

Business Analytics for Management Decision
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Lecture – 55
Prescriptive Analytics 3 (Contd.)

Hello everybody this is Rudra Pradhan here. Welcome to BMD lecture series that to we will continue with descriptive analytics and the coverage is on non-linear programming. In fact, we have already discussed several problems dealing with the both linear programming and non-linear programmings.

In fact, in the last lectures we have already highlighted the detail structure of non-linear programming. So that means, you know whether to use linear programming or non-linear programming that typically depends upon the kind of you know data structure and the kind of you know business problem. So, having the data and the kind of you know business problem; so we will have a model which can be linear 1 or non-linear one. If it is a linear 1 then you solve through linear programming problem and if it is in non-linear programming case then we can solve through non-linear programming problem.

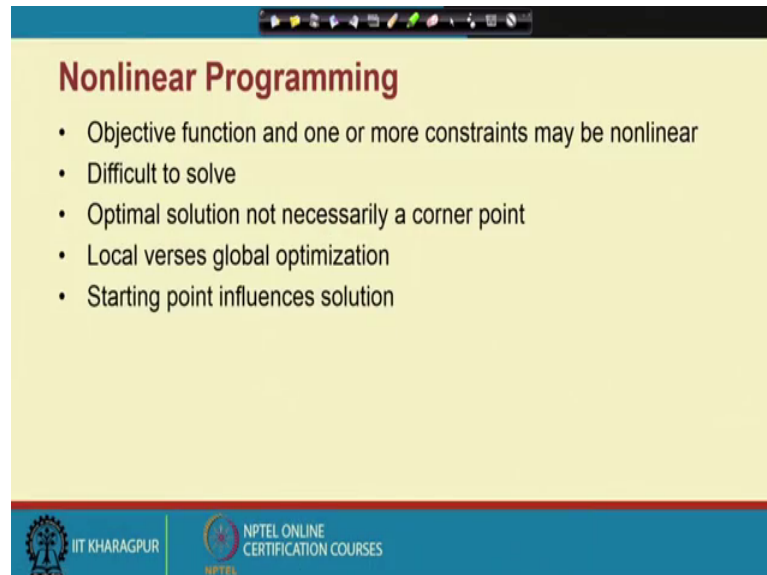
So, now, the kind of you know structure which we have here is the NLP; non-linear programming and in the NLP we have a different kind of you know structures. The typical structure which we have highlighted is a you know optimization that to optimizing objective functions without any constraints and in the second case we have the optimization with constraints and then we have a single variable case, multi variable case, we have a equality equality constraints case, inequality constraints case.

Then against we have a single a single constants case, multiple constants case. So, likewise we have highlighted the details about the particular you know structure and then we have actually discussed what are the ways we can you know get the optimal you know results. That is the optimality solutions and a look for the values of the decision variable through which you can address the business problem as per the particular you know business requirement.

And having the clue of the details about the NLP, so we like to go for you know some kind of you know problem structure and look for the kind of you know solution; that

means, a the structure through which you can get the optimal solution to address the business problem.

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The slide is titled "Nonlinear Programming" in a bold, dark red font. It features a list of five bullet points in black text. The slide has a yellow background with a blue header and footer. The footer contains the IIT Kharagpur logo and the NPTEL Online Certification Courses logo.

Nonlinear Programming

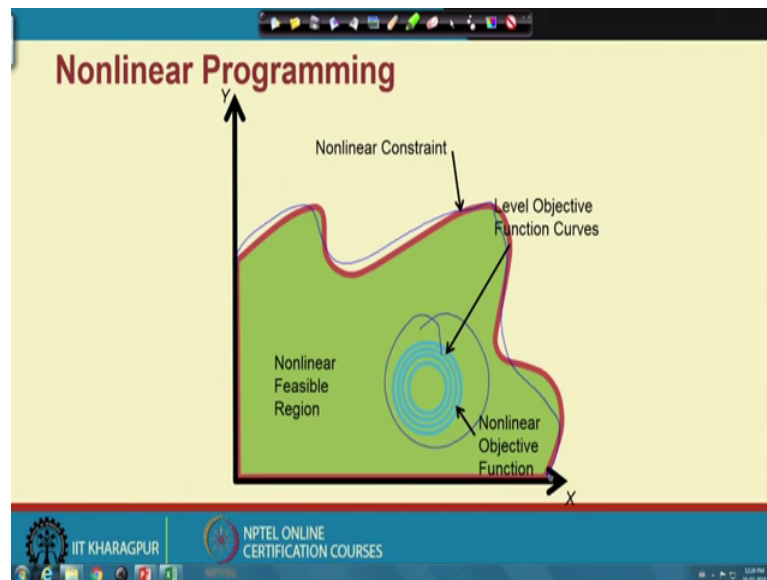
- Objective function and one or more constraints may be nonlinear
- Difficult to solve
- Optimal solution not necessarily a corner point
- Local verses global optimization
- Starting point influences solution

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So, now in the first hand as so for that non-linear program is a concern; so, what we have already mentions, objective function and then 1 or more constraints may be linear and non-linear. And sometimes depending upon the you know problem structures, you know there is a difficulty to or that means, there is a complexity to complexity to have this solution and where if the case is more number of variables and more number constraints and then the constraints is the non-linear incorrect or then the complexity will be start you know increasing.

Then as you know optimal solution you know here in the case of you know NLP not necessarily a corner point. And we have a local optimization and global optimization and starting point influences the kind of you know solution; that means, there is a high chance that you know we have actually local optimization and global optimization in a particular you know coin of process or you know business problem.

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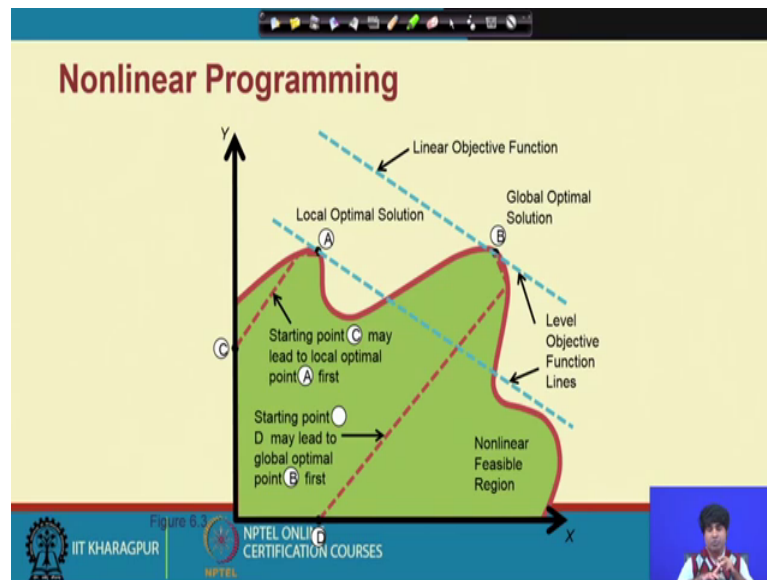


And in the non-linear programming structures the typical structure would be like this and here there, there is a kind of you know objective function; this is what the objective function and the typical constraints will constraints you know like this. So, this is what the type of you know constraints which is actually non-linear one. And like you know linear programming we look for the kind of you know feasible zones and the kind of you know corner points and look for the optimal solution.

This is also the case of you know non-linear programming and against we have objective functions with the constraints and look for the kind of you know solutions. In fact, most of the business problem you know means; you know around 90 percent of you know situations. We have both objective function and constraint and look for the optimal solutions and a you know very rare you know situation you will have only objective function and no constraints.

But we have already highlighted the kind of you know optimality or the kind of you know optimal solution structure whether it is a kind of you know objective function with constraints or objective function without constraint. So that means, technically we have you know we try to understand how is the case and then look for the kind of you know optimal solutions.

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So, now so this is another kind of you know structure through which you can actually analyze the particular you know structure and that is how the kind of you know requirement. And in fact, you know these are the type of you know structures, so where we have a constraints and then we look for the kind of you know solution. And what we have already discuss that you know it is not the optimal kind of you know solution within a particular you know optimal solution corresponding to objective function and constraint. Since it is a kind of you know non-linear constraints, so we will find we will find you know there are lots of you know possible points you know through which you can actually look for the optimality. So, what we have highlighted the local optimum which can be a visible here.

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Nonlinear Programming: Pickens Memorial Hospital

- Profit contribution and decision variables

Profit contribution per medical patient = $\$45 + \$2M$
Profit contribution per surgical patient = $\$70 + \$3S + \$2M$
Profit contribution per pediatric patient = $\$60 + \$3P$

where

M = number of medical patients admitted
 S = number of surgical patients admitted
 P = number of pediatric patients admitted

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And then we can have the kind of you know global optimum. So, in this case this is the kind of you know local optimum and this is what the kind of you know global optimum. Obviously, you know since it is a dynamic kind of you know environment, so we will find there is you know lots of you know ups and downs.

So, so we have actually multiple structure through which you look for the optimal solutions right. So, this is how the typical case through which you can address. And then you know what we can do? So, we can actually so, this is how these typical examples; and let us assumes that, you know the type of you know business problem. T


He objective is to you know maximize profit and here is there are you know three you know kind of you know profit structure. So, that is with respect to the first ones and the this is the first one, this is the second 1 and this is the third one; where you know M represents the a medical patients admitted, S represents number of surgical patients admitted and P is number of you know, you know that type of you know patients admitted in the particular you know process and then look for the kind of you know you know kind of you know optimality. That means, typically this is a health analytics problem and where the particular structure is to optimize the proper subjective to certain number of constraints and the constraints are here is like this.

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
Nonlinear Programming: Patient mix problem

	MEDICAL	SURGICAL	PEDIATRIC	AVAILABILITY
Number of x-rays/patient	1	3	1	560 x-rays
Marketing budget/patient	\$3	\$5	\$3.5	\$1,000
Number of lab tests/patient	3	3	3	140 hrs


Time required per lab test (in hours) = $0.2 + 0.001M$



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So, the typical constraints will be follow like this; so that means, so, we have a three constraints corresponding to three different you know profit structures. So, now the first constraints; first constraints is a with respect to this one. So, that mean say the typical problem will be with respect to three variable; let us say x_1 , x_2 , x_3 and there are three constraints; constraints 1, constraints 2, and constraints 3 and so, you know accordingly, so this these are all you know researchable ability and these are the inputs to the particular constraints and then we are looking for the kind of you know optimal solutions.

So, now corresponding to these informations so let us say how is the kinds of you know problems. So that means a like linear programming problem. So, in the NLP will also have the kind of you know transformation. So, having the information; so, the first step of the process is to transfer the information into a model and then look for the optimal solutions as per the particular you know business requirement or the kind of you know management requirement.

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Nonlinear Programming: Pickens Memorial Hospital

Objective function

$$\begin{aligned} \text{Maximize profit} &= (\$45 + \$M) \times M + (\$70 + \$3S + \$2M) \times S \\ &\quad + (\$60 + \$3P) \times P \\ &= \$45M + \$2M^2 + \$70S + \$3S^2 + \$2MS \\ &\quad + \$60P + \$3P^2 \end{aligned}$$

subject to

$$\begin{aligned} M + S + P &\leq 200 && \text{(total patient capacity)} \\ M + 3S + P &\leq 560 && \text{(x-ray capacity)} \\ 3M + 5S + 3.5P &\leq 1,000 && \text{(marketing budget, \$)} \\ (0.2 + 0.001M) \times (3M + 3S + 3P) &\leq 140 && \text{(lab capacity, hrs)} \\ M, S, P &\geq 0 \end{aligned}$$

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So, having the kind of you know information, then the final model will be means will NLP; a non-linear programming model is like this. So, maximize profit. So, this is what the a profit functions and a so, this is actually cluster of you know three different situations and then if will we simplify and then that by default will we come into a kind of you know non-linear structure. So, this is how the a typical you know business you know structure and then finally, a analyzer you know process the particular you know profit structures as per the business requirement or the kind of you know business creator. So, the ultimate to the objective function will we will we have is a here is the kind of you know non-linear one.

And then we have certain constraint here. So, what we have already mentioned there are three constraints, corresponding to these particular you know problem so that means, technically. So, this is what the constraints sides; so, 1, 3, 3. So obviously, this is how the 1, 3, 3. So, the typical constraints will be the M plus S plus P less than equal to 200, M plus 3S plus P a less than equal to 560 and a likewise we have actually five different constraints all together. So, constraints 1, constraints 2 and constraints 3; so obviously, this is constraints 1, constraints 2, constraints 3, constraints 4 and that is how the objective function to be optimize; a and with respect to all these you know for constraints.

And like the you know previous case; so, we have objective functions. So, what will we do? The typical structure you know since it is a actually three variable case. So, graphically a little bit you know complex to get you know the typical you know solution. So, if it is a two variable case, even the there is a question of you know nonlinearities, still we can you know look for the optimal solution that to biographical methods and when the particular model you know extends to multivariable case; so, like you know the linear programming case, we can use actually algebraic method through which you can look for the solution.

And here in the case of you know three variables you know you can go by algebraic structure, the way we have already highlighted the, that is the calculus base you know mechanism through which you can look for the particular you know solution. Of course, you know the solver can help us to get the optimal results corresponding to the objective functions; that is the non-linear incorrect or and subject to all these you know you know constraints and out of which we few constraints are you know linear type and few constraints are you know non-linear type.

So, now what is the usual procedure is; just you know if it is a let us a case of you know two variable case. So, first you plot the kind of you know constraints into equality. Then you know kind of you know draw the kind of you know indications and find out the feasible region, find out the corner point and a put these corner points to the objective function and check where is the value of the objective function is the highest and accordingly we can actually find out the optimality.

And where the value of the objective function will be highest and that by default that will be the optimal solutions and then check what should be the values of the decision variable corresponding to that you know optimal value of the objective functions. And this is what actually usual structure of you know graphics or graphical methods and a in the case of you know algebraic method. So you can simply actually just you know follow the procedures and then you know subsequently move on till with these the optimal structure where you can get the optimal solution and the values of the decision variable as per this, as per this particular you know business requirement.

Of course, you know solver is there has to help us. For instance, let me take you to the a solver case, where you can actually you can you know understand the particular situations.

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	A	B	C	D	E	F	G	H	I	J	K	L
1	DV	X1	X2	X1 sq	X2 sq	X1X2						
2			2	4								
3	Obj		2	3			16.0000002					
4	Constraint		0	0	-1	1	0	20.0000007				
5			0	0	0	0	1	7.99999966				

Handwritten notes on the spreadsheet:

- Max $Z = 2X_1 + 3X_2$
- $X_1 + X_2 \leq 20$
- $X_1 \leq 10$
- $X_2 \leq 10$
- $Z = 16$
- $X_1 = 2$
- $X_2 = 4$

So, this is what actually the kind of you know like a linear programming structures. So, which you we can have here; the kind of you know what we can called as you know non-linear structures. So let us say this is a problem here and the problem is with respect to two variables that is the X1 and X2. So, that means it typically the business is with respect to two variables. It is a two products case and since the model actually non-linear in character, so the objective functions and the constraints will be non-linear in character.

And in fact, in this case we have actually we have the two variable case, but the modeling structure is the non-linear in character. So that means, a this one is the objective function here and the objective function, the structure of the objective function is here is a like this. So, what will we do here? Let us say ok. So, what we can do here? So, we just specify the, what is the objective functions; so that means, a X1 is the decision variable, X2 is the decision variable and the objective function is the a maximizing Z equal to $2X_1$ plus $3X_2$ and subject to constraint.

So that means, in this problem a constraints are you know actually non-linear in character where objective is linear in character. But in real life scenario the objective function can be a you know non-linear in characters, which we have already highlighted

in the last example. And a so, so there is a case like you know objective function non-linear in character where constraints are you know linear in character or objective function is the linear one, constraints are a you know kind of you know non-linear one and in the third, third option is the both objective function and constraints are non-linear in character.

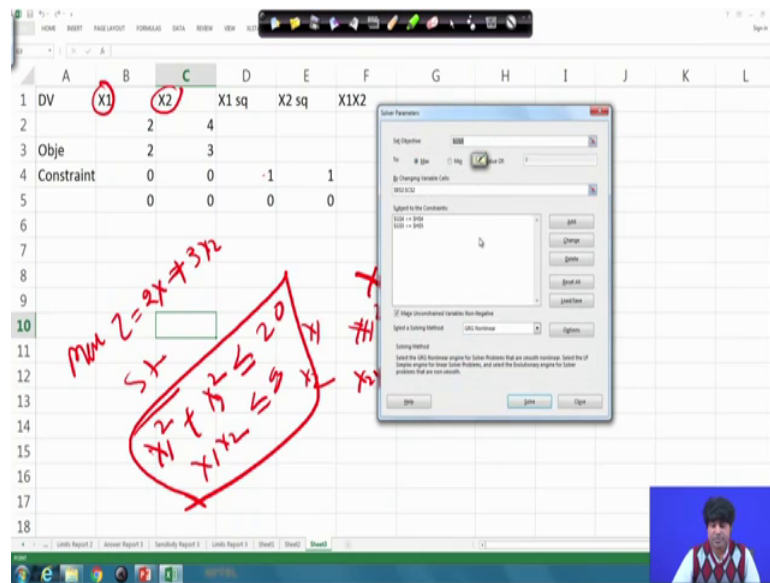
So, whatever may be the scenario we have to actually look for the kind of you know optimality and the optimal solution through which you can address business problem. So, now, here in this case; so the objective function is like this and the constraints will be you know you know there are typically actually two different constraints and in fact, corresponding to X_1 and X_2 , so if will we go by you know X_1 and X_2 here and $X_1 X_2$ here ok.

So, we have actually a dot product and cross product. So that means, this is a what X_1^2 squares and this is a what actually X_2^2 squares and $X_1 X_2$ and then this is $X_1 X_2$ and this is $X_2 X_1$. So, these are the kind of you know, that is called as you know what we can called as you know, in you know analytics language it is called as you know dot product and cross product.

And a sometimes in the business language equaled as you know interactive effect and a here the first constraint is with respect to X_1 square; that means, 1 into X_1^2 squares. So that means, X_1^2 square, then again 1 into X_2^2 square; that is X_2^2 squares and less than equal to 20 and the second one is actually X_1 and X_2 is less than equal to 8. So, this is what the actually constraints which are actually both are both are actually inequality type.

Now having the objective functions and having these two constraints, so we look for the optimal solutions. So that means, technically what is the procedure here? So, like you know the same procedure you know, which we following the case of you know linear programming problem. So, go to the data analysis and then we choose the solver the case.

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And here is we have actually already you know the entry. This entry is main for the kind of you know linear programming that to we have solve in the case of you know integer programming and (Refer Time: 16:43) programming case. But here is since it is a non-linear programming problem, so first of all you know you have to change the, you know requirement and so, what will we do? First we put you know reset; so, that you know the typical structure will be removed. So, now we have to enter the existing structure; which is more or less actually same. And what will we do here? So, or the typical requirement is here the objective functions.

So, the objective functions structure will be here ok. So, this is what the objective function and then the typical constraints, so that means, ok. So, this is so this is what the objective functions and then and a we have the kind of you know, see these are the options here. So that means, typically if you look here; so, this is the indication about the objective function like linear programming problem. So, we have a maximum, minimum and you know the typical you know the kind of you know nature of the objective functions. Since it is a maximization type, so we have already opted maximization type and then this is the box for decision variables. So that means, typically so, what will we do here? So, we can you know indicate, give the indication about the a decision variable here.

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So, here the case is the, that will be the first you know indications and since it is a sign is a less than types. So, we can put you know less than type and the constraint is the maximum availabilities; the 20 and again, so this is the first constraint indication. Then again put add on so, that is with respect to second constraints. So, you put the second constraint indications and then put it is lim. This is also less than type and put it is limit; which is actually 8 and so, ok. So, this is this is what the indication. So, put here 8 ok.

So, now what will we do? Since our requirement is in now NLP format. So, just you put you know non-linear indications. So, after that we just you know put you know ok. So

that means, technically you ask this solver now to analyze the particular you know a problems. So, and you know give the kind of you know optimal solutions. So, now the moment you put the indication ok; so, the solution is already available here. So, so now, it is ready to operate. So, this is how you have to give the indications. So, that means so now, so the operation is here.

So, this is what the indication. So that means, technically so this problem in the in the corresponding problem is like this and a still we can apply actually sensitive analysis and the kind of you know requirement; that means, you can put the restriction what kind of you know decision variables. Whether you need actually in a kind of you know positive you know means non negativity you know and agains like we have a discuss about the kind of you know integer type of you know solution etcetera.

You can put all these conditions again in the solver to have the final outcome. So, then what we will do ultimately. So, in this case corresponding to this objective function and constraint; so, we have X_1 equal to 2 and X_2 equal to X_2 equal to 4 and where Z equal to actually coming 16; so that means, this is what the final outcome.

So, it is a actually the process is the more or less same like you know linear programming problems. So, where we have already solve couple of examples through this particular you know solver mechanism and here also same mechanism; just here the typical you know difference is the just you have to create a decisions variables as per the particular you know requirement. So, so, what will we do in the particular you know process. So, you have to first you know you know formulate the problem and the typical structure is actually in the solver, such you know the first requirement is a how many variables all together; starting with you know let us say for these two variable; X_1 and X_2 . Then the next you know nonlinearity will come you know X_1 square, X_2 square, X_1 and X_2 .

So, these are you know possible kind of you know structures. So, then after knowing all these in you know variables indication, then you start entering the objective functions and the kind of you know constraints and the kind of you know requirement and after that you know just like you know linear programming structures. So, you just you know as the solver to give the optimality result and by default you will get the optimal results

and that to values of the decision variable corresponding to the particular objective functions and the kind of you know constraint.

Again is if you like to change the particular you know structure which is the kind of you know application of you know sensitive analysis. So, where we can actually change the you know coefficient of the constraints; left hand side, right hand side, change the coefficients of the objective functions. You can add one after another constraint and then again is you look for the optimal solutions.

So that means, like linear programming problems, we have also here flexibility; a to find out the you know optimal solution corresponding to the original problem and look for the you know further optimal solution corresponding to the changing scenario that to with respect to objective function coefficients, with respect to constraints coefficient left hand side and right hand side and then we may actually look for the optimal solution against by including one after another constraint into the system.

So, then that means, technically we have lots of you know flexibility here; like the linear programming case and the dynamics of the business. You know we cannot just advisory or arbitrary you can you know add ons variables or add ons constraint. Obviously, should be supported by the particular you know business problems and the kind of you know theory.

So, so either it will be supported by you know business problems or the kind of you know theory or else if you like to add up add ons you know you know extra, so it should be logically connected. So, that you know you can address the particular business problem more accurately and more effectively and then come with a kind of you know management decisions and that to as for the particular you know business requirement. So, having you know the kind of you know structure here. So, go back come back to the particular problem.

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Solution: Pickens Memorial Hospital

These are the only three decision variables in the model.

Entries in columns E to I are nonlinear terms.

Entries in this row are functions of the decision variables.

All entries in column J are computed using the SUMPRODUCT function.

	A	B	C	D	E	F	G	H	I	J	K	L
1	Pickens Memorial Hospital (NLP)											
2		M	S	P								
3		Medical	Surgical	Pediatrics								
4												
5	Number of patients											
6												
7		M	S	P	M ²	S ²	P ²	MS	MP			
8	Variable terms	=B5	=C5	=D5	=B5^2	=C5^2	=D5^2	=B5*C5	=B5*D5			
9	Profit	45	70	60	2	3	3	2		=SUMPRODUCT(B9:I9,\$B\$5:\$I\$9)		
10	Constraints:											
11	Total patients	1	1	1						=SUMPRODUCT(B11:I11,\$B\$5:\$I\$9)	<=	200
12	X-ray capacity	1	3	1						=SUMPRODUCT(B12:I12,\$B\$5:\$I\$9)	<=	560
13	Marketing budget	3	5	3.5						=SUMPRODUCT(B13:I13,\$B\$5:\$I\$9)	<=	1000
14	Lab hours	0.6	0.6	0.6	0.003			0.003		=SUMPRODUCT(B14:I14,\$B\$5:\$I\$9)	<=	140
15										LHS	Sign	RHS

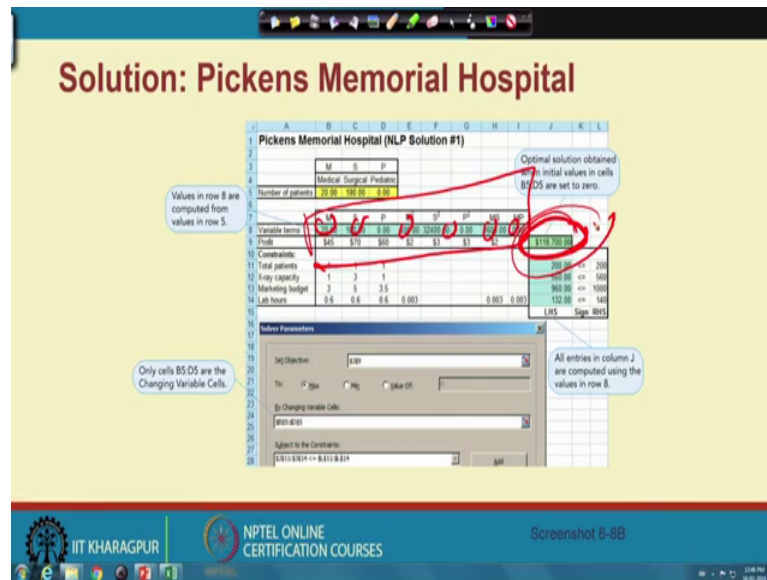
So, so now corresponding to this problem, so we have the solution again here. So, the we have we have already solved. So, this is also same case. So, we have a three variable case, but the problem which we have solve is a two variable case. So, here there are three variable case and the three variables are M, S, P and then corresponding to with the kind of you know dot product and cross product and the kind of you know problem. So, we have here you know dot product case, dot product case, dot product case and this is a cross product and cross product.

So, now so with what I have already mentioned; you cannot just you know mathematically connect, but all these you know inputs or you know the kind of you know problem formulation in the solver depends upon the existing business structure or you know business condition. Then actually we put the inputs and then look for the optimal solutions and the optimal solution which will we received through the solver, exclusively well connected with the original problem or the typical you know business problem. So, so, in the first hand we are not actually changing anything else; depending upon the problem we are just allowing the solver to have the optimal solution as per the particular you know business requirement.

And then if any change we need you know need to apply, then again we can change the particular requirement and look for you know further optimality structure and then we can compares; so the existing optimal solution and the change optimal solutions. So,

likewise you know so, the process which we have highlighted you know in the excel sheet, so this is also same case. So, now in this case corresponding to this a business problem, so that means, the typical business NLP problem is here.

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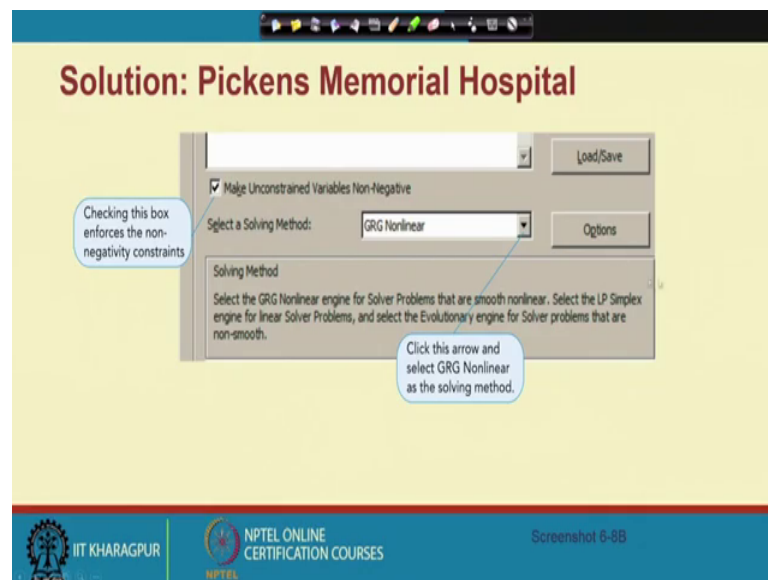
Now corresponding to this business problem, so, we have the kind of you know structure like this. This is how the input to this you know solver and then in the next process we will we have a inner solution. So that means, the solution is here so, this is what the solution to this particular you know (Refer Time: 26:13). So, this is how, what the value of the objective function and a again so, we are looking for the values of the decision variable.

So, the values of the decision variable is actually reputed here corresponding to the objective; that means, the objective function actually the objective function actually here is a let us see here. So, this is what actually so, M M square, S squares, M S, P and P square. So that means, we have a couple of you know decision variables so as a result. So, we will we find you know, so these are the these are the values of the decision variables and as a result if you put all these decision variables values of the decision variable to the objective function, the objective function will be give you the optimum value. And so that means, technically this is the values of the decision variables and this is what the value of the objective functions.

So, like that means, technically if you look into the solver. So, the solution mechanism is more or less same. So, it is actually the kind of you know objective function and the constraint. Only difference is actually the kind of you know functional formality. So, earlier we have solve actually all the objective function and constraints which are linear one, but here we are solving the similar kind of you know problem where either the objective function or the constraints or both objective function and constraints are non-linear in correctors.

Then we are you know looking for the a optimal solution; that means, technically there is these are all not you know highly distance. Of course mathematically we have we have actually describes different kind of you know structures to look or to look for the optimality and to have the optimality. But if you know apply this particular you know you know software's, then by default you know the process is more or less same and depending upon the kind of you know problem come you know requirement or the kind of you know problem inputs. So, the output will be also getting a accordingly. So, this is how the kind of you know solution corresponding to this particular you know problem.

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And now so, what I can say that you know this is what we have already you know highlighted how we can obtained this results from the solver.

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Solution: Pickens Memorial Hospital

Pickens Memorial Hospital (NLP Solution #2)

Solution now recommends admitting only pediatric patients.

Optimal solution obtained when initial values in cells B5:D5 are set to M = 100, S = 0, and P = 100.

	M	S	P	M ²	S ²	P ²	MS	MP
Variable terms	0.00	0.00	200.00	0.00	0.00	40000.00	0.00	0.00
Profit	\$45	\$70	\$60	\$2	\$3	\$3	\$2	\$132,000.00

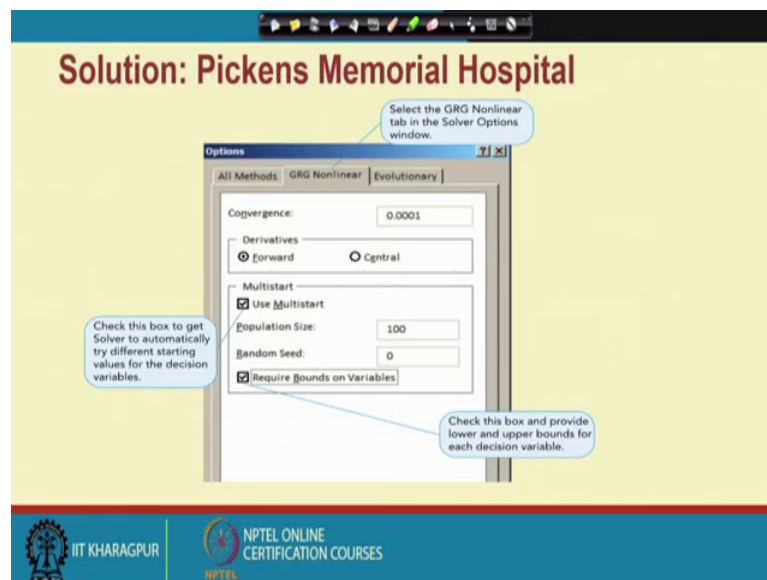
Constraints	M	S	P		
Total patients	1	1	1	200.00	<= 200
X-ray capacity	1	3	1	200.00	<= 560
Marketing budget	3	5	3.5	700.00	<= 1000
Lab hours	0.6	0.6	0.6	120.00	<= 140

LHS Sign RHS

Screenshot 6-8C

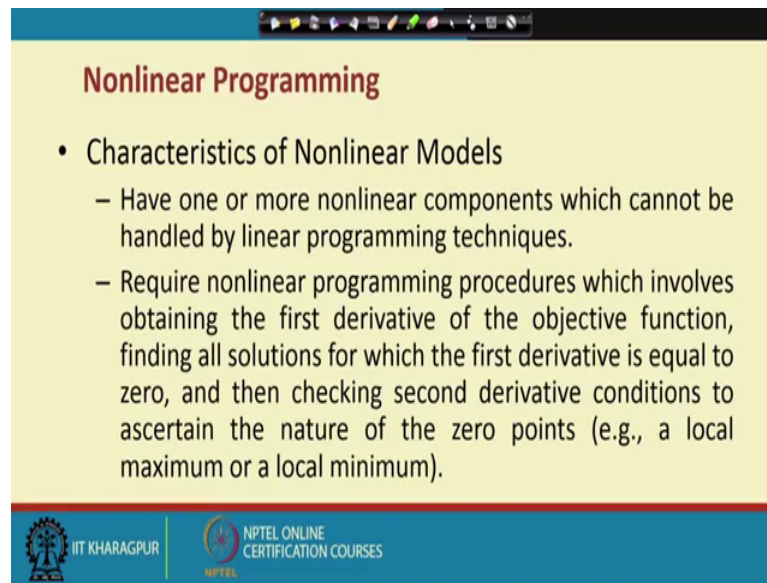
So, now any change (Refer Time: 28:28) so, the result will also change. So, we have actually a similar kind of you know structures.

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So; that means, technically what I like to says you know if I like to summarize.

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Nonlinear Programming

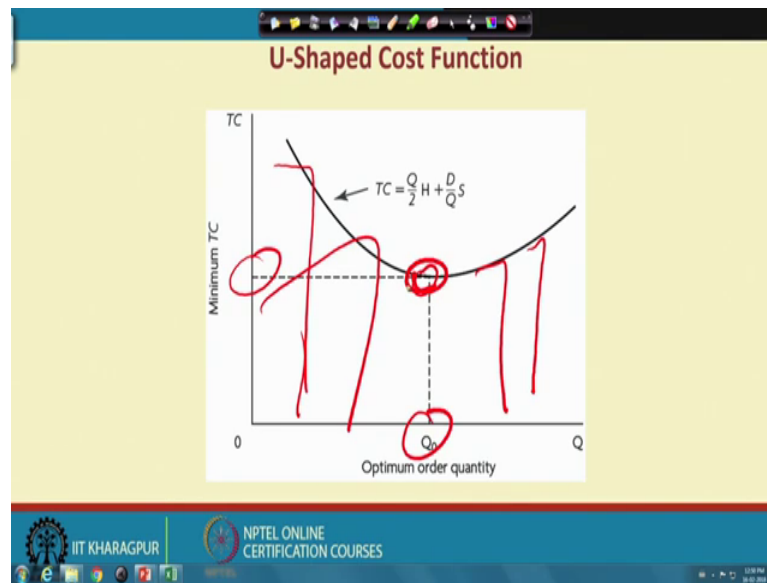
- Characteristics of Nonlinear Models
 - Have one or more nonlinear components which cannot be handled by linear programming techniques.
 - Require nonlinear programming procedures which involves obtaining the first derivative of the objective function, finding all solutions for which the first derivative is equal to zero, and then checking second derivative conditions to ascertain the nature of the zero points (e.g., a local maximum or a local minimum).

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So, we have actually the kind of you know problems, where we have objective functions, we have a constraints, we have a conditions and then the typically you know you know difference corresponding to the earlier discussion about the linear programming; is that you know the either the objective functions or you know constraints or both. They are you know somehow you know connected with the nonlinearity and then we are looking for the kind of you know solutions.

So, what I already mentioned that you know there is a kind of you know local maximum or you know local minimum. In fact, you know what we have already solved through the kind of you know simplest structure or dual simplest structure. So, it is a iterative process will we find actually a different kind of you know solution in every stage, the value of the objective function will be changing and there is a kind of you know improves. That is why in every a you know every after every iteration, so we called as you know it is a improve basic feasible solution and then finally we list of where you know, the particular value will satisfy the condition and constraint and then the value of the objective function is at the highest level.

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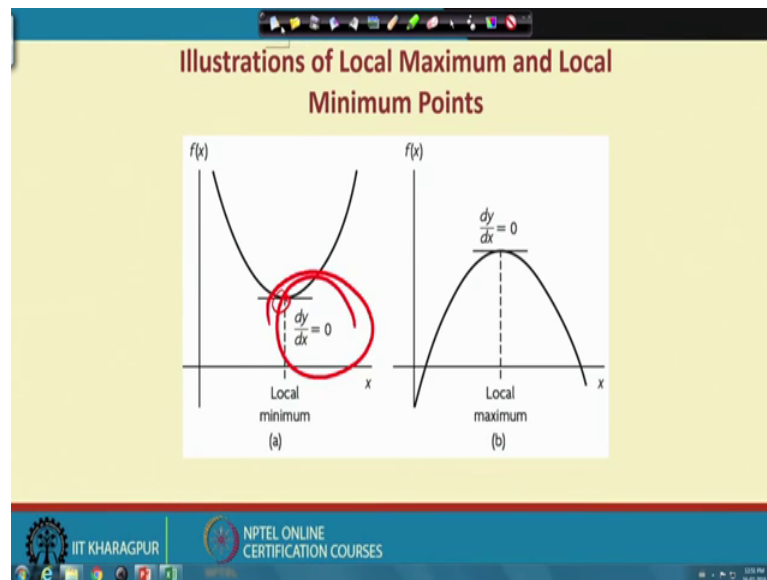


So, this is how the kind of you know structures and a usually so, if you will look for the you know you know graphical structures, where we have actually the kind of you know cost structure like this. So, minimum means you know so, we look for the particular you know structure like this. So that means, what we have already discuss the graphical structure, this is what the minimum point which we are looking.

So, that is why we are actually differentiating the total cost function with respect to the output and by using the first order condition and second order condition; then we can find out the optimal output where you know cost can be minimize. So that means, that is how these. So that means, if you look you know there are various options which you have actually.

Before the point before the you know optimal point and after the optimal point. So, so that is how actually the kind of you know you know structures. So that means, typically you will have lots of you know flexibility options; like you know different corner points we have in the case of linear programming. We have also here different kind of you know options, but ultimately so, there will be one corner point which can give you the kind of you know optimal solutions.

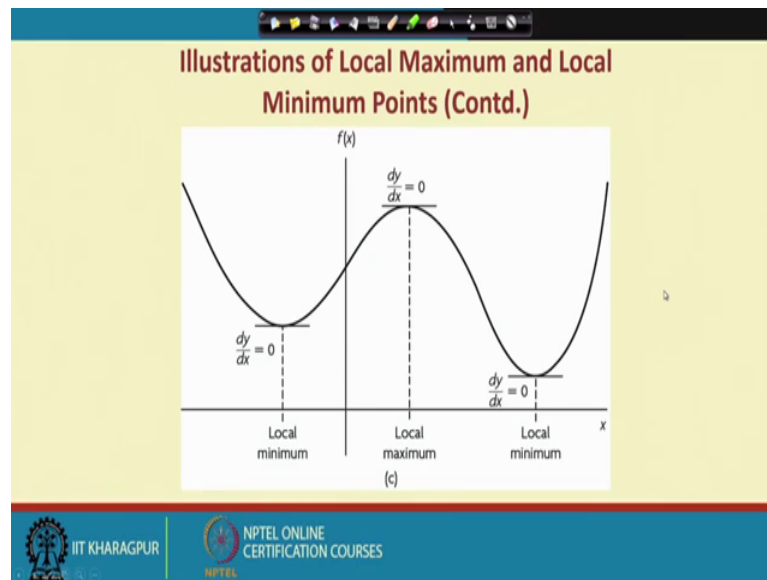
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So, this is this is the case of you know particular you know you know objective functions and corresponding to these objective function; this is how the a local minimum and in the case of you know local maximum it is just you know opposite. So, it is again between actually convex to concave only. So, in the convex kind of you know scenario, so we like to go for you know local minimum and in the case of case of you know convex type of you know situation, we look for you know local maximum for instance.

So, the typical you know structure which will we receive through the calculus base method and that is how we have we have already discuss that you know first derivative should be equal to 0, to get this point and second order sufficient condition will just to support the particular you know process. That means, it will just deleted the particular you know optimality and the values of the decision variable through which you can address the particular you know business problem.

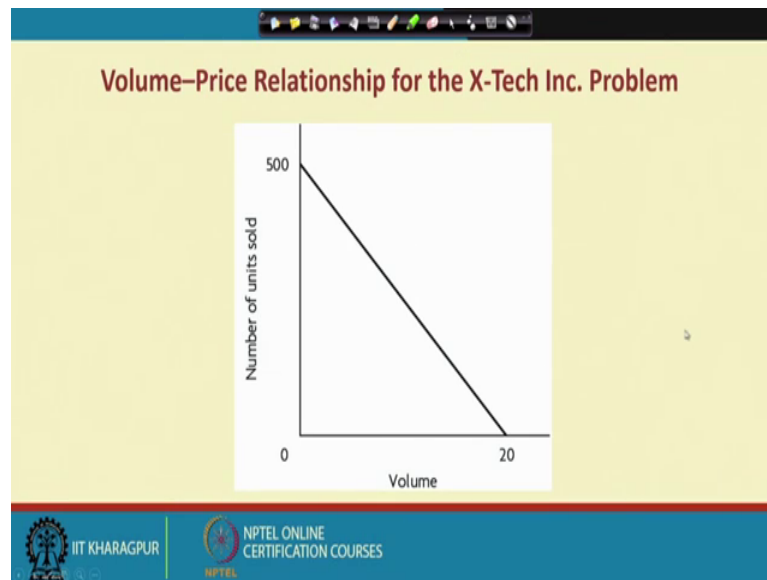
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And a likewise you know we have actually here the case about the you know complex process, where you know we have a three different points through which you can actually find out the a kind of you know optimal solutions. So that means, technically if you go back to the previous you know structures you will find you will find a there are you know there is only one single point which gives the kind of you know optimality structure. But now, if you will come here is you will find there are actually three different points where we can have actually the optimality structure.

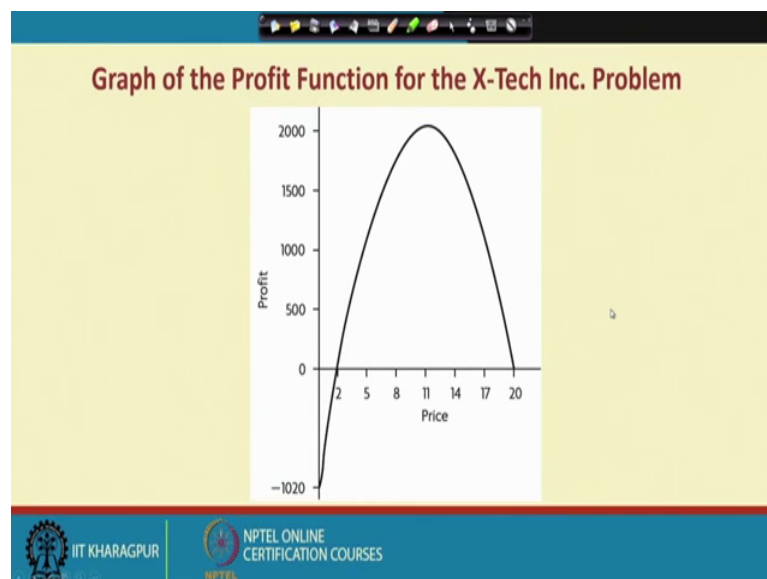
So that means, a this gives the kind of you know indication or signal that you know; there is a importance of you know robustness check that is what the second order sufficient condition is all about and this will give you the indication that you know out of these three different options which one is the finally the best options, to have the optimal results and to address the business problem as per the particular you know requirement ok. So, so that means, we like to highlight you the, here the importance of the second order sufficient condition about the a optimality, a requirement right.

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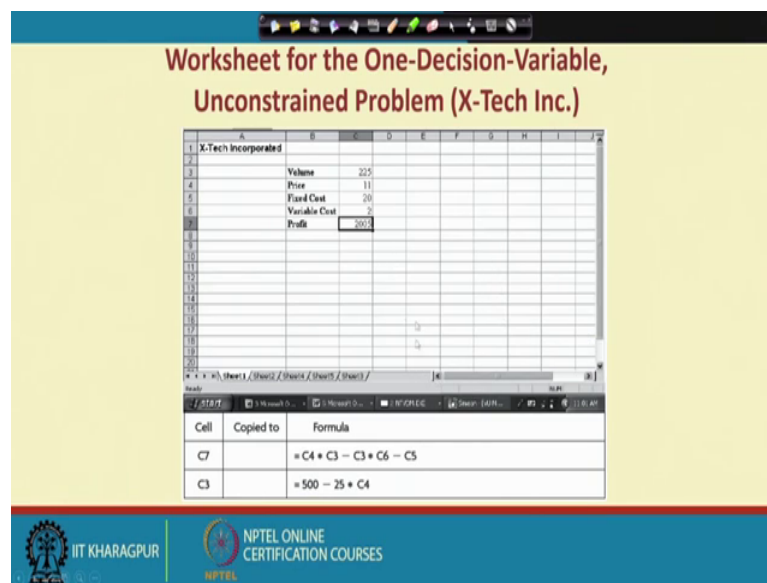
So, now corresponding to this is so, what will you have? Here and the here is the kind of you know structure called as you know constraint and this is what the kind of you know profit functions and then constraint will help you to stimulant the process.

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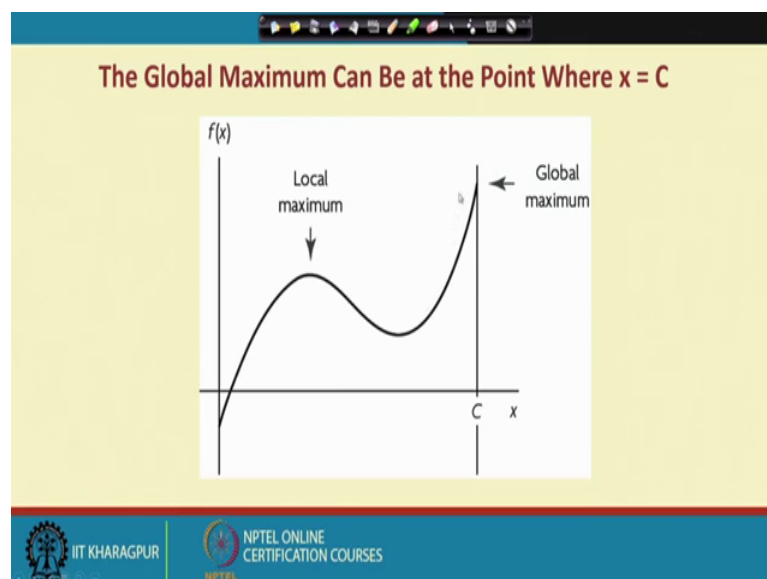
And then take you to the kind of you know optimality structures; where you know it will satisfy you know the kind of you know requirement of the objective functions and that to with respect to the conditions and the kind of you know requirement.

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And this is how the you know the one way to solve the problem through solver package again. So, there is a we have already solve. So, this is how the typical case.

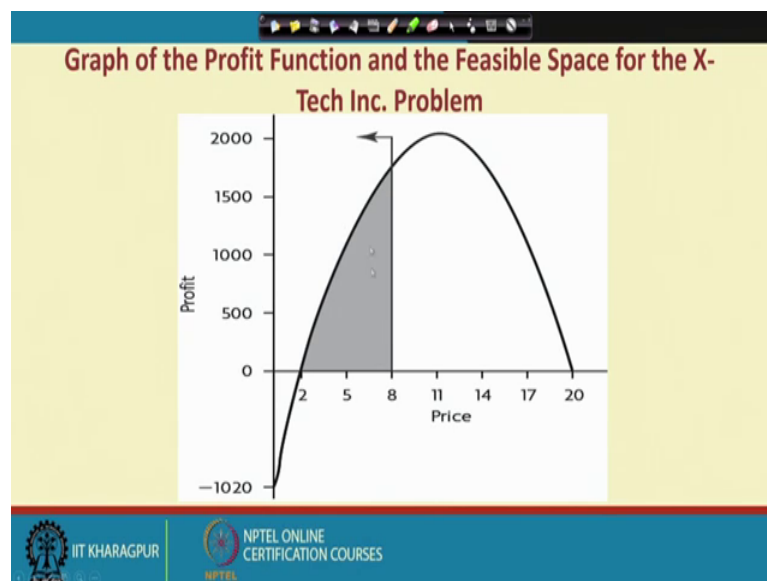
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Again this is another kind of you know nonlinearity; that means, technically if you look for you know the various models. So, in the case of nonlinearity we have a more flexibility and complex and in fact, it is more interesting than the kind of you know linear one. And in the case of you know linear one; so, you will find one kind of you know indications, you know either you know downward or you know upwards. It just

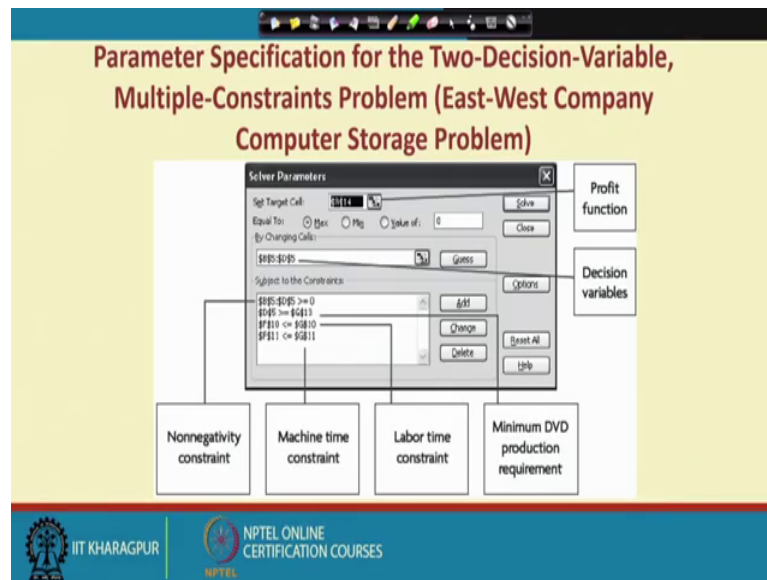
changes the angles from one, one point to another point, but here you know this situation is more you know flexible in nature where you know, you will find you know in a particular situation there are you know (Refer Time: 34:23) ups and downs and you will find you know more than you know one points where you can get the immense. We can have the kind of you know optimality, but ultimately the final optimality will be single one; through which you can you know address the business problem as per the requirement.

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So, this is what the feasible regions and that means, technically corresponding to the objective functions, we cannot you know specify the feasible region; until unless we have we can connect with the constraints. So, now with your constraints; so, we can actually have the feasible regions. Then we look for the kind of you know solutions right, this is another case. So, again solver can be help to get the particular you know regions.

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This is these are all solver outputs and a so, so, all together is what I have mentioned that you know the in that kind of you know prescriptive analytics we have discussed several problems; the kind of you know linear programming structure, non-linear programming structures; where we have objective functions, we have a constraints, we have a conditions, we have a kind of you know a restrictions, the kind of you know requirements; like you know integer type and the kind of you know pure integer, mixed integer.

So, means you what I can say that you know we have a plenty of you know options or you know different tools in the kind of you know prescriptive analytics structure, through which you can address the business problem; more effectively, more accurately as per the particular you know requirement. So that means, technically what kind of you know model we can use to address the business problem exclusively depends upon the data structure and the kind of you know types of you know problem and the history band the problems, some kind of you know extra innovatives you know third process you can add, depending upon the particular you know requirements and dynamics of the business.

Of course these are these are all supported you know should be supported logically. But ultimately we need to find out the optimal solutions and the values of the decision variable; through which you can address the business problem more effectively, more effect you know accurately and you know as per the particular you know business

requirement and that is what the beauty of the prescriptive analytics compare to you know descriptive analytics, inferential analytics and predictive analytics.

And this is you know something you know special case where you know we like to the we like to find out a particular you know position through which you can say that you know business is more effective and business is actually more efficient. So, that is that is the point actually you know means that is the point through which you can you know, take the management decisions or you can you know the apply the management decision very effectively and a prescriptive analytics will help you lot to come to the kind of you know situation and to address the problem; as per the particular you know requirement and with this will we stop here.

Thank you very much have a nice day.