Engineering Econometrics
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Lecture - 48
Time series Modelling – Volatility Modelling

Hello everybody, this is Rudra Pradhani here. Welcome to Engineering Econometrics that too on time series modelling will continue with volatility modelling. In the last lectures we have highlighted a lot about the Volatility Modelling in that too basically the structure of you know non-linear time series modelling.

We have slightly highlighted some of the non-linearity issues and then we have discussed basics of you know like (Refer Time: 00:56) test, then BDS, test then the artificial neural networks. And now we will come down to the kind of inner structure the most important models which actually used in the live scenario that too solve some of the volatility in engineering problems. Typically the structure is like this.

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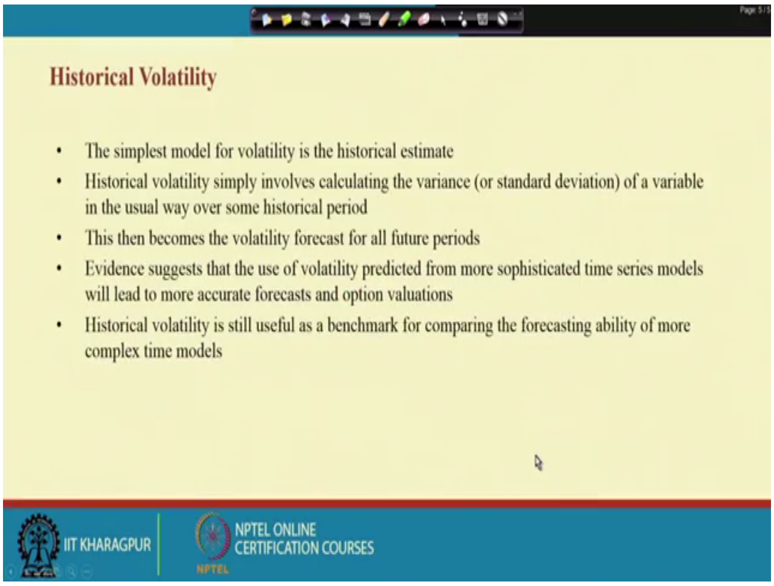

Let start with you know, basics about the volatility, historical which college in historical volatility. The simplest method for volatility is the historical estimate. Historical volatility simply involves reporting the standard deviations or the kind of you know variance of a variable. Basically, the use of you know historical data; that means, variables information recorded historically over the time.

The volatility forecast you know we do you know with the help of you know fast data and do for the kind of you know future requirement. Evidence suggests that the use of volatility predicted from most specific a sophisticated time series models will lead to more accurate forecasts and option valuations is in a kind of you know kind of you know financial markets. Historical volatility still useful as a benchmark for comparing the forecastability of more complex time periods ok; so; that means, this is the issue about the historical volatility.

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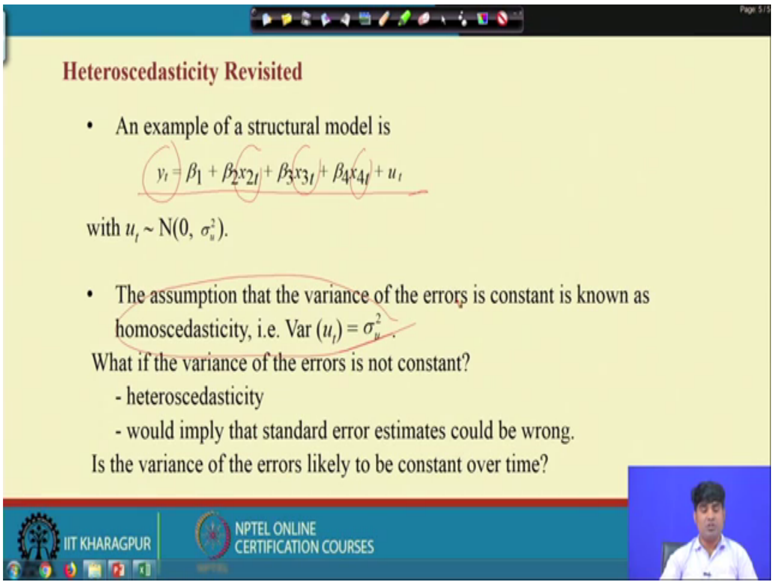

So, now, the question is you know how will go about it? And what kind of things you are suppose to bring so that we can address this problem more effectively ok. And as per the requirement of you know some of the engineering problems and the kind of you know decision making process. Let us start with a simple models. When will you discuss about the volatility modelling; so, one of the biggest component is the error term, which is actually derive through the actual structure and the estimated structure.

And when we have the estimated structure that that can be started with you know, one independent variable or many independent variables; that means, whether it is a bivarial structure or multi varial structure, we have no issue. We have means every time there is a dependent variable and series of independent variables starting with at least one or you know many.

Then ultimately end of the day we have the gap between y and y head which brings the error terms that is the difference between the y actual, that is the dependent variable actual and y estimated that is the estimated dependent variable. Now, whether the dependent variable is a function of you know one independent variable, so many independent variable that is not the big deal.

Of course, you know as per the problem description is concerned the kind of you know requirement is concerned we have to put one after another variables in the system that too in the form of every time dependent variables. Because, this kind of you know model it is a kind of you know one way clock structure where dependent variable is a kind of you know single.

And then other variables will be entered to the system as a independent variable. That is what the entry point and that is what the kind of you know logic and that is what the kind of you know structure about the ARCH and GARCH clusters.

So, now, basic structure is let us start with you know simple model here. We have a model here where y is the dependent variables and then we have a series of you know independent variables. Three independent variables we have taken x 2, x 3, x 4. And that we assume that you know the error terms which will derive by this estimation process by using some time series data so, which we have mean error 0.

And then variance sigma square u constant and that is the requirement of you know ordinary least square mechanism. And if that is the case and that happens in any problem in any contest in any situation that is what the declaration is called as a homoscedasticity which we have already gone through it.

However, if the variance of these error terms are not actually coincide or it means they are not same and not constant that is what the, this term called as you know heteroscedasticity. And that is the beginning of this volatility modelling that too ARCH cluster and GARCH clusters. So; that means, the entry or the understanding of you know ARCH cluster and GARCH clusters or the requirement of the ARCH cluster, GARCH cluster is the error term and the error variance typically.

So, once you find out the estimated models and then you derive the error terms and error variance, then the game will start here in a, another mode. And then try to check the kind of you know rules regulation and the kind of you know requirement about the prediction and forecasting. So; that means, we would simply say that you know standard error estimate could be wrong that is what the kind of you know is the variance of the errors like to be constant of a time.

So, that is what the questions will be usually expecting in the process of this you know modelling. So, now, basically this is the deal.

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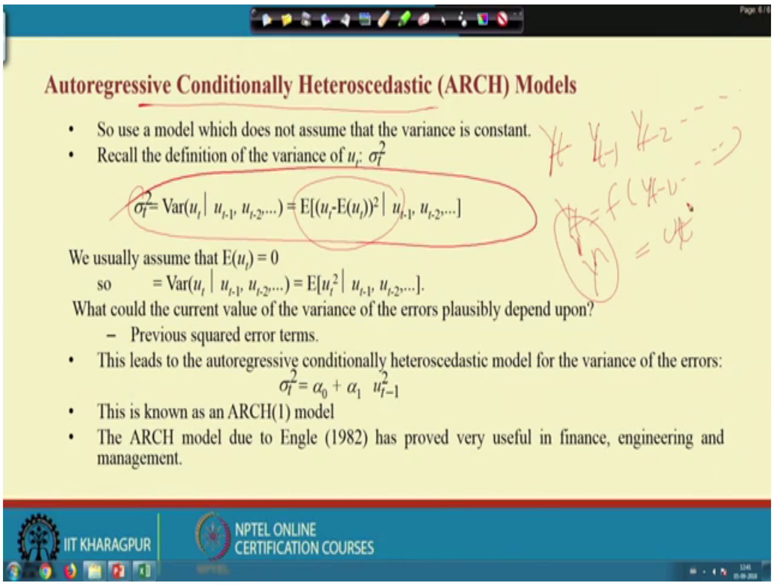

And then we start with you know after knowing the particular basic structure about the functionality, model estimation, obtaining error terms, error variance, then we will move to the kind of you know clustering called as you know volatility clustering that too volatility modelling starting with the ARCH modelling first and then GARCH modelling the later.

In the ARCH modelling it is the kind of you know game where error variance as a function of you know error terms and the GARCH modelling, it is little bit you know more comprehensive, much attractive and much broader by including the lag of these error variance in addition to error terms, square of the error terms. So, let us see how is this models. Let start with first ARCH and then moving to the GARCH model that is what the starting point of you know ARCH models that is technically called as you know autoregressive conditional heteroscedasticity.

So, we start with the usual structure about the estimation process whether it is a bivarial structure say in the kind of you know trivarial structure or the kind of you know multiple structure. Ultimately we need error term u which you called as u t and then the error variance sigma square u which sigma square which is called as sigma square u t. And that is what the actually error variance here sigma square and which is actually calculated by this process.

And then the conditionality is that as per the OLS requirement error term should converts to mean means mean of this error term converts to 0 and variance of this error term should be converts to 1, that is what the normally distributed with the 0 mean and unit variance that is what the requirement.

Sometimes it may not the case when the error variance is not unit unitary. So, that brings the kind of you know variations; that means, instead of saying sigma square u it is becomes now sigma square u i, that creates a kind of you know functions. If it is a i constant, then it will it will like you know single figures. Now, if it is not constant then it will more than the particular single figures that is how brings the functionality.

So, now, how is this game actually the ARCH clusters? The ARCH clusters simply connecting to the error variance to the square of the error terms and that too lag of the, you know error terms like this.

So; that means, technically the game will be like this we have y t then y t minus y 1 and y t minus 2 and so, on like this. Either the prediction will be like this you know y t with you as a function of y t minus 1 and something like that or y t as a function of another variable say x and something like that. And then we have estimated y and the difference between these 2 will give you the error component that is the big deal in a big requirement of the first instance and after knowing the error terms, ok.

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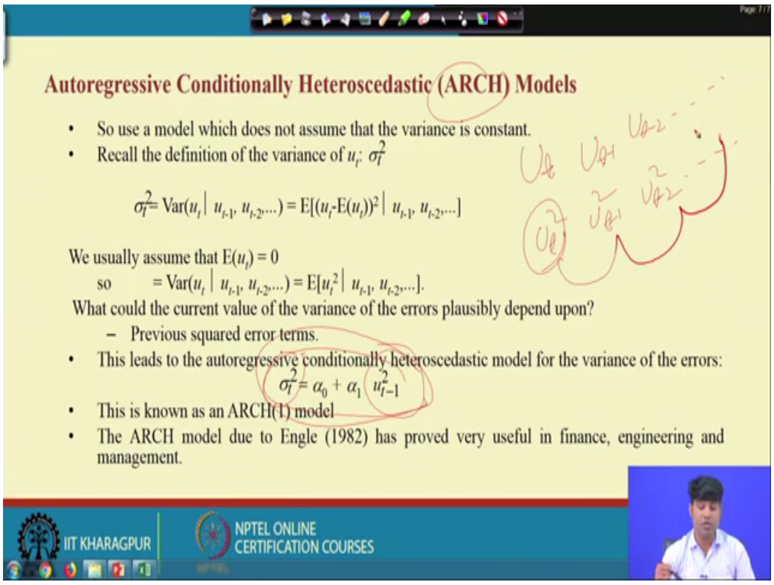

Now, the deal will be in this term u t minus 1, u t minus 2 and. So, on in another way it can be u t square, u t minus square t minus 1, u t minus 2 square.

So; that means, you say you know squaring all these you know error terms lag of these error terms. The advantage is that you know all these variables here now kind of you know positive in structure and which may have some kind of you know better indication compared to the you know the actual kind of you know error various.

So, now, the first models which we call as you know ARCH models starting with you know the general form is like one this is ARCH 1, where you know simply error variance is connected with this square of the first lag error variables that is what called as you know u square t minus 1.

So, that is what here is we are here u square t and we like to connect with u square t minus 1, t minus 2 and. So, on technically since their time series data and the error component by default will follow the time series. As a result every time there is high chance the lag of variable will be well connected to the previous lag variables. So; that means, technically u t minus t minus 2 depends u t minus 1 and something like that.

So, u t technically function of u t minus 1, u t minus 2; that means, every current variables will be predicted through the first you know that is that is the fake, that is the time series structure all together.

So, it has well connection with just their hand that is what the historical data is all about. So, when will call as you know H 1, then simply we are connecting with you know first square lag error term that is what is called as you know u square t minus 1; that means, this what the kind of you know structure and we like to just check the coefficient of these terms would be statistically significant.

If that is the case then you know it will give value about this model and then we will check the reliability, validity then use this model for the predictions in comparison with a simple linear estimations right. That is what the kind of you know gain and that is how you know it is very simple. So, go to the will go to the excel sheet.

And the process which you have actually to have these variables for instance; let us say.

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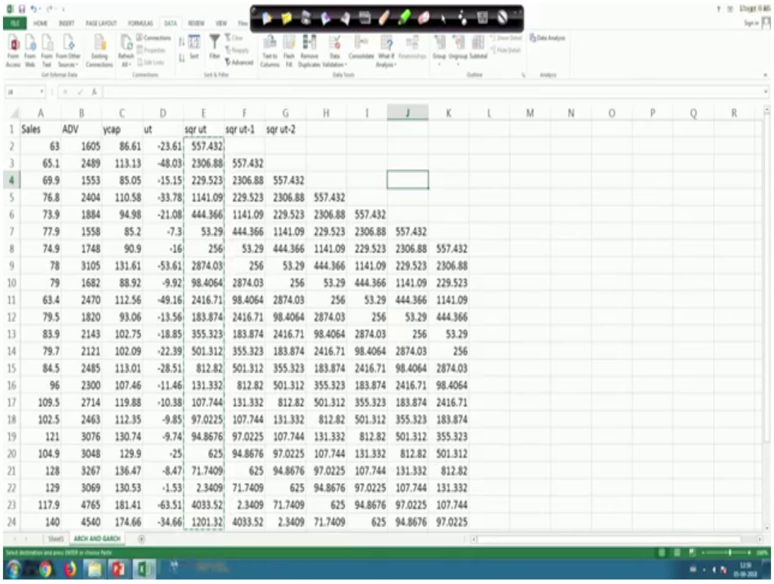

	A	B	C	D	E	F	G	H	I	J	K
1	Sales	ADV	ycap	ut	sqr ut	sqr ut-1	sqr ut-2				
2	63	1605	86.61	-23.61	557.432						
3	65.1	2489	113.13	-48.03	2306.88	557.432					
4	69.9	1553	85.05	-15.15	229.523	2306.88	557.432				
5	76.8	2404	110.58	-33.78	1141.09	229.523	2306.88	557.432			
6	73.9	1884	94.98	-21.08	444.366	1141.09	229.523	2306.88	557.432		
7	77.9	1558	85.2	-7.3	53.29	444.366	1141.09	229.523	2306.88	557.432	
8	74.9	1748	90.9	-16	256	53.29	444.366	1141.09	229.523	2306.88	557.432
9	78	3105	131.61	-53.61	2874.03	256	53.29	444.366	1141.09	229.523	2306.88
10	79	1682	88.92	-9.92	98.4064	2874.03	256	53.29	444.366	1141.09	229.523
11	63.4	2470	112.56	-49.16	2416.71	98.4064	2874.03	256	53.29	444.366	1141.09
12	79.5	1820	93.06	-13.56	183.874	2416.71	98.4064	2874.03	256	53.29	444.366
13	83.9	2143	102.75	-18.85	355.323	183.874	2416.71	98.4064	2874.03	256	53.29
14	79.7	2121	102.09	-22.39	501.312	355.323	183.874	2416.71	98.4064	2874.03	256
15	84.5	2485	113.01	-28.51	812.82	501.312	355.323	183.874	2416.71	98.4064	2874.03
16	96	2300	107.46	-11.46	131.332	812.82	501.312	355.323	183.874	2416.71	98.4064
17	109.5	2714	119.88	-10.38	107.744	131.332	812.82	501.312	355.323	183.874	2416.71
18	102.5	2463	112.35	-9.85	97.0225	107.744	131.332	812.82	501.312	355.323	183.874
19	121	3076	130.74	-9.74	94.8676	97.0225	107.744	131.332	812.82	501.312	355.323
20	104.9	3048	129.9	-25	625	94.8676	97.0225	107.744	131.332	812.82	501.312
21	128	3267	136.47	-8.47	71.7409	625	94.8676	97.0225	107.744	131.332	812.82
22	129	3069	130.53	-1.53	2.3409	71.7409	625	94.8676	97.0225	107.744	131.332
23	117.9	4765	181.41	-63.51	4033.52	2.3409	71.7409	625	94.8676	97.0225	107.744
24	140	4540	174.66	-34.66	1201.32	4033.52	2.3409	71.7409	625	94.8676	97.0225

This is a data and we like to first you know let us say this is a sales data and that is in the time series recording let us say n value for you know last 23 years. And then and the advertising expenses in this n organisation over the last 23 years; that means, it is a time series a you know structures.

And we need to predict actually sales by advertising in that too check the, you know check the kind of you know forecasting through you know volatility modelling. So, the first instance of this process of ARCH and GARCH cluster use this data and predict the either 2 way you can do.

So, I call it says you know y t and then advertising we call is you know s t. So, there are 2 ways you can enter to this process y t as a function of y t minus 1 or y t minus 2 and so, on. How many lags will you use that is depends upon you know through AIC starts t and AIC starts c which you have already discussed.

And or else you can just connect with u i and x and then find out the error terms and that error term can be used as a benchmark for ARCH modelling and GARCH modelling. For instance let us say these are the 2 variables and then we go to the data analysis. Of course, there are time series taken you know you use micro (Refer Time: 15:17) all this things.

But, since it is not here so, what will you do? Will you do this through excel, but do will do some kind of you know extra processing. So, let us say have the regression first and then this is what the dependent variable requirement and then the independent variables that is what the independent variable requirement k and then run the models.

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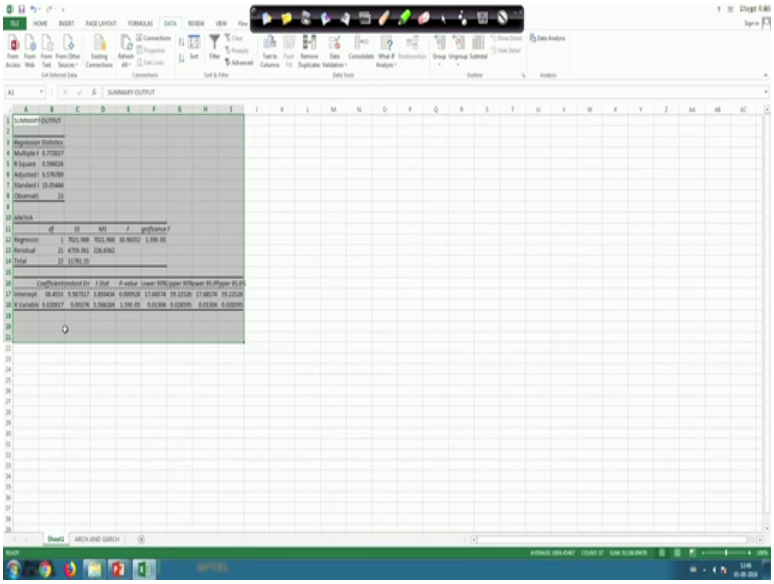

Now, we have 38.45 minus 0.02 so; that means, we will go to this you know you know structures then we will have actually this is y, y cap and that too y y t that too y y trans you know that the time period t actually equal to 38.446 ok, 38.46 that is k equal to 38.46 ok.

And then here it is a kind of you know 0.03 38. 46 and 0.03 this is plus 0.03 then multiplied by the kind of you know independent variables ok. If you connect with the simply lag variables then y t, y t minus can be regress and then the estimated kind line can be drawn. So, this is what the predicted information's ok. Alright now, with the error component that is the u which is actually the difference between the kinds of you know y actual minus the y estimated that is what the error term ok.

Now, will have like ok we can scroll it this once then it will be generated that is what the u t generated ok. So, what did you called you know square u error terms which is just equal to actually this multiplied by again this alright. So, this will actually the error term square right. This is what the , a square if we call is you know u t and then this will be actually square of the u t right. That is what the different variables.

And then we create actually lag of you know square u t. So; that means, s q r u t minus 1 so, ok. So, this is the first lag and likewise you can go about s q r u t minus 2 that is what the, that is what the kind of you know I adjust a little bit enlarge the view.

That is we know square u 2 so, now, what we can do you can just cleared this one. So, this is the square u t which you have already created, now we need to create actually the lag of this you know variables. So, you just go and then copy that is one ok, this is the other likewise you can create here you know n number of variables, that is very easy to do that ok. That is very easy to do that is very easy alright.

So, you have actually lots and lots of variables in the kind of you know case and then. So, we like to connect actually this with this. Again; that means, technically now this is the data set and technically since we have created lots of you know lag if you go up to all lags then this much you start this sampling from here only ok.

So, this much sample will finally, added into the process. Now, the first ARCH model will be this with you know square of this terms right that is what the kind of you know. So, now, again you go to the regressions and then this will be the dependent variable and this will be the independent and do the estimation that is what we are discussing here all about now ok.

So, when will call at you know first ARCH 1 so; that means, we are connecting with you know first square of the (Refer Time: 20:14) under u square t minus 1 right. And when it go for you know ARCH 2 then alpha 1 e square t minus 1, then alpha 2 e square t minus 2. So, 2 different estimates you will find.

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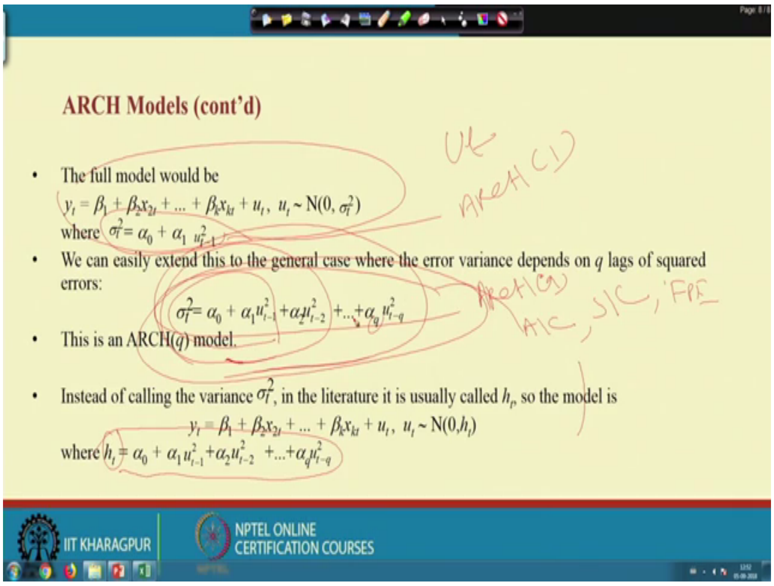

And then it can be also generalized it can be also generalized. So; that means, taking the clue from the previous you know you know discussions you see here. So, this is what the first and starting and then will have the error terms, then the first error variance with the first error term that is ARCH 1. And then if it actually says called ARCH 1 and then this is actually called as a ARCH p or q ok. So, it is a q here so, you can call it q right so; that means,.

So; that means, technically we can have the model of like this, we can model like this or we can model have like this. Now, the question is a, what is the correct one? So, that is for that you know you can go again AIC using the AIC statistics, SIC statistics and finance prediction errors something like that then you fix the lag length first so,.

So, since it is a time series model into that too in the lag modelling. So, fixing the optimum lag length is more important first, after fixing this one then you will go for other kind of you know requirements and the kind of you know estimation then the kind of you know testing, the reliability . So, everything will come you know one by one.

So, but this is very important how to fix the optimal that is what the you know one of the biggest challenge in the time series modelling and that too this type of you know models.

So, now, instead of calling variance sigma square the literature it is usually called you know h t.

So, so, that is means technically so, h t is a sigma square error variance which is actually connected with a this terms that is what the this is what actually generalize you know format of you know autoregressive condition heteroscedasticity.

And we can have little bit extensions by putting in the lag of these you know error variance into the system. Then extend this model to the ARCH clustering and something like that. But, it is this in the structure here for you to behave with respect to the kind of you know situation and the kind of you know requirement. So, that is the form of you know as we know so; that means, technically it is actually a second step process where you know the original variables are not here. Means technically we start with y and x, but ultimately the models are different variables, error variance, then square of error terms and lag of error variance.

So, these are all you know items which are derived from the original variables y and x, but ultimately the final prediction with respect to there you know the error terms which is the difference between the actual and the estimated. That is the beauty and that is what the difference you know different kind of you know look and start as compared to the kind of you know discussion which we have lr is in the lr case some of the cases we use actually models.

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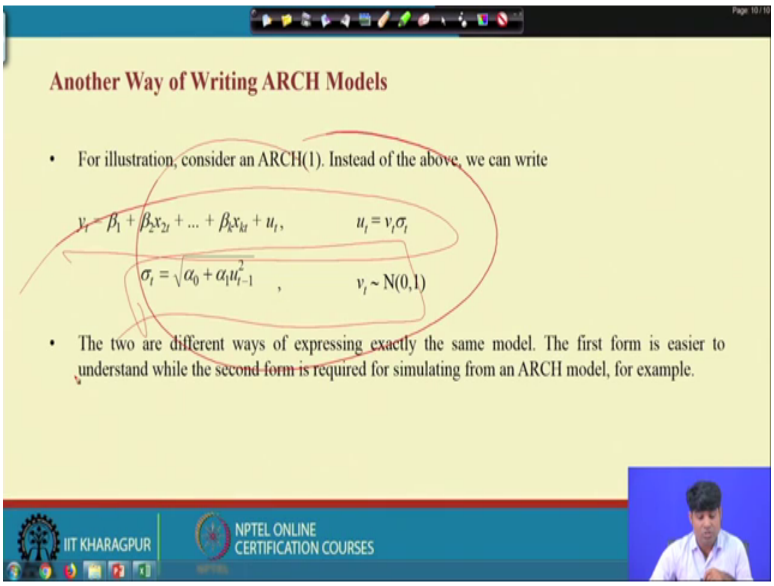

Every time dependent variable will be in a targeted. And error term can be used as a instrument to check the validate the model, but final prediction is with respect to the dependent variables, original dependent variables y t and then the dependent variable and through the error terms. But, here to predict this dependent variable through this independent variable we use actually error terms instead of the actual y and the actual x that is what the big deal.

And here is the, you know general structure of this you know kind of you know say the kind of you know say the ARCH modelling and we start with this particular forms and then finally, this is what the model about the first one.

That means, there are 2 different ways of you know expressing exactly the similar kind of you know structure. The first one is easier to understand while the second one is requires was simulating from ARCH models only ok. That is what the actual sigma 2 equal to square root of this one. Because, it is simply sigma square equal to alpha 0 as alpha square u t minus 1 and that too for ARCH 1 just you know keeping sigma 2 by default at the other side will be square root and then we can do this kind of you know estimation.

So, that is the big deal actually and we like to actually understand the structure.

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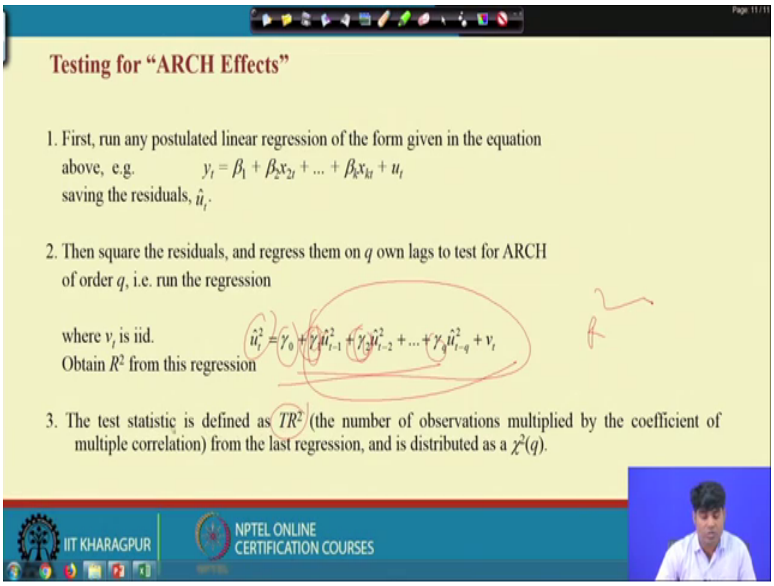

Now, after building the kind this is a first step and then bring ARCH means let us say you know actually 3 step process all together. In the stage one you have the actual problem set up with the clear cut identification of you know, dependent variable and independent variable.

And then we have estimated you know independent variable that is what technically called as a y k. And then in the second step you need to find out the error term which is the difference between the actual dependent variable y t and the estimated dependent variable y t y y k. and then we will get the error terms, after getting the error term then we will get error variance and then connect with you know square of these error terms ok.

And after getting the, is you know error term and the lag of these error terms then we have to set again you know the form of you know model. We say the form of you know set the form of the particular model. And again this model will we read on and to get the estimate you know estimated you know structure ; that means, technically we like to know these parameters build which ok. And against; that means, is completely new set up all together a where the is you know square of these terms right that is what here actually. And connecting with you know all these lag error terms that is what the big deal.

And then we connect with you know you know you know against the data structure, the kind of you know regressions and then finally, we can you know get the estimated values of all this parameters starting with you know at this one's this one this one. So, all these parameters actually so, we will find here.

And then finally, we get actually r square as usuals. Now, we need to first test the models by using actually tr square and that too through chi square test. Justifying that you know the r square is actually you know supporting the kind of you know requirement ok. That is that is very similar process all together only thing is the formulation of you know the first step and the second step that is the basic understanding and then rest of the things as usual actually structure.

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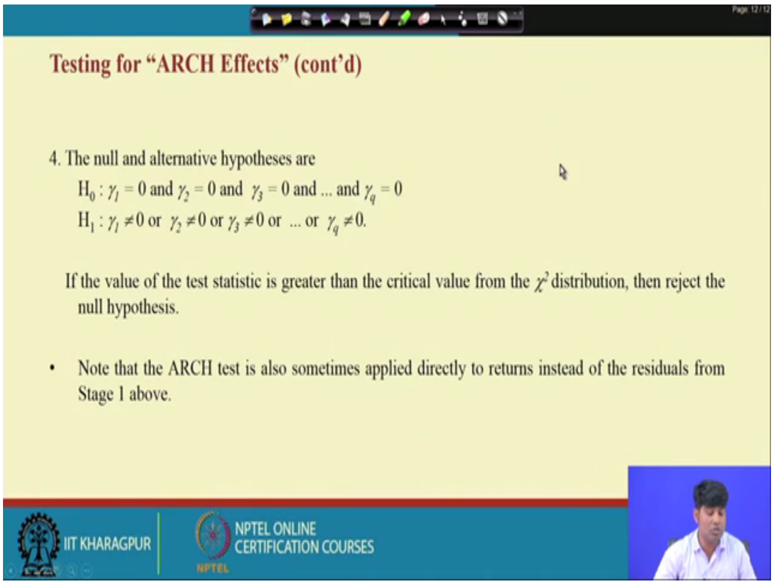

You see here what I have told. So, the null hypothesis is to test that in all these parameters you know to the statistically significant along with the r square and for that we use chi square and for this you know parameters we can use this test. And then we can actually justify the kind of you know requirement.

If the values of the test statistic is greater than the critical value from the chi square distribution then we can reject the null hypothesis that is the usual procedure again and not that the ARCH test is also sometimes applied directly to return instead of the residual form this single one that is the a another kind of you know.

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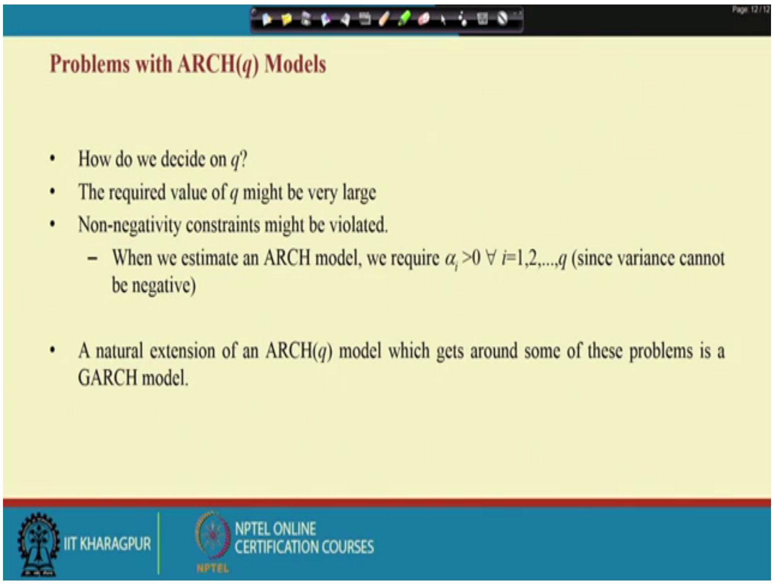

So, now couple of questions, first thing how to decide the q that is the number of flux and the require q value of q might be very large or it should not be too small ok. Then this needs actually optimum one if it is too large is an it may affect the process that too it may effect the degree of freedom and then you know the whole process will be in negatively affected.

So, that is why the first requirement is how to fix the q and for that I have already discuss and highlighted that you know we use usually use chi c statistic, SIC statistic, FFP statistics to decide the optimal lag length.

And then finally, non negativity in a negativity constants might be violated so; that means, technically a since we are squaring. So, by default actually negativity will be not there. When we estimated an ARCH models, we require you know you know this alpha i should be greater than equal to 0 ok.

So, since the variance cannot be negative that is that is the that is the big a big and big you know kind of you know you know kind of you know information that you need to have before you apply this model for the kind of you know engineering forecasting and the kind of you know decision making requirement.

And natural extension of ARCH q model which gets around some of the problems and that is actually connected with you know kind of you know say called as you know GARCH models right.

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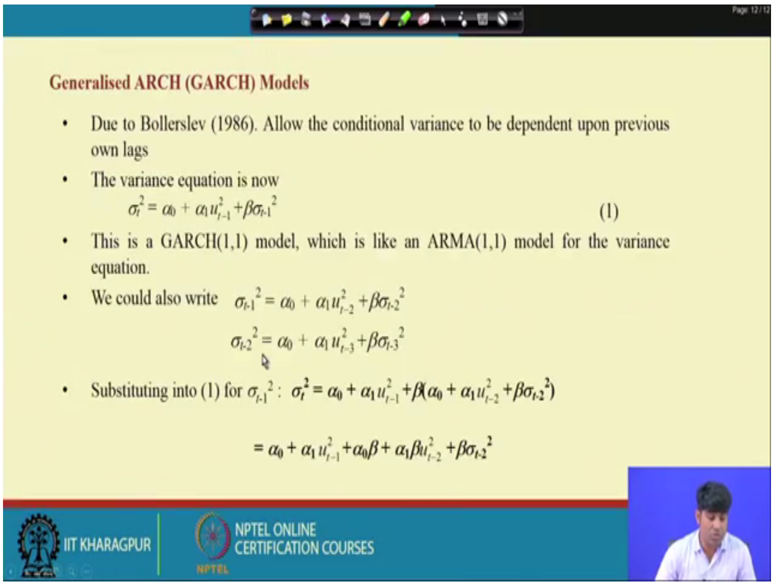

So; that means, the, this part is actually not so, complicated. So, we starting with you know simple structure of estimation then get the estimated model, error terms, then will get error variance then square of these error terms. And as usual again second stage you can just you know connect with a simple regression by a error term with you know square of these you know lag error terms. And that is as usual as a again regression then through that you know output we have to predict the kind of you know engineering output and you know engineering requirement.

And then come with a kind of decision making process. Similar kind of model, but it is the 2-3 stage you know extra things we are suppose to do. To being the scenario that you know which can have you know greater kind of forecasting's as per the particular you know engineering requirement and the a the kind of you know decision making requirement. This we will stop here and in the next class we will discuss the other the counter part that is the, a GARCH models.

Thank you very much have a nice day.