**Lecture – 34**
**Non-Linear Regression Modelling- Interactive Regression Modelling**

Hello everybody, this is Rudra Pradhan here, welcome to Engineering Econometrics. Today's discussion is on Non-Linear Regression Modelling we have discussed this concept in the last class and that to the issue of non-linearity in the context of regression modelling and that is the use of dummy modelling concept.

So, first of all we have discussed what is the dummy modelling and how you can develop a concept of you know dummy and that to how it is useful in the context of you know regression modeling. A dummy modelling itself is a kind of you know linear structure, but when we connect with you know multiple dummies and try to extract the interactive effect then it becomes a non-linear modelling.

So, what we can do here? So, we first like to know the dummy variables concept. So, how a dummy variable can be created, and how we can interpret, what is the importance and how it can be used in the estimation process, and then how to obtain the estimated output and the kind of you know interpretation. All these things are the already discussed in the last lecture, and today we will continue with this similar kind of you know flow and that to we will connect with interactive models.

So; that means, there are 2 dummies; just if you go for you know say a crush product or dot product, then it becomes a interactive effect. And before we go for you know dot product and cross product. So, it should have a kind of you know meaning full interpretations, that to there the there must be logic behind this interactive effect.

So, if the theory and logic will be supportive, then you can introduce the interactive effect, then as a result the particular model will be treated as a non-linear models.

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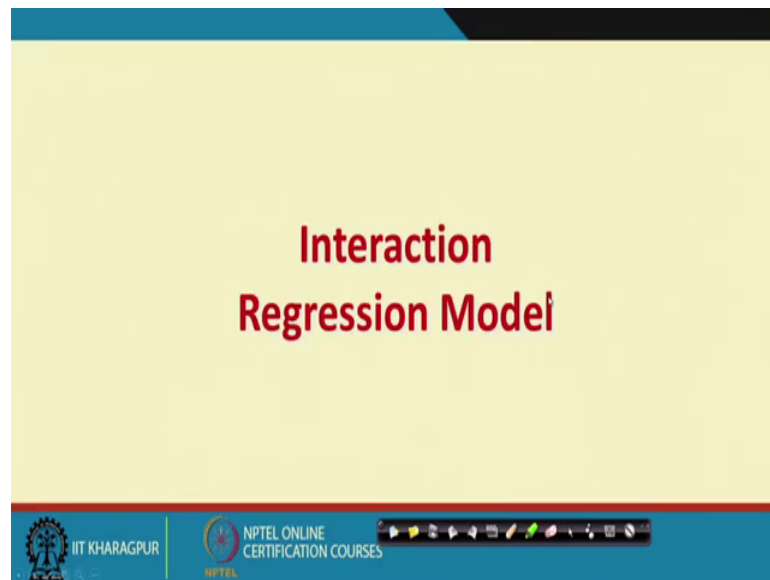


So, let us you know discuss on that aspect, and specifically we will discuss today the interactive effect and that to interactions regression modeling, and a for that a again you know just to have a our look a multiple regressions linear non-linear.
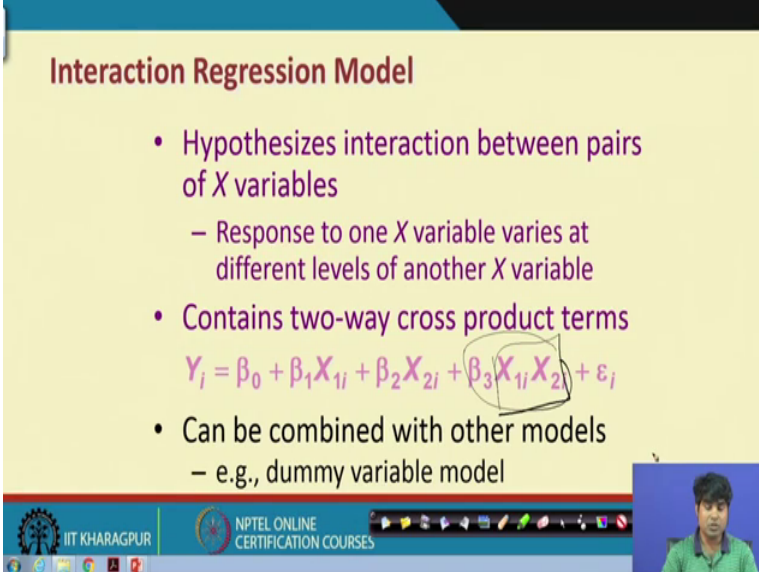
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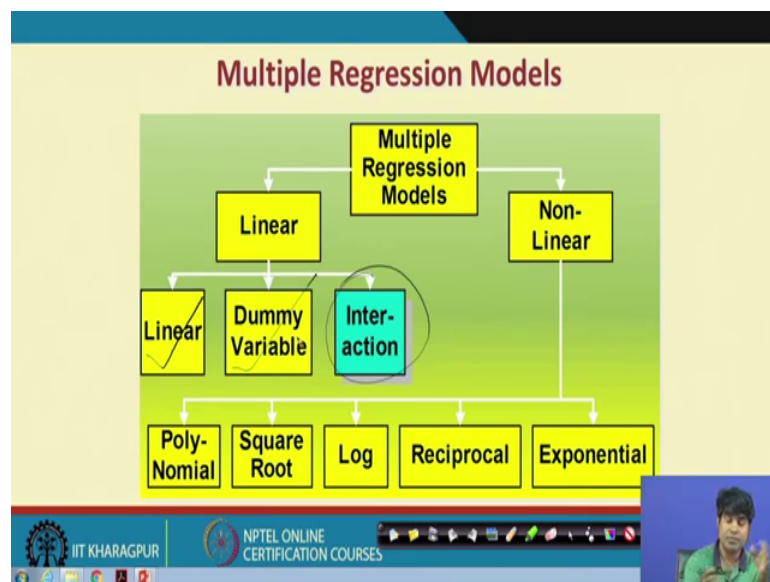


So, we in the last class we specifically discuss this aspect only linear modelling concept and that to the use of the dummy modeling. And then we are here to represent the interactive concept and after that we can directly go to the non-linear structures, here we have a different kind of you know functional form that automatically brings the non-

linear concept and that to the you know the behavior of the non-linear regression modelling.

So, let us first you know discuss the interactive effect, then we will move into the kind of you know non-linear structure.
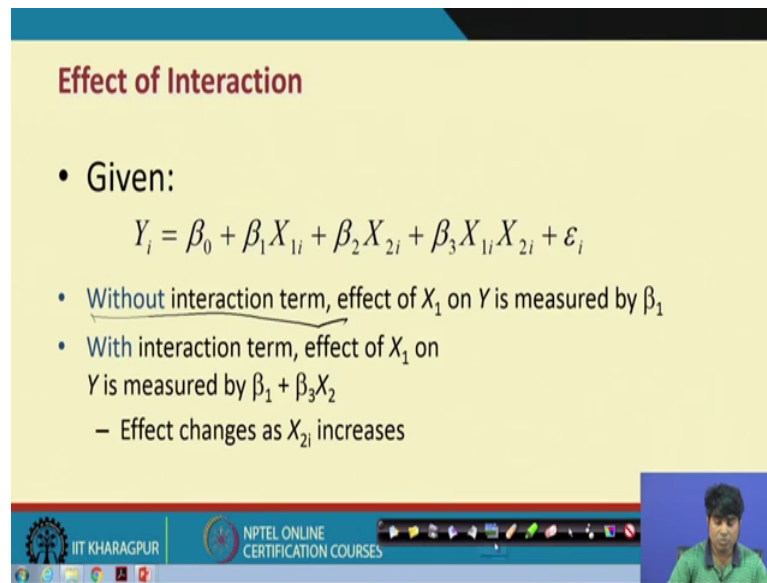
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So, here the interests you know in a you know simple way. So, we can start with the you know regression model like you know Y equal to Y equal to beta 0 plus beta 1 X X 1s beta 2 X 2 and then beta 3 X 1 and X 2. So, where X 3 and X 1 and X 2 is the interactive effect. So, means agains the issues you know X 1 is the independent variable X 2 is the another independent variable.

So, when you are actually multiplying then that becomes a you know interactive variables and for that you know the logic or the kind of you know theory, must be very supportive otherwise you cannot just add a variable and you know go through the kind of you know estimation process. It may bring the multiple issue or some other issues. So, as a result there should be a logic and there should be a kind of you know flow through which we can introduce the interactive effect. So, first of all we can explain this particular interactive effect with a kind of you know example and some examples, we can use in the context of you know dummy modelling which we have already discussed in the last class.

So, here in the same structure so; that means, a the model is the Y equal to beta 0 cross beta 1 X 1s beta 2 X 2 and beta 3 X 1 X 2 where you know we have you know without intercepts interaction term.

Then the effect of X 1 on Y is measured by either you know beta 1 or you know and then beta 2, but with interactive effect then the Y can be measured by the kind of you know the kind of you know the involvement of the beta 3.

So; that means, technically so, we have we have actually 2 variables X 1 and X 2 and the judgment will be on the basis of you know the involvement of you know X 1 X 1 and X 2 and the kind of you know intersections. So, which can bring the issue of you know Y that is the dependent variables. So, now so, what we can do here. So, we can just bring a practical example to highlight the this particular issue.

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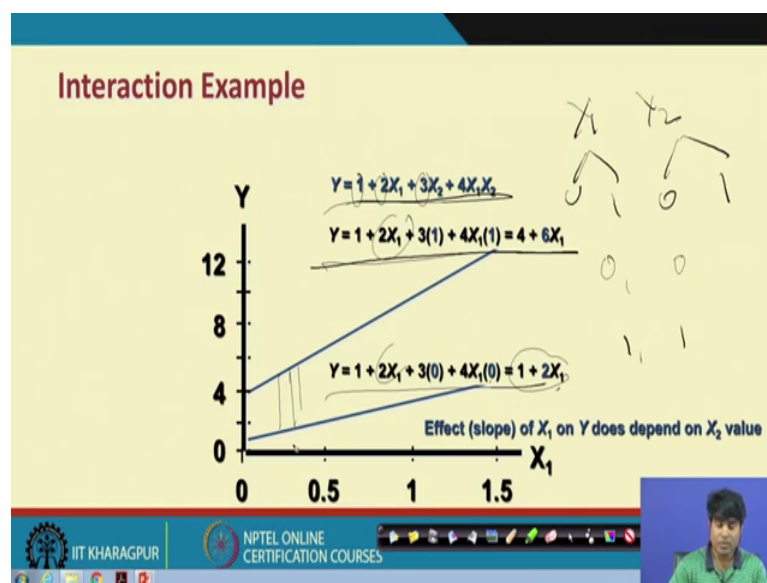


And graphically a this is what the structure and means technically if you go through the previous model. So, here beta 0 beta 1 beta 2 beta 3 are all parameters.

So, now depending upon the involvement so, when we connect with the data then we have estimated model, then the parameters can transfer into you know some kind of you know figures like in this case. So, where beta 0 is equal to 1 beta 1 equal to 2 and beta 2 equal to 3 and beta 4 equal to say 4. So, this is how the kind of you know case. So, now, what we can do you know once you put the X values let for instance X 1 X 2 and X 1 X 2 simultaneously, then you can have the kind of you know effect.

So, if you use you know kind of you know dummy representation then these variables can be represented as a 0 1 and; that means, like you know yes no kind of you know situation like a last class we have discussed the gender issue and the marital status. So, when gender and marital status will be log, then this becomes a you know this becomes an interactive effect. So, how we can actually address such kind of you know issue and what is the beauty of this model that you can actually reflect from this particular you know figure.

So; that means, technically it is kind of you know discrimination or different classifications. So, what is the like you know yesterday discuss about the salary predictions, with respect to the gender issue and the kind of you know marital status and both are represented by the dummy. So, obviously, it is a question of you know yes no.

So, after getting the estimated equations so, you just you know put X 1 or you know dummy variable d 1 either in the form of 0 or 1.

So, if it is a 0 then it will go to a particular levels let us say male marital status, and then female marital status something like that; and as a result you will find 2 different kind of you know flow. So, this is this is the case here. So, this is the estimated model and now we just have one kind of you know structure for instance, X 1 and X 2. So, we allow either 0 or 1 here also we will either you know 0 and 1.

So, in the first group of you know classification. So, you can have actually 0 0 or 1 1. So, now, if it is 1 1 then this model can be transferred into this one, and if it is 0 0 this model can be transferred into this one. So, here we are allowing the dummy with respect to X 2 and the other one is the constants then the imparity, interactive effect will also change accordingly.
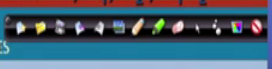
So; that means, technically here the if you put 0 then this becomes you know 1 plus 2 X 1 only, by default the X 2 X 2 will be removed in the process, but when actually put one then all will be there in the system. So, that is the difference between the kind of you know model with a kind of you now dummy and model without you know dummy; that means, technically yes no. So, one group with the other group so, there is a difference and in this figures. So, we find the difference is this much only.

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So you if you go to the excel sheet or you know any kind of you know spreadsheet. So, this is what the kind of you know spread spreadsheet and we need actually Y variables and that to the information will be like this and then X 1 X 2 and by using X 1 and X 2 you can create actually a interactive effect just to multiply these 2. So, for instance here actually the sample observations are cross sectional type, and Y informations are here, X 1 informations are here, X 2 informations are here then by using X 1 and X 2 interactive it is just you multiply. So, 3 into 1 so, that is 3 then this becomes 8 into 5 40, 3 into 2 6 and 5 into 6 30. So, this will be the another information which is created for the interactive you know variable and then finally, you have to estimate Y with respect to X 1 X 2 and X 1 X 2 simultaneously.

So, then we like to check how is the behavior of this particular models and the kind of you know interpretations.

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So, likewise a you know you can actually develop different kinds of you know models and that to signify the importance by using the dummy, you know dummy concept and the kind of you know interactive effect. So, now, come to the practical. So, connect to connecting to the last class examples.

So, Y equal to alpha plus beta 1 male and beta 2 married and then beta 3 a divorced so; that means, technically so, there are 3 variables and all will be all will be represented in the form of dummy. So, for instance you know in the case of you know male. So, we can

ensure is either yes or no. So, then we can put a c a 1 and 0 similarly marrieds. So, yes no again we can put 1 0 and then divorce yes no. So, yes no and then we can put actually 1 0. So, like this.

So, obviously, a the kind of you know structure in such a way. So, you can understand very clearly and then you can actually connect with the estimated model so, that you know you can get to know what is exactly difference between the you know these variables that to with respect to male married and divorce. So, interactive effect will be male married or you know female married or male divorce, female divorce. So, when we are connecting 2 variables simultaneously, then it becomes a kind of you know interactive.

So, now how we can actually quantify the impact all together? So that means, let us say its starting with you know male married and divorce that is what you know like the kind of you know interactive then; obviously, is say let us say it is a completely female and then default the beta 1 equal to 0, beta 2 equal to 0 and beta 3 equal to 0. So, alpha itself the vector which can be represent the kind of you know impact on Y, and then if actually it is in the case of say male then beta 1 will be added into the process and again beta 1 added into the process means now Y impact will be alpha plus beta. If Y equal to simply alpha then that is the male contributions and then now for female contribution.

Now, if you know add actually a male component then Y equal to alpha plus beta 1 where male will be considered as yes and the code which we use as you know one to represent the male with respect to the female impact. So, then beta 2 is a another dummy which may be represented as a married unmarried. So, if it is married then yes that can be you know represented as a 1, if not unmarried then it will be represented as you know dummy again the description will be on the basis of 0.

So, male you know married. So, then if male yes and married yes, then Y will be alpha plus beta 1 plus beta 2, and otherwise it is a Y equal to alpha plus beta 1 male only then; that means, it becomes a you know beta unmarried. So, where beta 2 coefficient will be 0 so, likewise you will find you know different kind of you know options you will have the male divorce female divorce and then married you know again divorce is you know there is a chance married, but not divorced.

So, likewise there are lots of interactive effect you can find a against when there are multiple dummies and multiple options are there. So, you will by default get. So, lots of interactive effect for instance having male married and divorce. So, just connect male and married you multiply then you know you will get; you know interactive effect and I am very sure that you know, this particular interactive effect for instance male married. So, is not a question of you know multi (Refer Time: 15:23) problem. It is the it is rather you know give high weightage and high value to this particular you know modelling process to investigate any kind of you know engineering problem.

So, it is not necessarily every times you can use male female or the kind of you know married, unmarried, divorce, not divorce kind of you know thing; it can be any kind of you know engineering problem or any kind of you know specific problems we can actually apply, and a you know develop models as per the particular you know requirement. So, it is the kind of you know you know structures through which you can understand the interactive modes, and that to with the help of you know dummy modelling.

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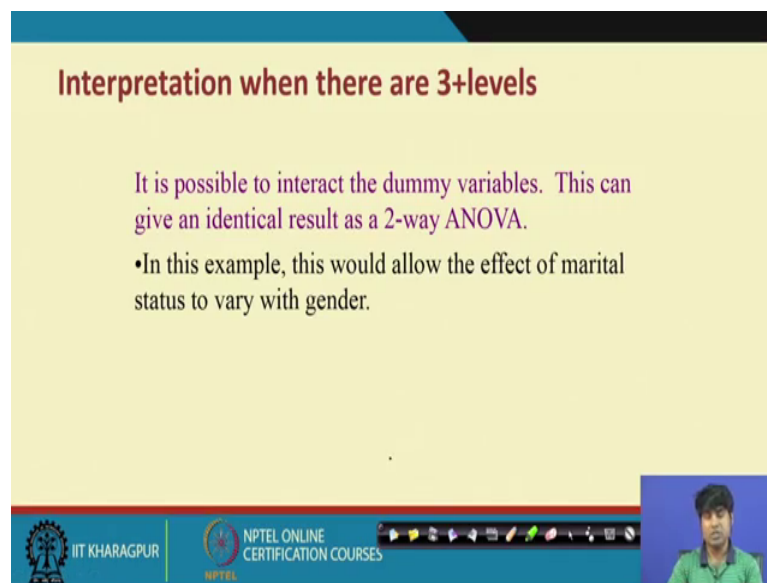


And by the way this I am in you know difference we can obtain through you know analysis of variance that is called as you know 2 way ANOVA because it is a 3 levels we are going genders, then marital status and then finally, whether divorce or not divorce.

So, it is a multiple layers through which you can find out. So, ultimately this sample will be you know sample will be divided into 2 parts so; that means, actually starting with the let us say like this a genders and marital status. So, where actually married and gender status will be male female, and then marital status yes no and then a divorce. So, it is also yes no.

So, now in the case of you know you know genders. So, male females it may be represent if you know say one and say 0 and this can be 1 this can be 0 and this can be 1 this can be 0. So, as a result so, every samples can be. So, for instance the cross sectional sample, which we have here in the previous you know kind of you know previous kind of you know situation for instance this one. So, these are all actually dummies. So, you just you know sketch 1 1 samples then ask the may be a you know the kind of; you know there you know marital status and the kind of you know whether they are divorce or not divorce.

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So, that is the kind of you know things you have to observe and then finally, you can go for the kind of you know classification or the kind of you know difference. So, this I am difference you can observe through analysis of variance. And the previous example which we have discussed earlier is one way ANOVA, and this is 2 way ANOVA likewise you can get lots of you know such similar kind of you know dummy and that to regression of you know interactive effect by using the dummy or by using non dummy,

but ultimately. So, it depends upon how many layers through which you like to find out the difference between these 2 groups.

So, ANOVA can be use and to check the similar kind of you know flow, and that is process technically called as you know 2 way ANOVA. And in this example this should allow you know the effect of marital status to vary with you know genders. So, that means, you know male the marital status and female the marital status, again you know say married with divorce, married without divorce right. So, this these are the kind of you know classifications you will find that can be very frequently or very easily can be used through dummy modelling and then the kind of you know interactive kind of you know effect.

So; that means, technically this kind of you know problem it is mandatory to know or to use the dummy modelling fast, then by using this dummy modelling or the theoretical kind of you know flow you can create interactive effect and then finally, the model becomes you know non-linear once. So, whether it is a linear one or non-linear one so, the estimation process will be more or less same, you are supposed to get the parameters value.

So, after getting the parameters values so, with respect to dummy. So, you can allow 0 1 and then finally, the model will they actually differentiate each other as per the kind of you know model feed or as per the kind of you know model requirement. And this same analysis can be checked through analysis variance if it is one kind of you know attributes, then we can go for 1 way ANOVA if it is 2 attributes you can go for 2 way ANOVA, if it is actually 3 such kind of you know attributes you we can go 3 way ANOVA.

So; that means, different layers we can all together go to represent this particular situation. And whenever there are different layers in the particular system then by default there will be the interactive effect. So, the same interactive effect will be study through regressions and then check the reliability of the regressions and if not then you can use actually direct ANOVA and check the difference whether these 2 groups with respect to a particular you know objectives are you know different to each other. And if there is a difference whether it statically significant or not and that is what the issue and that is

more important to understand the concept and you know investigate as per the particular engineering requirement.
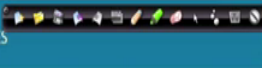
So, ultimately the model will be like this. So, till you know kind of you know till this particular you know issue of you know 2 ANOVA, the theoretical structure will be like this and here's (Refer Time: 21:04) dummy is used ones.

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So that means, technically in this models we will not find any interactive effectively, but by using this 3 dummies. So, we can have here actually a interactive effect.

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That means first male married and divorce, and that is the one way kind of you know investigations, then first 2 dummies male marrieds that can be one dummy then again male divorce this is another dummy.

So, male married and male divorce so, 2 interactive effect and that can be added in to the small link process and as a result this become this model becomes you know non-linear one, compared to this the particular model which is available here is linear once. Where we are just integrating Y with you know male married and divorce and male are the you know it is a 0 1 if you are male you can say yes, and that will be recorded as a ones. So, if not males then you can you say no then that will be recorded as you know 0 similarly married.
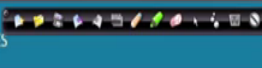
So, you are married or un married without in this you know issue of you know male and female, which we have already taken care in the first dummy and against the in the third dummy, we will be asking you know divorce or not divorce so; that means, this will be come just after you know married. So, if there is married marriage then the ratio of divorce or not divorce is coming into the picture.

So, if a if you if the issue is not you know male female, then marital status will not come into the picture so that means. So, the issue of married and issue of divorce will be part of this games and this attributes by default can be connected, and bring the kind of you know logical interpretations in the bring the reality, and then that need to be investigated and checked as per the model requirement.

So, here is the case and this 2 here, male you know if you are saying that you know male is the variable so, then; obviously. So, the information behind male will be one that is the yes otherwise by simple no and that will recorded as a 0. So, that the sample you know move while doing the dummy modelling and that to understand the concept of you know interactive effect.

Similarly, married; in the case of married you can come here in the case of you know married. So, answer will be defiantly yes or no; if it is yes then we can put 1, if no then we can put 0 and similarly divorce. So, whether it is a yes or no. So, if it is you know yes then you can put 1, if not then you can put actually 0. So, the now in the first end we have to fill the spreadsheet by asking in you know individuals, whether you know they

are married and whether they are divorce or not then a gender by default you can actually ask them whether they male and female.

So, against male and females, we like to check whether the guys or the particular you know candidate is married or un married and again next questions if you know they are married, then obviously, the question is whether you divorced or you are staying with you know family. So, that is the kind of you know things and then we can integrate with you know male married. So, that can be one integrations. So, then by default the other alternative will be female married, because here we are putting male equal to you know e s that to (Refer Time: 24:46) you know 1 if not it is a 0.

So, by default the deficit or you know the total or total effect minus this particular effect then the counterpart will be the female effect. So, that is the kind of you know things we can you know integrate and; that means, all these you know interactive effect will be artificially created concept and the variables. And which is actually very effective sometimes depending upon the particular you know engineering problem and this is what the summary of this particular you know case.

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So, we have actually technically 3 parameters depending upon the you know dummy variables we used, male, married and divorce. So, now, since it is a question of male the counterpart will be by default female. So, in that case so, it depends upon you know so; that means, basic starting is alpha, and then if it is the case of you know male then that

that will be that particular component will be 1 and as a result the impact will be move to Y equal to alpha plus beta ones right. So, similarly if it is the you know married, then this will be called you know multiplied with the 1 because that is the answer means that is the answer for you know yes, and as a result beta 2 will be added into the process.

Then ultimately the male impact will be a beta 1 plus beta 2 and then actually the kind of you know beta 4. Where married is yes and as a result so, beta for male when we are putting 1 so, that will be again added to this you know you know beta 2 impact. So, like this say. So, this is what the kind of you know case, and likewise we will have actually different kind of you know structures. So, this is actually married and a when we are putting actually let us say 1 so, by default.

So, this will be also 1 so, by default. So, the final impact will be beta 2 plus beta 4. So that means this is the male then male married and then the interactive effect. So, if it is if not then by default the married concept you know for female concept it will put 0. So, beta 2 will not be there and then when we put married equal to 0 so, by default to be. So, you know the particular component will be removed, and then only a one single component will be added into the process.

Similarly, in the case of you know divorce so, beta 3. So, when beta 3 is yes. So, beta 3 will be added into the process and then by default, the interactive effective will be come into the picture. So, as a result the final impact will be beta 1 plus beta 3 plus beta 5 and that to by using the divorce component and then again by using male and married. So, this will be the different. So, you see there is a big difference beta 2 here beta 1 and beta 4 extra, and again here beta 3 and corresponding to beta 3 beta 1 and beta 5. That is the counterpart between the male and female that to with respect to male marital status, and the kind of you know divorce case. So, these are the things which you can easily actually a kind of you know things we can apply and.

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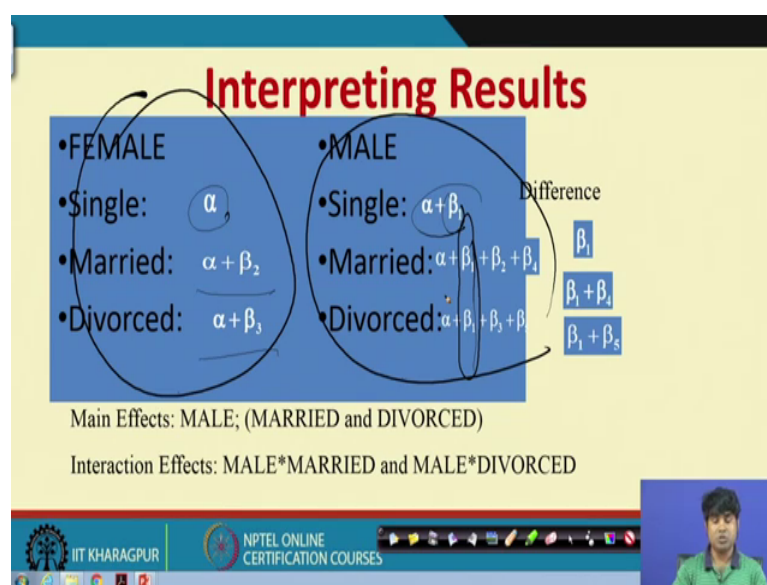


By the way so, we just summarizing the things and you know the model start with the Y and intercept, and then again genders male female representation yes no 1 0 then marital status married unmarried. So, again yes no 0 1 0 status and again divorce yes no and that to 1 0 status. So, if every time yes then you know. So, every component will be added into the process. So, alpha beta 1 you know beta 3 beta 5 alpha beta 1 beta 2 beta 4. So, that is what the difference and the male will go to the direct impact and the interaction effect will be through male married and male divorce.

So, for that actually 2 variables you know must and they should be logically connected. Because male married possible and male divorce also possible similarly the counterpart will female married female divorce again male unmarried and then in that case. So, divorce by default will not coming to the pictures, but the sample will be structuring in such a way that you know there will be a candidate, they must be married and then the question of you know divorce or not. So, the addition of you know divorce component will be coming depending upon whether the guy is married or not. So, if it is married then the divorce issue will be coming into the pictures, if they are not married then the question of divorce will not you know coming to each other.
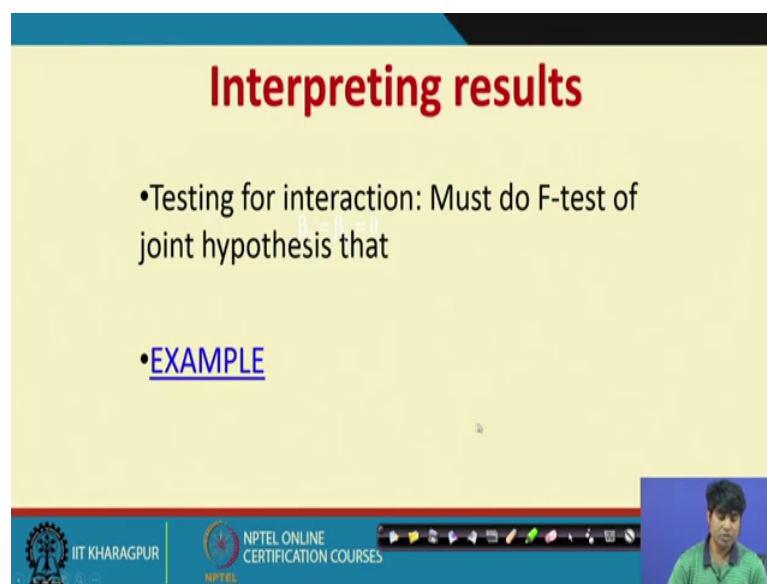
So, what is more important here that you know data re structuring. So, depending upon the data so, we need actually specific samples through which actually you develop this kind of you know model and do the kind of you know estimations.

So that means, technically a for instance if you like to keep all these 3 simultaneously say gender, marital status and divorce. So, you try to actually a bring such a way that you know so, all the components will be in a kind of you know there in the systems and then we can observe easily, and again with the help of you know estimation and the kind of you know dummy interpretation by default you will find the kind of you know difference between male and female in that to with respect to married unmarried and divorce not divorce kind of.

So, here is the clear cut you know kind of you know case, and after seeing this once you can easily understand the kind of you know a different. So, this is a female a impact and this is the male impacts. So, if female case only alpha, single and alpha beta 1 here. So, beta 1 is extra added here. So, now, here alpha beta 2 marital status case and divorce case, alpha beta 3 and by default here beta 1 will be added everything in the process because that is the signal about the genders.

And when we are saying male, then it will be actually yes and in that that will be like to you know you just multiply with u 1 and that competition will be added into the intercept. And if not then that will be removed from the particular process and only left out terms should be in the system, in that will be the interpretation for the rival, that too in this case it is the impact of the female you know female in that to weight of the female compared to male.

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So, these are all you know kind of you know issues, through which we can address this particular you know problem and you know what I can you know like to highlight here. So, we have gone through the dummy modelling structures, and how dummy variable can understand and can use you know for you know extra integrating with you know interactive (Refer Time: 32:26) that to bring the non-linearity concept.

And for that we have already discuss in details, what are the instances we can actually apply this and then what is the kind of you know interpretation the kind of you know classification, the kind of you know discriminations, and then we like to actually check all these details through a spreadsheet and the kind of you know estimated outputs. And for that we can you know have a dataset and create a dummy and then by using the data and dummy, we can create interactive effect informations and then finally, by estimation process and after getting the estimated output. Again you will find how they are you know different with respect to different attributes. So, we will discuss this particular problem again with a you know solid data set, and the kind of you know examples with this we will stop here.

Thank you very much.