

Engineering Econometrics
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Lecture - 42
Time Series Modelling- Trend Analysis

Hello everybody. This is Rudra Pradhan here welcome to Engineering Econometrics. Today we will continue with Time Series Modelling. The last lecture we have discussed this component that to knowing the understanding of time series data and what kind of models we like to use under the time series umbrella. Here some of the engineering problems can be solved or we can do the predictions, do the kind of forecasting's, different types of models time series models we can use. But one of the requirements is that data structure must be with respect to time only; it maybe annually it maybe day wise, it maybe monthly, it may be quarterly.

So; that means, having the data with respect to time. So, we like to do the prediction of course, different types of models can give different kind of setup, and structure through which you can do better and better kind of modelling and that to the way of or the kind of requirements of forecasting, or the kind of prediction.

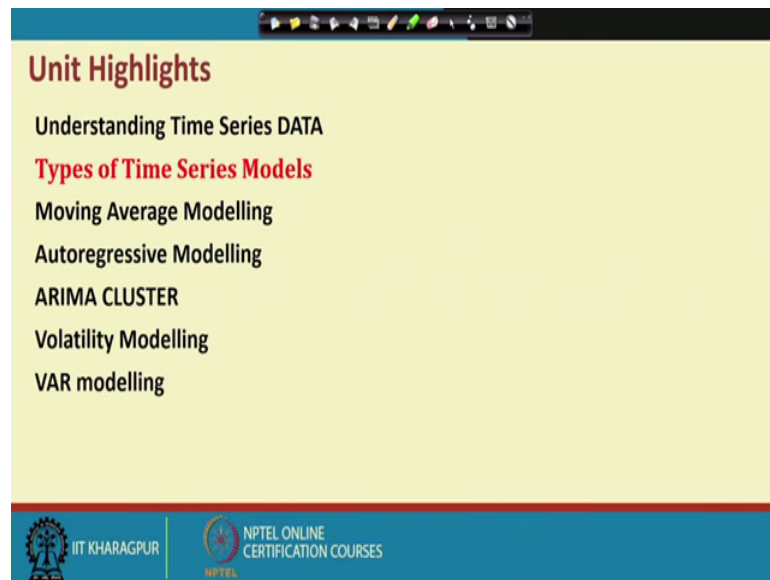
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So, let us see what are the kinds of structure we like to discuss.

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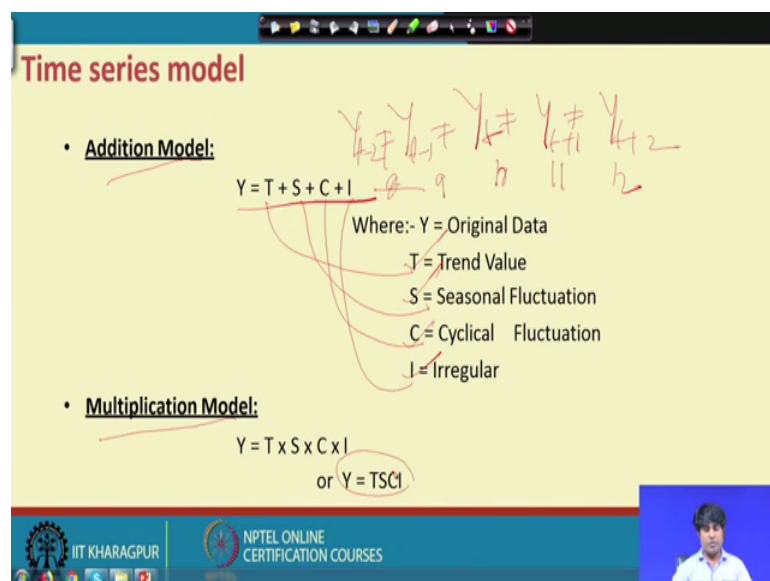
Unit Highlights

- Understanding Time Series DATA
- Types of Time Series Models**
- Moving Average Modelling
- Autoregressive Modelling
- ARIMA CLUSTER
- Volatility Modelling
- VAR modelling

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And of course, last class we have spent couple of time to understand the time series data, and slightly touch upon various issues of the time series data. Typically you know we have discussed about the kind of variation in the time series data, so over the time it cannot be stagnant. So, it will you know having up swing or down swing that is why we have discussed various factors you know responsible for this change. So that means if we start with a particular numbers. So, it the number may not constant.

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Time series model

- Addition Model:**
$$Y = T + S + C + I$$

Where:- Y = Original Data
T = Trend Value
S = Seasonal Fluctuation
C = Cyclical Fluctuation
I = Irregular
- Multiplication Model:**
$$Y = T \times S \times C \times I$$

or $Y = TSCI$

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So, it will go up or it will go down and for that we have discussed you know for different causes that is regarding secular trend, seasonal variation and may be due to cyclical variations and irregular variations. So that means, data over the time may change you know that to with respect to either the structure of secular trend, or with respect to seasonal variations, or with respect to cyclical fluctuations and the kind of in a regular pattern.

So, we have already discussed all these things in the last lecture. And if you look here is and all together's a time series data at a particular point of time involves many things. So that means, technically so means the time series data maybe like this Y_t , then Y_{t+1} , then Y_{t+2} . This is actually forward movement and then Y_{t-1} Y_{t-2} , and this is called as the backward movement. So that means, technically we expect that these are all not equals.

So, these data are having different over the time. So that means, if you say here a 10, then this may be 9, and this may be 8, this may be 11, this may be 12 just time in a giving the kind of structure not exactly the levels, or the kind of movement may not be actually completely up swing, or completely down swing. So, it may be kind of non-linear in characters like you know sometimes up, then slightly down again down and slightly up like this.

So, all these variations with mostly with respect to these factors the cyclical structure, then seasonal structure, irregular structure, like you know we may have you know any kind of structure.

So, ultimately, so four factors which we have identified T S C and I. So now at a particular point of time you know the variation of the data you know that is with respect to all these four component T S C I. So, ultimately they will be linked either in additory format, or in a kind of multiplicative format.

So, that is why its call as additional addition models or multiplication model so; that means, just Y equal to T plus S plus C plus I ; T stands for trend value, S stands for seasonal fluctuations, C stands for cyclical fluctuation, and I is stands for irregular fluctuation, and in the multiplicative you know mechanisms. So, it is simply actually you have to multiply each other so; that means, a it is a kind of additive model and otherwise you know you can say that multivity you know multiplicative model.

So, there are two different ways the variation can happen here. So, ultimately the idea is that to check whether data is actually varying with respect to time and if that is the case what are the factors responsible for that? And in the past times we expect that the variation is mostly due to this four factors trend factors, seasonal factors, cyclical factors, and irregular factors; so now, after understanding the kind of variations. So, let us see how is the kind of time series model?

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Measurement of Secular trend

- The following methods are used for calculation of trend:
 - ☒ Free Hand Curve Method:
 - ☒ Semi – Average Method:
 - ☒ Moving Average Method:
 - ☒ Least Square Method:

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Ultimately, we need to do some kind of forecasting's. And the beauty of this forecasting we have already mentioned here. So, we have actually historical data and means the time series otherwise called as the historical data since it maintains consistency.

So, we have a historical data from the left, or from the past and then with the help of this left. And past data we have to trend for the future that is what the structure called as predictions and forecasting. In fact, in the cross sectional type you know model we use the data, then have the estimated models, and then you check the estimated model.

If it is then you have to just you know extend this estimated model for the kind of forecasting. The same structure we are doing here, but the particular prediction and particular forecasting the kind of extension is the much easiest. and very easy to understand in the case of time series data; that means, just you enter the data is just you know pull the kind of information's. Then you will find some kind of trend lines or something like that.

By default you will get a some kind of predicted line, again without using any kind you know standard technique. In totals there are four mechanisms through which you can actually predict the particular engineering problem and that to with the help of time series data.

So, first one is the free hand curve methods that is the first mechanism. And then the second one is the moving semi moving average, and then the third one is the moving average and then least square methods. So, there are four main mechanisms. So, first one is the free hand curve, semi average then moving average, and least square mechanisms our topic least square mechanism is a is the best. Of course, the way we have written here free hand, semi average moving average and least square. There are all something called as better and better we have to not doing the forecasting that to with the help of time series data.

So, let us first go one by one and; that means, technically how we do the prediction with the time series data by using free hand curve. Against do we means how to predict the variables with respect to time series data that with the semi average methods, and again for by the moving average methods, and then finally, with respect to least square methods. So, let us see; how is this particular structure.

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Free hand curve fitting

- In this method the data is denoted on graph paper. We take "Time" on 'x' axis and "Data" on the 'y' axis. On graph there will be a point for every point of time. We make a smooth hand curve with the help of this plotted points.

Example:

Draw a free hand curve on the basis of the following data:

Years	1989	1990	1991	1992	1993	1994	1995	1996
Profit (in '000)	148	149	149.5	149	150.5	152.2	153.7	153

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And so first we start with free hand curve methods it is actually very simple one. And just to graphically means it is a kind of inspection method through which actually observe and draw a kind of predicted line.

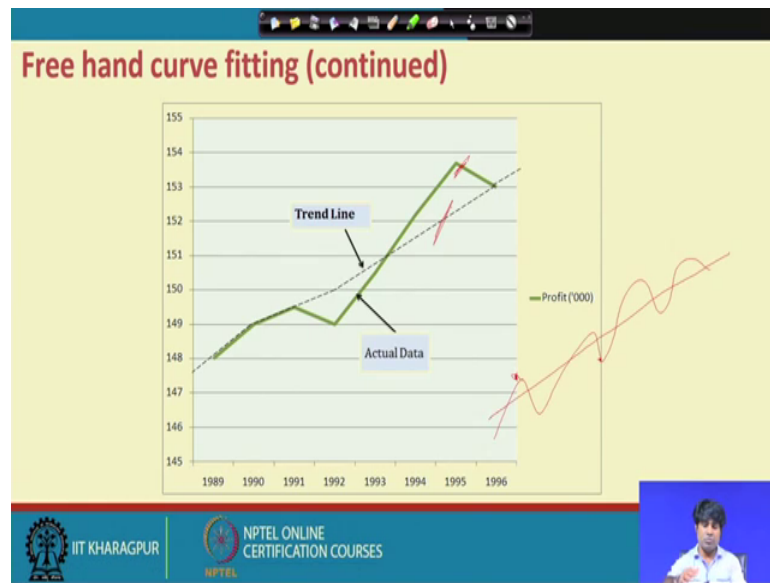
For instance so we have actually yearly data here. We will start with 1989 to say 1996. It may be any forms just I am taking you know few time series data and this time series data is the annual data, and starting with 1989 and ending with the 1996.

So; that means, you see here 1989, 90, 91, 92, 93, 94, 95, 96 and then it will go up to say 97, 98. So, it will continue up to its a long series data, but just we are giving a samples we know with how much for first you know few points only that is you know with respect to 1989 to 1996.

So, we have actually data up to 2017, but you know for simplicity we are just taking a few data, and then this time series you know focus with the profit data. So that means, it is the industry a profit data and for a particular industry and that to from the that is with respect to time and that to from 1989 to 2017.

And for instance 1989 the profit is 148 that to in 1000, then against 1991, 49 and then one 149.5. So, there is a little bit variations. So now, if you plot these actually data with respect to time then you will find there is a kind of fluctuation because they are not constant. So, they are varying, sometimes increasing, sometime decreasing; so with this particular variation; so how we can actually draw a predicted line.

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So, just go through it is a graphical mechanism through which you can use. So, you see here there are 2 lines and one is the dotted line and that is what the kind of predicted lines. And the green line that is actually the kind of actual data you will see the actual data is actually slightly deviating, little bit more deviation and then is the dotted line is the predicted line that is otherwise called as trend line.

So, what will you do actually? So, after knowing like this without any evidence I can plot a line like this ok. Let us say the moment is like this. So, if you say that free hand curve fitting that is means you just a drop by judgement that this could be the predicted lines.

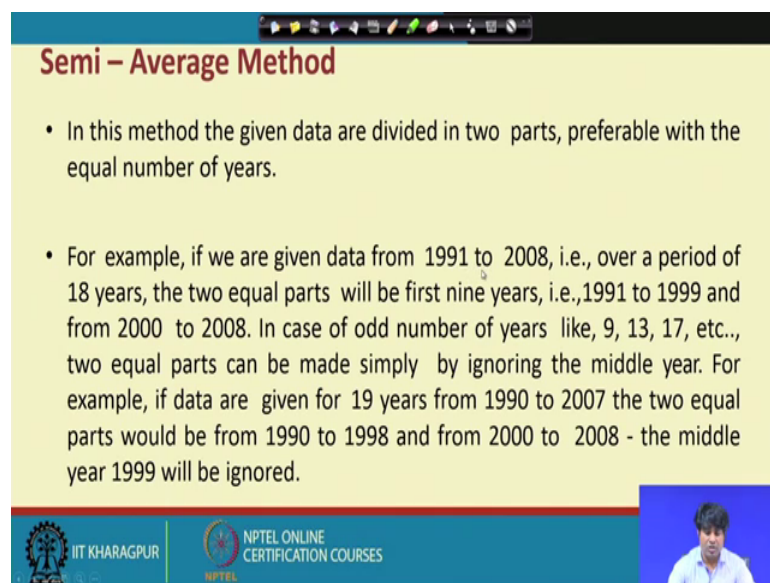
So that means, usually the predicted line will be in between all these average points; so that means, you check the peak this sides this sides and the down side peaks; so once the peak up and down. So, the predicted line by inspections should be in between actually.

So, this could be a this could be the very you know in our you know simple trend line. So means that is the trend line through which you can actually a predict the profit here, and that with available of this much of samples. So, if you increase the sample size again one after another then the prediction structural beam much more attractive again.

Here more data to know the clear picture, but whatever it is a small data or big data it is not a big deal. So, what will you do? In this kind of mechanism. So, you check the highest point and the lowest point and then few other peaks in the up side and down side.

Then after checking all these peaks up side and down sides then you draw a line in between and that is what called as free hand curve. And that is the curve through which you can do the forecasting. So, this is a simple mechanism through which you can actually draw generate a predictive line.

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Semi - Average Method

- In this method the given data are divided in two parts, preferable with the equal number of years.
- For example, if we are given data from 1991 to 2008, i.e., over a period of 18 years, the two equal parts will be first nine years, i.e., 1991 to 1999 and from 2000 to 2008. In case of odd number of years like 9, 13, 17, etc., two equal parts can be made simply by ignoring the middle year. For example, if data are given for 19 years from 1990 to 2007 the two equal parts would be from 1990 to 1998 and from 2000 to 2008 - the middle year 1999 will be ignored.

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Then second one is the semi average mechanisms. So, like free hand curve, so we have the data similarly from let us say 1991 to 2008.

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Find the trend line from the following data by Semi – Average Method Example

Year	1989	1990	1991	1992	1993	1994	1995	1996
Production (M.Ton.)	150	152	153	151	154	153	156	158

■ There are total 8 trends. Now we distributed it in equal part.
Now we calculated Average mean for every part.

First Part = $\frac{150 + 152 + 153 + 151}{4} = 151.50$

Second Part = $\frac{154 + 153 + 156 + 158}{4} = 155.25$

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And then how you can apply the semi you know average data that to again to predict the kind of predict the kind of structure. And a here the same data which we have used for the free hand curve again we are using for the kind of semi average mechanisms.

In the semi average mechanism most of the instances we divide the data into two equal parts. And if the data is actually having even numbers let us say 10 data points, then the first five points and the last five points is the first step process to identify. And so the first point you know in this case we have actually 8 data points.

So that means, technically so the structure moving average will be like this. So 1, 2, 3, 4 that is the one set. Then the remaining four will be the a second set. So, what is the semi averaging method you know the first half you take the average and then second half you take the average.

If it is even number then it is very easy to do that and if not let us say if it is a odd series like you know let us say 9, or 11 something like that. So, then from the actually the left hand side you take you know 50 percent and then the remaining 50 percent will be in the right side for odd numbers particularly for we will say the figure 9.

So, then first four points and the last four points then the middle one should be you know remain silent; for instance the fifth number should be silent then you have the series from 1 to 4 then 6 to 9. So, what you like to do here? So, first fours you take the average and

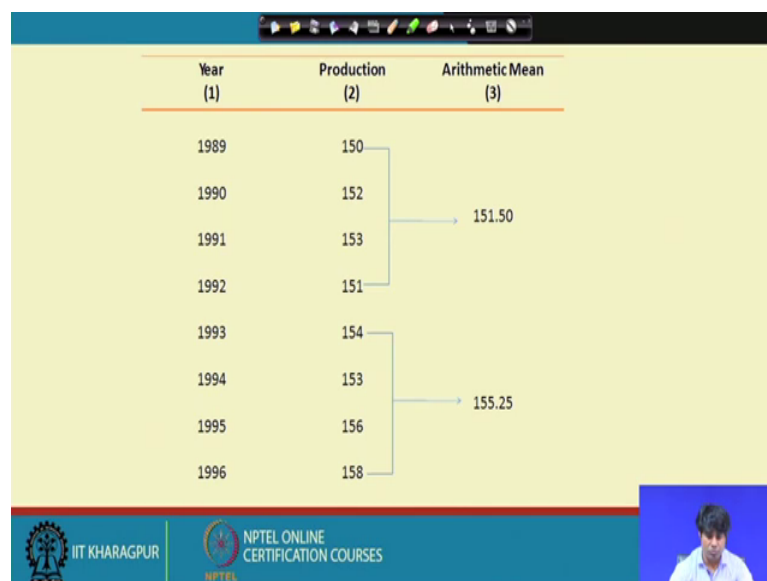
last four you take the average and then you will just join the points two points that will be the trend line, or the predicted line and what we called as forecasted line.

Like you know the free hand curve where we our target is to check which one is the top or top point? And which one is the bottom point? And then draw the line in between. So, that is how the object you know inspection through which we have to draw the curve. And against little bit you know what we can called as logic logically establish or something you know more accurate where divide the data into two parts. Then take the average, and take the average and join.

And then in between you can have a actual plotting and then the kind of if you join these two points then this will give you some kind of predicted line. So, now what we can do here is whether the prediction is actually accurate or not accurate. So, we can find out the forecasting error which is actually the difference between the predicted lines and the actual lines.

So, if the difference will be more or less very small then the kind of predicted lines. So, whether it is free hand curve or something kind of semi average mechanisms you know it is not an issue that lines can be used for the prediction and forecasting. So, that is the second mechanism through which do the prediction.

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Year (1)	Production (2)	Arithmetic Mean (3)
1989	150	151.50
1990	152	
1991	153	
1992	151	
1993	154	155.25
1994	153	
1995	156	
1996	158	

And again the similar structure we can have here. So that means, you see here whatever I have told you. So, just you put in the kind of you transfers the data then it will be in a particular order form like this. So, this is the first point and take the average here and the these are the last four points and take the average.

So, now you go through plotting 1989 to 1996 that is in the x axis. And in the y axis so you just take the point 151 point and then you take 155 point and then just join this two points see here.

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So, this is what the kind of game. So, you do the actual plotting first and then you find out where is the point actually 151.50. For instance here 150 and a then what will you do? So, you just check 151.50 here, then again so 155.52. So, you just take it here then with these two points you can just extend this line. And you refined a if there is a kind of difference and a and the predicted line will be very effective to draw the conclusion years.

Of course, this is that whether the particular line is very perfect to predict that that we can just through the forecasting error which we discuss in the forth coming lecture. But in the meantime this is the procedure through which you can get the trend lines, or predicted line through the semi average mechanisms.

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Moving Average Method

- It is one of the most popular method for calculating Long Term Trend. This method is also used for 'Seasonal fluctuation', 'cyclical fluctuation' & 'irregular fluctuation'. In this method we calculate the 'Moving Average for certain years.
- For example: If we calculating 'Three year's Moving Average' then according to this method:

$$= \frac{(1)+(2)+(3)}{3}, \frac{(2)+(3)+(4)}{3}, \frac{(3)+(4)+(5)}{3}, \dots$$

Where (1),(2),(3),..... are the various years of time series.

□ **Example:** Find out the five year's moving Average:

Year	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996
Price	20	25	33	33	27	35	40	43	35	32	37	48	50	37	45

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So, now move to another structure called as moving average. So, this is a relative concept and we can have a several kind of lines through different moving average mechanisms. So, you can means for examples we can have a three years moving average, five years moving average, seven years moving average, nine years moving average. That means, actually the entire data we will put some kind of normalisation. So, instead of using actual data you have to actually normalise that to standardization we can standardize the data.

Like has a such a beauty actually to do that. For instance a if you are saying that we have weekly data. So, then one week is nothing, but actually 7 days. So, the 7 data, 7 days data will be fluctuates, but if you take the average and the fluctuation will be little bit normalised.

So; that means, a first 7 days it will be one figure, then second 7 days there is another figure, third 7 days then another figure. So, then the way we are saying that first four points and last four points. So, here let us assume that 7 into 7 49 data points. So, as first 7 points take the average second, you know 7 points take the average. So, likewise you have a 7 different points and then using this 7 different points.

And of course, there is very I mean say you will get a standard line to reduce the kind of requirement. and forecast the kind of requirement. Of course, you know there are two ways here if you compare with the semi average and moving average.

In the case of semi average just you know the entire data you will divide the to two parts and then you just draw these two points means connect these two points and you will get the predicted line. But here the process is slightly complex and not exactly just to join the two points, but here what is happening? So, you are taking you are dividing the data into few clusters like we have discussed earlier the dummy modelling. So, one type of modelling is actually you have probability value will be only 0 and 1. And there are models where the probability value will be in between 0 and 1. So, this is also similar kind of case.

So, in the case of semi average the entire data will be simply transfer into two different parts and then take the average and join these join these two point to get the predicted line, but moving average mechanisms.

So, the entire data will be will be generated into few points instead of two it will be couple of points in between and then you join these couple of points and you will get the predicted line. So, we have here starting is the three years moving average. So, so the structure will be like this. So, what will you do?

So, let us assume that this is the first data point, second data point, third data point, fourth data point. So, if it is three year moving average then first three will be one average, then again 2, 3, 4 another average; then 3, 4, 5 another average; then 4, 5, 6; another average; then 4, 5, 6, another average.

So, likewise you will you will have a kind of chain actually. So, instead of having actually actual data; so now you have the average data where each starting with the first one; so every three items will be transferred into one.

So, that is the average value we are putting and then you are joining these average points. Once you do that then you will find your line and that is what the called as trend line and the predicted line. So, that is what the mechanism called as moving average mechanism. And of course, it is not necessarily every time the moving average will give with respect to three years only or three time periods and not exactly year wise data it can be weekly.

So, week wise data three; you know three days you can create a kind of structure again another three weeks, or three days you can create a kind of structure. So, means ultimately these the time series data no specification whether it is annual, or monthly, or

quarterly, or day. So, whatever maybe the form of data; so while doing moving average to three years moving average just you know start with the first. And then follow every three every three point is the average and then you join the average.

So, when you had just transferring the data every three into one. So, we have no business with the data structure that to whether it is the day wise information, or hour wise information. So, or the kind of course that is that understanding will be there definitely.

Otherwise you may not on you know do the kind of time series modelling. Once you had the records then after that the process; you know the process which you like to do you have no connection with this kind structuring.

So, likewise so we have a different kind of setup. So, likewise we can go for five years moving average. So, instead of three points, so we take first five, then second five that starting with second data points to six data point, so and go ahead with the all these possibilities. Then finally you join these average you will get the average lines and the predicted line through which you can do the prediction so, like this.

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Moving Average Method

Year (1)	Price of sugar (Rs.) (2)	Five year's moving Total (3)	Five year's moving Average (Col 3/5) (4)
1982	20	-	-
1983	25	-	-
1984	33	135	27
1985	30	150	30
1986	27	165	33
1987	35	175	35
1988	40	180	36
1989	43	185	37
1990	35	187	37.4
1991	32	195	39
1992	37	202	40.4
1993	48	204	40.8
1994	50	217	43.4
1995	37	-	-
1996	45	-	-

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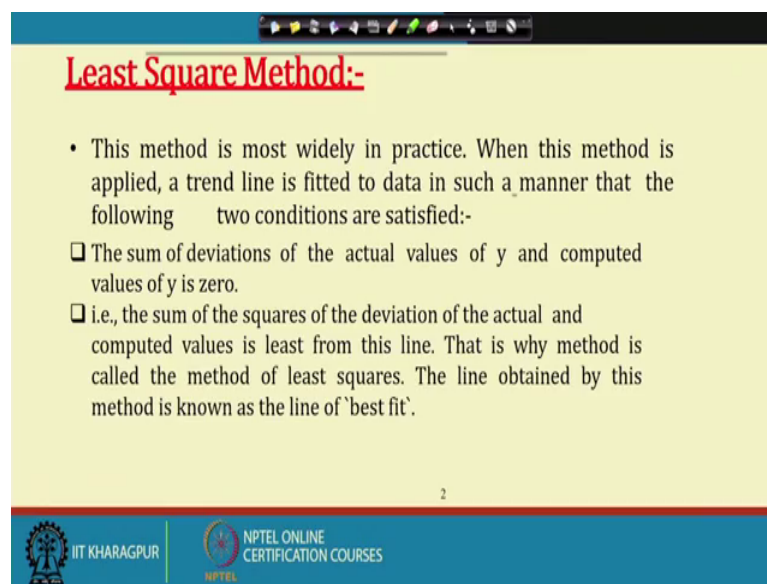
So, you we will just transverse the data. So, now this is the this is the this is the actual data here and starting with years say 1982 to 1996 and we have a price of sugar data that is the original. And then we have here predicted data and that to with three years moving average and this is with respect to five years moving average. Then you can go for seven

years moving average and so on. There is you know such a hard and fast rule only thing is you should know how we can have this average figure.

So that means here if it is three year moving average, then the first three will give you this one, then the second three will give you this one. So, this is how the kind of structure which you like to follow and what is happening here? So, what we do it means in this particular a tables, so we are just adding these first three data points to one then take the average and then you draw the kind of kind lines.

So, it is not actually big deals how to do this? So, after doing this then you just join the point and you will find a predicted line in that predicted lines which will be used for kind of forecasting right.

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Least Square Method:-

- This method is most widely in practice. When this method is applied, a trend line is fitted to data in such a manner that the following two conditions are satisfied:-
 - The sum of deviations of the actual values of y and computed values of y is zero.
 - i.e., the sum of the squares of the deviation of the actual and computed values is least from this line. That is why method is called the method of least squares. The line obtained by this method is known as the line of 'best fit'.

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So, then then the method is called as least square methods; that means, technically there are four standard methods through which you can actually do the the fast and prediction through the times series data.

Of course there are lots of HARKER models, like ARIMA models, then ach models, GARCH models, bar models. So, these are top class models in the time series circle, but in the first instance having the time series data. And these are the simple methods, or mechanisms through which actually you can do the predictions and then do the forecasting as per the kind of engineering requirement.

So; that means, technically in the first instance this simple mechanisms will be by free hand curve then semi average mechanism and moving average mechanism and the least square methods. We have already solved the problems through the kind of semi average, and the moving average and the kind of free hand curve.

So, we get to know that how is this particular structure of course, every you have the kind of option to compare and check which one is the best and then finally pick up. But you know method you know mechanically if you restart with free hand curve to the semi average and moving average and least square methods.

So, as per me know experience is concerned, or literature is concerned; so with all these methods or systematically improved versions. So that means, the prediction will be by least square method is a slightly better than the moving average. And prediction a moving average is better than the semi average and prediction with the semi average is substantially better than the free hand curve so that means, it a degree of improvement over the time.

So, till now we have discussed the free hand curve, semi average methods, and moving average methods. Under moving average methods we have a different forms like three years, five years and seven years. And then we know how we can do the predictions and that to with the availability of time series data only. So, when you are actually doing the moving average. So, since data are very consistent with time we can do this average. And then do the predicted line, but if it is cross section data which is not very easy to do the similar kind of prediction; and that is what the beauty of this time series modelling.

And the last one is the least square method which we will discuss in the next class with this we will stop here.

Thank you very much.