Business Analytics for Management Decision
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Lecture – 54
Prescriptive Analytics 3 (Contd.)

Hello everybody. This is Rudra Pradhan here, welcome to BMD lecture series; today we will continue with prescriptive analytics and that to coverage is on non-linear programming. So, we have discussed couple of business problems dealing with the linear programming and under linear programmings we have discussed several cases several instances and we find there are lots of flexibility we have in the prescriptive analytics to solve the business problems depending upon the particular situation or the depending upon the particular case or with respect to number of constants and the kind of requirement the kind of conditions.

So; that means, say a technically prescriptive analytics you know having you know lots of kind of advantages through which we can actually solve some of the business problems depending upon the type of requirement and the kind of business requirement. So now, connecting to the previous you know discussions that is it related to linear programming.

So, today we strictly you know highlighting the issue with respect to non-linear programming. So that means, technically whether we like to apply you know linear model or non-linear models that exclusively depends upon you know the type of data the kind of inner structure the kind of business problems usually we cannot actually build a linear models by default.

So, having the clue you know or you know getting the clue from the data set or the kind of the kind of know business problems; we can actually develop the model and then we look for the optimality that is the values of the decision variables and that too for the prescriptive analytics to solve the business problems and come with a kind of management decision as per the particular requirement.

So; that means, that is how you know in the previous couple of lectures starting with you know and descriptive analytics then the inferential analytics predictive analytics. So,

every times the typical suggestion is that; so, we have to go through you know very in depth the data visualizations, the understanding of the problems the kind of dynamics of the problems.

Then on the basis of data availability or you know informations then the kind of business theory or the kind of issues and the kind of challenges. Then we try to transfer the particular model into a model and that model you know since it is a kind of data transformation to the model or you know problem to model. So obviously, we cannot just you know transfer the typical you know problems or issues to you know kind of linear model.

So, the transformation of information and data to model whether it is a linear one or non-linear one; depends upon the situation and data itself will read the things and accordingly we can actually develop the model. So now, after you know knowing all these details you know from the data visualization the kind of problem requirement on the kind of dynamics of the problem.

So, assuming that the model is a kind of non-linear ones then and then we will look for the kind of solution that to optimal solutions and here we look. We look for the values of the decision variable through which you can address the business problem more effectively and then come with a kind of management decision as per the particular management requirement.

So, so, we have already discussed you know like you know in the case of predictive analytics. So, having the data having the problems we can first you know start with in linear kind of model, then go for the estimation go for the kind of optimizations and look for the solutions. And in fact, you know there are lots of test procedures or you know diagnostics or you know robustness checks through which you can give the kind of or the you can find out the kind of clues or the kind of inference; whether the typical model that is the linear model is perfect one for the business predictions or business a kind of for castings.

If not then we have the option to change the particular structure and again the estimate you know reoptimize then look for the final optimal solutions which can be very effective as per the problem requirement. And the kind of business requirement and then we can a come with a kind of effective management decision.

So, now with this you know basic backgrounds. So, we like to highlight what is the a concept of non-linear programming and how it is actually different to linear programming and how is the kind of solution that is the optimality and the values of decision variable through which you can address the business problem more accurately.

So, you know if you like to you know see the like to analyze the difference between the linear programming and non-linear programming. So, it is not you know highly you know different. So, they are you know more or less in this a similar kind of in a basket, but the difference is just you know the kind of functionality only.

So, for as you know structure is concerns you know it is more or less same because our optima means, our final or you know the immediate requirement is the optimal solutions. And that too values of the decision variable through which you can actually address the business problem as per the typical management requirement.

So, that is why, since that is why it is should not know highly different. So obviously, the concentration is on you know typical objective function that is how to address the business problem. So, like linear programming here a most of the instruction non-linear programming can be also having objective functions constants and the conditions and look for the optimal solutions and you know the values of the decision variable. So that means, you know the problems which you are discussing in the prescriptive analytics and the tools which you are using various tools which you are using mostly the kind of optimization tools.

So, now I will give you a little bit background about the details of the optimization structures; then we will come with a kind of nonlinearity structures and then really connect some of the problems. And then how we can actually obtain the optimality or optimum solution through which you can be non-linear programming you know in the similar way you know can analyze the business problem more effectively like linear programming.

So obviously, the issue is like this the kind of a structure is a. So, we have the kind of structure here; so, this is a kind of optimization.

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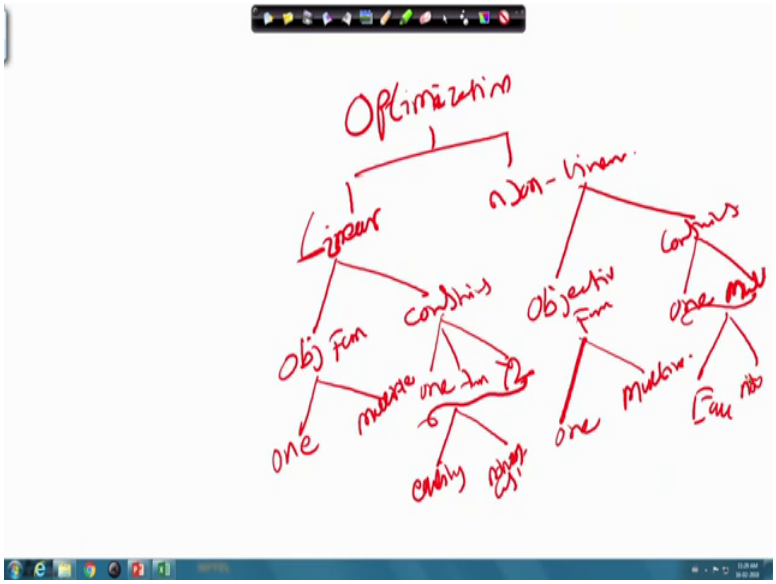

So that means, technically the optimization tool optimization tool is it is you know typically two types. So, it is you know the kind of linear optimizations the kind of linear optimization and the kind of non-linear optimization.

So, here in the case of linear optimizations we have objective functions and objective functions and we have constraints ok. So, in the non-linear you know programming and so, that is how the kind of we have again objective functions and then in the kind of constraints. So, it depends upon you know case to case for instance objective functions can be one and can be multiple one that is what multi objective structures and constants can be one can be two and can be more than 2 right.

So, this can be the kind of structuring and here also the kind of inner structure is we have objective functions we have objective functions one and it can have also more than two objective functions and then constants it can also have one constants and it can have also multiple constants multiple constants right. So, this is how the typical you know structure through which you can actually address the business problem.

So, now in the case of constants; so we can have actually any all the all the kind of situation we may have this you know structure like you know equality constants, and then non equality constants and non equality constraint right. So, here also in the constraint sides we have equality of options and we have non equality of options. So that means, you know this is the typical flowchart under the optimizations; so, in the in this

you know in this context we are looking for the kind of optimal solution and then we can address the business problem as per the particular requirement.

So, now that means, taking the clue from the optimization structure which can be you know it can be divided into two different ways. So, the linear one and the non-linear ones and in the linear case we have objective function we have a constants. And again in the non-linear programming problem we have also objective function and we have the constants. So, the objective function can be a single one or multiple one. And constants can be you know single one or multiple one again constants can you know have equality structure and inequality structure. And these are all equally true both for you know linear programming problem and non-linear programming problem.

So, this particular lecture is specifically you know or exclusively concentrate on non-linear programming problem where we must have actually a particular objective functions and then constraints. So, sometimes we have a problems having objective functions and there may be constraints and there may not be constraints.

So; that means, technically now we can you know again for formulate the non-linear programming problems corresponding to you know the typical optimization structure.

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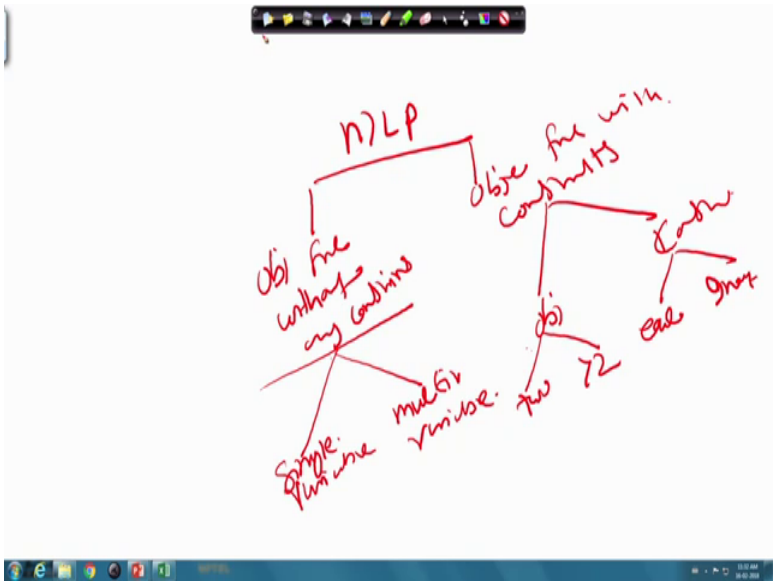

So, the non-linear programming structure will be like this. So, one with you know objective functions objective functions without any constraints without any constraints.

So that means, non constraint optimizations then here objective functions objective function with you know constraints objective functions with constraints right with the constraints this is what actually the kind of division.

So, objective function without any constraints and objective function with the constrains. So, this is what is called as a constraint optimization problem. So, in the in this case we have only objective functions and the objective functions can be optional two types single objective functions and multiple objective functions and that two with single variables single variables and the kind of multi variable case multi variable case right multi variable case.

Similarly, here we have objective functions and we have constraints and then the objective functions may be with respect to you know since its a constraints. So, it will start with you know I know or two variables and more than two variables and then constraints which can have actually one constraints or you know more than two constraints and again some are you know equality type and a some are you know inequality type.

So, why we know these classifications are required? Because we have actually different kind of inner structure through which you can actually apply the linear non-linear programming problems. Generally we use calculus based kind of solution through which you can address the business model and the business problem as per the particular requirement.

So, we have a different kind of techniques and depending upon the kind of problem depending upon the kind of inner structure; we look for the solutions and then address the business problem as per the particular requirement. So that means, technically starting with the a nonlinearity structures. So, you know without constraints and then with constraints.

So, let us you know see the particular structure where you know the problem is your of non-linear programming and where we have only objective function and no constraints. Then against we can analyze the situation where we have objective function with the constraints right. So, that means, technically we have a two different structure all together in the case of non-linear programming.

And then we like to see how you know the optimality structure in both the cases and how they are you know different to each other to address the business problems. Obviously, we can start with your objective function and then we look for the optimality then over the times, we will have really find you know there are lots of constraints which can affect the objective function or you know business objective. So, as a result; so, the unconstraint you know optimization can move into the constraints optimization. And then look for the optimal solutions and the values of the decision variable through which you can address the business problem more effectively.

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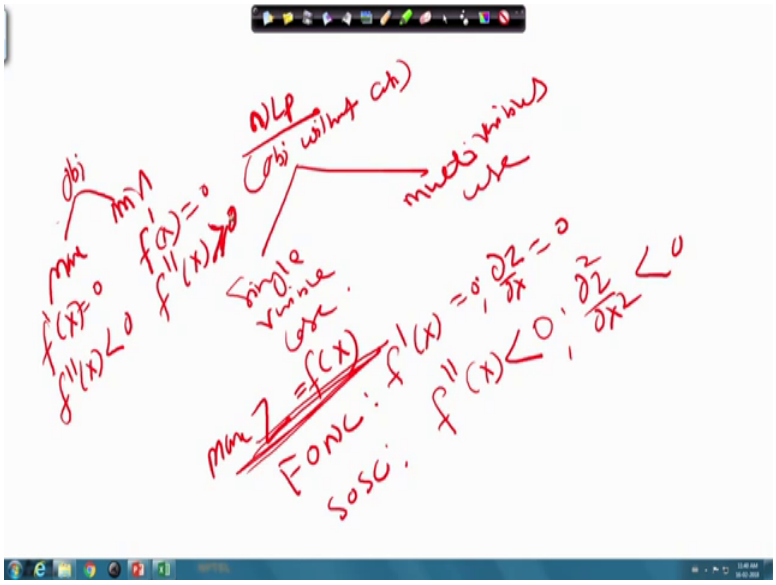

So that means, the in the case of non-linear programming typically, so that is you know objective functions without constraints. So that means, it can be have a single objective function means with that is with respect to a single variable case and then we have actually multivariate case multi a multi variable variables case right a single variable case and multi variable case.

So, its depends upon a particular business scenario of course, in the big business and the dynamics of business will be find you know a business problem which can be addressed within a multivariate environment. And you know a business problem with single variables is you know you may find you know rare in the real a real life scenario, but still we like to know how is the typically inner structure and then will move into the

multivariate case where you know most of the business problems can be addressed there and then look for the optimal solutions as per the particular management requirement.

So, now in the case of single variable case let us assume that you know Z equal to objective function which is nothing, but you know from f of x so; that means, you know it is a objective function with a single variables now we look for you know optimal solutions.

So, then in the case of non-linear programming; so, we like to you know go for again the optimality and the values of the decision variables at the optimal stage through which you can address the business problem for instance let us say this is a kind of profit functions; then we like to know what is the maximum level of output through which you know the business firm you know optimum profit. So, that is how a simple structure can be designs and then look for the a optimal solutions.

So, a typical structure is like this so; that means, here we look for the optimality and in the case of optimality that we in the non-linear programming problem. So, we wish to follow the calculus based you know structure you know through which you can look for the optimal solution and values of the decision variables.

So; that means, technically so, there are two different structure altogether and in the case of calculus based mechanisms. So, there is a first order necessary condition and second order sufficient condition and that we depends upon the a kind of types of objective function whether it is a maximization type or you know minimization type, but the typical structure is more or less same for instance. So, when I am writing Z equal to f x; so, the this is not a actually clear cut you know signal what to do exactly.

So, now we have to you know specify first the objective functions let us start with you know the maximization of Z then it is a kind of meaningful specification and as a result you can actually apply a particular technique through which you can look for the solution. So that means, a; so, there are two different you know structure first order necessary condition and second order sufficient condition.

So, far as a calculus base you know optimality is concerned and this is the kind of structure which we which we generally apply in the case of and non-linear programming you know problems and that to non-linear programming optimization and in this case

that is with respect to objective functions that is the maximization of Z and that to f of a x that is that is the case of single variable.

Then here the first order necessary condition will be like this f dash x would be equal to 0; so; that means, it is nothing, but actually a d Z by d x equal to 0 and then in the second order sufficient condition will be f double dash x would be you know less than equal to 0. So, that is with respect to maximizations.

So, and in this case it is a d square by Z by d x squares less than equal to 0. So, this is how the one you know one variable case and. So having the; that means technically what we like to do here like you know predictive analytics. So, we have a data corresponding to a particular business problems.

So, using the data you transport the data into a model and then you check whether it its coming in the linear structure and non-linear structures you know after you know lots of diagnostic check and robustness checks we will have a models which is actually non-linear type then we will then we can apply this calculus based methods through which actually we look for you know optimal solution and then and the values of the decision variable through which you can address the business problem.

So, this is how the typical you know structures. So, once you get the get the kind of structure you know the objective functions and that to the requirement is the maximization type. So, this is the structure which you can apply then look for the optimal solution and the values of the decision variable. So, now, in the case of minimization the typical structure is more or less says same that is the case of single variable.

So, just you have to change the objective function structure maximization to minimization. So, whether it is a maximization type or minimizing type the first order necessary condition is more or less same that is the first differential should be equal to 0 that is d x d Z by d x equal to 0. And the second order condition will be different and in the case of maximization; the second order differentiation will be less than to 0 while in the case of minimization it will be greater than to 0.

So; that means; so, if corresponding to objective functions if it is a maximization type and then f f f f dash x equal to 0 and f double dash x x would be less than equal to 0. And

in the case of minimizations same f dash a f dash x f dash x equal to 0, but f double dash x would be greater than greater than to 0.

So, this is how the typical case in the case of one variable case and that to without any constraints. So, now, let us you know see the situational in the case of multivariable case. So, if it is a multivariable case then the objective function will be with respect to at least two variables.

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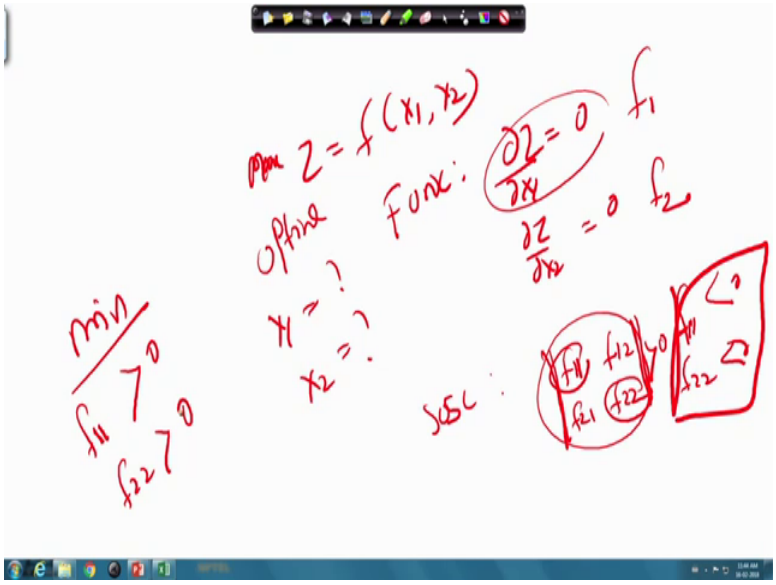

So; that means, technically let us you know maximize Z equal to f of X 1 and X 2 and then we look for the optimal solutions we look for optimal solution; that means, what should be the X 1 value and what should be the X 2 value through which the particular functions can be you know reach at the highest levels; obviously, a X 1 is the kind of decision variables may be with respect to profit may be with respect to revenue.

So, we like to you know optimize the revenue functions and we like to optimize the profit functions and in this case. So, the typical structure is again same. So, we have a first order necessary conditions and we have a second order sufficient conditions. And in the case of first order necessary conditions; so, we can have actually d Z by d Z by d x d X 1 equal to 0 and then d Z by d X 2 equal to 0 ok. So, this is you know first order necessary condition.

So; that means, so this is nothing, but called as you know f f f f 1 and this is nothing, but called as you know f 2. And then and this is also same in the case of minimization of the objective functions and in the case of second order sufficient condition. So, we have actually you know different kind of structure altogether. So, for that we can have a matrix (Refer Time: 22:34) determinant matrix.

So, this can be f 1 f 1 1 and f 1 2 f 2 1 and f 2 2 that is the double derivative; that means, second order differences which we can actually apply. So, now, the entire matrix should be a you know like this so; that means, technically in the second order sufficient condition for multivariate case little bit you know complicated. So, in this case; so, the requirement is a for the maximization structure. So, f f double 1 should be less than 0 f double 2 should be less than 0 so; that means, this diagonal elements.

And then the up diagonal elements and the kind of the kind of matrix combination this the entire matrix this entire matrix should be a greater than to 0. So, this is how the typical you know requirement and that is what the case of maximization of objective function. And in the case of minimization of the objective function the typical process is more or less same, but we the difference is actually with respect to the second order differentials.

So, in the case of minimization in the case of minimization of objective function; so the second order objective function that to the diagonal differentials will be greater than to 0 and then f 2 to also greater than to 0. So, rest of the things are you know more or less same and that too with the case of multivariate a you know situations; that means, a a particular objective function I can be specified with respect to more number of variables.

So, since you know we have actually a different kind of business issues so; obviously, the total revenue or total profit may be the sum of two or you know more than two different you know products. So; obviously, the typical objective function will be multivariate in natures; so now, having the multivariate in a kind of objective functions.

So, we look for the a optimal solution and using the calculus based methods. So, we can follow the typical structure like this so; that means, we have discussed the typical structure in the case of one variable case two variable case and that can be generalized for you know multivariate case.

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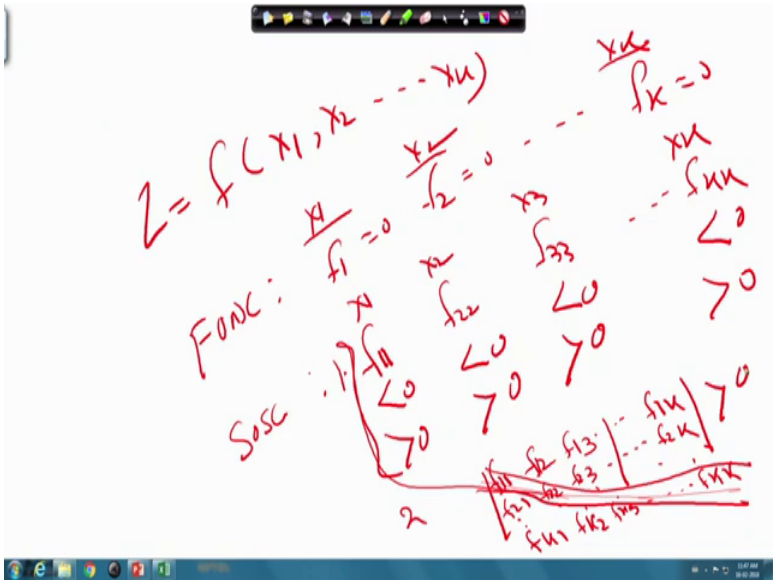

For instance in the case of multivariate case let us say Z equal to f of X 1, X 2 and so, on let us say X k.

So, then what should be the optimality structure you know with respect to multiple variables? And again that depends you know again with respect to objective function type of objective function whether it is a maximization type or you know minimization type. So; obviously, so, the first order necessary conditions every time will be f 1 equal to 0, f 2 equal to 0 then continue f k equal to 0 and that to with respect to X 1, X 2 and you know X k. So, these are you know number of variables which can be in the particular business process.

And the second order sufficient conditions as the two different you know structure in the first structure is with respect to diagonal elements that is f 1 1, f 2 2, f 3 3; then continue up to f k k and that is with respect to X 1, X 2 X 3 and X k. And in the case of a maximization, this should be less, than 0, this should be less than 0, this should be less than 0, this should be less than 0 and in that case of minimizes some it should be greater than 0, it should be greater than 0, it should be greater than 0 and it should be greater than 0.

And in the second order sufficient condition this is the first issue and in the second issue is the complete determinants. So, that is what let us assume that it is actually case of three variable; then f 1 1 and then f 1 2, f 1 3 or you can continue with you know up to

you know f 1 k; so; that means, technically this will be go up to f 1 k and then f 2 1 f 2 2 f 2 3 and then this will continue with the f 2 k.

And again this will continue, this will continue, this will continue then finally, f k 1s, f k 2 f k 3 and it will continue up to f k k. So that means, technically we have two different structure here the diagonal structure and the off diagonals and on diagonals; so regarding the diagonal structure which we have already specified here with the case of or with the requirement of maximization type and minimization type.

And in the case of second structure with respect to sufficient conditions a second order sufficient condition this matrix should be greater than equal to 0. So, this is how the kind of optimization principles with a objective function and that to have no constraints. And we have highlighted three different situations in one situation we have a single variable, in other case we have a two variables and then we can extend it to multivariable case.

So; that means, depending upon the type of business problems which you like to analyze. So, we try to see what is the typical you know case. So, whether it is a one variable case or two variable case or multi variable case then you transfer the particular problem and you know information to a models and that too in a kind of a situation where the model is a kind of non-linear in character of course, we cannot you know just arbitrarily design the particular non-linear models. So, the data itself will give you the particular structure.

So, once we get the particular structure assuming that you know the particular structure is coming in the form of a non-linear format and that to whether it is you know one variable case, two variable case or multiple variable case. So, these are the procedure which you can follow then look for the optimal solutions.

So; that means, say the typical requirement of the non-linear programming is that you know you must have been know knowledge about the calculus and then manually you can actually go for the solution like this. And you know otherwise you know it is very difficult to get the optimality depending upon the particular business requirement.

Of course, we have a excel solver where we can also you know without any you know manual calculation we can just you know give the kind of an instruction to the server. So, server will give you the optimal results and that two values of the distant variables or

depending upon the particular business problems or the kind of business you know conditions.

.So, now, also these are the cases under you know objective functions without any you know constraints; that means, we have discussed three different situation under non-linear programming problems and that to we have objective functions without any constraint. So, first case with the single variable, second case with the two variables and third case with you know more than two variables and every case we have know typical constraints to address the business problem. But in real life scenario or in the kind of dynamics of the business we will we find the particular business problems may have been a constraints and then we look for the kind of optimal solution depending upon the particular business requirement.

So, in this case again we will you know see the particularly you know optimality structures and look for the optimal solutions. So that means, the second part of the model is ecology non-linear programming problems.

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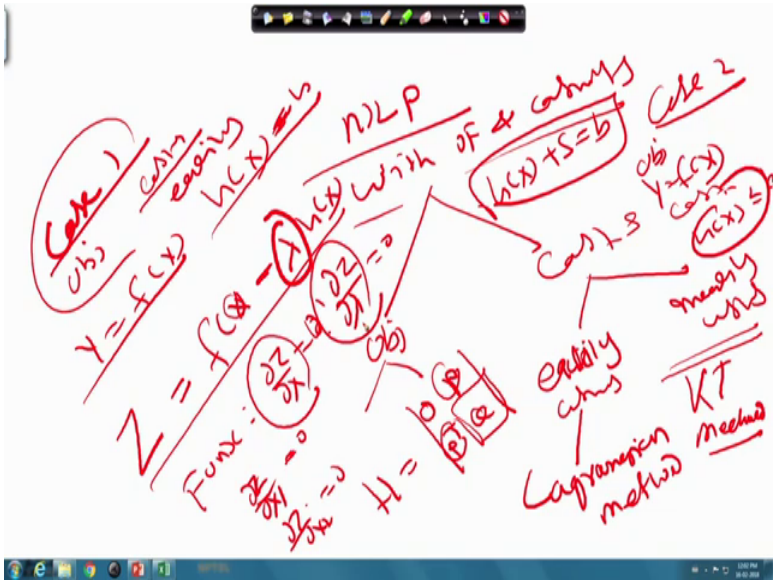

You know with objective functions objective functions constraints right constraints. So, this is how the typical case; so, in this case you know we have objective functions and we have a constraints. And obviously, we can have a objective functions single objective or multi objective or single variable case multi variable case.

And then we have a constraints and the constraints can be of one constraints case and two constraints case and a then in the first instance really have actually a the a type of constraints. So, one type of constraints is called as a equality signs or equality constraints and then this inequality constraints; inequality constrains like which you have already solved in the case of linear programming problem.

So, now you know the typical non-linear programming with you know constraints; that means, constraints typically it is a kind of constraints optimization problem, where we have objective function and constraints and that to one variable case and multi variable case. And then the with respect to constraints we have these scenario called as you know equality type you know constraints and inequality type of constraints.

So, depending upon the constraint structures equality type or inequality type we have a two different mechanisms. So, the first mechanisms under the particular constraints optimization problem that to under the non-linear programming format is called as you know equality constraints and further the standard method to look for the optimal solution is called as a Lagrangian method.

So; that means, this in this case we use actually a Lagrangian; Lagrangian methods to solve the optimum optimality and values of the decision variable. And if there is a kind of an inequality type of constraints we look for the kind of solutions and that to we apply Kuhn Tucker conditions. So, Kuhn Tucker conditions you know mechanisms ok; so, mechanisms to solve the problem.

So,. So, this is how the kind of game here in the case of a constraint optimization. So, we have objective functions we have a constraints and a depending upon the number of constraints a the typical structure will be the equality type of constraints and inequality type of constraints.

Then in the case of if the particular problem is a type of equality type of constraint; then the standard mechanism which you can apply to have the optimal solution is called as you know Lagrangian multiplied methods. And then the if the constraints are you know typically inequality type then Kuhn Tucker condition or Kuhn Tucker mechanisms can be applied to look for the optimal solution and the values of the decision variable through which you can address the business problem.

So; that means, technically we like to know how is the typical you know structure in the case of Lagrangian mechanisms and in the case of Kuhn Tucker mechanisms let us see here. So, in the case of first case or case one case 1; it is a objective functions with you know constraints and that too equality type and in that too equality type; that means, we have let us say Y equal to function of X and subject to there is a kind of constraints the constraints will be let us say you know kind of say h of x that is actually the kind of constraints type which is actually nothing, but equal to b and because sometimes it is called as you know budget constraints.

And you know depending upon you know objective function if the objective function is profit type, then the first hand constraint will be equaled as you know budget constraints. And then we look for the optimal solutions the first and you know trick of this particular process is to you know have a you know single functions which can be actually obtained by clubbing the objective function and constraints and the mechanism through which you can club the objective function and constraint is through multiplier that is through Lagrangian multiplier.

That means, technically having the objective function like this and the constraint like this. So, the first and the requirement or first ones first step of this particular process is to have and this is actually f of x and then f of x then minus lambda you know lambda h of x. So, this is how the objective functions.

And then again we like to you know look for the optimal solution. So, so the a typical difference is here actually you try to build a single functions which is the combination of the objective function and constraint. So, now, once we get the single function which is the a cluster of objective function and constraints then the procedure to have the optimal solution by calculus based method is more or less same the case which you have already highlighted where we have no constraint all you know at all.

So, in this case the typical structure will be like this again. So, the first order necessary condition will be a d Z by d X equal to 0 and then d d Z by d lambda is equal to 0. So, this is the first and first and necessary condition and of course, I am just putting d Z by d X equal to 0, but you know it depends upon you know number of X. So, this X is a vector and it may be with respect to one variable it may be with respect to two variables it may be with respect to you know three variables.

So, now having the kind of structures; so, then you can extend; that means, technically. So, d Z by d X equal to 0 means it may be d Z d Z by d X 1 d X 1 equal to 0 d Z by d X 2 equal to 0 and so, on. Again you know there may be situation that you know we have actually more than more than you know one constant.

So, in that case; so, you can put you know lambda 1 h 1 X lambda 2 h 2 X and so, on. So, then again means accordingly; so, d Z by d lambda 1 equal to 0 d Z by d lambda 2 equal to 0. So, depending upon the particular requirement so; that means, corresponding to objective functions and you know with a single variable and that with you know multiple variables; we may have you know single constraints and we may have a multiple constraint.

So, having more number of variables and more number of constraint; so, you will find you know the problem will be more complex in nature. And in that case you know we look for the optimal solutions and then against the first and requirement is with respect to d Z by d X equal to 0 and d Z by d lambda equal to 0, then we create a again Haitian determinants and which can be a with respect to you know 0 P if P and then this is the P transpose and Q.

So, the P is the kind of kind of the first differential of the budget constraint and this these are all called as you know second order matrix which we have already discussed like you know f 1 1, f 1 2, f 2, f 1 3 and so, on. And so, your idea is actually to calculate the Haitian determinant and check how is the kind of structure.

So, in the case of multivariate case. So, the typical structure depends upon you know the sign of this you know Haitian determinants with respect to one variable, two variables and you know with the order of the matrix with respect to 1 into 1, 2 into 2, 3 into 3 so; that means, the value of the square matrix. So, you know it depends up you know situation to situation is the nature of this particular Haitian matrix may have been on different options; it can a follow a particular trend increasing trend or decreasing trend and ups and downs. So, this will give you the signal that you know how is the behavior of the objective functions and that to address the business problem as per the particular requirement.

So; that means, typically the non-linear programming problem structure is very complex and the first hand complexity will start from a functional forms which is actually a non-

linear in character. And then again the complexity will be added into the process you know by integrating the constraints and again further complexity will start by adding one after another constraint into the process.

So, now, this is the typical problem through which actually the kind of constant optimizations and that too in the non-linear you know problem programming we will address and then look for the optimal solution as per the particular business requirement.

So, now corresponding to the case 1 where you know we have objective functions and equality constraints and the standard method which you can apply is called as you know Lagrangian multiple. So, the combination of the lambda which can actually a club the kind of objective function to constraint that is how it is called as an Lagrangian multiplier methods,. And in the case two the typical difference is the constraints are you know less than type situation.

For instance you know in the context of case 2; so, let us start with you know case 2 here and this is a objective functions and where Y equal to function of X and then the constrains will be h upon X is less than equal to b or you know greater than equal to b. So, in that case like you know the case of linear programming where you know most of the problems which you have discussed is inequality type.

And here this is a special case till now whatever we have discussed where the constraints are you know inequality type. And here we have objective functions, we have actually inequality constraints and the idea is just to club the objective function with the constraints inequality constraints and to come with a kind of final functions through which you can look for the optimal solution and to address the business problem.

So, in this constraints what will you do just you know club and then again the same procedure we have to follow, but before clubbing. So, we use actually slacks and surplus variable so; that means, technically is; so, this instead of this one. So, first and transformation is the h the kind of in a constraints side h X less than equal to b.

So; that means, technically it is nothing, but you know h X plus S equal to b. So, this is what the kind of constraints. So, now, again; so, when we like to add all these things. So, then the standard equation will be h upon X plus S minus b equal to 0 then that will be just connected with lambda and then look for the solution. So, then in the kind of

optimality structures earlier we have d Z of d X equal to 0 depending upon the number of X.

And then d Z by d lambda equal to 0 depending upon the a you know number of constraints if there are 2; then lambda 1, lambda 2, if it is 1 then simply lambda and then we since we have used actually slack variable or surplus variable the h type; then in this case one more requirement is d Z by d S equal to 0. So that means, the further addition is the d Z by d S equal to 0 and again we look for the Haitian determinant and then we check the kind of optimal solution.

Ultimately, whatever you know cases we which you have already discussed here and in every instances you know or every instance you know means you know all the instances we find the typical structure is like this you know we have objective function and we have a kind of calculus based structure through which you look for the optimal solution. And that too where what we have done actually we highlighted the first order necessary condition and second order sufficient condition.

Ultimately the optimal values and optimal results and that two values of the decision variable exclusively depends upon the first order necessary condition. And the second order sufficient condition is just to support the particular output that is one we can you know one way you can address that it is a kind of you (Refer Time: 43:14) check to support the or to validate the a optimality results.

So, you know; so, we have a different kind of situations or depending upon the typical you know business problem, you can actually apply a particular model. And then look for the optimal solutions and a look for the values of the decision variable through which you can address the business problem more effectively and as per the management requirement and so.

With this we will stop here.

Thank you very much.