Engineering Econometrics
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Lecture - 46
Time Series Modelling- Volatility Modelling

Hello everybody this is Rudra Pradhan here. Welcome to Engineering Econometrics. Today we will continue with a Time Series Modelling and that too the aspects of Volatility Modelling.

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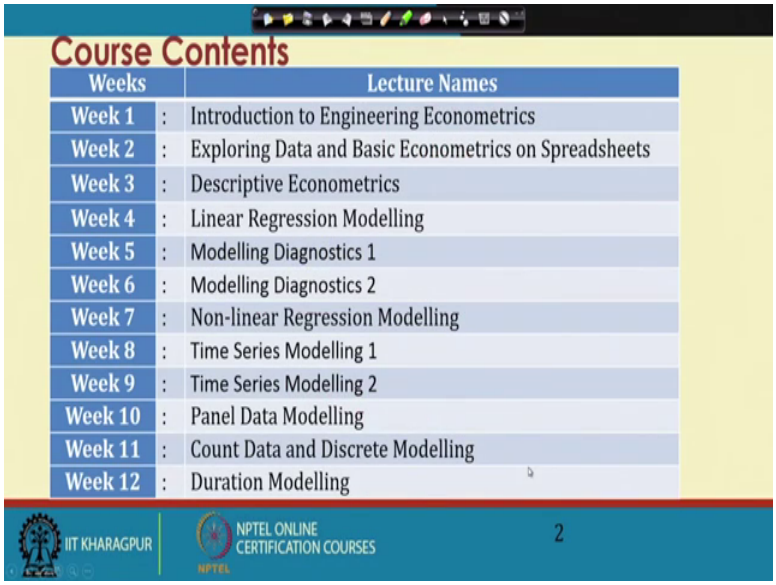

So, let us see how is this kind of you know structure.

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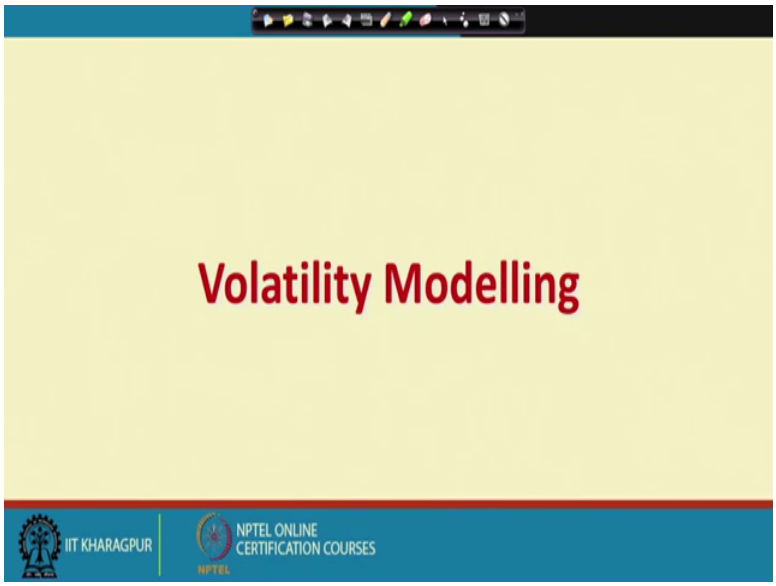

So, this is what the kind of you know discussion technically before I start.

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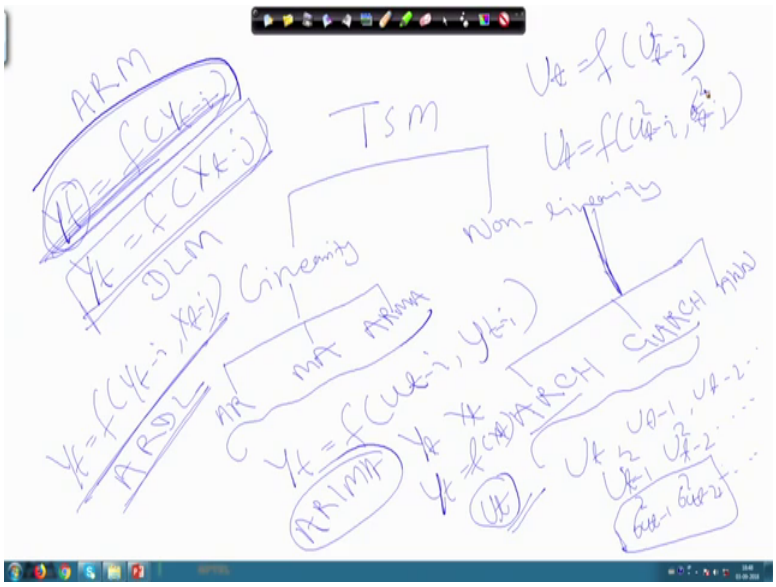

So, basically the time series modelling can be of you know two types whatever we have discussed earlier. And one is called as you know the Linearity structure. And the other one is non-linearity structure.

Under linearity structure we have discussed. In fact, three different forms; one is called as Autoregressive scheme, then moving average scheme then Auto autoregressive moving average scheme. So, that is the kind of you know item which we have discussed

earlier. And basically autoregressive schemes it is the structure of you know lag concept. And we have two different types of you know lag modelling, one is a autoregressive the other one is a distributive lag modelling.

So, which we have already discussed and then the kind of you know moving average. We start with a simple estimation process and then have the error term then finally the dependent variable can be connected with a lag of the error terms. So, technically so, if you go through this ARIMA structure completely the it is taking it is basically depends upon Y t as a function of u t minus i and y t minus i. So, this is how the kind of you know structure. So, one instance we are just connecting Y t with lag of you know Y t that is what the autoregressive scheme.

And sometimes if there are more number of variables the Y t can be connected with another variables and that too lag variables. For instance we can have like this Y t as a function of Y t minus i. And in other case we may have like this Y t as a function of X t minus j so; that means, if Y t is a kind of you know time series variables it may be connected to any engineering problem, but the requirement is that we must have a time series data. And that too with the time series must be consistent whether it is a day wise informations or monthly information or weekly information or annual information.

So, it must have consistency with a time a particular time that is with reference to our day, week month and the kind of an annual even in quarterly data and then we from that you know you know data structures we can create a lag variables.

So, now having actually a particular variable say Y t. So, we can create you know a n number of you know lag variables provided sample size should be very you know high and in the same times there should be you know you know they should maintain some kind of you know consistency. So, means technically there should not be any a gap in between or you know there should not be any kind of you know inconsistency in the process.

The other form of model is you know we have a two different variables like Y t and X t, but the dependent variable Y t can be function of you know lag of another variable say X t minus j. So that means, X t is a variable and by default the lag will be X t minus 1 X t minus 2 and. So, on and if the if the model is a with respect to the first one where Y t as a function of lag of Y t and that is what the structure called as as a Autoregressive

structure. And we call as you know autoregressive model and which we have already discussed.

And the second one is Y t while function of you know X t minus j that is called as a Distributive Lag model DLM. Now we can have a structures like this Y t equal to function of Y t minus i and X t minus j if this is the kind of you know model which we called as you know ARDL model. So that means, technically it is called as a Autoregressive Distributive Lag Model.

And means this is one kind of you know clustering the time series setup simple Autoregressive model which we have already highlighted. And then a distributive lag model which. In fact, also highlighted and then by you know if you combine the ARM scheme and DLM scheme then we have a structure called as you know ARDL scheme that is called as a Autoregressive Distributive lag model the structure of this you know ARDL is a Y t as a function of you know Y t minus i and then another variable X t minus j.

Not necessarily it will be bounded with only two variables it can be extend to you know multiple variables, but every times there is a dependent variable and we connect with a you know lag of this you know dependent variables and lag of some other independent variables, that is what the ideal structure is all about.

Ah the other form is the autoregressive then moving average and then finally, ARMA like a ARM DLM and ARDL. So, we have here AR then MA then ARMA so that means technically autoregressive then moving average and then finally, autoregressive moving average. So, technically here the dependent variables Y t as a function of Y t minus you know I like you know the case of you know ARDL. Where you know we are connecting Y t with lag of Y t that is the autoregressive part as means like the case of you know a ARM and the kind of you know ARDL. And another for m is a Y t as a function of you know u t minus i.

So; that means, technically we first connect with a Y t with a one of the lag terms or any other variables. Then trying to find out the error terms then finally, Y t will be connected with the lag of this error terms. And then we will connect a scheme where you know dependent variable as a function of this you know lag of this error terms. So, if that is the

case that is called as you know moving average scheme. Which we have again discussed in the last lecture.

So, whether it is a scheme of you know ARM, DLM or ARDL or the scheme of ARM or ARMA. So, it is exclusively depends upon you know time series setup and variables must be clear and problem must be correctly identified and the fit of this kind of you know models or the use of this kind of you know model you know typically you know requires the time series data. Whether it is a kind of you know simple management problem or any kind of you know engineering problems you know, but usually you know in the management having the financial data. So, we have lots of you know time series structure.

But similarly in most of the engineerings we have a time series data. And then this models can be very easily fitted there which we have already discussed some extent and there is no issue about you know connecting to any engineering problems while you know you know time series data is there in the system. And in any form so, once it is clear then you can use this models subject to availability of you know time series data.

The other form of you know time series modelling which you are interested to discuss in this lecture is the non-linearity part and that too again the time series you know structure. So, whatever we have discussed this side. So, the starting with a ARM DLM ARDL and AR MA and ARMA. So, that is the structure called as you know linear time series structure and the counterpart will be by default called as a non-linear time series.

Here we have also you know different kind of you know models, but typically we will hired three types. One is the structure called as you know ARCH and GARCH cluster ARCH clusters then GARCH clusters and then artificial neural network clusters artificial neural network cluster and ARCH clusters again ARCH and GARCH depends upon you know error term typically. So, like you know the kind of you know moving average. So, we start with a simple model first that means, technically both a for you know ARCH and GARCH that is one cluster then artificial neural network is the another cluster, but both these techniques are in the basket of you know non-linear time series modelling.

So, in this case we start with a let us say two variables, but not necessarily every time two variables, it can be extended to you know multivariable case. So, here if you start with you know Y and X the two variables. So, first you connect with you know Y t as a

function of you know X t or vice versa X t as a function of Y t. And after you know estimations you may have the component called as you know U t that is what called as you know error terms.

And we have discussed you know lots and lots problem you know to connect a how dependent variable and the independent variable connected and how to get the a estimated output and then how to derive the a predicted dependent variables that too let us say Y t is a dependent variable then we need to have you know y estimate that is what the predicted variable. And then with the help of you know dependent variable Y t and the estimated dependent variable Y t k f will find the error terms right.

So, after getting the error term so, we can have actually the structure called as you know log of the error terms. Starting with you know U t then U t minus 1 and then U t minus 2 and. So, on like this that is one kind of you know cluster. Again you know we can have an non-linear structure like you know U square t minus 1 u square t minus 2 and so on. So, in the previous case we are using the linearity structure. Now we will you know connect with a non-linearity structure where instead of you know U t u t minus 1 U t minus 2 we like to connect with u square t minus 1 u square t minus 2 and so on.

The beauty of this particular structure is that you know if you connect with you know U t with U t minus 1 etcetera. Then you know there is a chance that the mean of the error term will be close to 0, but when we go for you know squaring these you know error terms then by default we have a significant impact. And that impact can actually you know considerably you know having effect on the dependent variable.

So, compared to linear structure the non-linear structure has actually more kind of you know infirmity because we are transferring this information in a kind of you know setup where you know it can produce some kind of you know a significant impact to the system. So, technically using simple you know the lag variables we like to use the you know square of this a lag variables that means, technically U square t minus 1 U square t minus 2 and so on

Simply the first means this type is like this. You can have first error term then create lag of this error terms for instance if U t is the error term then the lag of this error terms will be U t minus 1 t minus 2 U t minus 3 and. So, on and finally, you can create a you know square U t minus 1 U square t minus so that means, we are just you know bringing into a

some kind of you know structure so that means, the advantage is that you know if you square then. So, the value will be coming differently.

So as a result so, it can produce some kind of you know you know significant impact to the system and again it is not the question of, you know U t with the square of the error terms that is what the cluster called as you know ARCH cluster and technically it is called as a Autoregressive conditional Heteroscedasticity. Means a just it is a counterpart of you know how much skedaticity where we have you know uniform error variants. Or here in this case we have a heterogeneous you know error variance that is what the term called as a heteroscedasticity.

So that means, technically before you start analysing the cluster called as ARCH and GARCH clusters. So, simple understanding is about you know heteroscedasticity and that is the first hand you know requirement and your understanding must be clear there and then we will come to the ARCH cluster and the GARCH cluster we have a series of models on this particular cluster. Typically you know you know we start with you know error terms then trying to explore the heteroscedasticity issue of course, we have already discussed you know heteroscedasticity issue which is actually typical virus before you know start doing the, means doing the kind of you know estimation and the kind of you know predictions etcetera.

So that means, technically once you get you know estimated model and the error terms. So, that error terms behaviour should be you know should be analysed properly in starting with you know heteroscedasticity and a homoscedasticity. Homoscedasticity is good for the modelling, but heteroscedasticity is not you know is a just negative impact to the modelling. So, we try to minimize the heteroscedasticity issue, but the structure of the time series data is that you know heteroscedasticity means when we have some kind of heteroscedasticity problem to the system then we will have the kind of you know different system where we can use this kind of you know model that too non-linear time series model and that too ARCH cluster and you know GARCH cluster.

So, first of all the kind of you know understanding of the heteroscedasticity then the autoregressive scheme and then the some of the condition we have to site. And as a result we can have the model called as you know Autoregressive conditional heteroscedasticity. So, autoregressive is a one part, heteroscedasticity is another part and the kind of you

know condition we which we are you know setting there is called as you know ARCH clusters. Like which we have already discussed in the case of you know ARMA cluster. Where we are taking care of you know autoregressive scheme and moving average schemes, but ultimately in between there is a technical term called as you know order of integration.

So, that is a instead of saying ARMA. So, we called as you know ARIMA. So, like this you know this is what the kind of you know structure called as a ARIMA. So, autoregressive integrated moving average what is that typical name so that means, before we start the ARMA process. So, we are supposed to check the stationarity of the variables. And that means, technically we need to check the order of integration through which the variable, you know the typical variable will reach the stationarity.

And until unless you know the stationarity and the kind of you know order of integration then this kind of you know model will not give you solid kind of you know structuring or the kind of restructuring to predict certain things. So, that is why unit root is very much required to justify the situation and the kind of you know requirement.

A similar kind of you know flow is with actually ARCH and GARCH cluster and the artificial neural network cluster. So, here the ARCH cluster so, we are just connecting you know U t with a error of this you know square of this error terms, that too in the with the help of you know lag structure. And the typical cluster is called as ARCH cluster. And it is extension called as a generalized autoregressive conditional heteroscedasticity. Where we like to connect error term with the square of the error variance that is what called as a sigma square U t.

So; that means, technically. So, we can have U square t minus 1 you can have U square t minus 2. This is one kind of you know cluster the another kind of you know cluster is you can called as you know sigma square U t minus 1 sigma square U t minus 2 you know. So, like this is another cluster. So that means this is what called as you know error variance as we have already discussed this one. Sigma square is the error variance which is actually a sum of the error errors sum of the squares of the errors with respect to the degree of freedom.

So that means, when we have a time series data. So, we can easily get the you know lag components and the square of these lag components and then finally, the lag of this you know error variance.

So, now, every time in this ARCH and GARCH clusters the typical error term need to be predicted with it is lag variables and that too square of these lag variables and the square of the error variance. So, means a error variance with respect to the lag part.

So, technically the first cluster will be U t as a function of you know U square t minus i that means, technically the way we have write written here Y equal to function of U t Y t minus i and Y t equal to a a function of X t minus you know t minus j.

So, here one particular cluster will be like this U t as a function of U square t minus i. So, this is one particular cluster and then another cluster will be U t as a function of U square t minus i and then sigma square t minus j. So, sigma square of course, sigma square U t minus j. So, that is what the basic structure of you know ARCH versus GARCH so, the autoregressive conditional heteroscedasticity and generalized autoregressive conditional heteroscedasticity.

So, typically you know we put lots of you know condition of course, GARCH has lots of you know extension and. In fact, ARCH also having similar kind of you know extension for instance. You can have a ARCH cluster of you know one to one and the kind of you know one to two like this and then GARCH clusters again like you know ARMA clusters.

So, we can have the GARCH clusters. So, 1 1 2 2 3 3 like this. So, then we can design the models. Mathematically it is you know well established you can just start with a game with you know 1 1 structure 2 2 structure it looks you know very decent to you know do the modelling a econometry modelling and the that too for engineering econometrics.

But ultimately the use of this model and the kind of an experiment the kind of you know used to predict and the kind of a forecasting. So, a problem should be exactly on that particular framework. And most important is the data part and which must have been a time series setup then you can go ahead with this kind of you know models and do the

kind of you know prediction and forecasting as per the particular you know engineering requirement and the need of the decision making process.

And then we will discuss in details in you know about the ARCH cluster and GARCH cluster because the ARMA part we have already covered in the last lecture. Since we are discussing non-linear time series just I am connecting how there is a kind of an extension which you can have here in the form of you know ARCH cluster and GARCH cluster and then finally, we connect with a concept called as you know neural network.

Because we are connecting again with a linear to non-linear setups and by default artificial neural network will be in that particular similar kind of you know cluster. So, where you know the lag component is not a big deal, but what is the big deal is the variables will be connected with a different functional form that too in a non-linear structure. And typically for artificial neural network is same two variables can be connected instead of simple you know linear equation, for instance y verses x we can put you know y equal to simply alpha plus beta x that is what the linearity structure. If not you know neural network structure will be follow the sigma functions, which is actually non-linear in character like you know exponential functions or logarithmic function.

And then do the processing and then you know do the estimation to predict the dependent variable with respect to the independent variables. So, overall structure is that. So, we have the system called as you know as non-linear time series modelling and that too we will cover three things the ARCH cluster GARCH cluster and artificial neural network. Of course, more important is the ARCH cluster and GARCH cluster which is very much you know important and that too to maintain the continue with the autoregressive distributive lag, ARDL, autoregressive and a moving average and then ARMA then similar cluster we are just extending to ARCH and you know GARCH clusters.

And in between you will find you know there are many such you know GARCH models like you know (Refer Time: 24:33) so on. So, many things are there and again every time the extends of extension of you know ARCH and GARCH and that in between this you know ARCH clusters you will find plenty of you know different type of models and that too again with respect to different conditions.

So, we put you know different condition and then the model extension will start and that will be you know having more flexibility and you know to the moulding basket how we can predict any kind of you know engineering problems subject to the time series data and the then the kind of you know the use of the non-linear time series models. Now, with this actually we are just highlighting how is this kind of you know volatility modelling.

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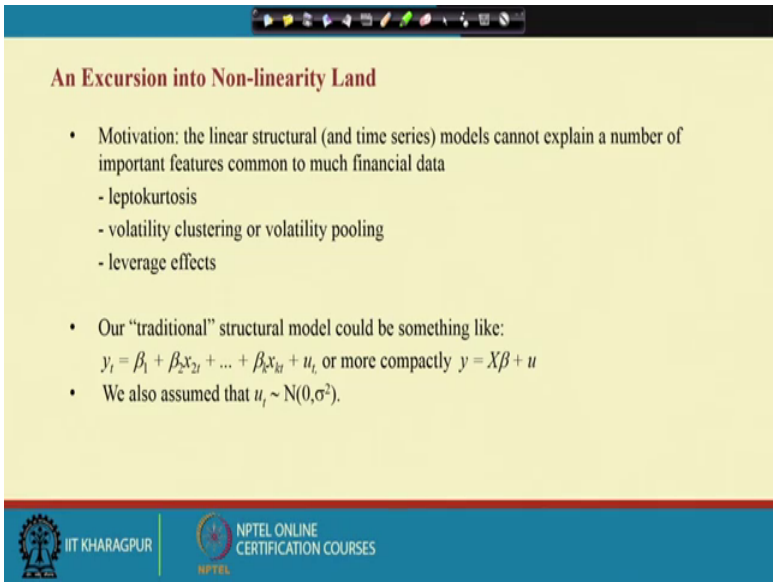

And technically I will just highlight the issue of you know non non-linearity first. And then I will come back to the ARCH and GARCH cluster and again slightly I will touch up on the artificial neural network. Then here the motivation is actually you know we have gone through the linear structural framework and that too in time series model that too we some of the important you know functions or features we cannot actually you typically analyse while handling financial data and engineering data.

For examples the kurtosis and volatility clustering and volatility pooling. And there is also leverage effects and coming to you know traditional you know form of models that too in you know multivariate framework start with independent variable y equal to simply beta 1 beta 2 x 2 beta 3 x 3 and so on. And where we are assuming that you know error term will behave with you know mean or mean of a you know 0 and then error region sigma square.

Now, the issue of non-linearity will come that too for a ARCH cluster and the GARCH cluster. Where the error term will not actually follows you know sigma square u that is what the homoscedasticity it will be forming a structure called as a sigma square u i that means, technically instead say instead of saying you know U t.

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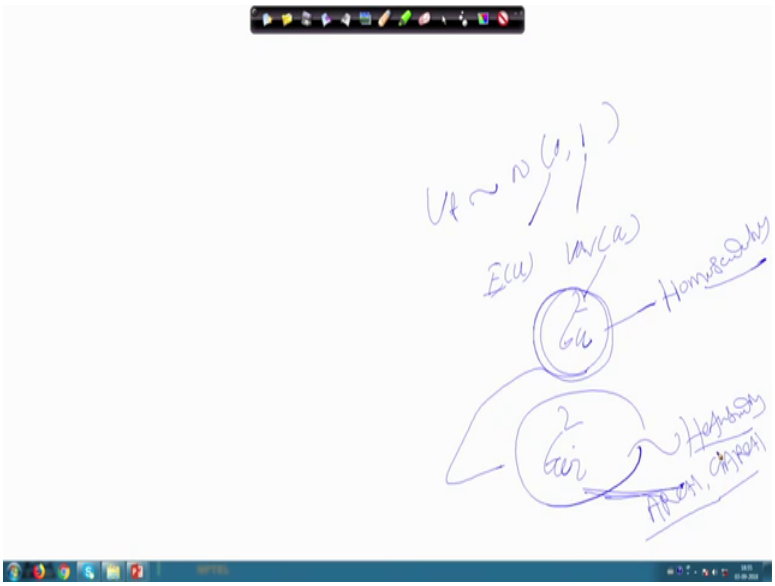

Which is actually let us say normally distributed with 0 and unit variance that means, this is actually mean of the error term and then this will be called as you know variance of the error terms. So, that that is what called as a sigma square u and. So, when we have sigma square u like this that is what the game of called as a homoscedasticity which we have actually discussed earlier.

But if not then the structure will form you know kind of you know problem like this sigma square u y so that means, error variance will not same over the time. So, it will be having you know different values with respect to different time frame. And if that is the case what is called as you know heteroscedasticity problem.

So, now, we are doing the extension of the heteroscedasticity problem. And that too in the form of you know ARCH cluster and GARCH clusters.

So, what we can do. So, how is the ARCH cluster and how is the GARCH cluster and that is under the basket of you know non-linear time series modelling. So, we can start with a simple you know problem and then connect with a time series data. And once we

do the estimation and then get the error term and this kind of model by default will be coming into the picture and they do the kind of you know predictions and the kind of you know forecasting as per the engineering requirement and the kind of you know means as per the decision making process.

So, we have you know beautiful structure through which you can you know generalize this issue and typically the structure is you know yes this is what the error variance.

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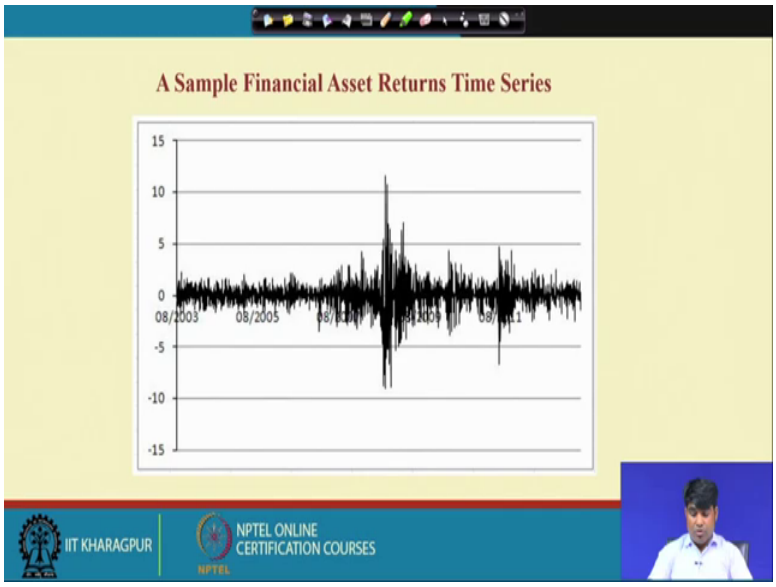

And if you look into this particular you know non-linearity structure and that too ARCH and GARCH cluster this is how the kind of you know structure where we can exactly apply the a non-linear model and that too ARCH cluster GARCH cluster and you know artificial neural network.

So, that means, to we like to just you know plot these variables with respect to different time periods. And check whether there is a kind of you know volatility signal or not. So, volatility means (Refer Time: 29:19) you know very simple language there are lots of you know ups and lots of downs. And as a result the situation is not so stable. So, the if this situation is not so, stable that is what we called as you know volatility.

So, within the volatility structure in stability case how we can bring you know better and better and better prediction to the particular you know engineering problem and as per the requirement. So, ARCH and GARCH cluster can give better you know such kind of

you know clue to deal with this problems and do the kind of you know better forecastings and the kind of you know engineering requirement.

In the next class we will start from here and then we will connect with a problem with this particular you know models that too ARCH cluster and GARCH cluster with this we will stop here.

Thank you very much.