Business Analytics for Management Decision
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Lecture – 48
Prescriptive Analytics (Contd.)

Hello everybody this is Rudra Pradhan here welcome to BMD lecture series, today we will be continuing with Prescriptive Analytics and, that to coverage is on duality problem. In the last couple of lectures we have discussed so, many problems connecting to prescriptive analytics and in fact, what we have you know done is we have 2 different kind of new clusters, linear programming and non-linear programming and, the kind of you know problems which you have discussed in the case of linear programming is nothing, but called as you know primal problem.

And the same problems which you can solve or you can analyze in a kind of in a different angle, that is what it is called as you know dual problem so; that means, the problem in it is original form is called as a primal problem and, it is counterpart is called as a dual problems so; that means, we have discussed several you know business problem and, the whole idea of linear programming problem is you know to transfer the business problem into a model; that means, in the form of linear format, and that to called as you know LPP linear programming problem.

And the moment you will transfer the problem to a model in a LPP structures, then the first hand interpretation or the first hand introduction to the problem is called as you know primal problem and, then the same problems which you can analyze in a different angle is called as a dual problem. So, like you know you know in the case of you know primal structures that is the first an introduction to LPP.

So, we have objective functions we have a constraints and, we have a conditions and, we are looking for the optimal solutions or the optimality that to values of the decision variable, through which you can address the business problem more effectively and more efficiently. And infact in the dual problem the same business problem can be addressed in a different angles, where we have also objective functions, we have also constraints, and we have also conditions through which we can look for the optimality that is the values of the decision variable through which, we can also address the business problem more effectively and more efficiently.

So, this lecture specifically highlights the LPP that to in a primal setup, and how we can actually analyze the same problems in a dual dual setups so; that means, like we have gone through lots of you know robustness checks, and sensitity, you know analysis. So, this is another kind of you know structure through which the same business problem can have a 2 different structure through which you can analyze the issue and, then we can come with a kind of you know management decision or through or through which we can you know address business problem more effectively.

So, let us see how is the kind of you know primal structure, and how is the you know dual structure corresponding to the primal structures.

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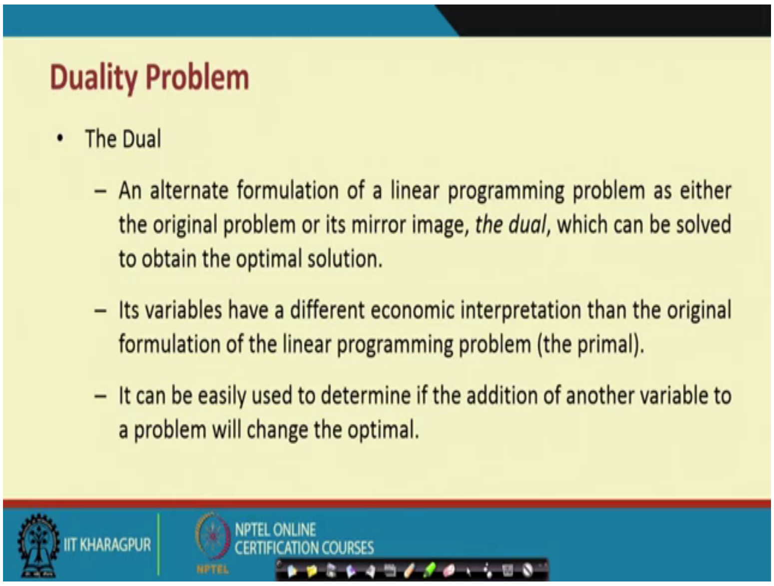

Let us you know start with the primal structure first and in fact, in the dual problem you know corresponding to the primal, it is an alternative formulation of LPP that is either the original problem, or it is mirror image that is the dual, which can be solved to obtain the optimal solution. it is variable have a different kind of you know economic interpretations corresponding to original formulation of linear programming problem, that is the primal you know a structure so; that means, we have LPP, LPP in it is original for called as a primal problem, and the counterpart is called as a non-dual problem.

It can be easily used to determine, if the addition of another variable to a problem will change the optimal kind of inner structure so; that means, if the primal has a kind of in a solution, then dual can have also solutions, but it will give you another kind of you know structure or you know flexibility through which, we can get the optimality and, then we can address the business problem as per the particular you know requirement.

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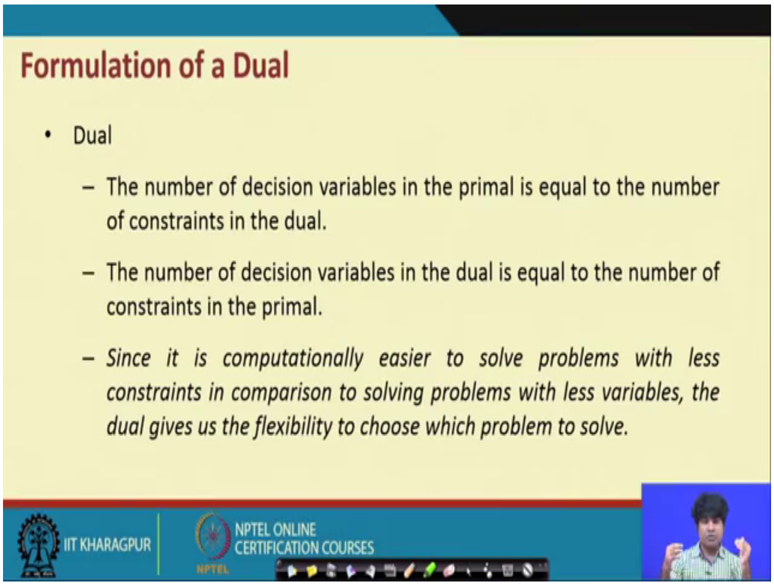

So, in the dual set of the typical structure is like this, the number of decision variables in the primal is equal to the number of constraint constraints in the dual. So, the vice versa is also true, the number of decision variables in the dual, is equal to number of constraints in the primal.

So; that means, there is a close connection you know in correspondence to primal and dual. So, the this is the number of decision variables in the dual is equal to the number of constraint in the up you know primal. Similarly the number of decision variables in the primal is equal to the number of constraint in the dual so; that means, there is a you know high integration you know with primal and, dual since it is a you know in you know computational issue, you know depending upon depending upon a particular structure where we have a multivariate variate frameworks.

We have more number of variables is more number of constraints, then the structure of primal dual can address the problem more effectively, and that too in a kind of in a simpler structure. So, sometimes if the primary is a complex kind of you know structure there is a high chance that dual can be little bit simpler than the primal, and the vice versa is also equally true.

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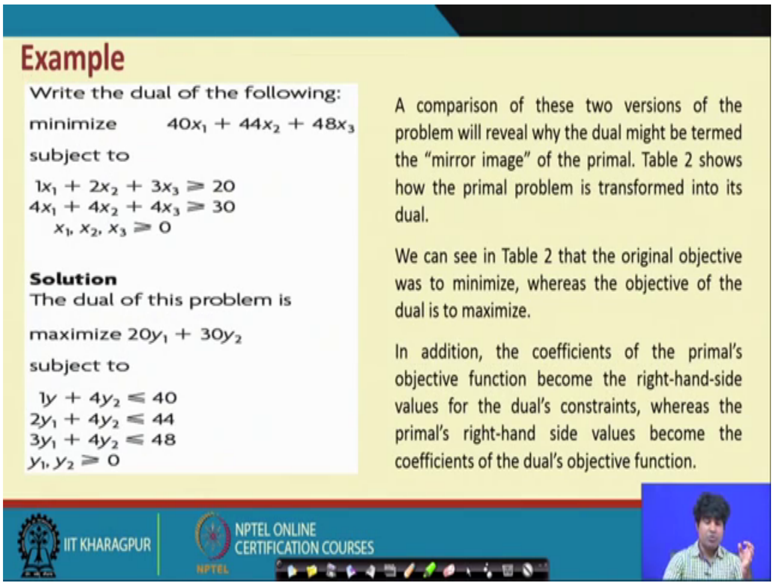

So, once you start with the kind of in example, then we can effectively address the issue for instance. So, let us start with the kind of you know structure.

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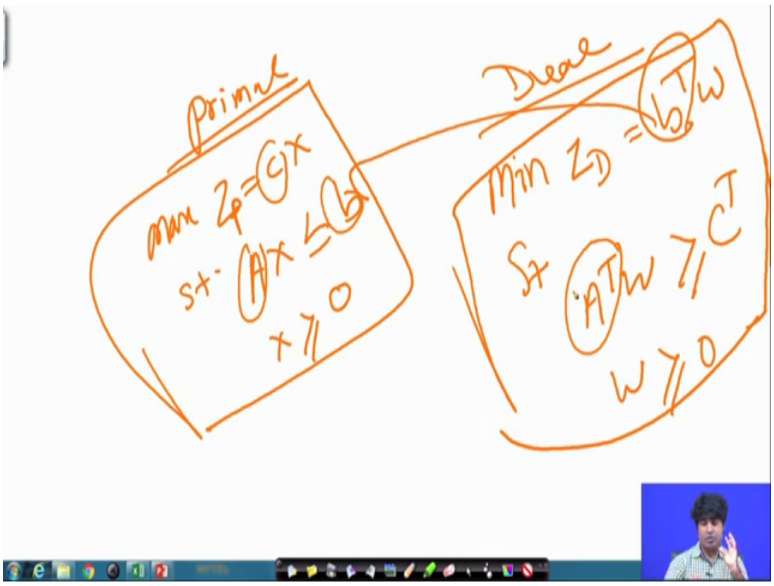

So, let us say let us start with the in a fast and primal structure, in the primal structures let us you know set the objective functions maximize, Z equal to c X and subject to the constraints are X less than equal to B then X greater or than equal to 0. So, this is for the first and LP linear programming problem and, that to we called as a primal you know problem and, then this is a model and then in the model requirement is to find out the optimal values of X that the values of the decision variable through which, we can maximize the objective functions, and then address the kind of in a business problem as per the particular requirement and, then we will go for the kind of you know management decision.

Then the same problems if you it if will come in with a kind of dual structure, then the objective functions will be transfer the kind of you know you know safe that is you know since it you know it is a maximization type in the case of you know primal. So, in the case of duals it is the minimization I know of minimization function that to minimize the Z this is called as you know Z P that is the primal, and this is called as a Z D that is a dual, and minimize Z D that is with respect to b T, then there is a variables college and W that is the dual variable, and subject to A T then W greater than equal to C T, then W greater than equal to 0.

So; that means, if you if you look this is what the dual structure, if you will look here then, you will find you will find the kind of you know structure are you know more or less same. So, we have objective function here we have a constraints and we have conditions, and in the dual case it is also having objective function. So, we have a constraints and, we have a conditions. And the thing is that you know the same problems, we are you know addressing more you know you know effectively, and in a kind of in a different structure that to so, in the case of in objective functions.

So, the b T is nothing, but you know a b T from here only so; that means so, b T stands for you know it is a transpose. So, that means, technically these are all vectors. So, c A b r you know vectors. So, now so the mean all right hand side of the constraints will be the input to the objective function that to the coefficients of the objective function, and again the input of the copies you know constraints coefficient that is the left hand side is nothing, but you know transpose of you know original input coefficients and, then the right hand side of the constraint will be the transpose of objective function coefficients and, W is the dual decision variables, where X is the primal decision variables.

So, then you know; that means, technically the coefficients of in objective function the coefficients of the constraint in the left hand side and, the coefficients of the coefficients of the constraint in the left hand side, and these are the kind of you know you know you know inputs and, that to we are just you know changing the particular you know structure so; that means, you know the original problem. So, we can you know put in a kind of in a matrix format, then we can transpose the matrix so; that means, the angle we are you know just changing, and then we are you know addressing the business problem as per the particular requirement.

So that means, the coefficients of objective function, and the coefficients of constraints are you know changing it is role from primal to duals and, that to address the same business problem in a kind of you know different shape and, the kind of you know the kind of you know requirement. So, you know this is how the typical difference between primal structure and the dual structure.

And let us you know start with a simple examples and, then we like to see how is the primal structure and, how the counterpart the dual structure. For that we can go to the particular you know structures and, then only see the kind of you know requirement let us start with this particular you know structure here a so, this is what the original linear programming problem and, we have objective function minimize 40 X 1s plus 44 X 2 plus 48 X 3 subject to 2 constraints, and that 2 greater than type so; that means, this is as usual in a general case.

Since we have a minimization you know objective functions, then the constraint constraints you know structure is greater than type and, that is what you know the first hand problem is like, the and the corresponding counterpart is the maximizing objective function corresponding to constraint and that to less than type so; that means, technically if you see here is all the coefficients of the objective functions and constraints are you know derived from the primal structure so; that means, the if we will move from dual 2 primers same thing the inputs will be transferred to the dual structure.

So; that means, technically so, we start with the minimizing objective function 40 X 1 44 X 2 48 X 3 so; that means, there are 3 decision variables, and 2 constraints. So, what we have already addressed. So, in the case of you know dual the number of constraints depends upon the number of decision variables of the primal problem and, number of the decision variables will be the number of constraints for the dual problem. So, as a result here we have 3 variables this is some variables X 1, X 2, X 3 and as a result.

So, the is so, there are 3 constraints first constraint, second constraint, and third constraints in the dual problem, and again we have a 2 constraints in the primal problem. So, so as a results so, we have 2 decision variables in the case of you know dual so; that means, the number of decision variables in the primal will give you the number of constraint in the dual, and number of constraints in the primal will give you number of decision variables in the duals that is what the you know they are called as you know counterpart.

So, the cited example is here. So, we have 3 decision variables in the primal. So, as a result. So, there are 3 constraint in the dual, and we have a 2 constraint in the primal. So, as a result we have 2 decision variables in the dual. So, this is how the you know typical structure about the primal and dual and, then a whatever you know the constraints are there. So, in the in the case of you know dual so, it will be just opposite. So, for instance in this case we start with all greater than type. So, the counterpart dual it will be all you know less than type and, then the condition, condition is more or less same in the case of primal and dual.

So, the. So, far as a comparison is concerned, so, the these 2 decision are you know it means these 2 (Refer Time: 13:51) the primal and you know dual has a different kind of you know structuring that 2 minimization of objective function to maximization of objective function and, again counterpart minimization of dual is the maximization of you know primal and, then the constraint only a you know change as per the particular you know objective requirement.

So, how is the particular in us you know structure and restructure through which you can address the problem, then we will go to you know you know another example through which you can address more effectively.

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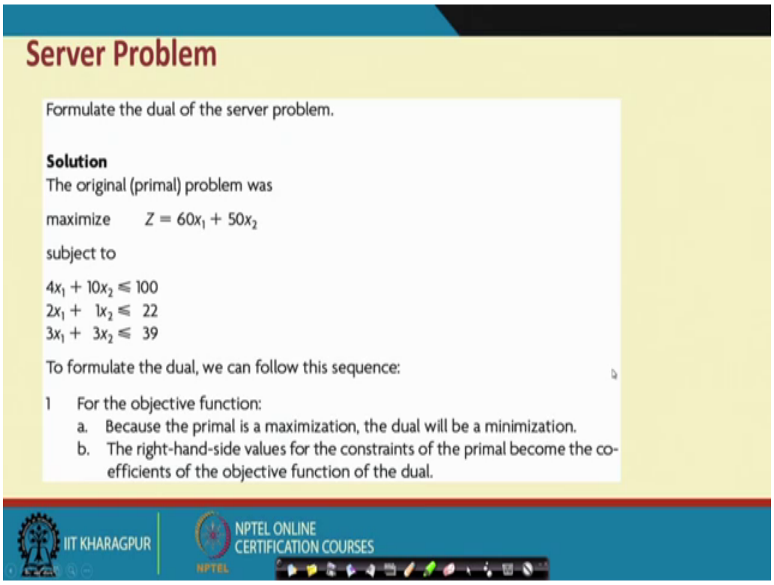

So, now so this is how the kind of you know example here. So, the original problem that is the primal. So, maximize Z equal to 60 X 1 plus 50 X 2, and subject to 3 constraint 4 X 1 plus 10 X 2 less than or equal to 100, 2 X 1 plus X 2 less than equal to 22 and 3 X 1 plus 3 X 2 less than 39.

So, now here the objective functions that is the primal is the maximization, and the dual will be the minimization so; that means, typically if you look into the business problem. So, usually if the objective function is a profit type, then our structure is to optimize profit or maximize profits subject to cost constraints right. So, now, when we are transferring the kind of you know problem into the dual, then the simple interpretation for the dual will be minimizing the cost subject to profit requirement right.

So, that is how the kind of you know variation between primal understanding, and dual understanding. In fact, same business problem here the objective followed by certain constraints, and now the particular constraints will be the objective kind of you know structure through which you know the particular you know objective will be the constraints right. So, that is how there is a close connection between the primal objective and the kind of you know dual objectives.

So, if the primal starts with the maximization, then dual will be the minimizations, and if the primal starts with minimization then dual will be the maximization and, accordingly the constraints will you know change the particular you know structure less than to greater than and greater than to less than, and then conditions are more alert more or less aims that is what you know the values of the distant variable should be in a kind of you know positives positive shapes that is how the particular you know linear programming structure.

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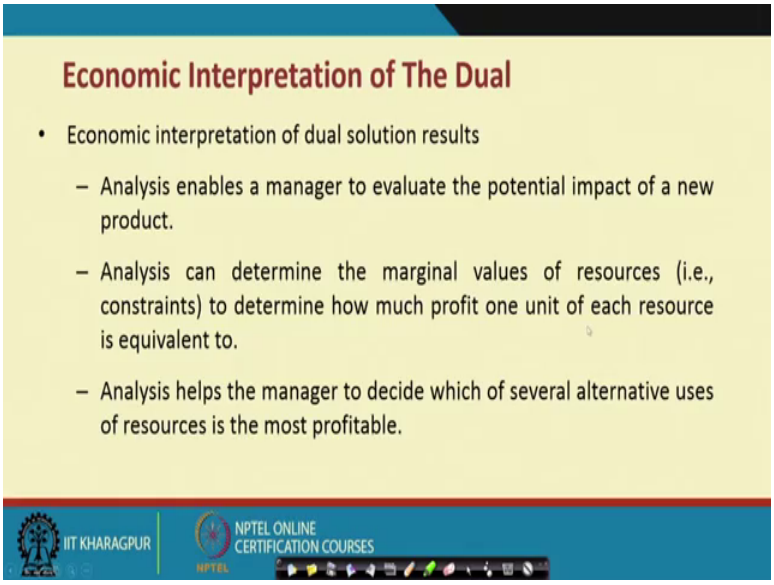

So, now corresponding to this particular you know problem. So, the economic interpretation is you know like this. So, means corresponding to this problem. So, you know the analysis you know give you the kind of an idea that you know to evaluate the potential impact of a new product, and the analysis can determine the margin marginal values of the resources that is the constraint, to determine how much profit one unit of each resource is equivalent to and, this analysis can help the manager to decide which of several alternative uses of resources is the most profitable. So, this is how the you know economic interpretation of the dual corresponding to the a primal requirement right.

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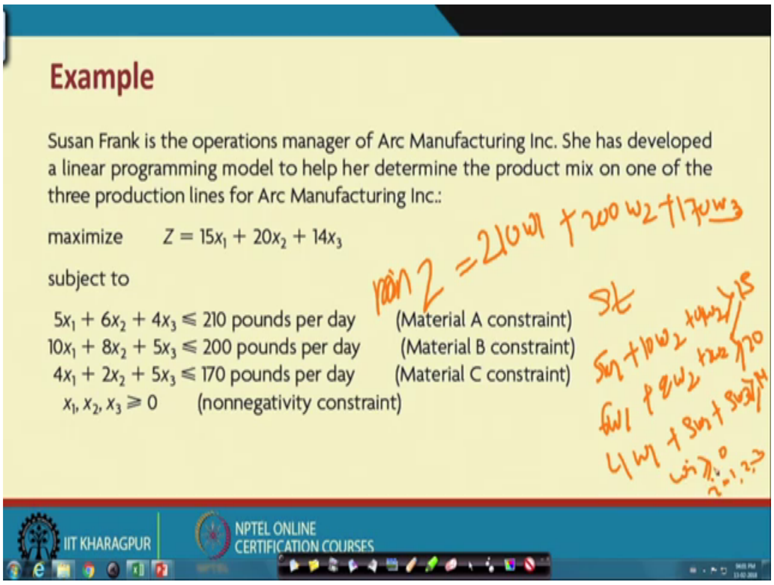

So, now there is another example here. So, the idea is here to maximize Z equal to 15 X 1 plus 20 X 2 plus 14 X 3 subject to 3 constraints, and that to less than type. So, the first constraints is the material constraint that is material A, second constraint is material B, third constraint is material C.

So, now again the non-negativity restrictions, then corresponding to this primal problem, then the you know you know we can have the kind of you know solutions; that means, tip typically since it is a 3 variables case that is the multivariate structure. So, usually a a graphical solution and graphical structure may not help you. So, you can go to these simplex structures, then you can solve the problem since it is the maximization type and less than type a simple simplex methods can be apply that to get the values of the decision variable that is the kind of you know optimality and, they then look for the optimal solutions.

So, now you will get the values of the decision variables, you know setting the objective functions the kind of you know constraints, and the same problems you can take you know you can take you know you can solve through the kind of you know solver package, just to you know it put the inputs and give the kind of you know indication about the objective function the kind of you know constraint, then by default it will give you the optimal results, the way we have already highlighted the particular you know solution through solver package.

So, then by default you will get the optimal solution and that 2 values of the decision variable through which you can address the business problem more effectively. And again so, transferring this problem into the dual, then you can also look for the solutions and then you know again so, either you can actually use the solver package or the kind of you know method through which you can get the optimal results and, then to address the business problem. So, the unique of this problem is we have objective function with the 3 variables and, then there are 3 constraints so; that means, technically this is 3 into 3 you know model and, then the duality constraint dual in the case of you know duality problem.

So, see number of constraint will be the number of decision variables, since there are 3 constraints in the primal. So, here in the case of you know there are 3 constraints in the primal. So, by default so, there are you know the dual problem will be also with respect to 3 variables that to w 1 w 2 w 3, and by default the dual problem will be a 2 210 w 1 w 1s so; that means, typically it is the it is actually 2 210 w 1 plus 200 w 2 plus 170 w 3 that is actually this is maximization, then this is minimization of Z equal to this much.

Then subject to subject to constraints and that to the input constraints, you know will be interchange the positions so; that means, technically it will be it will be like this a 5 w 1 plus 10 w 2 plus 4 w 3, then greater than or equal to the first you know coefficient of the first decision variable. So, that is 15, similarly and then 6 w 1 plus 8 w 2, then 2 w 3 greater than or equal to 20 that is the second objective function coefficients, and then 4 w 1 plus 5 w 2 plus 5 w 3, and greater than or equal to 14 that is the third objective function coefficient.

And then w i greater than or equal to 0, where i equal to 1 2 3 right. So, this is what the typical structure of the dual problem corresponding to primal problems so; that means, if you go to the solver package. So, you put all these inputs and. then give the kind of in a command to you know get the values of the decision variable and, same way what will you do you go to the actually dual sides again is we said the inputs corresponding to the a problem structure here is of course, this you know these inputs are derived from the primal structure, with you know as per the dual requirement.

And then again is we look for the solution of the dual problem. So, so; that means, technically if every primal kind of you know problems and we have a solution to the primal then; obviously, we have a dual problem and again we can have a dual solution. So, this is how they are you know you know closely connected to each other to address the business problem and, then come with a kind of you know management decision right. Now in order to address this problem.

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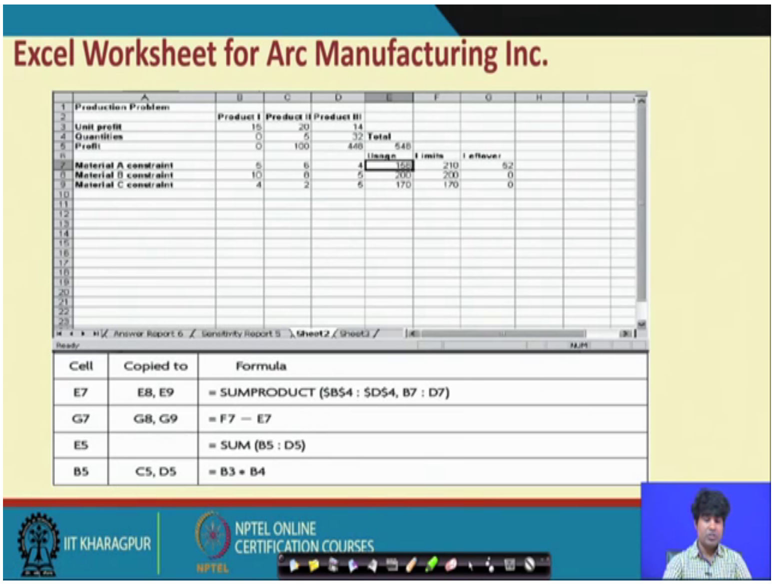

So, we go to the kind of in a solver package, this is what we have already you know got the kind of you know results. So, here is with respect to this 1s, then we have the unit profit 15 20 and 14 and then after the solutions.

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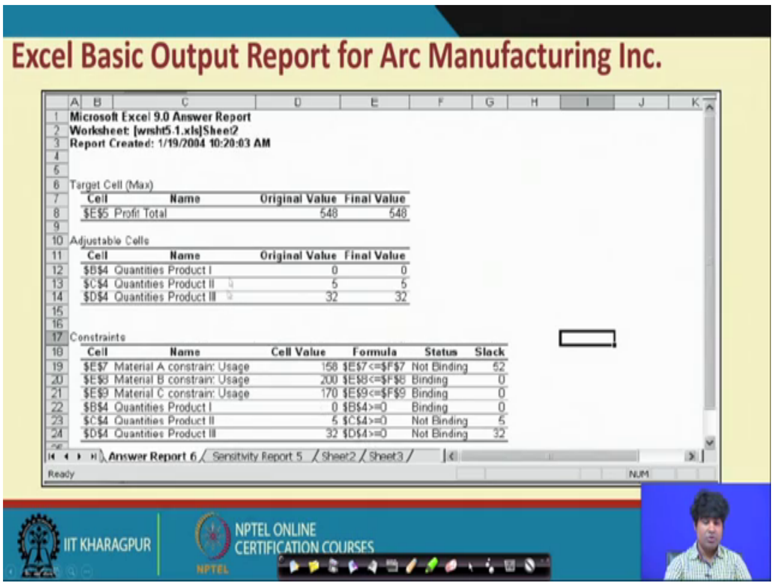

We have the kind of you know typical structure like this, and then these are all sensitivity kind of you know a structure.

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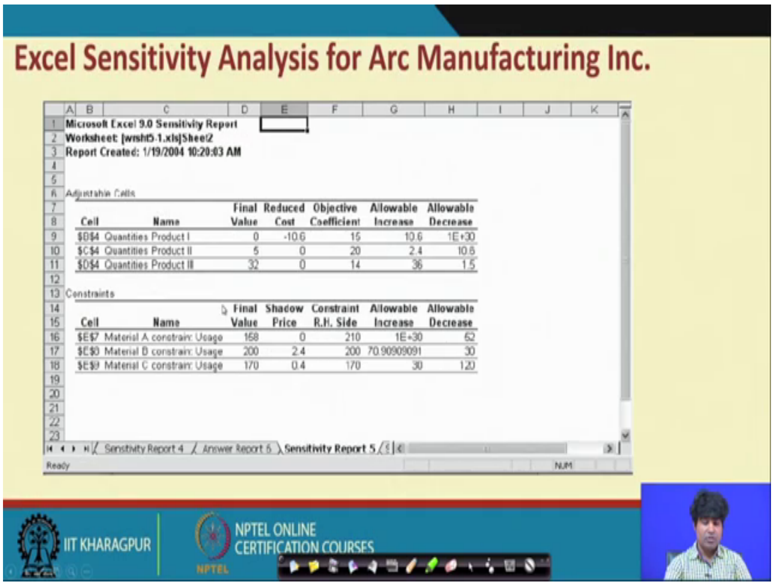

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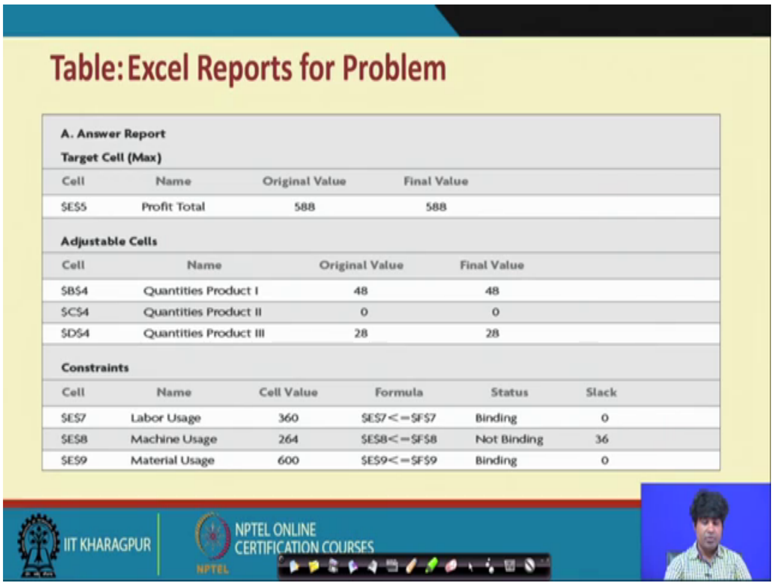

And then finally, the optimal solutions so; that means, to summarize this particular you know problem. So, the total profit the original structure will be 588 and the final value will be 588 then that is what the you know the kind of you know structuring.

And as a result the original values of the decision variable will be 48, and 28, and that to coming to the constraints, and 1 is the 1 is actually a first one is the binding second one is binding, and the sorry third one is the binding and second one is not binding so; that means, so the first constraint and third constraints are you know resources are fully utilized and optimized, and the second constraint are not fully utilized, where the values of the decision variables you know x 1 and x 1s are being the kind of you know kind of you know value, and that to optimizing the profit, and where the you know value of the profit will be evaluated. So, this is how the kind of inner structure to address the particular you know business problems, and agains they look for the kind of you know solutions, and then we will see.

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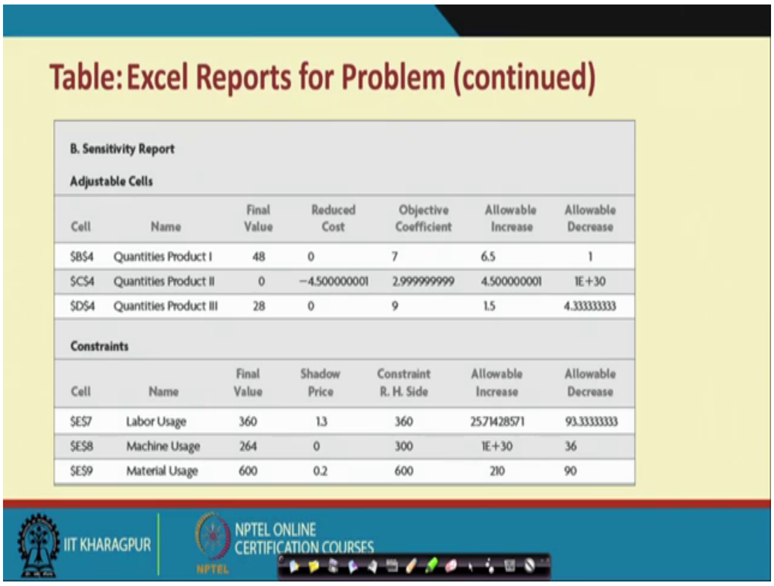

B. Sensitivity Report

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B4	Quantities Product I	48	0	7	6.5	1
C4	Quantities Product II	0	−4.500000001	2.999999999	4.500000001	1E+30
D4	Quantities Product III	28	0	9	1.5	4.333333333

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R. H. Side	Allowable Increase	Allowable Decrease
E7	Labor Usage	360	1.3	360	25.71428571	93.33333333
E8	Machine Usage	264	0	300	1E+30	36
E9	Material Usage	600	0.2	600	210	90

So, this is what the kind of you know final you know solver report through which you can address the a particular you know problem, and that to the kind of you know x 1 product and x 2 product and x 3 product, where the final optimal solution is a you know with respect to x 1 and x 3 only where x 2 is coming 0, and that is how the kind of you know optimal solution.

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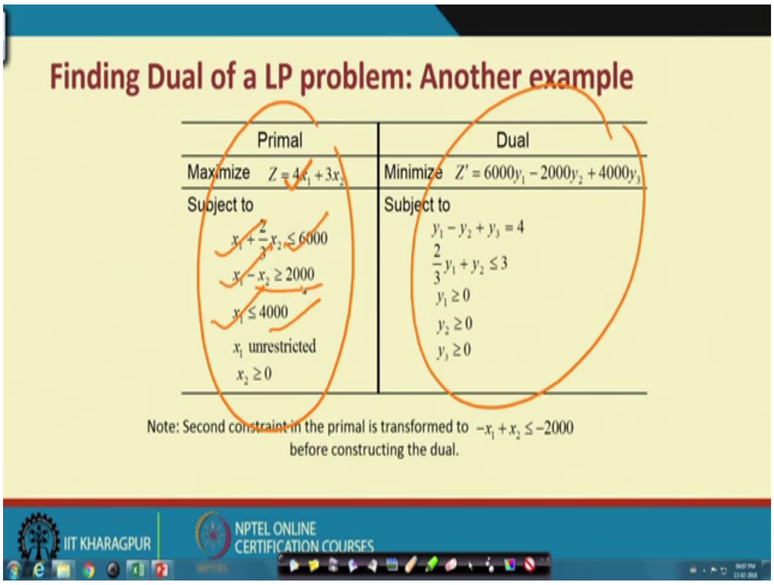

And accordingly we can have a kind of you know comparisons. Similarly, what you can do you can actually transfer this problem into dual what the wave I have just highlighted, then again look for the kind of you know solution in the same way of you know solver package you know. So, then finally, what is happening here you know, if you kind of you know make a kind of in a comparison then you find the typical difference. In fact, here there are you know typical difference.

So, this is what the primal structure and this is what the dual structure, and the problem which you have already highlighted to 3 problems. So, that is actually in a kind of you know simple structure and generalized structure, where if the objective function in the case of primal is the maximization the constraints are less than time, easily we can convert into dual problem, where objective function is minimization type, and the same less than type of constraint will convert into greater than type of you know constraint constraints and, then you know then we will address the problem as per the particular you know business requirement.

However in real life scenario we have objective function that we in the primal structure may be minimization type may be maximization type, but the constraints are not actually exactly in a kind of in a simple format so; that means, technically it is not typically less than type with respect to or you know maximizing objective function or greater than type where the objective function is minimization type.

So, there are certain problems where corresponding to a particular objective function constraints are you know mixture type, where we have a less than type, we have a greater than type, and we have actually equality kind of you know situation, and then what we did do actually a to go for the primal to dual transformation, and dual to primal transformation. So, we have to first you know simplify a or we have to bring a kind of you know a unique structure generalized structure, then after that we can actually go for the transformation.

For instance in this case, we have a primal problem where there are 2 decision variables x 1 and x 2 by default, in the case of you know duals we will have a 2 different constraints, and agains we have here 3 constraint, constant 1, constraint 2, constraint 3 and it is a maximization type. So, the first hand requirement is all the constraints should be less than type, but unfortunately the first and first one is the a then type second one is the less than type, but the sorry third one is the less than type, but the second one is the greater than type. So, now to maintain a kind of you know uniqueness. So, what will it do. So, the second constraints can be transfer into to less than type and, then can solve the problem. In fact, you know if you are looking for the primal solution in the case of you know pick is software package like you know solver.

So, there is no requirement of you know making you know uniqueness or unique structures. So, by default it will adjust and get you know give you the kind of an optimal solution, but here in the case of you know dual transformation, we need a kind of in a unique structure and further. So, what will you do in the second constraints, we transfer into less than constraint by multiplying minus sign in the both the sides, and as a result. So, this is what the typical structure. So, initial structure is x 1 minus x 2 rather than 2000.

So, now the typical constraint will be minus x 1 plus x 2 less than equal to minus 2000, then a then all the constraint will be you know unique kind of you know character that is the less than type then finally, we will go to the dual transformations. So, now maximization will transport to minimization, and by default all the coefficient of the constraint in the right hand side will be the input coefficient to the objective functions. So, that is 6000, 2000 and 4000 you see here 6000 will go here, 2000 will go here is, and 4000 will go here is.

So, now a corresponding to you know primal problem, then the dual problem will be y one minus y 2 plus y 3 so; that means, so these 3 inputs 1 1 1. So, it will go y 1, y 2 y 3. So, this will go to the first constraints and second; obviously, a this and this and this will go to the you know second constraints and, then the values of the decision variables; obviously, the coefficients of the body sides will be derived from the objective functions. So, now or like you know primal case, we have actually the kind of you know conditions. So, we have a condition about the duality.

So; that means, in the primal case we have x 1 is unrestricted so; that means, you know typically greater than type. So, it can have also negative, and in the case of you know duals, but we transport into such a way. So, that is that all variables should be in a kind of you know positive you know sides. So, that depends upon the kind of you know business requirement the kind of you know business structure, but we can we can also look for these solutions, where you know the values of the decision variable may have a kind of you know unrestricted kind of in a structure.

But when will go for you know transformation primal to dual or dual to primal. So, you can actually normalize and, then you can simplify in a kind of you know in structures as per the general linear programming problem structure, and that is somehow duality is a kind of you know beautiful structure through which, the normalization you know are we can bring some kind of you know normalization to address the business problem more effectively and, that to as per the business requirement, or the kind of you know management requirement.

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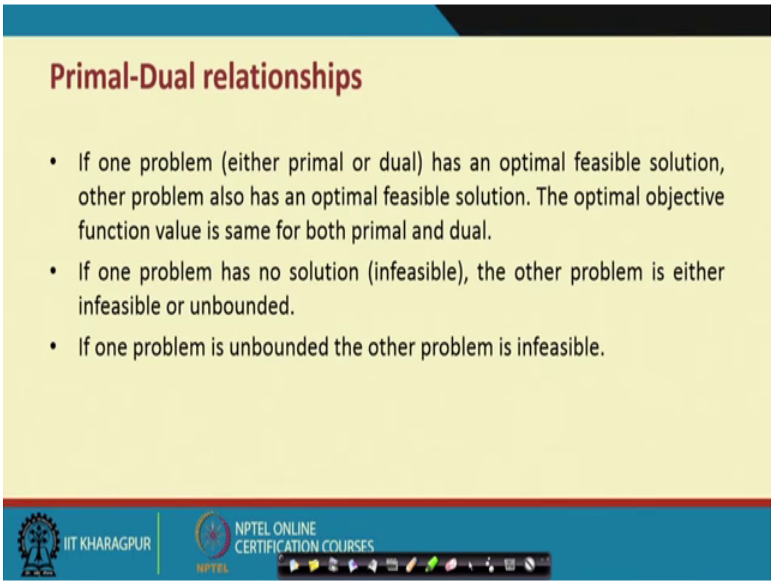

So, now you know of corresponding to this kind of in a structure. So, what we have here the primal dual relationship so; that means, if 1 problem either primal or dual has an optimal feasible solution, the way I have already highlighted the other problem also has an optimal feasible solution; that means, we start with the primer, and look for the dual, and when we have a primal and we have a primal solution primal optimal solution, then by default the counterpart dual must have actually optimal solutions

So that means, typically the format or these you know structure of the problem will not you know interchange, but the kind of you know you know flexibility we only have here to address the same business problem in a more effective way so; that means, there is a high chance that you know having primal problem, we may not get the unique solution or you know feasible solutions.

There is a question of you know unbounded earnest kind of in a situation, there is a possibility of you know infeasible solutions, sometimes we may not have actually you know solutions where the constraints are you know inconsistent. So, so in this kind of in a scenario if the primarily done bounded then; obviously, the dual cannot have actually optimal solution, if the primary is having you know inconsistency, then you know dual can have also inconsistency.

So, if the primal has a multiple solution, then dual can have a multiple solution so; that means, it is a very you know close connections, whatever the structure of primal dual will have a same kind of you know structure, but the kind of interpretation or the address of the business problem will be you know different. It will just change the angle, through which you can address the business problem as per the particular requirement, and the kind of you know you know management decision.

So, if one problem has no solution, the other problem is either you know infeasible or you know kind of you know unbounded, and if 1 problem is unbounded the other problem is infeasible so; that means, it is the kind of you know specific issue or you know so; that means, technically we have 2 different structures, in one case we have optimal solution, in the case of you know primal by default, we have been optimal solution in the dual, and the second case is in the case of you know primal. If you have a, you know if you have no solution then the other problem will be either you know infeasible or unbounded.

And against if one problem is unbounded the other problem will be actually invisibles, in total so; that means, we have optimal solution we have no optimal solution. So, when primal is a having optimal, then dual will have optimal solution, if primal has no optimal; that means, either infeasible or unbounded or whatever maybe d, then dual problem will also have no optimal it will also have a similar kind of you know direction or different direction may be unbounded may be infeasible.

So, there is a kind of you know consistency; that means, so solving the same problem in your dual angle we are not doing any you know mistakes rather, we are having alternative kind of an options to address the business problem more effectively and, then as per the particular you know business requirement. So, having a problem. So, you can actually transfer into the model and, the moment you will transfer into the model in your linear programming format, then the first hand expressions are first of first hand address is the primal structure, and go for the economic interpretation. And the same problems we can transfer into the dual structure, then go for the economic interpretation and, then look for the kind of you know optimal solution.

So, also so; obviously, if you have a primal optimal solution we have a dual optimal solution if primal optimal we have no primal optimal solution, then we may not have actually you know or we will not have actually dual optimal solutions. So, we have some kind of you know extra kind of you know kind of you know flexibility through which, we can address the same problem same business problem you know more you know more attractive way or in a kind of you know different structures, compared to sensitive analysis and you know kind of nervousness.

So, here we are not applying any kind of you know sensitivity or some kind of you know changing the structure of the problem like you know adding extra variables, or you know adding you know constraints, or dropping any constraints or something like that or change of the coefficients. So, here we are not doing anything.

So, we are in a same input same structures, just we are you know changing the direction of the problem. Against if there is a kind of you know sensitivity, then this will also apply accordingly, and a corresponding to the primal problems, then we use a kind of you know math mechanisms that is called as you know simplex, then primal simplex mechanism through which you can get the particular you know solution.

Again corresponding to the dual so, we have a structure that too called as you know dual simplex structures and, then we will look for the optimal solution and; that means, a particular problem primal type we can solve the problem in a kind of you know simplex mechanism in the discolors a primal simplex mechanism and, corresponding to the duals we can also apply the duals you know you know a simplex structure and, the particular mechanism is called as a dual simplex a mechanism, or dual simplex; simplex structure and of course, you know the kind of you know entry or the kind of you know iterations are more or less same, but you know like you know structuring, and restructuring some we will ago you know you know kind of know differently to get the optimal solutions.

So, how will it reach the kind of an optimal solution through dual simplex methods, really address in a in a in a kind of you know another kind of you know structure, and that will a addressing in the next lecture. So, with this we will stop here and.

Thank you very much have a have a nice day.