Business Analytics for Management Decision
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Lecture – 49
Prescriptive Analytics (Contd.)

Hello everybody. This is Rudra Pradhan here, welcome to BMD lecture series; today we will continue with prescriptive analytics. And that to coverage on dual simplex mechanism; we have solved couple of problems and that too highlighting primal structure, dual structure and looking for the optimal solution and the solution which we have already highlighted in the context of primal problem.

So; that means, corresponding to a particular business problem whether it is a bivariate structure or multivariate structure that would depending upon the number of decision variables or number of constants in a kind of system or in a kind of problem. So, we like to choose a particular technique; in fact, programming and the solver will not actually the kind requirement of whether it is a bivariate structure or multivariate structure.

But when we manually handle the business problem then we have two different structure to you know to get the kind of optimal solution. For instance if a particular primal problem is with respect to two variables and two constants or two variables which you know more number of constants; then you know simple graphical structure you can apply and look for the optimal solution that will address the business problem as per the particular management requirement.

So, now if a problem is a if a problem you know having let us say you know more than two decision variables, then a graphical structure cannot you know have a kind of kind of structure through which you can get the optimal solution. In this in this context we can apply simplex method through which you can you know get the optimal solution or the values of the decision variable.

That means, simplex method means it is a algebraic method through which you can look for the optimal solution that is the; that means, we will move for you know one iteration to another iteration you know and continue till you get the optimal solution. So, now, what is happening here is if we have a to variables and more number of constants then

you know you can start with you know graphical structure through which you can get the optimal solution.

But when you have a more number of variables then irrespective of the kind of constants number of constants, then by default will go to the simplex mechanism through each look for the optimal solution whether you know of course, you know through solvers there is no such requirement. But still you have to you know understand the situation and look for the optimality.

Now means the point which I like to address is like this why there is a requirement of duality corresponding to the primal you know kind of structure. For instance you know if a primal structure is having you know two variables with the 3 constants, then easily you can actually you can apply graphical structure to look for the solution.

And then you know its counterpart dual will be again you know 3 variables with it two constant which may be little bit you know complex. Now if your problem is having let us say 3 variables with the two constants then the counterpart dual problem will be two variables with the 3 constant. In this case we can simply you know apply the graphical structure so; that means, technically the primal structure where we cannot easily apply the graphical mechanism to get the optimality or to get the values of the distant variable.

But in that if you go for the dual transformation the same problems can be you know give you the optimal solution and that too by the application of graphical mechanism. So; obviously, we will see how is the particular structure and then look for the optimal solution. Sometimes if the primal is complex then the dual may be little bit simpler to address the problem or if it is it dual is a kind of complex then the counterpart primarily give you the kind of simple structure through which you can address the business problem more effectively and.

So, we have two different structure altogether corresponding to the prime also so, the mechanism which you can apply ecology in a simplex that is called as a primal simplex mechanism and then corresponding to the dual then the mechanism which will apply called as you know dual simplex mechanism. So, here is we like to see what is the dual simplex structure through which you can you know look for the optimal solution.

In fact, in the first and whatever problems will have. So, we can start with you know primal simplex structures you can start with you know dual simplex structure. But there is a typical you know difference ultimately will reach the optimal solutions here the start of the particular process you know in a kind of set up, but the counterpart will be having you know different kind of setup.

For instance in the case of primal simplex methods, we start with you know the means you set all these things then after setting the initial basic feasible solution or initial basic feasible matrix; then you know you will you will check for the optimality. If you reach optimality of stop and you know address the business problem accordingly if you have not reached the optimal solution then you go to the next iterations.

And the procedure of movement from first iteration to second iteration or initial iteration initial step to next step, depends upon the requirement of incoming indication and the outgoing vector indications. So, in the primal structure the first hand requirement is to find out which variable will be the incoming into the basis matrix and then looking for the outgoing vector.

So; that means, if there are 3 variables and the basis matrix depends upon which particular variable will be replaced with which particular variable. So, that is how we look for you know incoming vector fast that will give you the particular indication that you know that particular variable should be into the basis matrix; then against we will look for the outgoing vector where that particular indication will give you which particular variable will be out in the basis matrix; that means, the incoming vectors will be in place of the outgoing vectors.

So, that is how the structure of primal simplex method and the corresponding dual simplex methods we start with you know just opposite you know opposite direction. So, where we first look for the outgoing vector and corresponding to the outgoing vector we look for the incoming vectors. So, also that is how there is not typical drastic difference between these two mechanism, only the entry of incoming to outgoing and then outgoing to incoming.

So, then the incoming outgoing intercetions will give you the pivot you know element and through which the entire structural will change; you know then we need you know finish of the particular iteration and again look for the optimal test. If you reach the

optimal you know structure then you stop and you know address the business problem and come with a kind of management decision. If not then again if continue the particular procedure.

Go to the second step again you follow the similar kind of structure, go for the testing if you reach the optimality that is; that means the indication of the optimality then stop there and again address the business problem. If not again you know repeat the same process; that means it is a kind of continuous process. In steps you will get the improved basic feasible solution that to values of the objective function you know moving from first iteration to second iteration and so on.

So, that is how the beauty of the primal structure and the kind of dual structure that two primal simplex mechanism and dual simplex mechanism. Let us see how is the dual simplex structure altogether and how is the indication of incoming vector, how is the indication about the outgoing vector? The way which you have already addressed in the case of primal simplex method in the last lecture and in fact, in the dual simplex methods we have also similar kind of set up.

But only thing is that you know we like to know what is the indication of incoming vector and they are kind of outgoing vectors. Of course, this start and this starting process is from the outgoing to incoming vector and then we look for the optimality and then finally, will proceed till you get the optimal solution as per the particular LP structure or the kind of prescriptive analytics structure.

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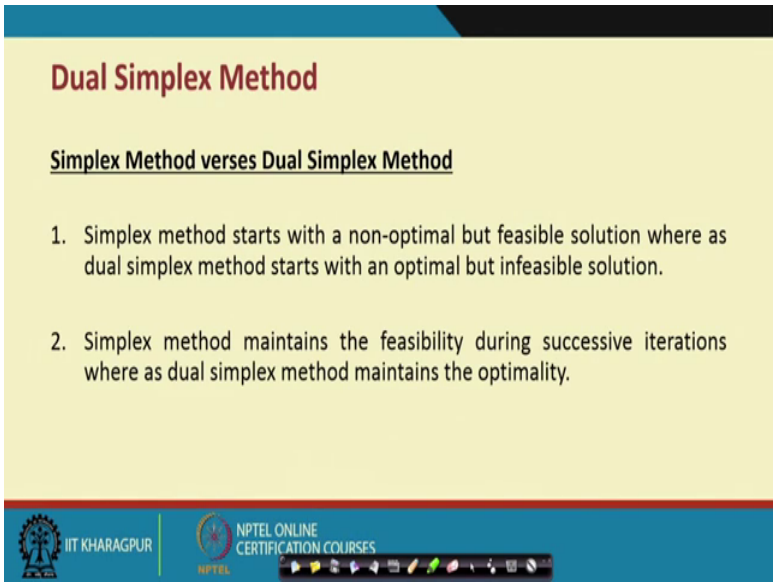

So, let us see here the kind of dual simplex mechanisms. So, simplex mechanism and dual simplex mechanisms; simplex method starts with a non optimal, but feasible solution where as dual simplex method starts with an optimal, but infeasible solution. So; that means, it is just you know counterpart kind of things through which you can address the business problem. And simplex method maintains the feasibility during the successive iterations whereas, dual simplex method a kind of optimality/.

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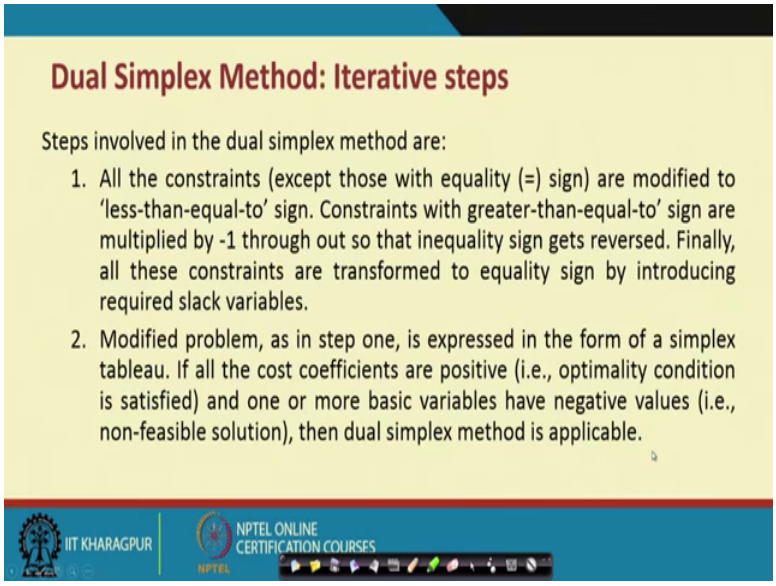

And so, far as a stepping is concerned; so, the first hand requirement that you know all the constraints except those with you know equality signs are modified to make you know less than equal to sign and constants with a greater than equal to sign are you know multiplied by minus 1 through throughout so that you know the inequality sign get you know gets reverse.

Finally, all these constraints are transferred to equality signs by introducing the kind of slack variables. So; that means, this is the typical requirement of dual simplex mechanisms. And once you acquainted then you can actually follow the iterative process you know process till you get the optimal results right. So, in the modified problem like you know with respect to the step 1 is expressed in the form of a simplex tables.

If all the cost coefficients are you know positive and one or more basic variables having negative values then the duals then dual simplex method is applicables. So, that is the first hand requirement of entering to the a you know dual simplex mechanism; for instance like you know primal to dual transformations; so, we do the transformation and while doing the transformations you maintain a kind of consistency having the with respect to the kind of constants.

And the procedure of dual simplex method is like that you know at least you know one of the right hand side of the constants should have a negative signs; after the transformations. If that is the case then dual simplex method can be applied and then you follow up the iterative process you know process and the kind of search for optimality till you get the optimal solutions or the kind of indication about the infeasible or unbounded or multiple.

So, whatever may be the kind of scenario, but ultimately this is what the starting procedure through which you look for the optimal solutions.

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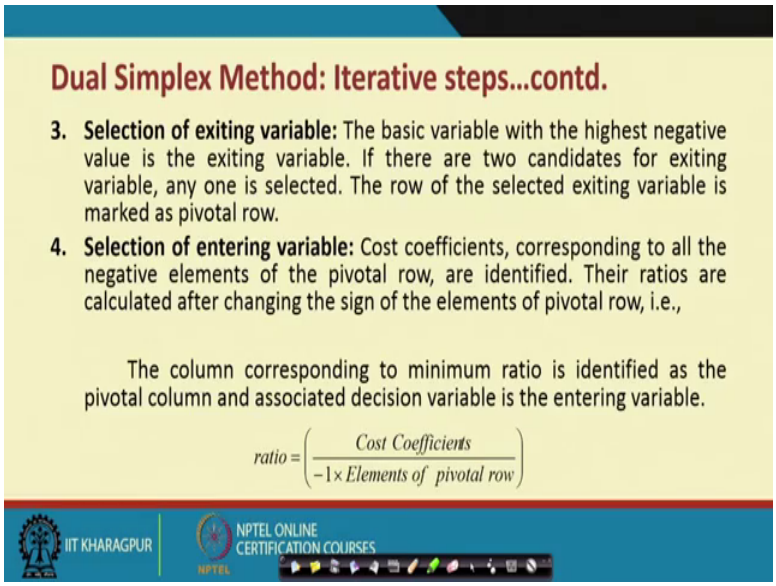

Then accordingly the selection mechanism will be like this selection of existing variable the basic variable with the highest negative value is the; you know what is called as you know outgoing variable. And if there are two kind of situation for you know kind of living vectors and new one can be selected; the row of the selected you know exiting variable is mark as you know pivot kind of row. That is what actually the kind of indication and then selection of entering variable that is you know what is called as an incoming vector.

So, that typically depends upon the cost coefficients corresponding to all the negative elements of the pivot pivotal row and that to or that to be identified. And their ratio are calculated after changing the sign of the elements of pivotal row; that is the column corresponding to minimum ratio is identified as the pivotal column and associated decision variable is the entering variables.

So, this is what the kind of transformation which we like to have here. And so obviously, it is the first hand requirement is the a transformations as per the dual simplex requirement. So, after doing the dual transformation requirement and then check the coefficient of the; you know constant that to right hand side and then a look for the outgoing indications.

And corresponding to the particular outgoing indications we look for the incoming indications and for that we need to have actually cost coefficients and the a values of the

a decision variables corresponding to the incoming vector in a row ok; so, that is how the kind of requirement. So, then we look for the kind of solutions let us see here how is the kind of solution.

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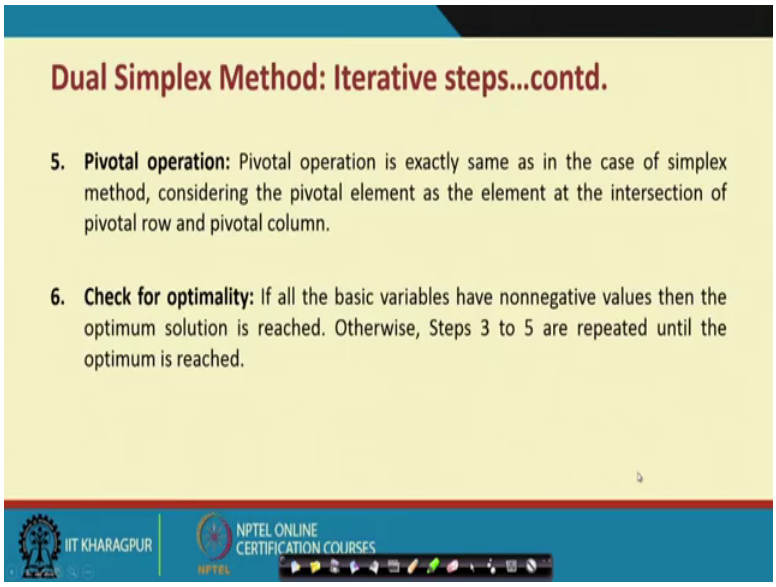

So, the key operation is a exactly same as in the case of simplex mechanism. So; that means, technically in the case of primal, so incoming vector and then outgoing vector then the key element that key element will be the unit matrix format. And accordingly will change the ordering and look for the optimality and again in the case of dual simplex method; this is also similar procedure. Only difference is we start with a outgoing past and then in coming second where in the case of simplex mechanism; we look for incoming past then looking for the outgoing second. Ultimately the pivot element will be in the similar kind of indication.

So, you start with incoming and outgoing again we start with outgoing and incoming. So, the inter intersection point will be tetras you know pivot point through which the entire you know iterative process really again give you the kind of structure through which you can look for the a improve optimal solution or improved basic feasible solution through which you can address the business problem provided so, that particular steps would be the optimal step.

If not then you will again move forward till you get the optimum solution as per the particular business requirement. So, now, check for optimality is the if all the basic

variables have non-negative values then the optimum solution is reached; otherwise will you know continuously move from step 3 to step 5 and repeat the process till you read reads the optimum solution or some kind of indication; about the unbounded unboundedness or some kind of infeasible solution or some kind of multiple solution.

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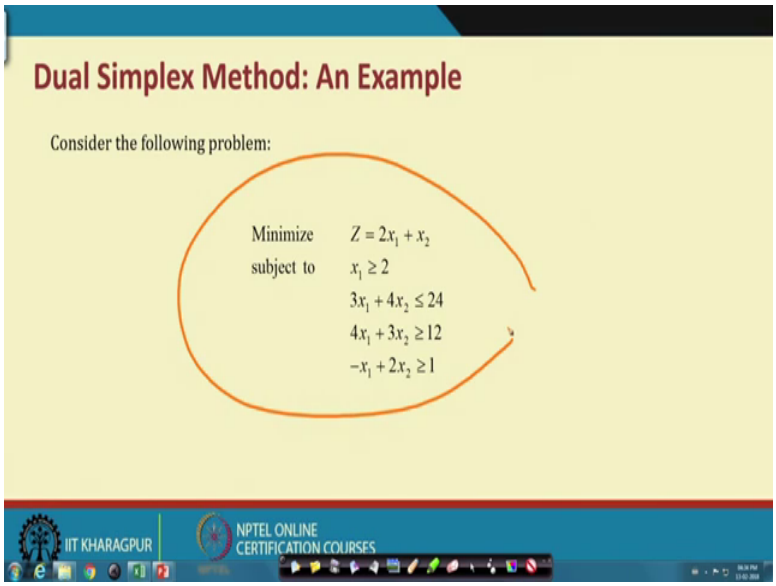

So, now, let us start with a simple kind of examples let say this is a problems which you have actually minimization type. And that too minimizing X equal to 2 x 1 plus x 2 subject to x 1 greater than 2 and 3 x 1 plus 4 x 2 less than equal to 24; 24 x 1 plus 3 x 2 greater than 12 and minus x 1 plus 2 x 2 greater than 1. So; that means, we have two variables and 4 constraints and you know some are you know less than type and some are you know greater than type.

So, now, we like to solve this problem in a kind of dual simplex mechanisms. In fact, we can solve this problem in a kind of primal simplex mechanism, where we will start with the initial basic feasible you know structure. Then look for the incoming vectors and the outgoing vector; that means, the kind of basic matrix structure through which you can reach the optimality.

And we will continue the procedure by searching incoming, outgoing and then we reach the optimal solution and the values of the decision variable through which you can address the business problem and the kind of management decision. And in the case of dual simplex mechanism; the procedure is more or less same, where the iterative process

will follow you know from the outgoing to incoming; then the kind of structuring and restructuring till you get the optimal solutions as per the particular business requirement. So now, corresponding to this business problem and the particular firsthand problems.

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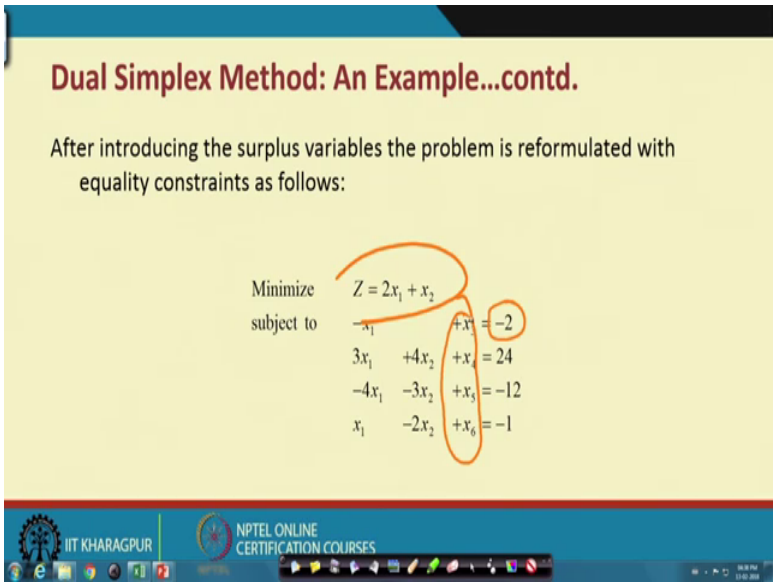

So, we will move how you can go to the dual simplex procedures. See here see this is actually minimization type that is the with respect to two variables and subject to 4 constants and where 3 are greater than type and 1 is less than type.

So, now after you know transformation so; that means, the first hand transformation is make the constant into equality by introducing slack and surplus variable wherever it is required and then look for the solutions. So, what we have done here? So, we actually transfer all the constant into a particular safe by changing the sign of the structure for instance you know here x 1 greater than 2.

So, what we can do? You can put you know minus x 1 less than equal to minus 2. And then we can address you know slack variable to address the kind of situation; that means, we transfer all the constant into less than type and then introduce or the kind of slack variable through which you can actually address the business problem as per the particular requirement.

So, now the objective function structure is more or less same here and then the constraints these are all called as you know the kind of added variable through which

actually we can address the a dual simplex mechanism. So, the first and dual simplex mechanism as after the transformation is at least one of the b coefficient that is the right hand side of the constant should be negative 1 and that will give you the indication about the outgoing vector and after that will calculate the cost coefficient.

And then look for the incoming vector then look you know check the optimality if you reach the optimal optimality you can stop and you know address the problem or interpret the problem as per the management requirement. If not then again you move to the second step, again you look for the outgoing vectors incoming vectors and again check the optimality and then you know look for the optimal solution and values of the distant variable.

So; that means, again it is a kind of continuous process like you know simple primal structures that is the primal simplex mechanism. And a dual simplex mechanism also more or less similar kind of iterative process or the kind of continuous kind of improvement through which you know will proceed and to get the optimal solution to address the same business problem as per the typical requirement. So now, corresponding to these; so this problem; so this is how the first step of the process for the dual simplex method.

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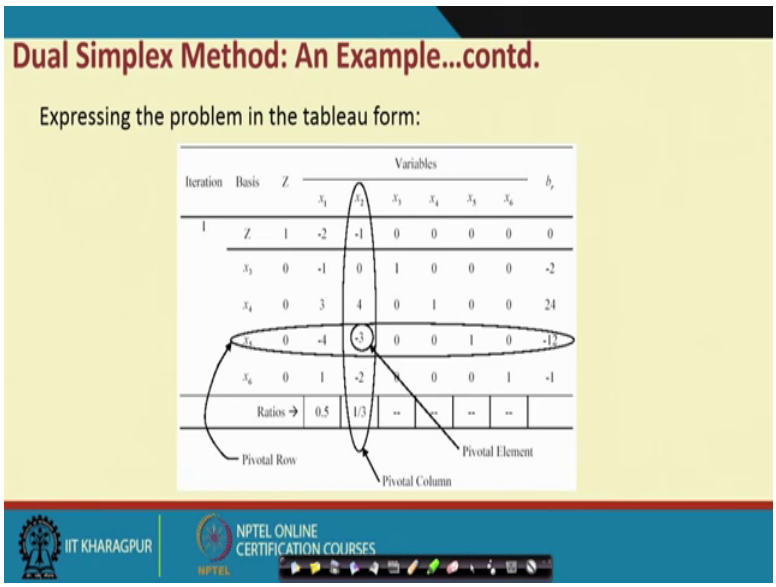

Then we develop the kind of kind of matrix and that to. So, these are the following variables.

So, means with which one will be the basis matrix that depends upon the unit matrix so; that means, technically. So, x 3, x 4, x 5, x 6 these are all; so, here. So, these are all actually designed a to have a you know 4 basic matrix and that is the unit matrix. So, as a result in the first hand you know initial setup. So, these are the entries to the basis matrix and to the kind of optimal solution and as a result here x 3, x 4, x 3, x 3 the kind of structure and the kind of structure here ok.

So, this is what the kind of structure. So, the x 3, x 4, x 5 and x 6 and then look for the kind of solutions; obviously, the solution will be here ok. So, obviously so, x 3, x 4, x 5 and x 6 these are the basic variables and then accordingly these are the input coefficients and x 3 x 4 x 5 x 6 are all unit matrix. And then this is what you know unit matrix structure and; obviously, first hand requirement is here the kind of outgoing variable indication.

And then we can look for the kind of solutions that to highlighting the kind of indication about the incoming vector and ok. So, this is what the; that means, technically x 5 is the indication about the outgoing vector. So; that means, we have actually 3 you know coefficients b coefficient that is negative.

So, we will take you know a most negative value that is here you know x 5 that is minus 12 compared to minus 2 and minus 1. And then you know means that is the indication about the outgoing vector and again we look for the incoming vector for that we like to find out the ratio which depends upon the cost coefficients divided by the kind of elements of the a outgoing vectors. And then look for the kind of result in this case most negative and in this case again maximum value.

So, means this is the kind of indication through which you can get the kind of indication. So, here it is a kind of most negative and this is how the kind of minimum ratio through which you can get the pivot element and once you get the pivot element. So, that is the indication about the a kind of unit matrix and again the adjustment will be x 2 will be coming in the place of x 5 and x 5 will go to in go to the kind of position of x 2. So, this process will continue till you get the optimum solution here you know really you know address the business problem.

And accordingly the second iterations; so, this is what the first hand iterations and so, after the first hand iterations the a typical structure is x 2; it will be coming in place of x

5 and x 5 will go to the place of x 2. Then you will then you will address the kind of structure again look for the optimality and then accordingly you move to the second step here.

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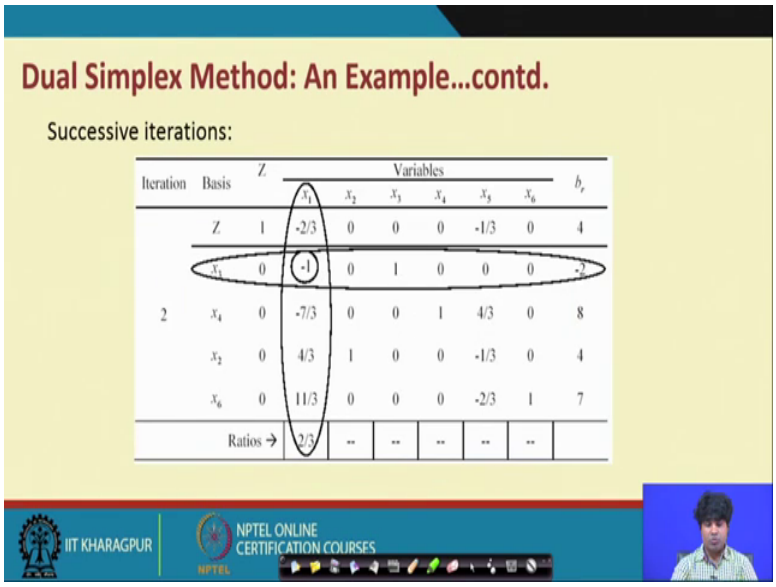

In the second step that is the improvertions after the first you know iterative process procedures and here we have only one negative v that is with respect to x 3. So, by default that will be the outgoing vectors and that will be the outgoing vectors and then we look for the incoming vectors and here there is only one case, where you can get the kind of incoming vector because in other case the coefficients are not actually negative.

So, as a result by default this will be the incoming vector so; that means, this is a kind of unique scenario here only one negative. So, by default that will be the outgoing vector indication and here it is the only one ratio through which you can choose the incoming vector. So, by default the indication is that you know x 1 will be coming in place of x 3 and x 3 will go to or in go to the place of x 1s then again you know change the typical structures and complete the iterative process ok.

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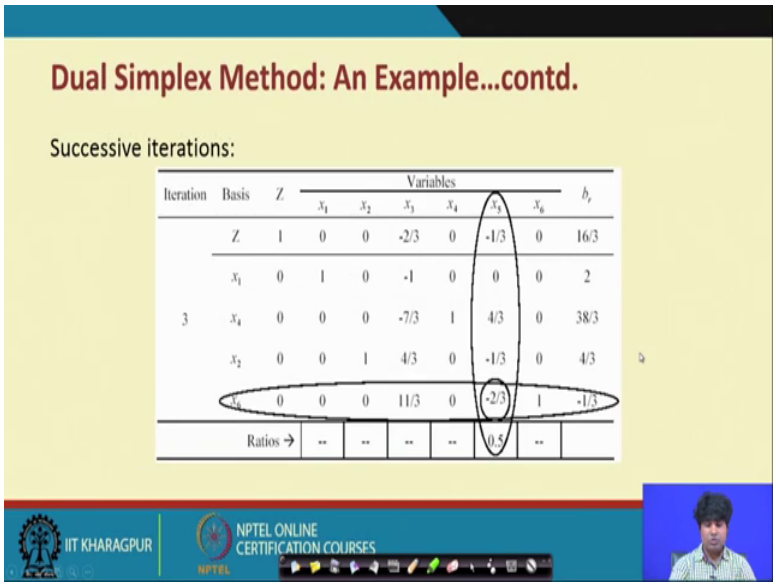

So, after doing the iterative process then we move to the next table that is the a third successive iterate iterative procedure. And here again like the previous case we have only one negative you know b coefficient and that too in place of x 5. So, by default x 6 will be the outgoing vectors and again corresponding to the minimum ratio or you know means the cost ratio between cost coefficient and the outgoing variables indications. So, only one ratio is actually coming here because x 5 is having only negative value. So, again it is a kind of unique here no extra kind of effort required. So, here this is the indication about the outgoing and this is what the indication about the incoming.

So; that means, technically x 5 will be coming in place of x 6 and x 6 will go from the basis matrix and that to in place of x 5. Again complete the iterative process by putting the pivot event to 1 and remaining element 0 and then you know move to the successive iterate iterative process.

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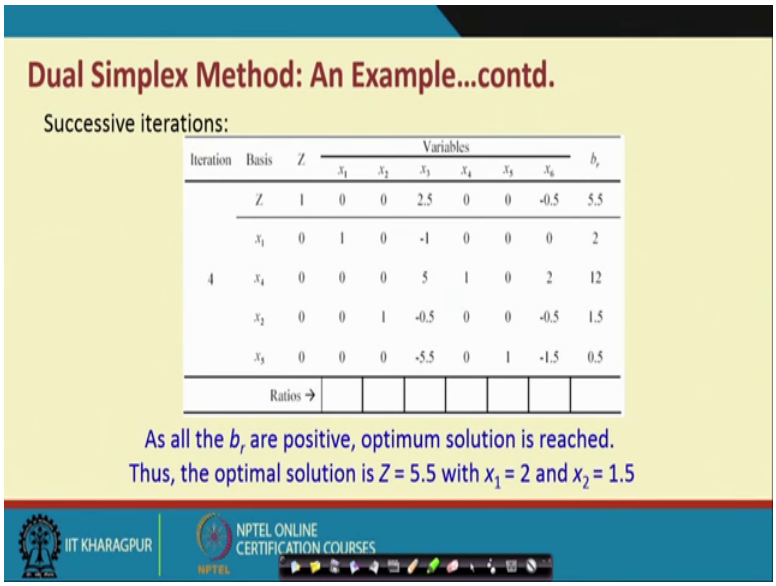

And here you know after you know third iterations will be fine all the coefficients of b are you know coming positives. So, that is the indication of dual simplex optimality so; that means, we start with the initial basic feasible solution and then move in a kind of iterative process where all b will be coming positive.

So; that means, we can apply dual simplex after the kind of transformation first hand transformations. And we can move to this particular process that is the dual simplex process. So, with the condition that you know at least one of the b coefficient that is the right hand side of the constraints should be negative; then you move on you know in a kind of iterative process till you get optimal solution. So, the this point of the optimal solution is like that you know all the b coefficients that is the right hand side of the constant should be positive.

And a corresponding to these problems so, the problem is here. So, the corresponding to this problem this is the first hand transformations where we have a you know two negative values of the b coefficient; then in the first this is two in the first you know iterations. So, we have actually 3 negative you know coefficients and in the second iterations we have one negative constants coefficient. And again in the next step we have again one negative b coefficient.

So, that is how we can we are means; that means, we are in a position to find out the kind of outgoing and the kind of incoming. So, again in the final so; that means, in the this

particular step which you get all as a final step or the; that means, declaration of the optimality. Because we have the it is the condition where you know all the b coefficient that too right hand side of the particular constants are positive.

So; that means, this indicates that you know we have reached the optimal solution and; obviously, in the basis matrix in the final optimal solution we have x 1, x 4, x 2 and x 5 and corresponding to the particular b indications. So, the x 1 value is equal to 2 and x 2 value equal to 1.5 because x 4 and x 5 originally not in the original you know problem.

So, these are all you know kind of slacking or surplus variable through which you can you know start the process to get the optimality. And since the problem is with respect to x 1 and x 2 and; obviously, x 1 is giving to you know optimal value of 2 and x 2 is giving the optimal value of in 1.5. Now you put this value in the objective functions and then the value of digit will be coming you know 5.5 which you really have here actually that is the Z indication. And; that means, technically we have reached the optimal solutions and that is in the process of dual simplex mechanism.

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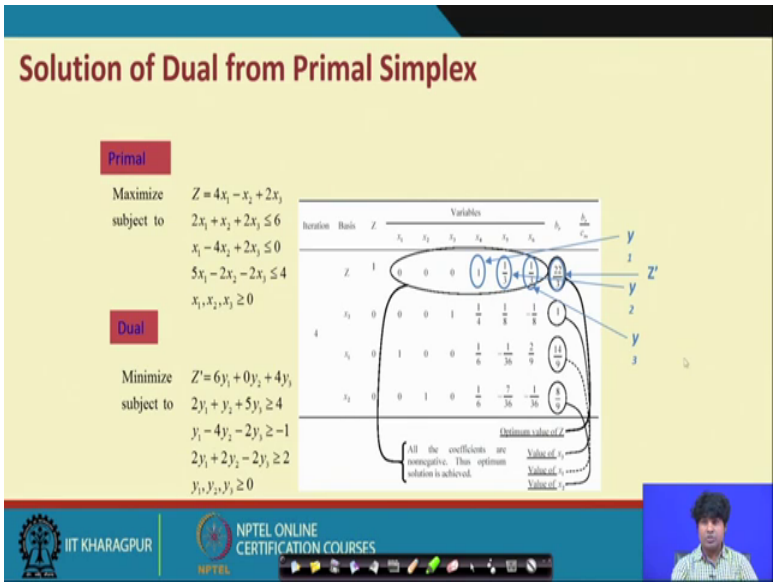

So, solution of dual you know from primal simplex is you know so, that the that is to start with you know maximizing objective function with you know less than type up you know constant 3 less than type of constant. Then the typical structure will be like this and now corresponding to the dual simplex structures you can do the kind adjustment again look for the optimal solution.

In fact, more or less the typical you know kind of iterative process is more or less same. Only thing is you know the choice of the incoming vector and the outgoing vectors; obviously, there is no issue about the kind of incoming and outgoing mechanism here the idea is you know corresponding to a kind of business problems, we look for the optimal values of the decision variable through which you can address the business problem more effectively more efficiently.

And whether you apply actually simples you know kind of primal simplex structure or dual simplex structure; that is not a kind of issue, but ultimately we should have a kind of optimal solution through which you can address the business problem. Because our ultimate you know idea is a you know to look for the solution to a kind of business problem, whether you follow the primal simplex structure or dual simplex structure that is actually not a kind of issue, but ultimately we should have optimal solution through which you can you know solve the business problem and come with the kind of management decision as per the particular in the requirement.

So, obviously corresponding to this problem primal problem; so our dual structure will be like this and again so, the final solution will be lying here only and that to x 1 is equal to you know x 1 equal to 14 by 9 and x 2 equal to 8 by 9 and x 3 equal to 22 22 by 3. So, this is what actually the kind of game between if you know primal. So, this is what actually the kind of transformation see here y 1, y 2 and y 3.

So, the first hand is the dual a primal indication x 1, x 2, oh x 3, x 4, x 5, x 6 and corresponding to this you know primal maximization Z and that to subject to constant since there are 3 constants. So, we have a 3 unit matrix that is x 4, x 5 and x 6 and again corresponding to this dual problem will be y 1, y 2, y 3.

And as a result this x 4, x 5, x 6 will be having actually kind of dual transformations. And that too so, the value of the objective function and the Z is coming actually 22 y 3 and that is how the kind of game between primals simplex method and the kind of dual simplex method.

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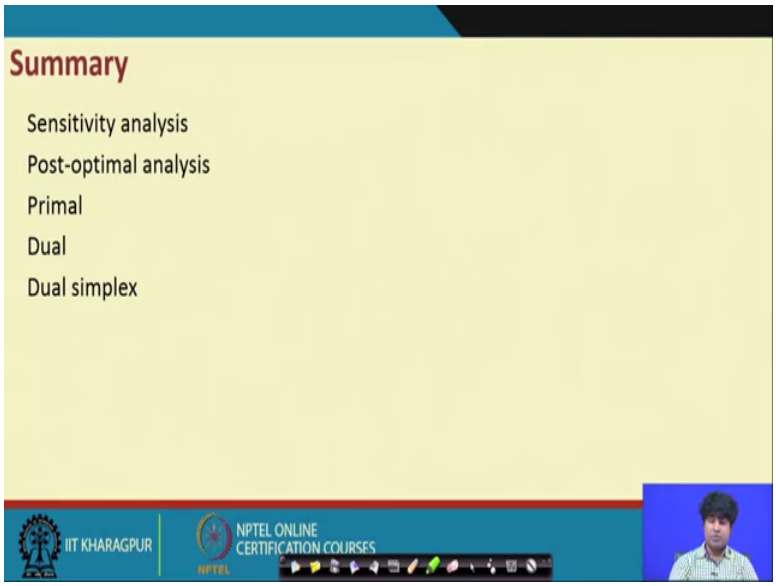

So; that means, typically I know in this particular lectures; we have means we have discussed a kind of concept that is the dual simplex method which is a kind of alternative tool to a kind of problems where we have already applied primal simplex method and look for the kind of optimal solution to attach the business problem.

Now dual simplex method is a kind of alternative mechanisms which can address the same problems in a kind of different angle altogether. But ultimately will have a solution for a particular business problem and that to look for the kind of management decision. The way which you have already highlighted if the primal is having optimal solution the dual can have optimal solution so; that means, if you have a primal problems and applying simplex methods. And if the problem is consistent and getting the optimal solution; obviously, the same problem if you will apply through dual simplex methods; you can also have the optimal solutions and whether you are getting the solution through primal or getting the solution from the dual simplex method we have no business ultimately.

We need actually solution to which you can address the business problem and then come with a management decision as per the particular business requirement. And separation of post stock analysis is concerned, we can also go for the sensitivity structure then the kind of change of the particular requirement like you know the change of the coefficients of the objective functions you can go for you know adding you know more constants

dropping few constraints and you know change of the right hand side of the coefficient left hand side of the coefficient.

That means, technically since you know business is very dynamic. So, these changes are you know always there and depending upon the original situation and the change situations; we can we can also have the kind of flexibility and look for you know various you know alternatives depending upon the a changed situation. And we through sensitive analysis will get you know optimal solution again with respect to the changing situation. And then, that will give you more practical or you know more value addition to the business problem through which you can analyze the business problem more effectively and more efficiently. And with this we will stop here.

Thank you very much. Have a nice day.