Engineering Econometrics
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Lecture – 50
Time Series Modelling - Volatility Modelling

Hello, everybody. This is Rudra Pradhan here. Welcome to Engineering Econometrics. We will continue with Time Series Modelling again and that too the same Volatility Modelling.

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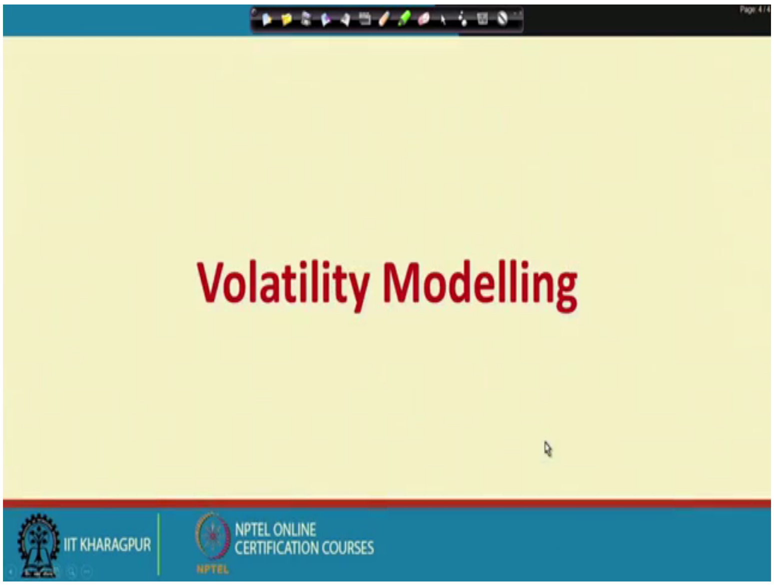

We stopped earlier that too the requirements of ARCH and GARCH estimation. We have already highlighted the ARCH structures and the GARCH structures and the kind of you know difference, the kind of you know need and the kind of you know flow how to start the process and then we like to know what is the kind of you know estimation mechanism.

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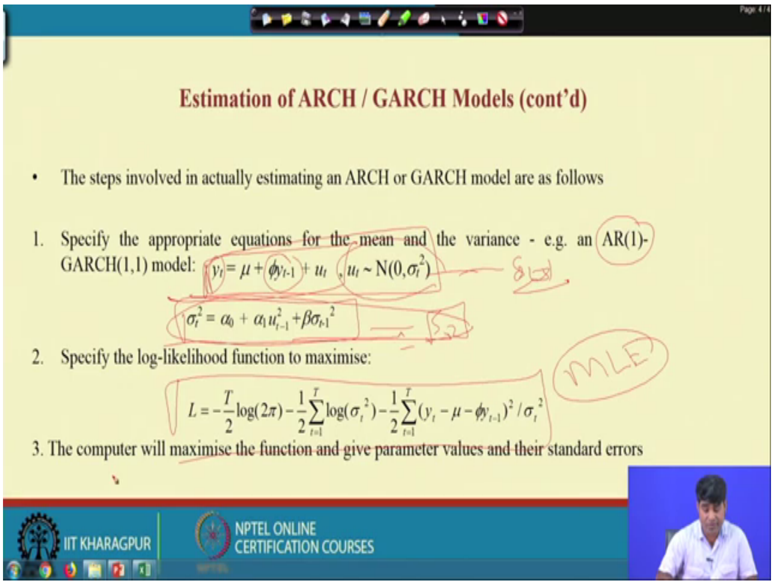

Because compared to autoregressive model then moving average models and ARIMA model these two clusters are non-linear structure that is why this slightly different from the estimation process compared to ARIMA model. So, it is technically step by step process like we do in ARIMA cluster. In specific we start with first mean equation that is here with respect to one variable and that to let us say AR 1. AR 1stands for autoregressive one and which is the first end entry to the ARCH and GARCH modelling and for that we like to estimate these equations.

y t is the original variables and y t minus 1 is the log variables and as usual we can run the model like simple regression modelling that too through OLS mechanism and error must behave like this and after getting the estimated y t we can have the error term u t and square of the error term and error variance and then we can connect like this sigma square equal to alpha 0 plus u square t minus 1 and the coefficient alpha 1 again beta coefficient with respect to sigma square t minus 1.

Now, this is step 1 and in the step 2 this is what the model we need to estimate and in the first end this is the mean model and that to can be followed with the OLS and this is non-linear model. So, we cannot actually directly go through OLS rather we use a technique called as you know maximum likelihood estimations that is what technically called as a MLE maximum likelihood estimation technique and the general framework of maximum likelihood estimation is like this starts with the l equal minus T by 2 log 2 pi to that is actually collage with the normal density function and minus 1 by 2 sum of log sigma square and minus 1 by 2 y minus mu minus psi t minus by sigma square that is what the kind of you know starting and the software with actually maximize this function and give the parameters values and their standard errors.

So, that means, technically we look here the values of you know parameters that too alpha 0 alpha 1 and beta that is that is the kind anything we are supposed to have here.

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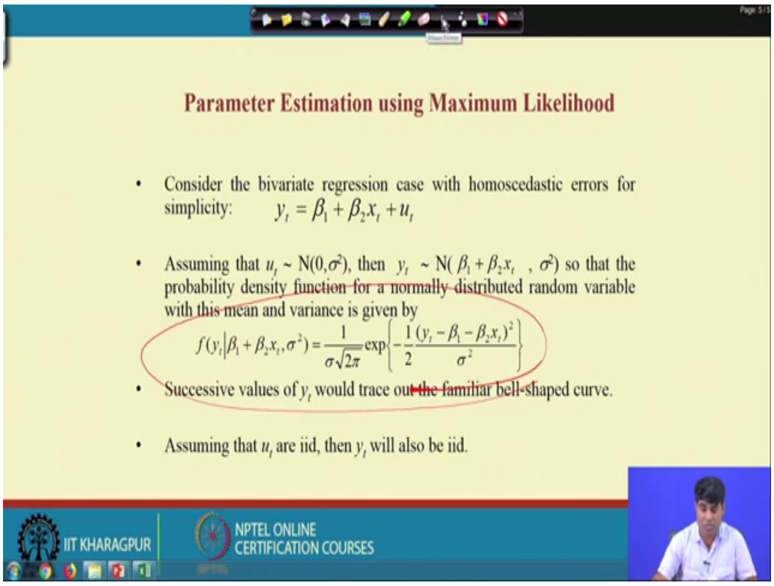

So, accordingly moving forward will have the equation like this is another kind of you know structure where we are connecting the bivariate regression y t as a function of x t. If you compare with the previous ones it is y t as a function of y t minus 1 and here y t depends upon another independent variable and as usual you can go for this is this is the mean estimations and that to have through OLS mechanism.

Assuming that error term is normally distributed with 0 mean and unit standardization as a result the probability density function for a normal distributed random variable with this mean and variance is given by this particular structures, ok. So, that means, technically this is what the exact you know which we have already highlighted the previous slide that is with reference to the mean equations this one that is what you y t minus. So, this is what the mean equation and here the same things this is what the mean equation.

Now, successive values of y t would trace out the familiar bell shaped curve that is the n d f assuming that error term u t r identically independent distributed which is 0 mean and unit variance and then y 2 will also similar structure followed by normally distributed with the mean 0 and unit variance.

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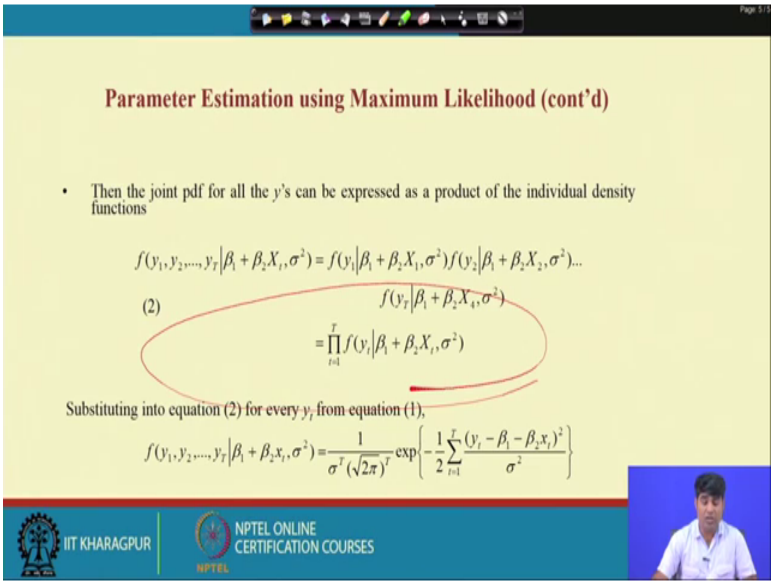

And, then we have a joint probability density function for all the y's and that can be expressed as a product of the individual density functions. So, ultimately you will have the structure like this. And, then after simplification the particular setup will transfer into the maximum likelihood estimation process that is what the maximum likelihood estimation.

So, now again taking the clue from this equation we can proceed further.

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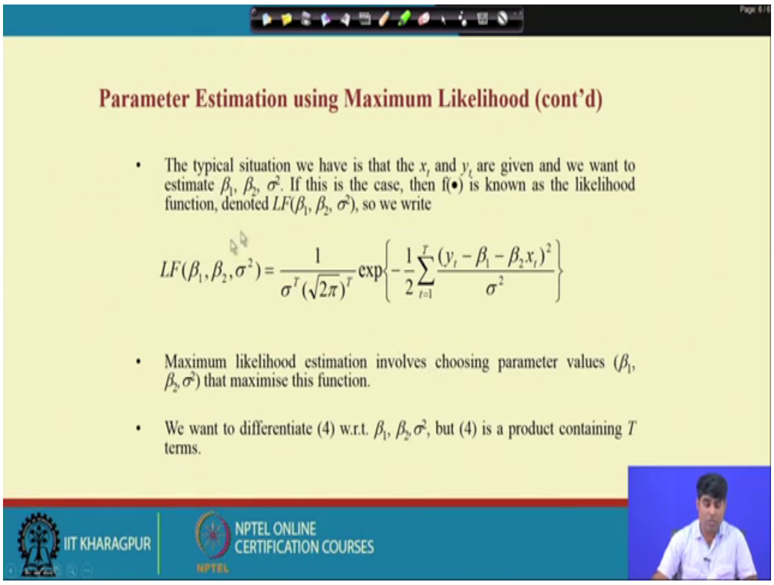

And, the whole transformation will be likelihood functions that is with respect to beta 1, beta 2 and the standard deviation sigma square and maximum likelihood estimation involves using parameter values that maximize this functions; that means, technically as usual like OLS MLE procedure is also similar where we can actually optimise with respect to beta 1, beta 2 and sigma square.

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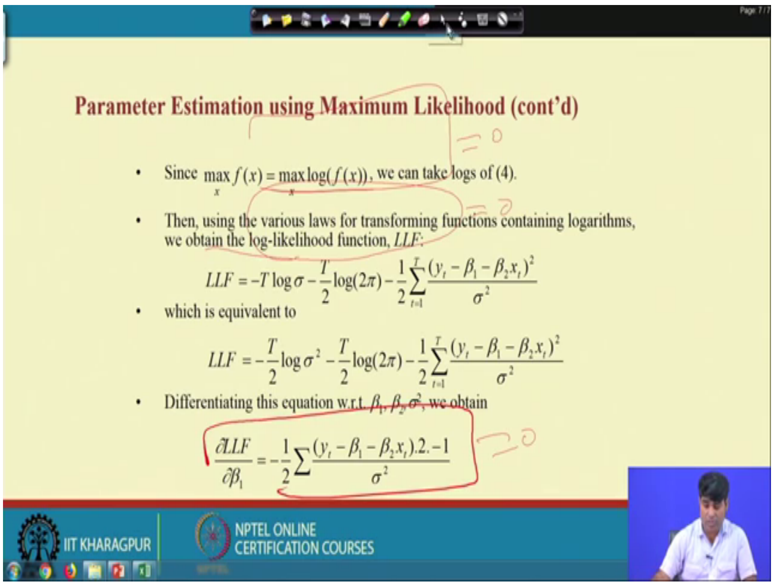

So, that means, technically. So, we like to differentiate with respect to dLF which d beta 1 with respect to d beta 2 and with respect to d sigma square like you know as usual OLS mechanism and by setting these three equation to 0 and simplify will have the values of the parameters beta 1, beta 2 and beta 3. So, that means, technically if you move further as for the you know given instructions so, will have like this and the first one that to differentiate is with respect to dLLF by d beta 1 is this much which is equal to 0.

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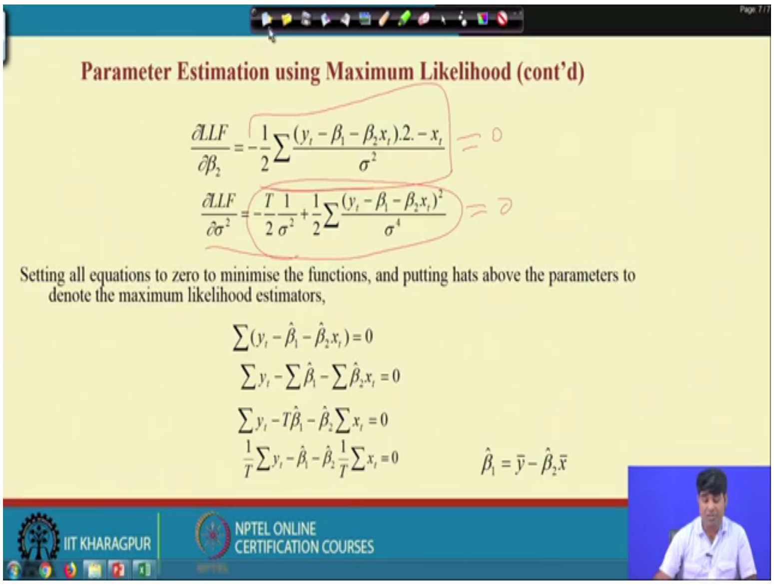

Again dLLF by d beta 2 which is this much, that means, this value should be equal to 0 and again dLLF by d sigma square which equal to this much. So, all will set to set to 0. So, that means, technically we have three parameters as usual actually like you know trivariate a simple linear modelling and we have here this is the first one which we can actually which we can have here you know with respect to beta 1, ok. So, that means, technically this 1's which is also equal to 0. And, then so, technically so, this equal to 0 this equal to 0 and this equal to 0.

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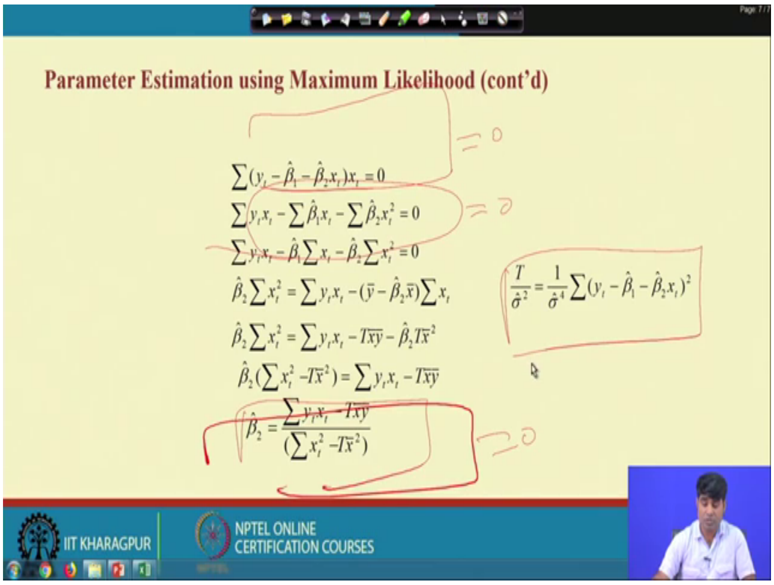

And, if you simplify then we have three different equation here, ok. So, the first one, so, this is this is what we have received that is with respect to the first equation this one and then this second equation and after doing all the simplification finally, we will be have beta 1 beta 2 and sigma square. So, that means, technically you will have beta 2 like this and sigma square like this.

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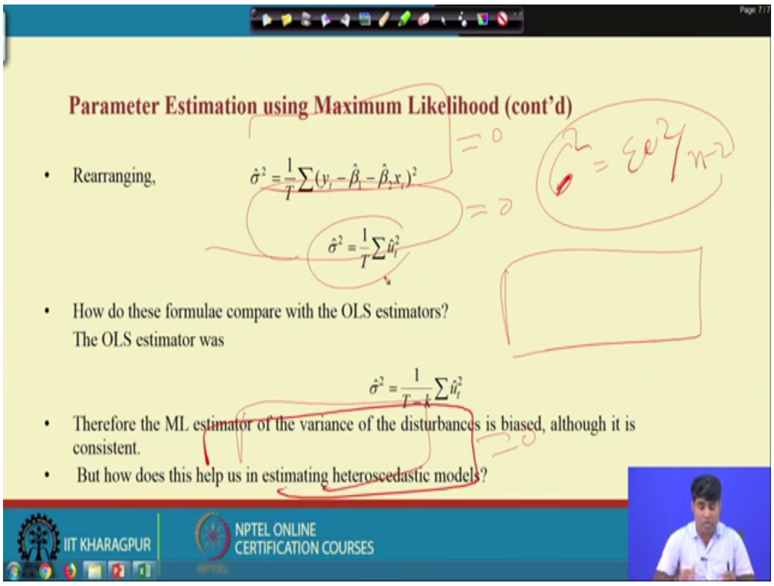

Then, again after simplification if you move further then sigma square equal to simply u square t that is what called as you know error variance. Compared to the OLS mechanism which we have already received sigma square, ok. So, technically that time we reported sigma square equal to summation e square y n minus 2 for y variate, n minus 3 for trivariate and this is a trivariate structural together. But, maximum likelihood estimation is having only this much sigma square equal to summation u square by e. Now, the question is the how do these for kind of you know the kind of you know value you know have a connection with you know OLS estimators?

That means technically what I have written here so, it is here all. So, it is actually t minus k that is what the degree of freedom. So, therefore, maximum likelihood estimator of the variance of the disturbances is biased although it is consistent. Now, the question is the how does this help us in estimating heteroscedastic models that is the big deal which you need to highlight here?

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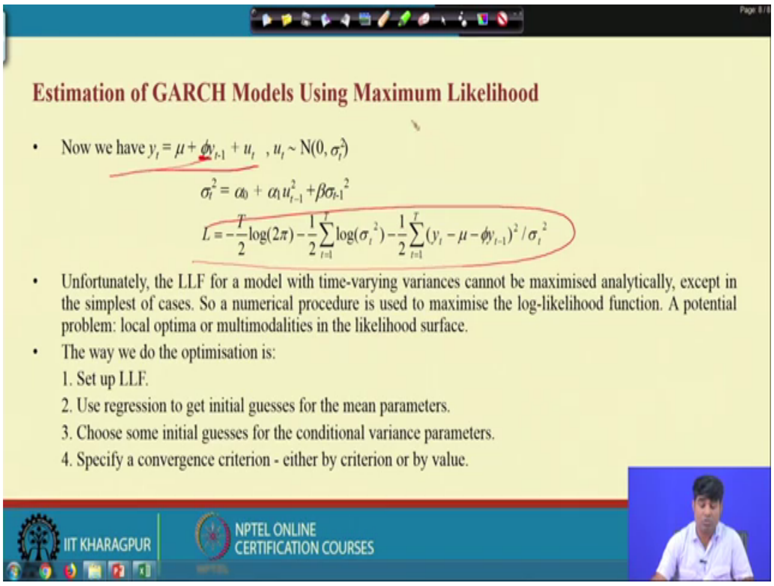

Now, coming to the you know mean equation again which you start from beginning that to y t as a connected to log of y t minus 1 where error term follows mean of the error term and you know unit variance and followed by sigma square equal to alpha 0 alpha 1 u square t minus 1 and beta sigma t minus 1 and then the likelihood functions, ok.

However, the likelihood function for a model with time varying variances cannot be maximized analytically, except in the simplest of you know cases. As a result, the numerical procedure is to maximize the log like log likelihood functions. Technically, the potential problem is to look for local optima or multi modalities in the likelihood surface;

that means, technically the optimisation procedure will follow to setup the likelihood functions use regression to get initial values of the parameters mean parameters which use some initial structure for the conditional variance parameters.

And then finally, specify a convergence criteria either a criterion or by a value, ok. So, that is what the procedure you have to follow.

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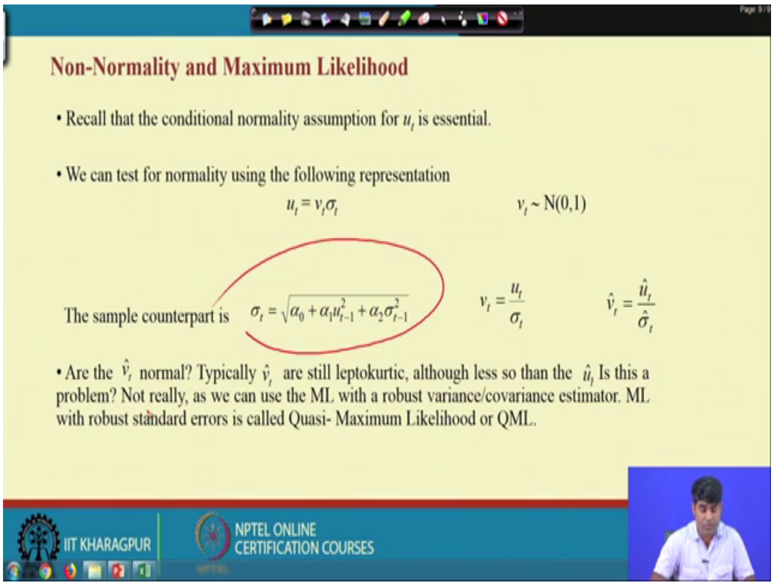

Now, to extend these ones we can recall that the conditional normality assumption for u is essential. So, we can test for the normality using u t equal to another a term v t and the standard deviation sigma t, where v t will follow normally distributed mean 0 and unit variance. And, the some sample counter parties sigma t equal to alpha 0 square root of alpha 0 plus alpha 1 a square t minus 1 and alpha 2 sigma t minus 1.

So, as a result v t equal to simply u t by sigma t and hence the estimated of v t will be estimated of u t by estimated of error you know sigma. Now, the question is whether the estimated v is normal? In reality v estimation are still leptokurtic although less so that hence compared to u t. So, now, question is this a problem? Frankly it is not really, as we can use the maximum likelihood which are robust variance and I think or you know can say covariance estimator. So, maximum likelihood with robust standard errors is called quasi maximum likelihood or simply called as you know QML.

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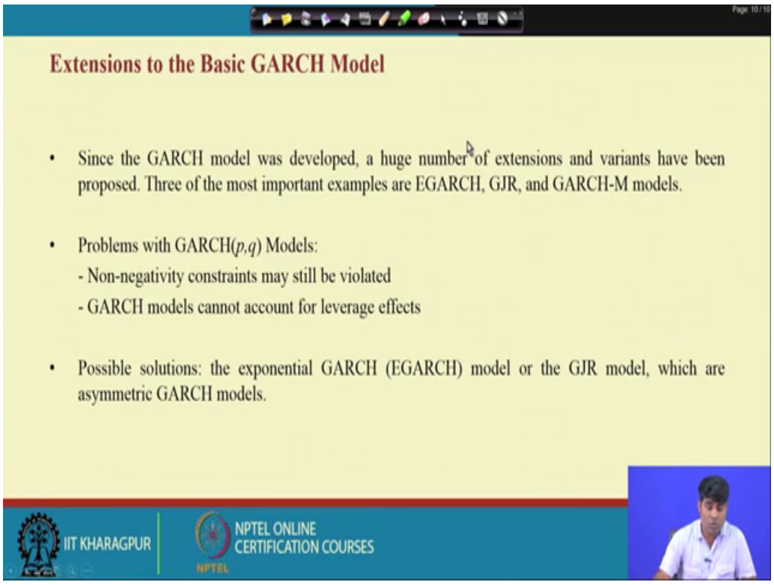

Again taking the clue, we have the GARCH model and where a huge number of extension and variants have been proposed, that is what I have already highlighted. That means, the GARCH clusters we have actually a basket that is starting with you know simple GARCH, then EGARCH, GJR and GARCH-M models. So, problem you see problems within a GARCH p, q models first the non negativity constraint may still be highlighted and GARCH models cannot account for you know the leverage effects which you can take care through EGARCH or GJR. Possible solution actually that is what the you know kind of you know structure through EGARCH that is called as exponential GARCH and the kind of you know GJR models which are actually simply called as you know asymmetric GARCH models.

So, let us see how is this particular you know structure that is with respect to EGARCH, GJR and GARCH-M models and in between we have already highlighted a component called as a IGARCH. So, that means, we have actually plenty of baskets.

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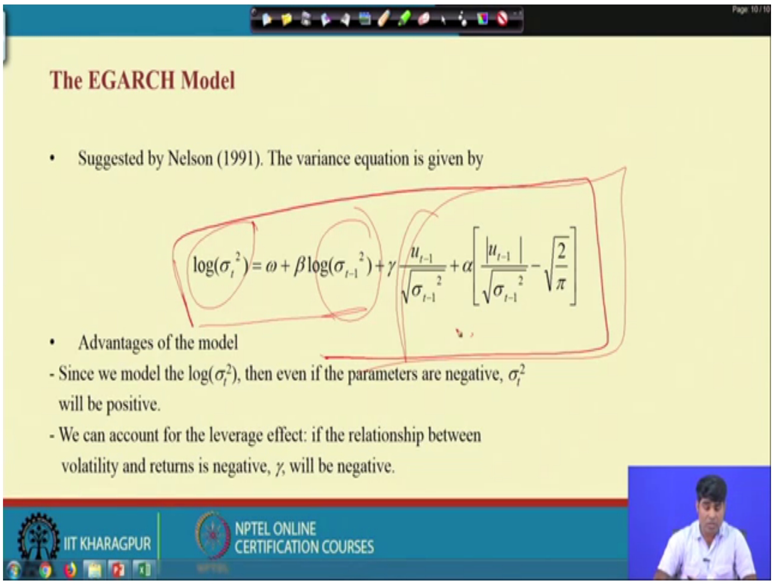

And starting with the first EGARCH that is actually the extension of you know GARCH model. So, here the general framework of you know EGARCH is like this just you know extension, where error variance is connected with the square of the error variance and then the additional part which will we have here is like this that is just you know extension to this case. Earlier you know we have y square t minus one and g square t minus one now it has some kind of an extension with respect to u t and g t sigma t. So, the advantages of this model are since we model the log sigma square even if the parameters are negative then sigma square will be positive. So, that is why some adjustment need to be taken care.

We can account for the leverage effect if the relationship between volatility and the returns is negative then that will be the negative one. So, for that means, this particular component. So, the there is the you know technically another parameter which can actually bring the kind of you know difference in the case of you know EGARCH.

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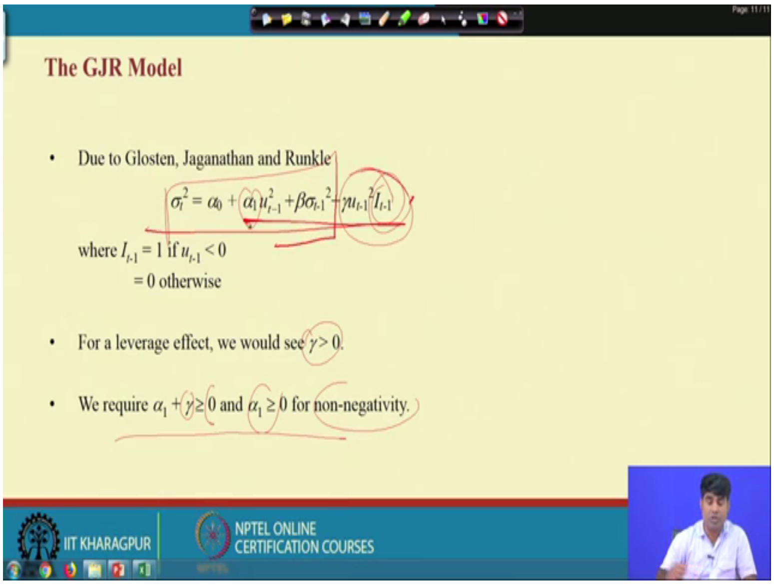

And, then the other form is called as you know GJR model it a actually developed by two three great statistician according to their name. And, the typical structure of this model is like this, it is again extension this part is actually GARCH and then this is the extra extension which we like to have and the I t minus is the kind of you know connecting point. It is it is like you know two different break ups, that is what the kind of you know kind of you know leverage effect 1 is the I t minus 1 equal to 1 if u t minus 1 less than 0 if not it is equal to 0; that means, when u t minus 1 greater than greater than or equal to 0 then I t minus 1 represents 0 value.

That means by default we can have another you know variable which we can include into this particular you know GARCH model and for a leverage effect we would see whether the particular parameter only positive one. The need is that alpha 1 close the leverage parameter should be greater than equal to 0 and alpha 1 for you see know non-negativity that means, that is this component that is the structure of you know GJR.

Of course, you know this is actually a technical kind of an understanding, but ultimately if we use the software; software by default will give you the kind of you know results very easily just we to have to set the mean equation and then you go to the variance equation by setting these you know ARCH and GARCH models. If you ARCH the software to give the ARCH estimations and that to you have to just fix the log length of course, you know manually you can put one after another and you know test, but you know software by itself will give you different results if you starting it putting you know log variance.

Similarly, in the case of you know GARCH and again you can have a different GARCH outputs by changing the log order 1 1, 2 2 and so on and again we can ask the software to give EGARCH results and GJR results and the kind of you know GARCH and types of you know models. So, all are you know it is there so, just you have to follow up it and then you can have the estimated results.

So, that means, technically the model is very clear or here the major you know kind of you know structure is that you know we are predicting a kind of you know engineering variable that to through a error component and the starting is actually the variable can be first linearly connected with you know the log variable and then have the error term. And through the error term again the kind of you know prediction we are doing.

So, you like to check how effective is this particular process to predict the y variables or we have a structure y t which can be influenced by some of the independent variables not necessarily that log variable and then again that will be the mean equations and you can have the error term and the error term can be used as a instrument again to predict or to you know you to forecast the particular you know dependent variable. The way I have cited the example you know error penetration or something kind of you know rho density that to with respect to different variables like road investment in other transportation and so on.

So, that is what the kind of you know different structure of the models through which you can generalize, ok.

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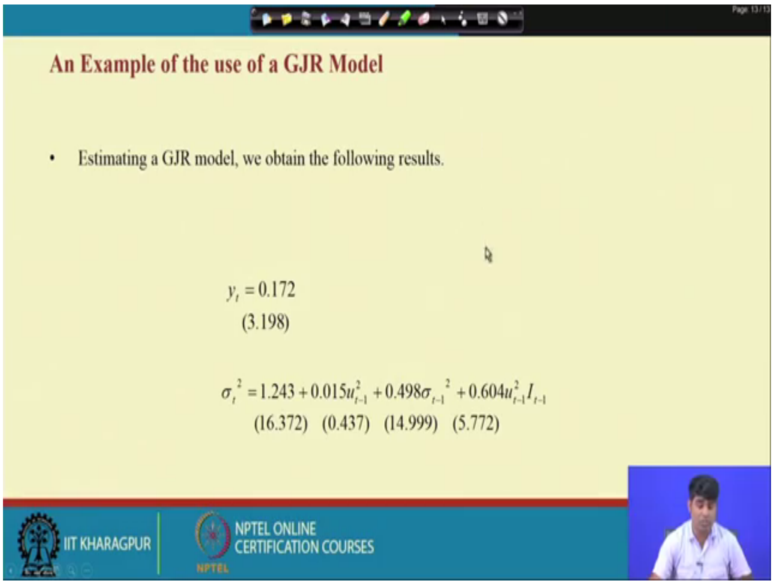

So, for example, if you go through you know estimation process the mean equation will be like this and the variance equation will be like this. Of course, there are lots of you know reliability checks or you know robustness checks to do this and first hand requirement is actually as usual to check whether the parameters are statistically significant and then the overall fitness. And, as a result the particular error variance can be used as an instrument to predict the kind of you know engineering variables.

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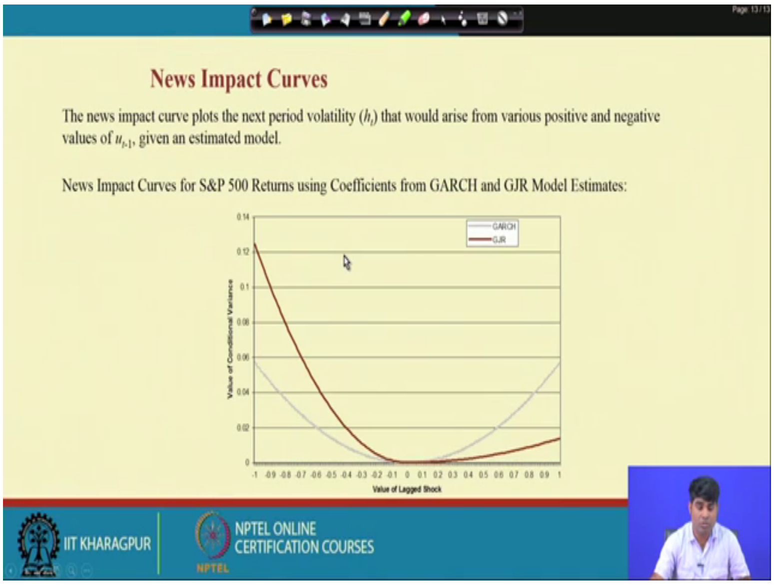

Now, this is another kind of you know you know see kind of you know market information which can be used as instrument in the case of you know ARCH and GARCH model and you know different kind of you know GARCH model. This is stock market problem technically and usually this kind of you know models more frequently used in financial engineering rather than you know say for other engineering like you know civil engineering or some kind of you know mechanical engineering etcetera. So, that is that is what the kind of you know structure.

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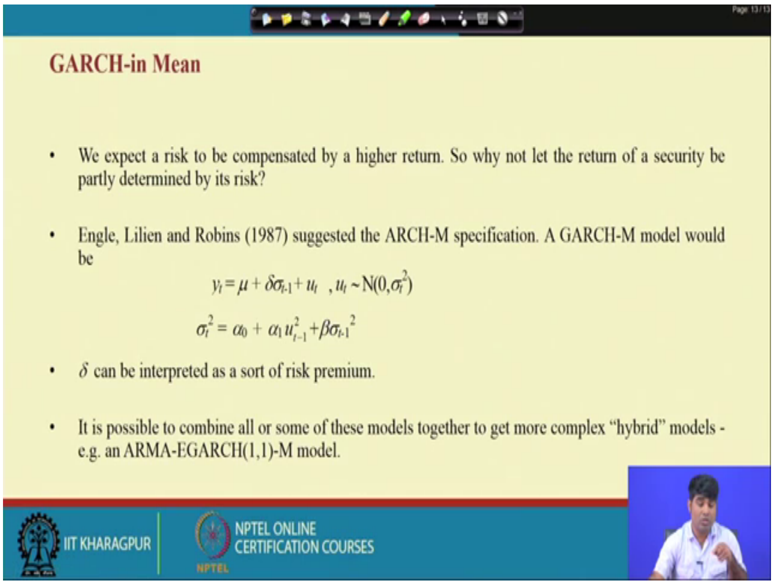

And, coming to the kind of you know structure because by this you know we use this one because it this one will actually create a two different you know structure altogether. So, that means, if you look into the kind of you know shape there is one structure declining the another structure is increasing. So, by default you like to find out the kind of you know structural change which can affect the ultimate you know dependent variables. Since the behaviour of the variable will changing route from one direction to another direction. So, the essential structure of GARCH model can be you know can be modified accordingly provided if the variable behaviour is like this otherwise you can simply you know estimate with you know ARCH or GARCH model.

Then GARCH you know GARCH in mean kind of you know structure we expect a risk to be compensated like again in the financial engineering you know scenario. So, why not let the return of a security be partly determine by its risks. So, technically so, the model is actually ARCH-M specification then GARCH specification; that means, technically we start with y t again mean equation and the kind of you know variance equations. But, here delta can be interpreted as a sort of you know risk premium. Again it is impossible to combine or some of these models together to get more complex that the hybrid models like you know ARMA EGARCH 1, 1-M model. But, that is the thing which you can have in the process; means it is a kind of you know more complex kind of you know character through which you can you know do the you know processing.

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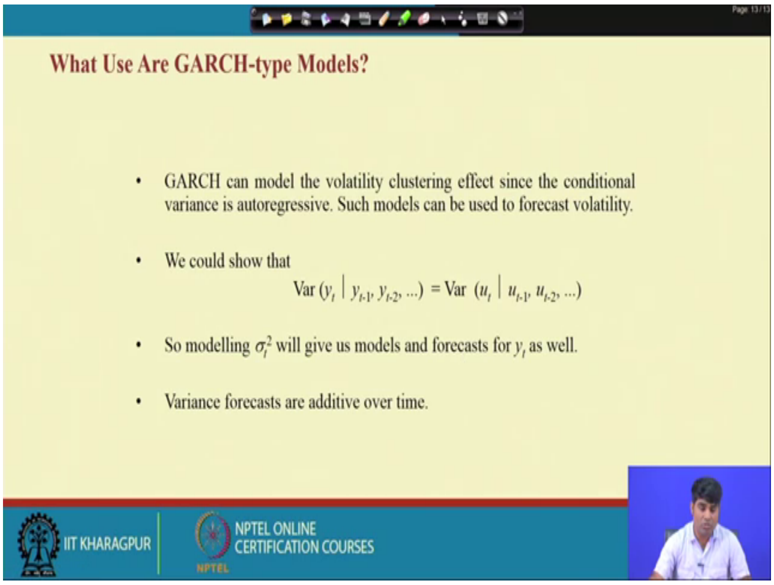

Now, so far a use is concerned what type of you know GARCH models we need all these things. So, GARCH can model the volatility clustering that too study the effect since the conditional variance is autoregressive. Such models can be used to forecast volatility simply. So, the procedure is actually you have to find out the variance factors and so, modelling will be done through actually error variance and that itself will be used to forecast the main variable, let us say y t. So, variance forecast are you know additive overtime. That is what the you know means we can say that it is the big advantage of this particular you know process, ok.

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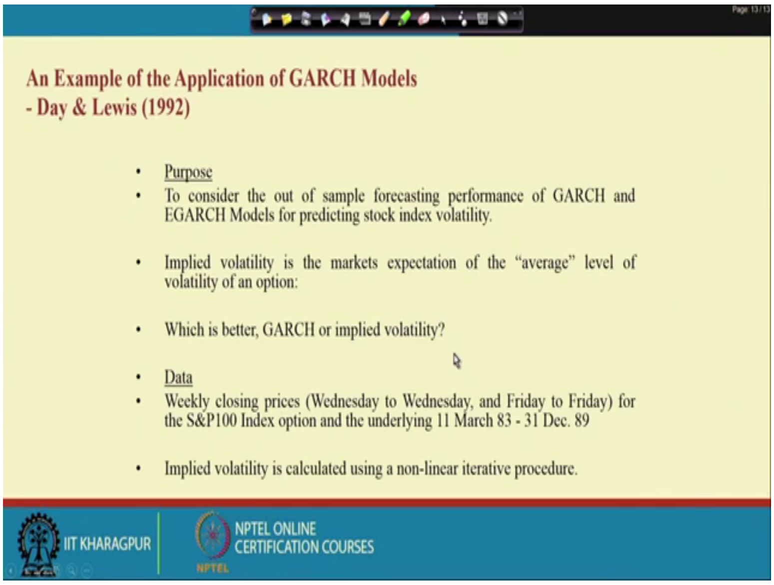

So, now you know some of the kind of an example which you can bring here that you know usually the main aim of this particular you know cluster is you know to consider the you know out of sample forecasting performance that of you know GARCH and EGARCH models for you know predicting the stock index volatility, particularly the case of you know financial engineering, an implied volatility that is the market expectations or the average level of volatility of an option. Now the question is which one is which is better; GARCH or implied volatility?

So, actually it is the kind of you know situation you to check and over the time you have to compare and one example which you have citing here it absolutely depends upon you know data structure.

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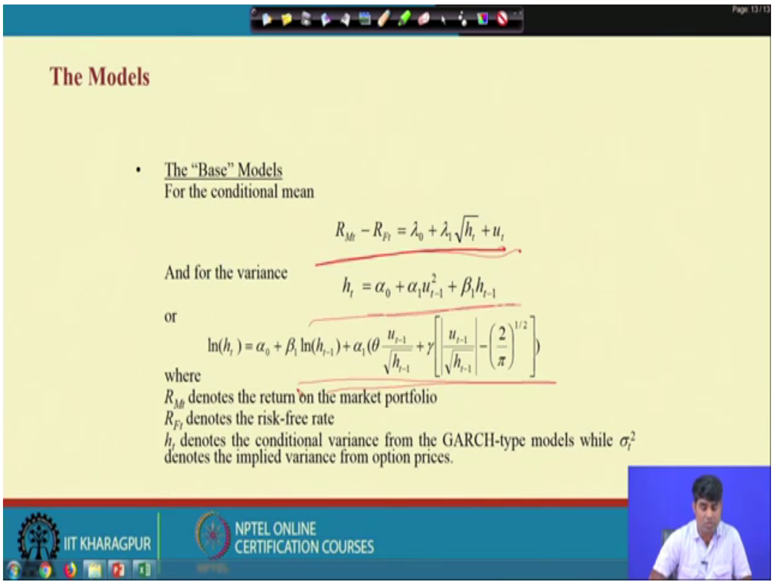

And, starting with this you know data structure we start with your know mean equation and then the variance equation here. So, this base models will be like that you know the mean equation and then we can have the variance equation and then the you know kind of you know its extensions.

So, now it is actually extension of you know capitalized pricing model, ok, sorry written on the market portfolio and denotes the risk free rate. So, h t denotes the conditional variance from the GARCH type models while sigma square t denotes the implied variance from the option prices. That is the kind of you know say actually, this kind of you know model is highly useful for you know financial engineering you have already highlighted and so, that means, technically we must have a time series data and some of the kind of you know you know kind of you know extensor or information which may be highly actually useful for this kind of you know modelling.

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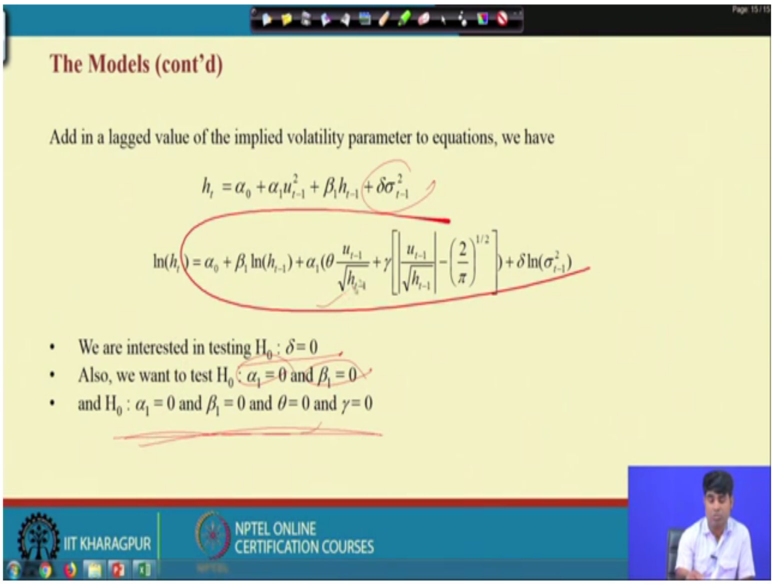

Again, adding a lagged value of the implied volatility parameter to equation we have the following again and that is what the kind of you know you know the extension. Here, we have these are conditions null hypothesis to set the delta equal to 0 corresponding to this one and then at rest alpha one equal to 0 and beta 1 equal to 0 and further we have actually the condition with respect to this model. So, these are all you know as usual procedure to set the null hypothesis and then as usual go by the estimation process we can have the following you know outcome.

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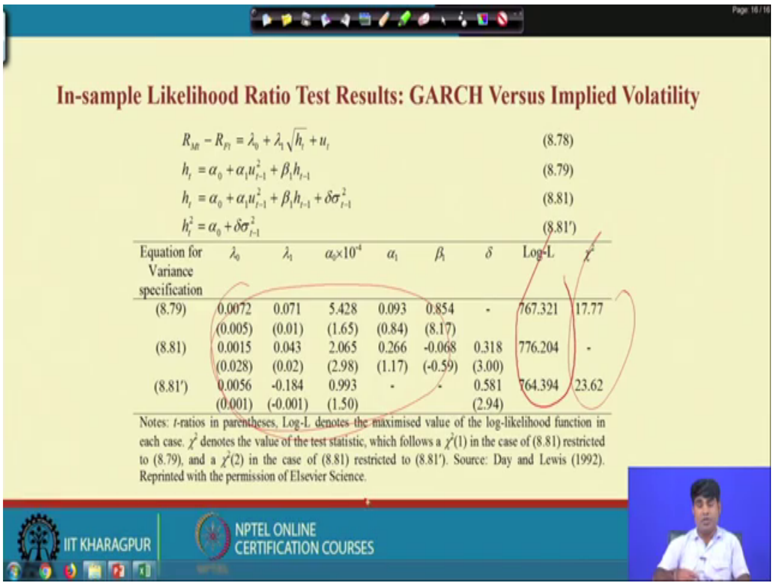

And, since we have used a particular data set and then you know various types of you know model which we have here starting with the mean equations, variance equations and the extensions, EGARCH and a the kind of you know implied volatility. So, we have the estimation process and then you are suppose to check whether the particular parameters are statistically significant and then the log likelihood ratio the chi squares. So, these are all actually will support the particular you know model.

Ultimately, so, we like to check whether these parameters are you know statistically significant of. That means, we can say that you know the volatility component will be very useful for you know predicting a kind of you know engineering variables. So, that is about the kind of you know case.

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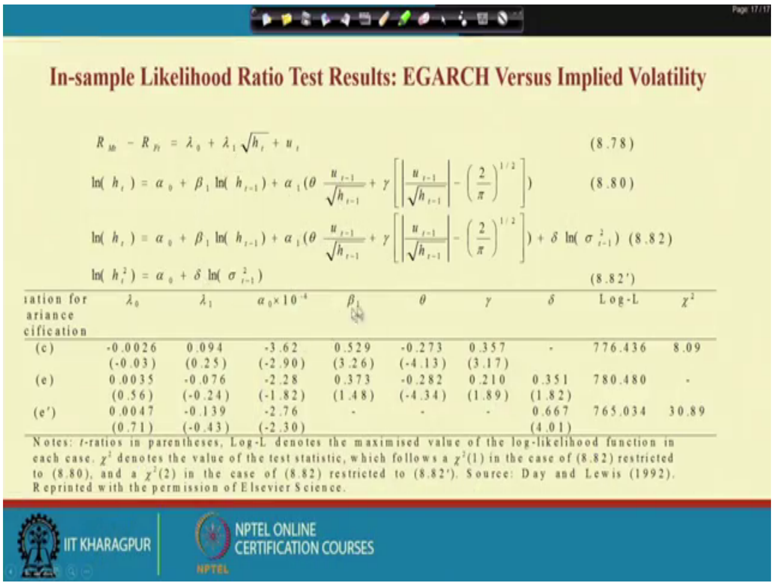

And, again if you extend this models we have actually plenty of results, that is that is the kind of you know extension to EGARCH versus you know implied volatility, ok.

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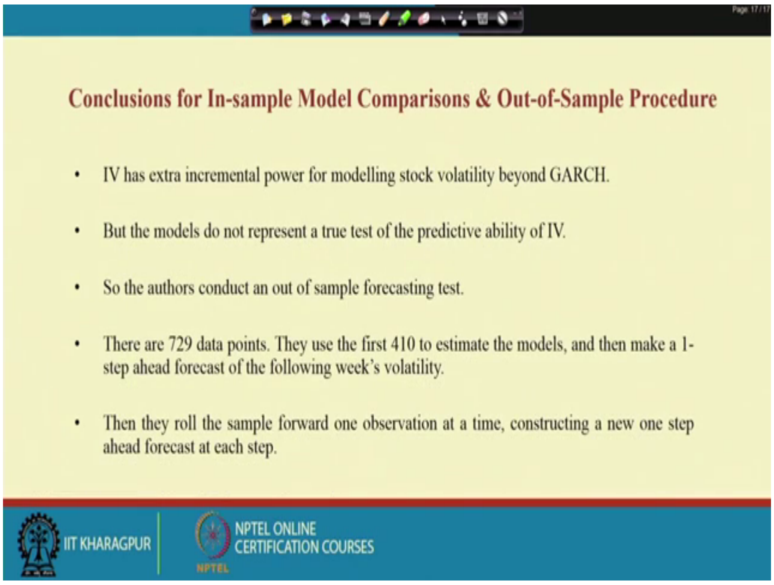

So, that means, technically we have actually variety types of means various types of you know volatility modelling; that means, we have lots of variety. So, far so far as ARCH and GARCH clusters are concerned and then on the basis of this you know models you can actually predict some of the engineering variables more specifically this kind of you know models are more frequently used in actually financial engineering what I have already highlighted categorically.

So, that means, on the top of this discussion that too in time series modelling, so this is a series of models which can be frequently used to predict some of the engineering variables, where actually volatility is an issue you can simply find out the standard deviation and then in a check whether you know there is you know high kind of you know unstability or something like that so that you know you can use this models and you know predict the kind of you know engineering output as per the particular requirement and the kind of you know need. With this we will stop here.

Thank you very much.