS 1 DS 2 DS 3 DS 4 DS 5 with reference to cost 0.6 13 13 34 34. So, these are the priority vector with reference with reference to cost for the design solution similarly, you have to find out with reference to reliability, with reference to maintainability and with reference to operability. Now, overall score you have to find out because, the weight priority vector also to be used, then if you do 0.08 multiplied by 0.06 plus 0.22 multiplied by these plus 0.08 multiplied by these plus 0.42 multiplied by these this one this sum will be this.

So, then DS 1 rank is 0.08, now same thing for DS 2 that mean 0.8 multiplied by 0.13 plus 0.22 multiplied by 0.09 plus 0.22 multiplied by 0.08 where 0.42 multiplied by 0.09 it will be 0.09. So, again DS 3 also as these values are same the rank is also same and DS 4 that is 0.34 into 0.08 0.22 0.50 0.28 into 0.38 0.42 into 0.21 this is 0.33 so, in this manner it is 0.41.

So, now, what happened basically which one is the best then DS 5 is the best as per as this calculation. So, this is what is AHP Analytic Hierarchy Process for finding out the best design solution please keep in mind that the calculation whatever given here you please check if there is any mistake anywhere in computing through calculator so, just rectify it. But what I hope that you understand the procedure and using this procedure you will be able to find to the best design solution.

So, few important thing here, that please note that the assumption from safety point of view any one solution is sufficient is too restrictive. Though the DS5 is the best it may not be sufficient in that case another complementary DS needed to be chosen. So, all we said it is sufficient, but there is no the experts only the design engineers, the safety engineers having design knowledge, process knowledge and the, and what is this? The hazard knowledge they will be able to tell that yes it is correct or not.

So, if you find out that node that DS5 alone is not sufficient then maybe you may go you may choose again DS4 also or if it is cost wise vary not vary not permits; permitive then maybe DS1 if you think . So that is what I mean to say this gives you the idea of that

which one is the best which one is the next best like this, again that AHP as good as the data provided by the experts.

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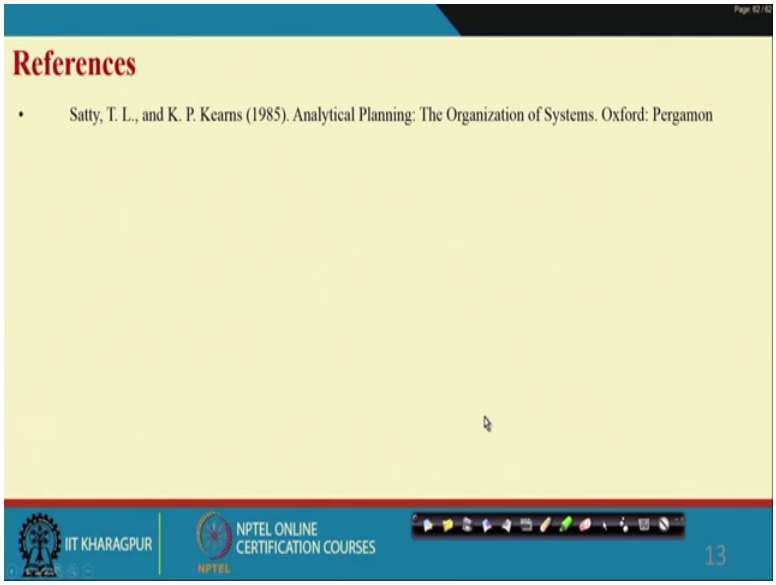

So, this is the reference so, I hope that you understand the use of AHP please that take a problem or the same problem you solve and practice it.

Thank you very much.