Engineering Econometrics

Prof. Rudra P. Pradhan
Vinod Gupta School of Management
Indian Institute of Technology, Kharagpur

Lecture – 49
Time Series Modelling – Volatility Modelling

Hello everybody. This is Rudra Pradhan here. Welcome to Engineering Econometrics. Today we will continue with the Time Series Modelling, and the coverage is Volatility Modelling again. And in the last couple of lectures we have discussed various types of time series modelling, starting with the autoregressive model, moving average models, ARIMA; Autoregressive Integrated Moving Average Model. And again taking the clue from ARIMA, we have discussed the ARCH cluster and GARCH clusters. In fact, in the last lecture we specifically highlighted the ARCH modelling, that is the autoregressive conditional heteroscedasticity model.

And the structure of this particular ARCH model is that, we like to connect error variance with square of the error terms. That is what the simple structure of this particular ARCH modelling. And the error is derived from a particular engineering variable, or from a group of variables; where as usual we have a dependent variable, that is what called as let us say y t.

And if it is only one variable that need to be predicted through this ARCH modelling, then we have to create a lag variable corresponding to i t, the lag variable will be y t minus 1. And then we just regress y t upon y t minus 1 and get the error term. That is the entry point to heteroscedasticity and the kind of ARCH modelling.

Now over the time, we will have estimated model by integrating y t and y t minus 1; where y t is a particular engineering variable. Let us say error transport coverage; that means, technically a road length over the time or a road penetration over the time, and that need to be predicted forecasted as per the particular engineering requirement or the economic requirement. On the other side, we will take this road penetration is a variable, and corresponding to road penetration we can have an set of other variables, independent variables; like road investment, the kind of investment in other sectors.

So, this may affect the road penetration technically. So now, whether to predict a road penetration with the lag of the road penetration, or we like to predict the road penetration through road investment, and investment to other transportation sector like port investment, airport investment, railway investment and so on; with assumption that the other investment may affect the road penetration.

Obviously, for a particular economy the investment amount of investment is a limited component, if more and more investment will go to port sector or you know airport sector or railway sector by default the less investment will happen in the road sector. Again in the road sectors, so the means the transportation sectors and again in the road sector the impact will come, and that to for the road penetration. Because it will go to less money for the investment purpose, and as a result the penetration rate will get affected. So, that is what the problem background, and keeping this type of problems we can predict particularly the road penetration with lag of road penetrations, or road penetration with lag of the road investment, and the lag of the investment towards other sectors like port sector airport sector and so on.

In any case we have 2 different structure. In the first instance we like to connect y t upon y t minus 1; where road transportation or road penetration will be connected to lag of the road penetration that is y t versus y t minus 1. That is the first structure first setup. And the others setup maybe where road penetration may be connected to road penetration; that means, lag of the road penetration and lag of the investment towards the road transportations, then investment towards the other transportation like port sector and airport sector. And by the way both the cases we have a simple model and we have a multivariate model.

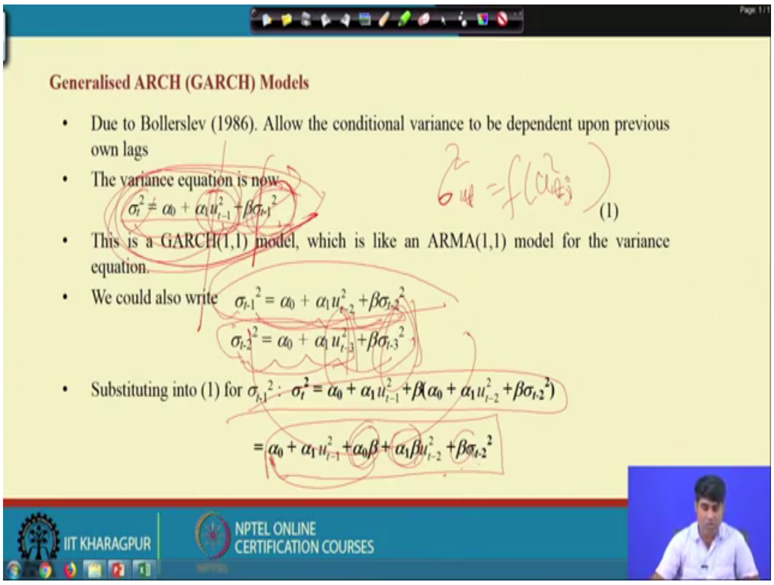
And ultimately we need to estimate and have the error term. Whether the error term is derived from the particular variable or this setup variables, depending upon the problem structure and the kind of requirement and the kind of prediction structure. In any case in the first instance we need to have error components that is what called as u t. And with the help of u t you can create series of lag u t; starting with u t minus 1 u t minus 2 and so on. And then again we can start with a square of the lag error terms, that is means technically u square t minus 1 u square t minus 2 and so on.

And again over the time we can have a variance of the error term, and again variance of the error term with respect to their lags. That means, technically sigma square u t sigma square u t minus 1 sigma square u t minus 2 and so on, that is what the procedure all about. So that means, technically in the ARCH and GARCH clusters we have to step procedure all together. The first step is the mean equation, that is what the u t connected u t minus 1. So, u t connect means y t connected to y t minus 1. Or y t can be connected with y t minus 1 and x t minus 1 set up other independent variables, and that 2 corresponding to their lags that is what the mean equation.

And after having the mean equations you can have the error component and then error variance. That is the second step of the process. And in the third step we like to have ARCH clusters by connecting error variance that is what the term called as a sigma square u t; which is the function of square of the error terms, such as u square t minus 1 u square t minus 2 and so on. And then, will we go up again reestimate the particular set up in the structure, and try to check whether the particular parameters are statistically significant, and the overall fitness will be as per the particular requirement. And for that we used to have chi square test and corresponding to coefficient of determination.

And I think we have already discuss about this set ups in the kind of ARCH modelling. I am just bring in the background once again to enter into the GARCH modelling. In the GARCH modelling, it has a just extend versions; that means we will do one extension compared to ARCH modelling. In the ARCH modelling, we are connecting sigma square u 2, sigma square u t with square of the error term that to with the lag variables. Technically sigma square u t equal to u square t minus 1 u square t minus 2 and so on. That is what the kind of structure; that means, technically we can have like this ok.

(Refer Slide Time: 10:07)



So, we can start here and technically so, the ARCH model is the cluster of sigma square, sigma square u t as a function of u square t minus t minus h j. This is what the simple structure. And then if you connect then you start with you know alpha 0, alpha 1 u square t minus 1 alpha u square t minus 2 and so on. Obviously, one of the first and requirement is to fix the optimum lag length. Because the particular lag can vary up to q, but which is actually unknown or the kind of infinite structure. So, you make it finite so that the particular structure can be bounded. If not bounded, then it is very difficult to estimate in any contest you can you know bound it then you go for estimation.

The usual procedure is you start with the 1 lag, 2 lag, 3 lag and so on, and every time have the estimated results and check the validity. That to you know fixing the optimum lag length, depending upon the AIC and SIC statistics. Once you have that then you can proceed accordingly. That is what the usual structure of ARCH cluster. Now taking the clue from ARCH clusters; so now, the GARCH model is like this. Where the beginning will be sigma square u t is equal to u square u t minus 1 and sigma square t minus 1. So, that means, we are allowing the error variance lag of the error variance. Now we have ARCH 1, ARCH 2, ARCH 3, and so on.

In the case of GARCH, that is what called as generalised autoregressive conditioner heteroscedasticity; which like ARIMA cluster autoregressive moving average, which has 2 parts y t and it is lag and u t and it is lag. The part is you y t versus y t minus 1 y t minus 2 and so on, and moving average part is the u t minus 1 u t minus 2 and so on. So, we are clubbing and have a ARIMA structure. And exactly GARCH is the similar kind of set of compared to ARIMA the first difference is the in the GARCH cases, it is a non-linear model, and we need square of the error term and the error variance.

That is what the requirement, and then in the second step you have to fix the lag component. So now, in the ARCH case we start with ARCH 1, ARCH 2, ARCH 3 because every times square of the error term is the independent variable. Since it is only one variables and that one cluster, as a result you can say ARCH 1, ARCH 2, ARCH 3 and so on. If it is ARCH 1 then the model we be restrict to simply alpha 0 plus alpha 1 u square t minus 1, if it is actually ARCH 2 then it will extend again; so, alpha 0 alpha 1 u square t minus plus alpha 2 u t minus 2 squares and so on.

And again it will continue, but we need to fix the optimum lag length by setting q equal to 3 for something or 2 even it can be also 1. If it is 1 then the model have 2 parameters only alpha 0 alpha 1. If it is actually a 3, then a 2 lags then it will be restricted the 3 parameters. Now exactly same in the case of GARCH, and we call it actually GARCH 1 1. So, that means, first one is related to the square of the error component, and the second one corresponding to lag of the error variance.

So now when we call GARCH 1 1, then the model by default will be restrict to this much. So, this is the ARCH part and this is the GARCH part. That is what the technique technical actually difference. So, that means, once you have error variance, then you can create lag of the error variance and then you can connect of course, because both are connected to mean equation.

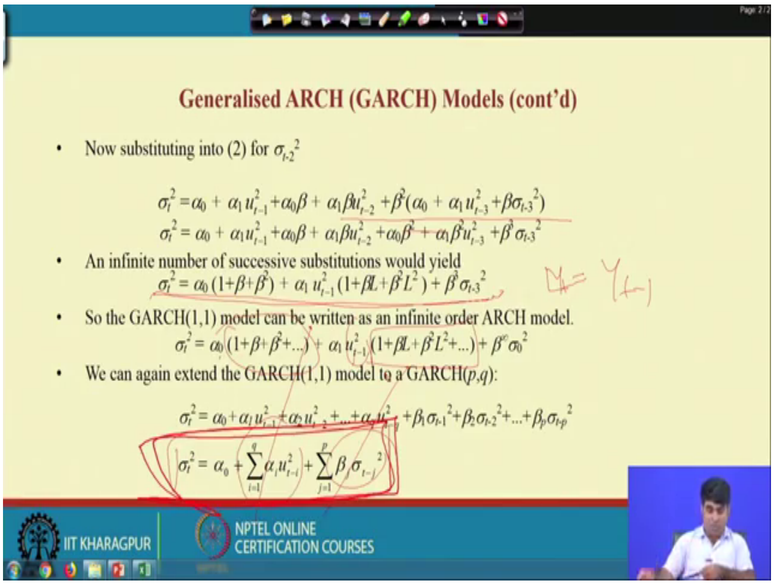
In the sub tails very easy to deal, for instance if you go through e views, then there is a option first, you will find 2 box and the first box will give you the mean equation estimate. And in the second box will give you the ARCH model output and GARCH model output depending upon your particular requirement and the particular command, which you like to apply. Now what is happening here? It is a kind of outer regressive; that is why the structure is called as generalised autoregressive. See here, if sigma square t is like this and sigma square t minus 1 can be like this ok.

So, here see sigma square t then followed by t minus 1 t minus 1. So, if it is start with the t minus 1 by default it will be t minus 2 and t minus 2. If it is start with the t minus 2 then it will connect to t minus 3 and t minus 3. So, t minus 3 this part is the error part and this part is the error variance part. So, a square of the error terms and error variance both will go parallely, and which can predict the current volatility. So now, since it is a kind of autoregressive scheme, then ultimately it will have a kind of extension. So, you put all these value, for instance you start with actually let us say sigma square t and square of the error term with respect to first lag.

And sigma square t minus 1 that too first lag variance. And then keeping other things remain constant. If sigma square t minus 1 will start varying like this and again sigma square t minus t will vary like this, then by default first you can put sigma square t minus 2 here, and again sigma square t minus 1 we will put here. And then finally, if you simplify then the whole equation will come in this format. That is what the final simplification with respect to the first equation where sigma square t equal to alpha 0 alpha 1 u square t minus 1 and beta sigma square t minus 1. Then instead of putting alpha 0 beta which may be or which can be written as you know.

Let us say delta, you know delta component and by default this part can be component come to the side. So, we called as delta s here and delta 0 and then alpha 1 y square t minus 1. Then alpha 1 beta can be called as mu 1 u square t minus 2, then similarly mu 2 is the t minus 2 and so on. So, we need some kind of simplification before you go for the estimation. So, it is not a big deal because ultimately in the excel sheet we must have the data you know a e square of the error term and square of error variance then you can create a lag and then simply you can connect. So, after that you can have the estimated value. That is what the kind of structure.

(Refer Slide Time: 18:57)



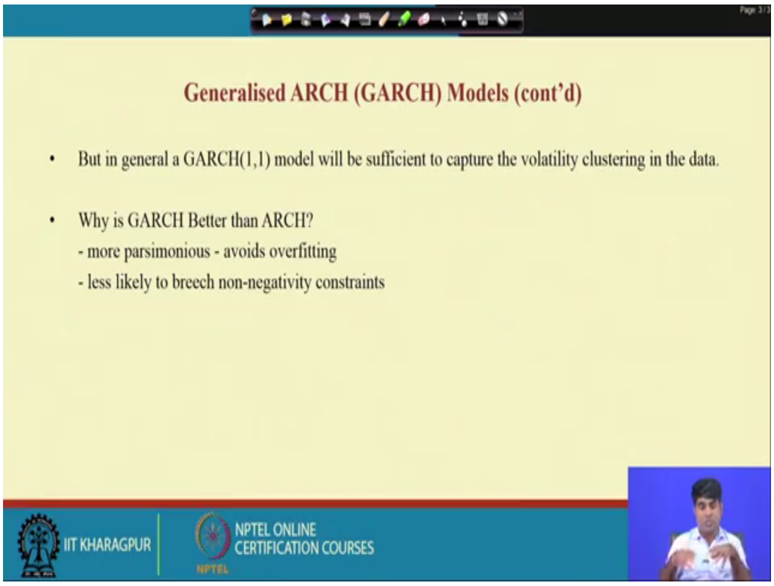
Now taking the clue if you actually further proceed the model can extend like this finally, we have model like this ok. And again if you will simplify it will give you a cluster like this. Where L represents for instant lag L y t if I will start, then it will be simply y t minus 1. So, instead of saying beta beta square actually y t and; that means, it is a L square y I t so, it is a 2 lag technically. So, as a result the entire component can be transferred into this form ok.

So, it is another kind of a structures; where the alpha 0 is connected with the this much of extra weights. And alpha 1 followed by u square t minus will connected with this much of weight and so on. Then finally, if you extend the whole model by default will be transferred into this format. That is what the generalised format of GARCH model. So, that means, we have square of the error component one part, and square of the error variance the other part. Now you can start with GARCH 1 1, GARCH 2 2, GARCH 3 3 and so on.

So, the procedure is start with the GARCH 1 1; so, one square of the error term when one error variance lag means that is with respect to lag. Then if it is start with GARCH 2 2 so, alpha 1 u square t minus 1 then alpha 2 u square t minus 2, similarly beta one sigma square t minus 1 plus beta 2 sigma square t minus 2. That is that is how the kind of extension you can do. And then finally, you have to fix the particular set up or this particular structure depending upon the clue of optimum lag length; that is, derived through again AIC statistic SIC statistic or final prediction error. That is what the kind of structure you have to follow then you work out the particular issue ok.

So, this is what the generalised autoregressive scheme.

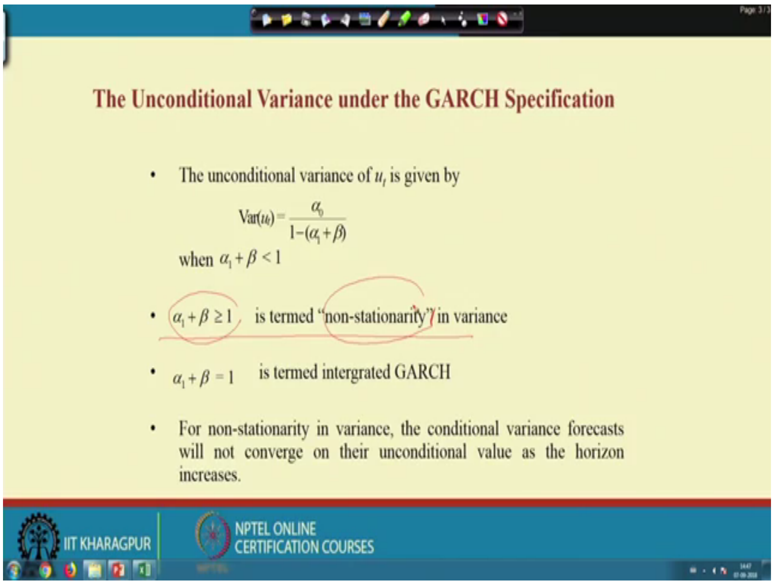
(Refer Slide Time: 21:51)



And then the issue is actually the GARCH model and then the kind of ARCH model. So, what is the basic actually difference so; that means, if you say that ARCH is a simple one, then GARCH is a little bit complex and that too if it is a bivariate structure then this will be a multi variate structure. Now the question is why is the GARCH greater than arch. Obviously the issue is, it is not issue of actually over fitting or the question of under fitting, we may we may expect that there is a question of over fitting.

And under fitting, but what is exactly requirement is the exact fitting, what is the need of the particular process. But the advantage is that it is a multi variate scheme. And then we may a more accuracy to predict the particular engineering variables. Because of the endogeneity issue and we are adding the kind of error variance to that particular system. And finally, there is a high chance it has a likely to be mu is non negative t constants. That is the advantage of GARCH modelling.

(Refer Slide Time: 23:35)



So now, the unconditional variance under the GARCH specification is that, variance of u t is equal to alpha 0 upon 1 minus alpha 1 plus beta.

Now, we have 3 different possibilities. Either alpha plus alpha 1 plus beta less than 1, alpha alpha 1 plus beta greater than 1, and alpha 1 plus beta equal to 1 ; however, if alpha 1 plus beta greater than 1, it is called as you know nonstationarity in variance. And if alpha 1 plus beta exactly equal to 1 it is called as a integrated GARCH model. So, that is what the case here that is what the case here. So, that means, technically if this happens that is alpha 1 plus beta greater than 1 then it is issue of non stationarity. Non stationarity is just opposite of stationarity which we have already used in the case of ARMA model. Where we call you know instead of ARMA we call ARIMA, autoregressive integrated moving average. So, the I stands for integrations, and we call it order of integration. So, the variables behaviour that too with respect to stationarity, this is actually good signal of a modelling, if not it is called as non stationarity.

That means in the entire time series starting with a ARIMA to VAR modelling; which we like to highlight in this particular course, every time we will be checking the stationarity issue. In technical, this particular issue is called as a unit root problem. We like to know; what is the order of integrations where the variable will reach stationarity.

The procedure is like that, we can start with the original data and check the stationarity, if the variable reaches stationarity at the original levels, then the order of integration by default we will call 0. If not, we transfer the variable that too we will go for difference equation and that too we start with first difference, second difference and so on. And in the first difference if that variable will again turn into stationarity, provided that is non stationarity in the original form, then we call as stationarity of the variables. And in that case the order of integration will be 1.

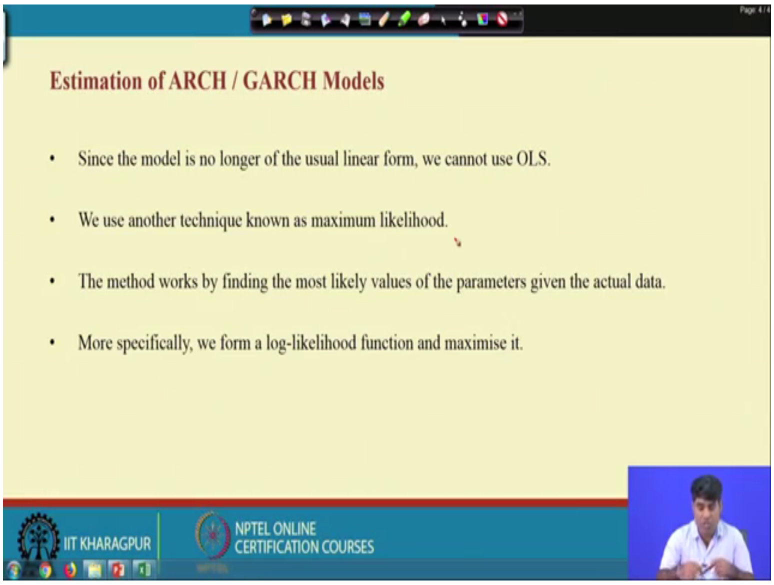
So, likewise you can proceed and we like to test till you get the variable stationarity. And we are always keen to know; what is the level where the variable reaches stationarity. And that particular level technically used as the indicator to use further setting for setting the ARIMA model, ARCH model, GARCH model, VAR model. That is the technical point through which you can proceed and do the needful.

So, obviously, the issue is actually to change the stationarity. We have a separate lecture after this GARCH clusters we will discuss the VAR modelling. And in the VAR modelling we will specifically highlight this unit root problem, because it is one of the mandatory component in the VAR modelling; which you we mess some extent skip in the case of ARMA and the kind of GARCH. But technically you should not actually ignore that one, but if you could not actually check then that maybe not so much big deal, but in the case of VAR modelling it is a big deal, because that itself will give you 2 different specification; which we will discuss in details in the later stage.

So, in this case so, we have 2 different options technically compared to the first case; where alpha 1 plus beta less than 1. The first one is the alpha 1 plus beta greater than 1, where you called as you know term non stationarity in variance. While the counterpart alpha 1 plus beta exactly equal to 1 is called as a integrated GARCH.

However, for non-stationarity in variance, the conditional variance forecasts will not converge on their unconditional value as the means as the horizon increases. That is the kind of structure ok.

(Refer Slide Time: 29:59)

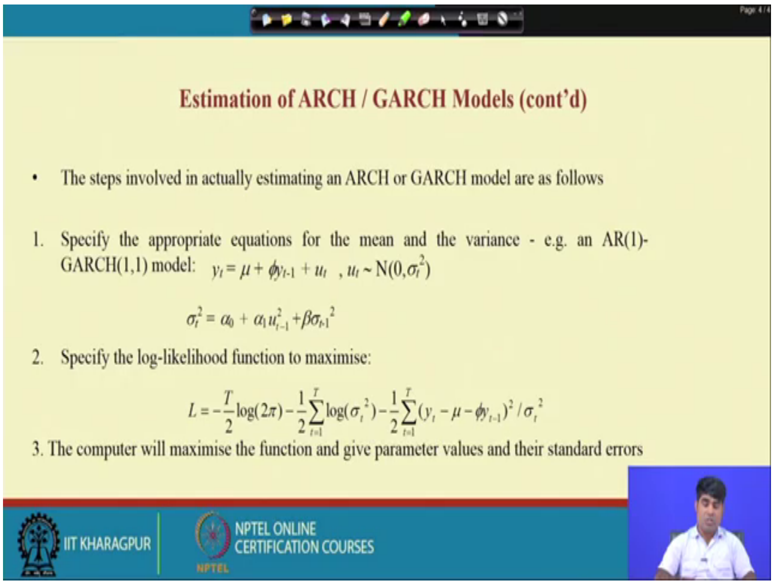


So, taking the clue; so we have here, the estimation process that too GARCH and ARCH and GARCH model.

Compared to ARCH model, in the GARCH model case since the model is no longer of the usual linear form we cannot simply use OLS. That is what the big deal again, but we can use OLS in the case of, like the case we have did in through autoregressive moving average and ARMA, a here some extend actually it is a non-linear in character. So, what we used to do here? We deploy another similar technique which is called as MLE, Maximum Likelihood Estimation and the method works by finding the most likely values of the parameters given the actual data.

More specifically we form a log likelihood functions and then maximize it ok. So, the procedure is a the procedure is a like this, where we have been know step by step process; particularly for ARCH cluster and GARCH cluster.

(Refer Slide Time: 31:15)



And what will you do here? I will just highlight the step by step procedure how to have the ARCH estimation and GARCH estimation. And then I will give you little bit extension to GARCH models, because GARCH basket itself has lots of variations; that means, technically we have a couple of different models under the GARCH clusters.

So, let us see first how we can have the estimation in the case of ARCH cluster, and GARCH cluster, and then we will go about the kind of different types of GARCH models. And sometimes they are very essential to highlight some of the engineering forecasting. And that too as per the requirement of some decision making process.

With this we will stop here and we will continue in the next lecture.

Thank you very much.