Industrial Safety Engineering
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Lecture – 13
Fault Tree Analysis (FTA) - Gate by Gate Method

Hello. Now we will discuss Fault Tree Analysis Gate by Gate Method. Actually this is under the fault tree quantification.

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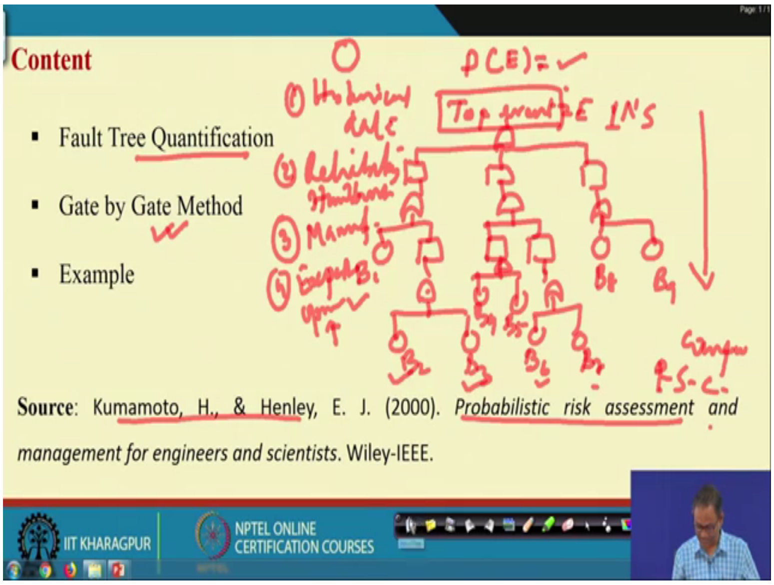

In last class you have seen how fault tree is constructed. So repeat this, so what happened? You first find out the top event, then find out the first level contributors, then find out some logic gate to link with the first level contributors, and then find out the second level contributors, and then find out gate again. Again you find out gate here, there may be the fourth level contributors. Suppose here one and gate, another contribution and here may be or gate and contribution for this top event is finally link to bottom events these are the component level events B 1 B 2 B 3 B 4 B 5 B 6 B 7 B 8 B 9.

So, when you are at the top level we say that you use the concept call ins find out the immediate necessary sufficient that intermediate events immediate bottom events which will cause the top events, put a particular gate and then that gate is a logical gate, where this logic gate is. And so, this mean, this 3 immediate event must occur for the top event to occur. Now here this one may be the subsystem level that failure event. Then you require to find out further event at the lower level, and in this process from system level event to the component level event you find out. And at the component level you use PSC concept and this is what is known as fault tree construction.

Now, once the fault tree is constructed you require to quantify the top event that is what is the quantification here probability of top event; if top event equal to E then; what is the probability of E. So, how can you get it? You can get if P E or you can quantify P E given the basic events. As basic events of the component level events their data is available or data can be obtained from reliability handbook or data can be obtained from historical records, or at the last may be you will go by expert opinion from subjective probability to objective probability computation you can do.

So, that means what is required for fault tree quantification? In fault tree quantification input is first is the for quality tree fault tree. This quality tree fault tree links to the top event to the bottom event or component level event. Then you required to know the component level probability or basic event probabilities. Basic event probabilities can be found from historical data historical data, basic event probabilities historical data; number 2 can be reliability handbook reliability handbook; number 3 you may be your manufacturer will specify this one manufacturer specification; number 4 many cases you rely expert you rely on expert opinion. So, expert opinion can be subjective in nature, it can be it can be objectively quantified the probability.

So, once you have the bottom level or basic event probabilities, then you can compute the top event that is probability of top event by different methods one of the method is gate by gate method. So, in this lecture we will discuss this gate by gate method ok. So, another method will be cut set method ok, so that we will discuss in next lecture. So, for your reference I you follow this book probabilistic risk assessment and management, for engineer and scientist this is by Kumamoto and Henley and you can go through any of good reliability engineering handbook or safety engineering book, you will find out the quantification part fault tree quantification

So, then what is our topic in this lecture fault tree quantification by gate by gate method.

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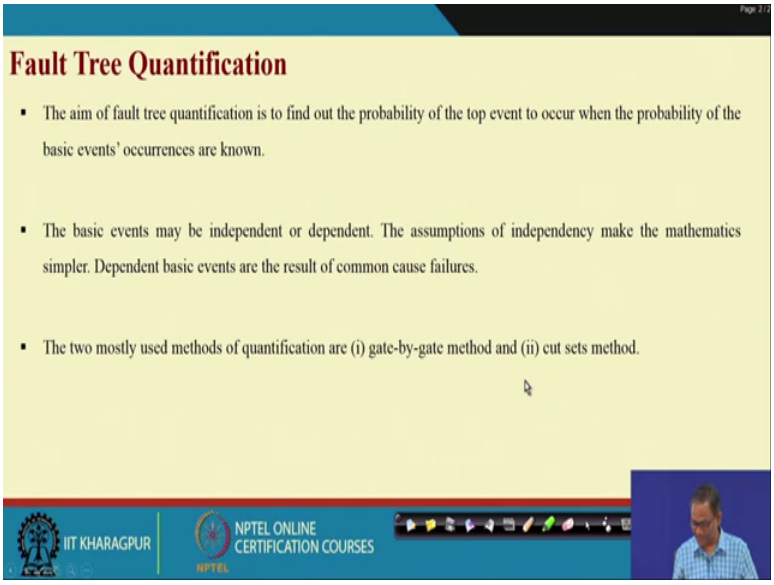

So, let me repeat the aim of fault tree quantification is to find out the probability of the top event to occur when the probability of the basic event occurrences are known. The basic event may be independent or dependent, the assumptions of independency make the mathematics simpler dependent basic events are result of common cause failure.

The two mostly used methods for quantification is gate by gate method and cut set method we will see both this lecture gate by gate method followed by cut set method.

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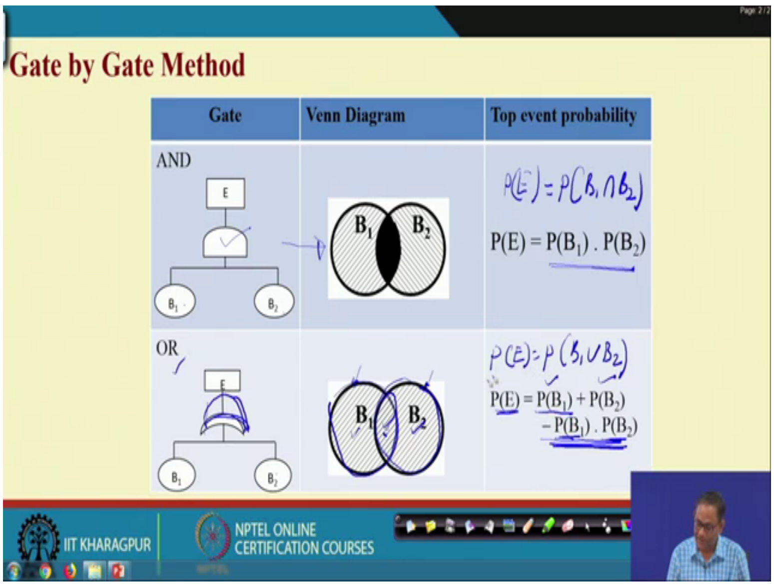

So, when we talk about gate by gate method, this is what is the issue

First is when there is AND gate, then the Venn diagram that mean probability B 1 and B 2 these 2 events their probability P B 1 and P B 2 are known now here the top event is E. So, what is the probability the top event will occur given that the bottom event or component event occurs? This is by the simple one

So, now this equivalent Venn diagram is this, this first circle talks about probability of B 1 and second circle talk about probability of B 2. What is AND gate? AND gate means both B 1 and B 2 should occur then only the top event will occur. So, that mean in this diagram you can see what is the common portion. This common portion the black portion, this is the probability that both B 1 and B 2 occur.

So, the common portion when is basic logically represented by AND gate. So, that is why probability of E equal to probability of B 1 into probability of B 2 this is the common portion. So, intersection that is probability of E equal to probability of B 1 intersection probability of E equal to probability of B 1 intersection B;. B 1 intersection B 2 is this overlap portion. So, suppose this is the simplest fault tree. So, you have one top event and 2 basic bottom event and your calculation is like this. Now what will happen if you have B 1 B 2 B 3? So, if there independent ultimately P B 1 into P B 2 into P B 3 like this.

Suppose the second this is what is gate by a gate quantification. But in a real fault tree there will be many gates. So, many you have to compute every gates. So, that is why gate by gate method. So, first basic gate operation or quantification you see second one; second one is OR gate. What is OR gate that mean if any of the basic event occur top event will occur, this is our OR gate if any of the basic event occur top event will occur

Now, the probability of B 1 is represented by this first circle, B 2 represented by the second circle. So, then if I say that both either B 1 B 2 or both will occur, then this overlap portion and the non-overlap portion all portion. So, that mean the probability is this portion, this portion plus this portion that mean non-overlap portion of B 1 non overlap portion of B 2 plus overlap portion.

So, then what is P B 1? P B 1 is the first circle; so probability of probability of E equal to probability of B 1 union B 2. So, this union means the total here what happen this equal to probability of B 1 so, this portion. Now probability of B 2 again this portion, so by adding probability of B 1 and probability of B 2, you are adding this overlap portion twice what it is this overlap portion? That is P E inter joint probability intersection P B 1 and P B 2.

So, as by adding P B 1 and P B 2 you are adding this overlap portion twice. So, one deduction; that means, subtraction is possible require, where is P B 1 plus P B 2 minus P B 1 and P B 2 this is basically if it is or gate you will use this formula.

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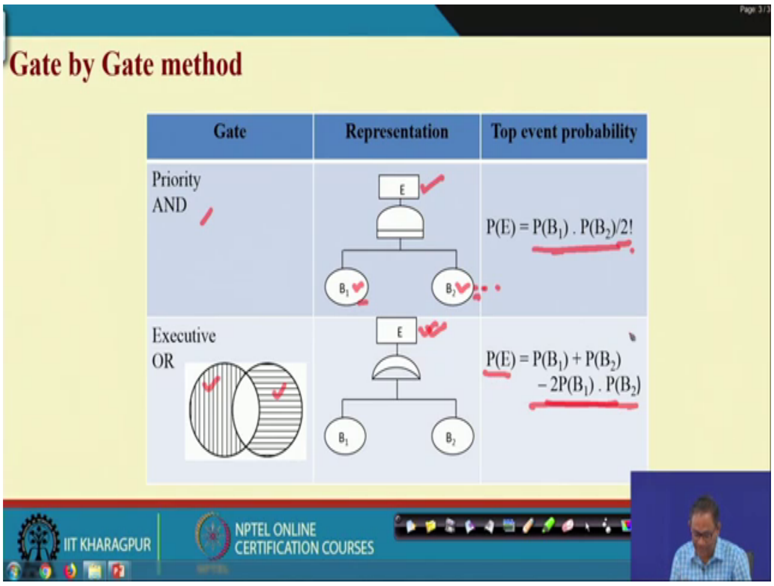

What will happen if it is a priority? If it is a priority AND gate. So, priority AND gate means B 1 B 2 should occur, but the order. So, then first B 1 occur followed by B 2 occur. So, the probability of E is definitely the intersection probability of B 1 intersection B 2 into something what is this? That how many order possible and how many is in favour. So, as there are 2 input event. So, either B 1 B 2 B 1 first occur followed by B 2 or B 2 first occur followed B 1 so, that mean it is factorial 2 way this will occur

How many combinations are favouring this to happen? Only one; so what is the probability of that priority ordering one by factorial 2? So, that is why that P B 1 into P B 2 into probability of that combination suppose C and then that is 1 by 2. So, this is the probability.

Now, what will happen if you have something more? So, what will P B 1 into P B 2 into P B 3? Like this into P B k divided by factorial k because this order is very very important. So, this is priority and gate similarly exclusive or gate exclusive or gate means what happen? Either this occur this occur, but not both. So, this is the symbol

Now, if we write probability of E the top event P B 1 first part plus P B 2 second part then twice we have taken this 2 P B 1 P B 2. So, you either use Venn diagram or some logic, but you have to have for every gate you have to find out the gate the output probability of the gate given the input probability. Other it is and gate or gate or exclusive or gate or priority and gate that is what we have to do.

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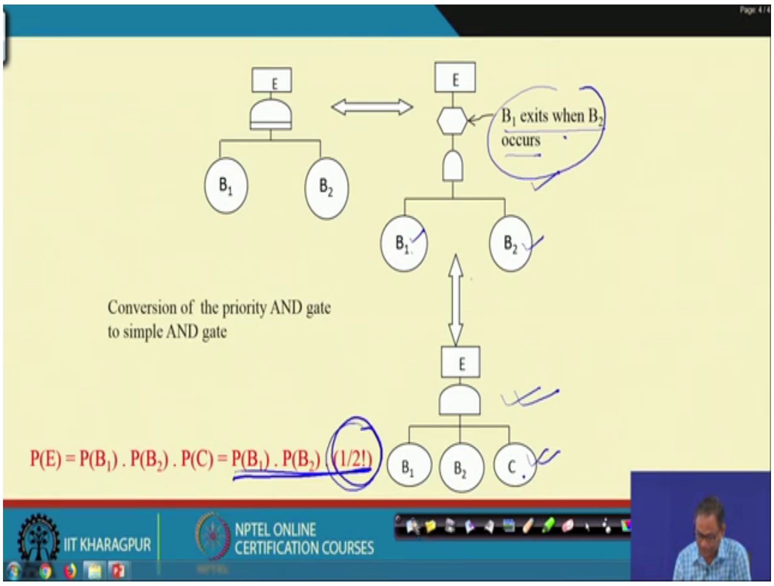

Now here I am showing you that how this formula is derived this formula. You see this is our priority and gate its equivalent representation is this that P B 1 or B 1 B 2 should occur, but at the same time the condition is B 1 exist when B 2 occurs that mean B 2 already B 1 occur before B 2 occur. So, these equivalently represented by this. So, B 1 B 1 B 2 B 2 and this condition we are representing by c. So, that mean it is an AND gate with 3 inputs B 1 B 2 and C. The C part probability of C is this. So, that is the derivation.

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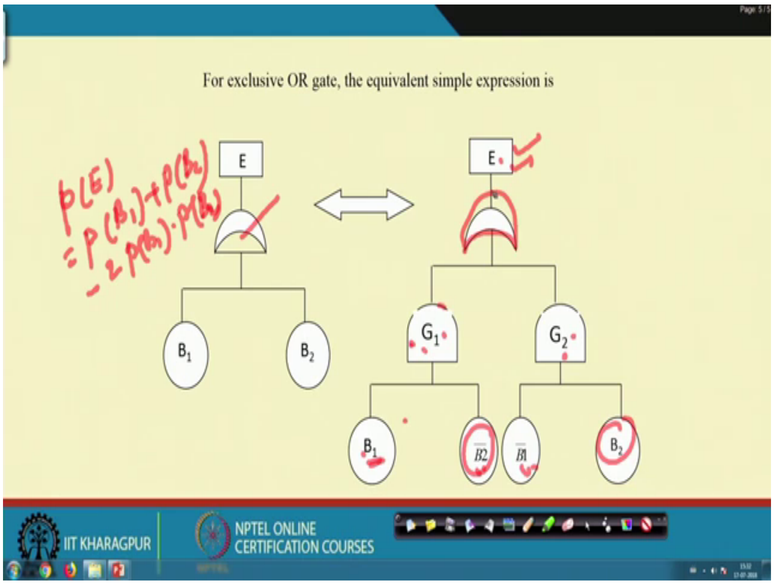

Now exclusive OR gate we are seen that P B 1 p p probability of E what you have seen probability of E P B 1 plus P B 2 minus 2, P B 1 P B 2 what is exclusive OR gate that, either of the 2 one should occur, but not both. So, its equivalent this is exclusive OR gate its equivalent or and representation is this. So, B 1 should occur B 2 should not occur that is why B 2 bar. Here B 2 occur, but B 1 should not occur that is B 1 bar.

Then B 1 occur B 2 does not occur then this AND gate if this condition satisfied E will occur, other way B 2 occur B 1 will not occur this condition satisfy E will occur. So, that is why these 2 conditions with in terms of in the gate of G 1 and G 2 are linked with E by an or gate this is an or gate. So, if G 1 condition and G 2 condition either of the 2 occurs then a 1 will occur

So, it is equivalent probability representation; that mean this is equivalently represented by this. Now if you want to compute the probability then this is what is computation.

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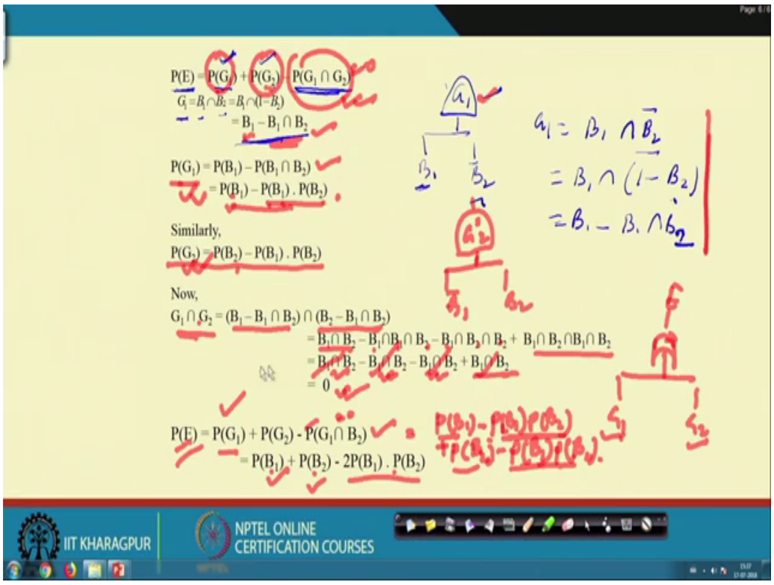

Probability of E probability this is linked with gate G 1 and G 2 by an OR gate. So, that is why the top event probability is this immediate child event probability that input probability G 1 plus G 2 minus their intersections that we have seen in OR gate

Now, what is G 1? G 1 is G 1 you have seen that G 1 link this is basically an AND gate with B 1 and B 2 bar. So, B 1 and this side B 2 bar B 1 should occur B 2 should not occur. So, that is why G 1 because of AND gate it is B 1 intersection B 2 bar. So, G 1 we can write in terms of Venn diagram B 1 intersection B 2 bar.

Now, this can be written as B 1 intersection 1 minus B 2 because B 2 that base that event either occur or does not occur. The occurrence if it is B 2 then one B 2 bar is 1 minus this. So, so then this can be written as B 1 minus B 1 intersection B 2 B 1 intersection B 2 that is what is written here.

So, as a result what happen when you are writing P G 1? You are writing P B 1 minus probability of B 1 intersection B 2; that is what you written here. Now, P B 1 P B 1 and this intersection as they are independent; that is P B 1 into P B 2; so the first gate is probability is quantified similarly for G 2 also for G 2 that is also an AND gate, and you will find out that it is B 1 bar and B 2 these are the input. So, B 1 bar is 1 minus B 1 and B 2 B 2. So, when you multiply that that you do similar derivation you will get P G 2 equal to P B 2 minus probability of B 1 into probability of B 2 P B 2 is probability of B 2 ok

So, that mean these 2 gate G 1 and G 2 are quantified then what is E? E is linked with an OR gate to what? That is G 1 and G 2. So, when it is OR gate your see P G 1 probability of G 2 minus this. So, we have quantified P G 1 here by this method P G 2 by this method, but the intersection is not quantified yet. So, if you write intersection G 1 intersection G 2. So, you have already seen that G 1 equal to B 1 minus this. So, you write this and intersection again you write G 2 like this,

So, if you do the algebraic manipulation here what happen? B 1 B 2; then B 1 B 1 B 2 B 1 B 1 B 1 B 2; then minus B 1 B 2 B 2 plus B 1 intersection B 2 B 1 intersection this B 2 again; so B 1 intersection B 2 is ok. Now, B 1 B 1 B 2 is nothing, but B 1 intersection B 2 then B 1 B 2 B 2 nothing, but B 1 intersection B 2 and this also nothing, but B 1 intersection B 2 so; that means, B 1 intersection B 2 B 1 intersection B 2 cancel out B 1 B 2 intersection cancel out this is 0. So, this particular part will become 0.

Then PE probability of E, this is P G 1 P G 2 P G 1 plus P G 2 with an [FL] or gate this one same formula we have written here, but this will become 0. So, then P G 1 is what? P B 1 then P B 1 into P B 2 and P B 2 like this. So, this can be written like this P B 1 minus p B 1 P B 2 this is for this part P G 2. So, plus P B 2 minus P B 1 P B 2 then this part is 0.

So, then P B 1 P B 1 P B 2 P B 2 minus P B 1 into P B 2 minus P B 1 P B 2 P B 1 into P B 2 that is what is the derivation. So, if you go back you see this is; what is the derivation exclusive or gate P B 1 P B 2 minus 2 P B 1 into P B 2.

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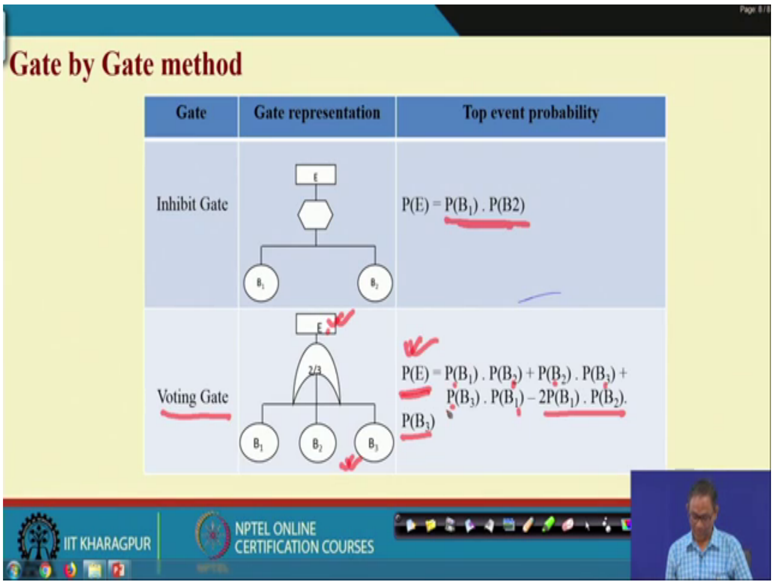

Now the last one from the last 2 inhibit gate and voting gate. So, we have discussed that inhibit gate in the last class and you have seen that it is nothing, but that an AND gate equivalent AND gate. So, that is why what happen in case of inhibit gate the top event will occur only both the event will occur and it is the intersection. Please remember we are basically talking about all the cases independent basic events.

Now, in case of voting gate that you have seen that m out of n. So, if we consider 2 by 3 the 3 inputs are there, if 2 of the 3 inputs occur then the output will occur where is the situation. And under 2 by 3 voting gate case then your probability of top event or the output event will be P B 1 into P B 2 plus P B 2 P B 3 plus P B 3 P B 1 minus 2 P B 1 P B 2 and into P B 3. So, that is what is the formula

Now, we will see that how this formula is derived.

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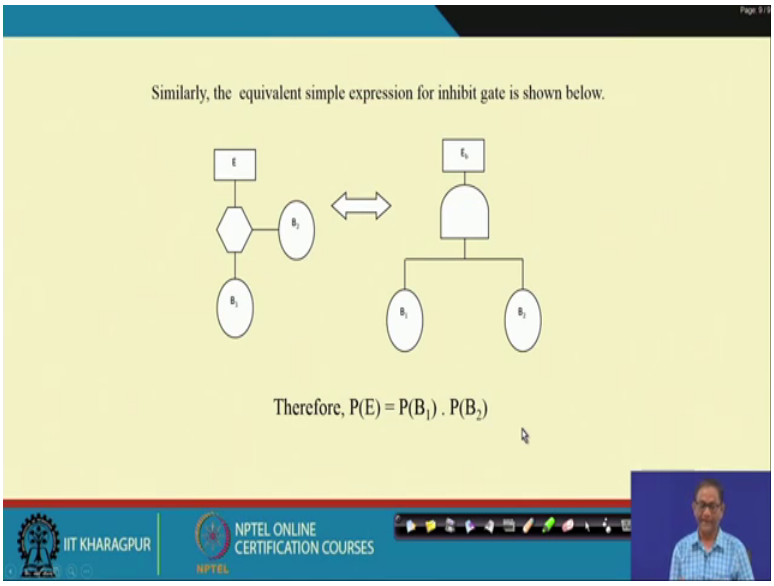

I told you about this is simple one that it is the inhibit gate and it is equivalent to this and then probability is this. So, we have discussed this no need of discussing further but this one let us discuss

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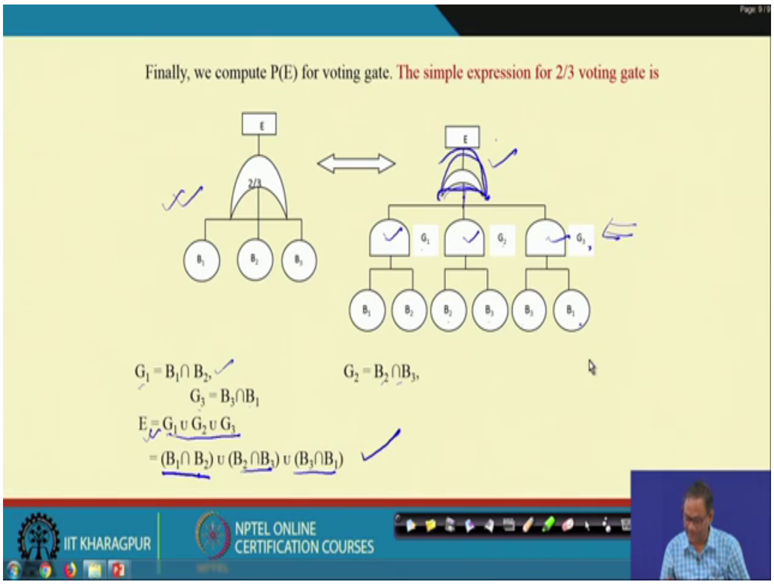

So, what we are doing here first this voting gate 2 by 3 gate is represented in equivalent or and AND gate this is AND gate or and AND gate. So, if 2 out of 3 occurs top event will occur. So, then if B 1 B 2 occur top event will occur B 2 B 3 occur top event will occur or B 3 B 1 occur top level will occur.

So, what that is why what happened that mean this condition this condition or this condition are in an AND gate that gate number G 1 G 2 and G 3 and then any of this conditions occur lead to top event occur that is why the or gate is given here.

So, the quantification gate by gate means first you quantify gate 1, second quantify gate 2, third quantify gate 3 then you quantified this gate 4. So, what is the first gate there AND gate. So, it is B 1 and B 2 and G 2 that is B 2 B 3 and G 3 B 3 B 1. So, G 1 and then what is the E? Anyone of this; so that mean the union part G 1 union G 2 union G 3; so as a result B 1 G 1 is B 1 intersection B 2 G 2 is B 2 intersection B 3 G 3 this. So, you do this ok.

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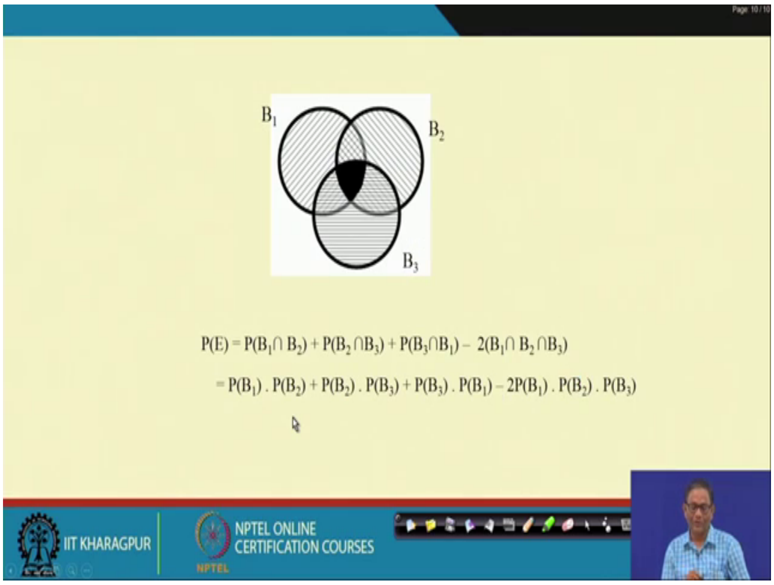

So, if you proceed in this manner you see your ultimate image basically to find out the probability of E. Now this is the equivalent Venn diagram. And you see that ultimately the probability from here what is required; probability of B 1 and B 2 probability B 1 and B 2 this intersection B 2 B 3; so B B 2 B 3 this intersection sorry this intersection and B 3 B 1 B 3 B 1 this intersections.

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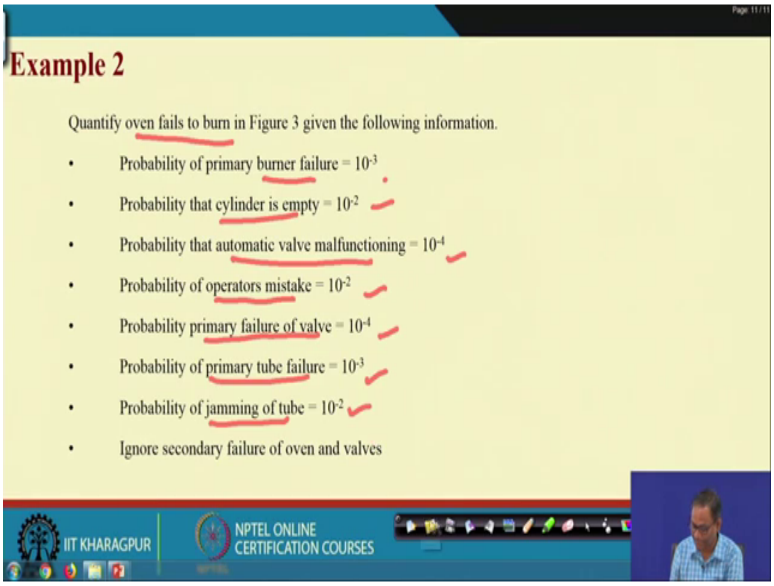

So, that mean this one this intersection this intersection and this intersection, and then minus the joint one this one B 1 B 2 B 3 in this black one. So, that will be because when you are taking place these and this you are basically considering 3 times the overlap portion. So, 2 times you have to subtract and you certain and this is your formula

So, now let us see the same principle we will use to quantify a fault tree. And the fault tree that example that oven fails to burn what we have seen in the last class qualitatively, we have developed this. And suppose the primary failure or the basic events burner failure given cylinder is empty is given automatic bulb malfunctioning was also given operator mistake, given primary failure of bulb is also given tube failure is given, jamming of tube is given ignore secondary failure of oven and valves we are ignoring this.

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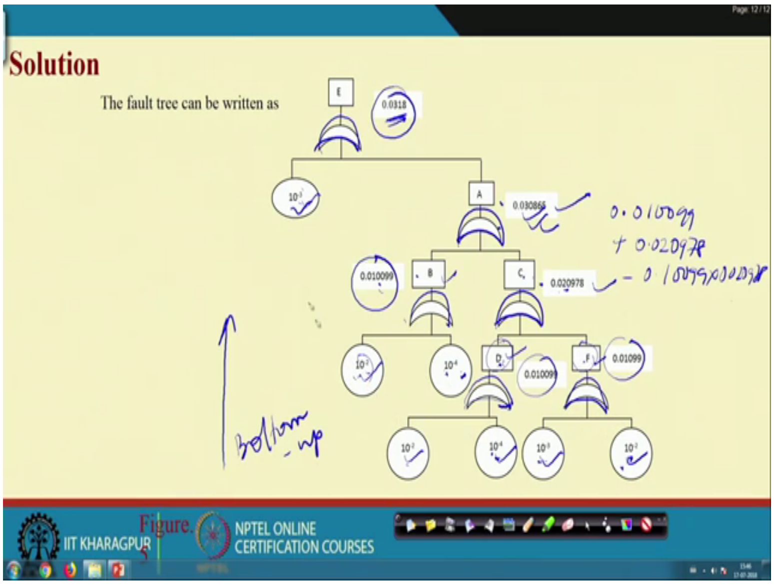

So, under this condition your fault tree is this. So, interesting thing is that this all the basic event probability you see basic event probabilities are given. Now, what you required to d? You start gate by gate method you cannot you cannot start here, because you do not know this probability. So, you have to start from the bottom. So, it is basically bottom up. So, you find out that which of the gates that can be quantified these 2 probability available.

So, and this is and OR gate. So, this can probability can be computed similarly this probability can be computed similarly using these 2 this probability can be computed, but this is not computed cannot be computed because unless we know these 2; so if these 2 with or gate. So, what will be the probability of d? These plus these minus this into this

What is the probability of this? These plus these minus this probability this into probability of this; so if you calculate you will get the probability of D is this probability of F is this again it is another OR gate. So, so then what will the probability of c probability of d plus probability of f minus probability d into f will give you this one.

Similarly, this side probability of B will be the probability of 10 to the power minus 2 plus 10 to the power minus 4 minus 10 to the power minus 2 into 10 to the power minus 4 this will give you this probability now. So, that mean now probability B is known probability of C is known. So, probability of a will be known because again this is AND gate. So, these plus these minus this into this gives you this means what is probability of a one for example, then it will be probability of 0100 9 9 plus 0.0 2 0 9 7 8 minus 0.100099 into 0.02 0 9 7 8 this is nothing, but this value.

So, similarly now you know this probability, you also know this gate a probability again and gate or gate is there. So, this plus this minus this into this will give you this. So, that mean in this particular oven example the probability that burner will fail to run, or fail to burn is 0.0318. So, that mean what we are expecting that it at 100 times if you do if you run burn the oven 3 times it will not burn it is quite high in that sense.

But m, but those probability values basic event probabilities values are not the true values it is basically hypothetical values higher given some probability values ok. So, that mean these probability values will be much low. So, that this will also be much low, that is actually what will happen. But suppose if these are the probabilities bottom level probability come basic event probabilities are like this is the procedure you follow

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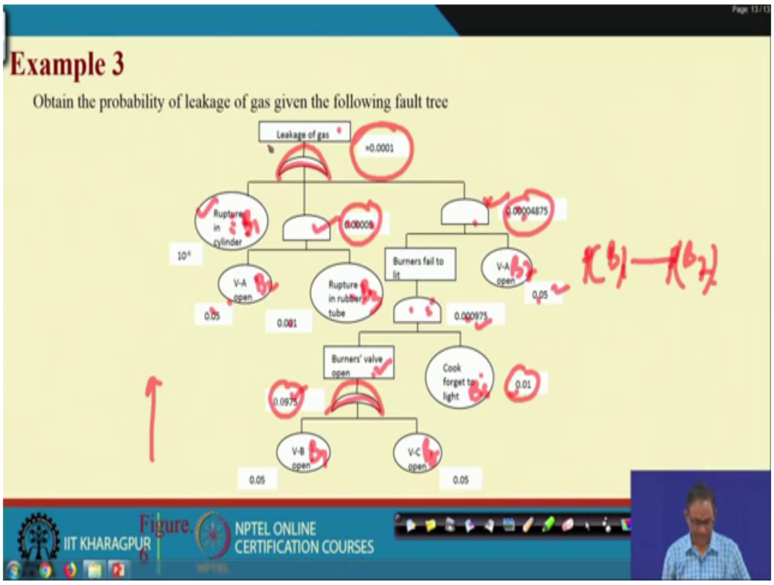

Now leakage of gas case see here. So, these are the basic events suppose this is B 1 this is B 2 then this one is B 3 B 4 B 5 B 6 let it be B 7. So, you have data one B 1 B 2 B 7. So, B 1 to B 7 these probability data that mean P B 1 to P B 7 this data are available with you.

So, you the same manner as these are available here OR gate. So, you know how to do suppose this probability will be this one, now this probability and this probability is known, but it is an AND gate, so this into this. So, that means 0.00910 and 20. So, 0.000 9 7 5 will the probability of this probability is known this probability is computed now this is an AND gate. So, this into this will give you this probability, this side again this and gate and gate this into this probability

So, now what happen you have an OR gate, and you have one probability here another probability another probability. So, probability that mean union of the probability of leakage of gas equal to probability of B 1, this union these union this. So, then use the formula I mean this into this plus this into this plus this into this. So, like this all the intersection minus the common part 3 intersection and minus the common part ultimately will give you this one.

So, which formula we will use here we will use this formula last case this formula. So, this is what is the gate by gate method. So, what we have done in gate by gate method? We started with the bottom most one then bottom most gate is first quantified and we will go up and finally, the top most gate will be quantified, which will nothing, but the quantification of the top level event or top event this is known as gate by gate method.

So, I hope that you understand it fully, because this is a very simple one and compute computation is also there by simple. And essentially what happen whatever gates you have you just if it is not and AND or OR you transfer this or transform to equivalent AND and OR gate. So, that this Boolean algebra case will be able to use otherwise you have to use the straightway formula, but it is difficult to remember, but AND and OR gate formula is easy to remember ok.

Thank you very much.