

Business Analytics for Management Decision
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Lecture – 45
Prescriptive Analytics (Contd.)

Welcome to BMD lecture series; this is Rudra Pradhan here and today we will continue with predictive analytics and that too again coverage on linear programming problem. We have already discussed couple of problems, and we have also highlighted different kind of you know situations, and different kind of you know business problems through which you were looking for the optimum solution with you know simple kind of you know structure, complex kind of you know structure, various issues, and you know several you know complexities till we are actually having a kind of you know optimum solution as per the business requirement or the kind of you know management requirement.

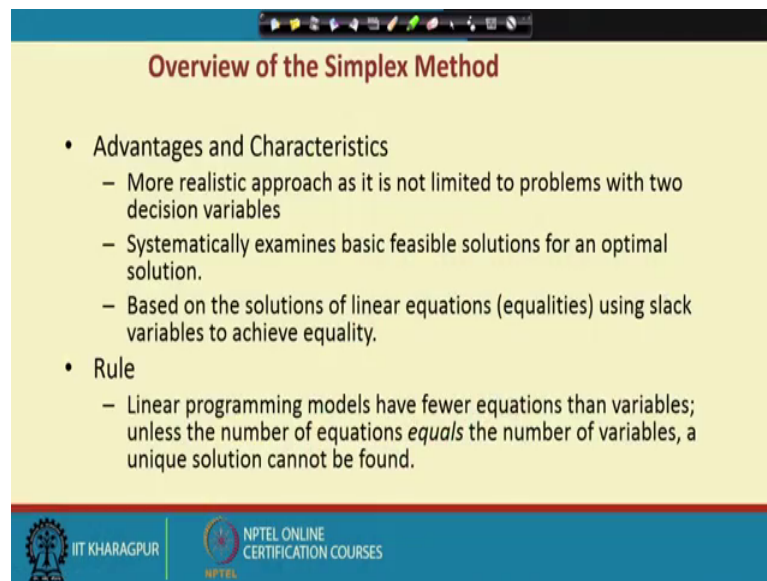
And in fact, till now whatever problems we have discussed and we are looking for the kind of you know solution by applying a you know typical structure called as you know graphical structure. And in this kind of you know environment there may be a kind of you know situation where more number of decision variables may be operative and again with respect to objective function and constraints; we are looking for the optimum solution as per the business requirement or the kind of you know management requirement.

Now, when a problem is complicated with respect to more number of decision variables and more number of constraints and then the graphical methods may not be very effective to you know come to a kind of you know solution in that context analytics you know you know has a kind of you know big rules either through a kind of you know computer programming and or through a kind of you know some you know typical software's through which you can solve the problems more you know effectively as per the management requirement and the business needs.

And in fact, without any software's or the kind of you know programming's till we can you know solve the problems as per the particular business requirement, and the solutions you know which will like to address here today is the kind of you know

simplex procedure and this is otherwise called as you know simplex methods or algebraic method through which we look for the optimum solutions corresponding to a particular objective function and the constraints where we typically use this particular you know mechanisms a with respect to more number of decision variables in the systems.

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Overview of the Simplex Method

- Advantages and Characteristics
 - More realistic approach as it is not limited to problems with two decision variables
 - Systematically examines basic feasible solutions for an optimal solution.
 - Based on the solutions of linear equations (equalities) using slack variables to achieve equality.
- Rule
 - Linear programming models have fewer equations than variables; unless the number of equations *equals* the number of variables, a unique solution cannot be found.

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And let us start with the particular you know structure and after highlighting the particular you know simplex structure then we will solve some of the problems by the kind of you know software use may be server we can use here to solve this kind of you know problem. And this is this is a very interesting technique we have lots of you know advantages to solve some of the problems corresponding to constraints and the kind of you know consistent the kind of you know conditions.

So, what is happening here in this; you know algebraic method and the kind of you know simplex method it is basically a iterative kind of you know environment and the procedure is to move you know one kind of you know iteration to another kind of you know iteration and then we will continue till we get the optimum solution which can which can satisfy all the constraints the conditions and the kind of you know objective function requirement.

And; that means, typically it is a kind of you know method of you know you know improvement. So, one iteration to another iteration like you know it is a kind of you

know chain and the beauty of this particular you know mechanism or the kind of you know structure is in each you know stage. So, you know means each subsequent stage we will find you know improvement in the objective function subject to the kind of you know constraint.

Then finally, (Refer Time: 04:23) the optimality where the value of the particular objective function can reach the highest level with respect to maximization type or minimization type after that you may not get the you know anything best corresponding to the objective function and constraint. So, the that is how this is a very standard kind of you know structure and most of the big problems and big you know kind of you know management kind of you know scenario where you know algebraic method will be very useful to look for the means very useful to you know use and then look for the kind of you know solutions.

So, linear programming models have a fewer equation, then variables unless the number of equation equals the number of variables a unique solution cannot be found this is like you know means all together it is a kind of you know simultaneous a equation systems where you know we have you know more number of you know decision variables with respect to you know constraints so; that means, in a kind of you know simultaneous equation system you know set of where you know; if the you know constraints are not consistent with you know number of variables, then we may not get a kind of you know unique solution and that is also equally applicable in this process of you know simplex methods and as a results we may not get optimum solution.

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Developing the Initial Simplex Tableau

- Notation used in the simplex tableau:

c_j = coefficient of variable j in the objective function
 a_{ij} = coefficient of variable j in constraint i
 b_i = right-hand-side value of constraint i

Handwritten notes:
 $A = (a_{ij})$ opt $Z = cX$
 $Ax \leq b$
 $x \geq 0$

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So, now some of the notations we like to you know, then accordingly we can look for the kind of you know solution typically a any linear programming problems. So, the we like to optimize Z equal to $C X$ and X less than equal to or greater than equal to b X greater than equal to 0 .

But here A is nothing, but actually the a_{ij} coefficient. So, that is actually a input to the you know decision variables and b is the kind of you know vector which is actually a you know the kind of you know balance or the kind of you know restrictions towards the particular you know constraint and, then C is the coefficient of you know vector involving the weights or you know coefficient to the decision variables.

So, accordingly the typical understanding is c_j coefficient that is the objective function coefficients a_{ij} is the coefficient of the variable corresponding to constraints and that too left hand side of the you know model and then b_i is the right side of the constraint and that too reflecting you know the vectors which represents v_1, v_2 and depending upon the number of constraints in a particular you know problem. So, now, knowing all these this things. So, you know we can move to understand; the particular you know requirement.

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Comparison of Server Model and General Simplex Notation

Server Model

maximize $Z = 60x_1 + 50x_2 + 0s_1 + 0s_2 + 0s_3$
subject to

$$\begin{aligned} 4x_1 + 10x_2 + s_1 &= 100 \\ 2x_1 + x_2 + s_2 &= 22 \\ 3x_1 + 3x_2 + s_3 &= 39 \end{aligned}$$

Symbolic Model

maximize $Z = c_1x_1 + c_2x_2 + 0s_1 + 0s_2 + 0s_3$
subject to

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}s_1 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}s_2 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}s_3 &= b_3 \end{aligned}$$

And then you know moving forwards we will have a problem like this is a you know kind of you know structures and the kind of you know structure is like this to understand the particular you know requirement and ok. So, this is this is what the typical problem and corresponding to this problem.

So, this is actually first hand problems where we have objective function corresponding two variables and that too maximizing Z equal to $60 X 1$ plus $50 X 2$ and subject to we have a constraints $4 X 1$ $10 X 2$ less than equal to 100 , $2 X 1$ plus $X 2$ less than equal to 22 and $3 X 1$ plus $3 X 2$ less than 39 . We have already solved this problem and here the first hand you know you know issue is you know to make the constraint into equality; that means, we try to put in a kind of you know simultaneous equation system where you know all the equations must be in a equality format.

And in reality this kind of you know problems we may have less than constraint; we may have a greater than type you know constraint, but the first hand requirement to apply the particular you know methods that is the simplex method is to make the constraint into equality a after you know you know using or introducing (Refer Time: 08:42) variable and; that means, there are two different structure we have to follow to make the constraint inequality constraint to equality constraint and that is the first hand requirement of any kind of you know simplex mechanism.

And when the when a particular constraint is less than type then the variable which can use to transfer the a less than type to equality is called as you know slack variable and when a constraint is greater than type to make the you know to you introduce a new variables through which that you know less than type constraint; that is the inequality type of constraint can be a you know moved to equality constraint. And in that case the particular adjustment is through a variable called as you know surplus variables.

That means when a constraint is less than type we introduce slack variable to make the constraint into equality and when a constraint is greater than type we use surplus variable to make the inequality constraint into equality constraint as a result S 1 is the kind of you know since all are you know less than type. So, now, S 1, S 2, S 3, three different slack variable is used for you know three different constraints and that too make the inequality constraint into equality constraints.

And, actually the simplex procedure follows directly a kind of you know mechanism called as a matrix structure that is the process cross elimination procedure and here the particular mechanism is we have a original input matrix and then we have to create a kind of you know identity matrix unit matrix a depending upon the a actual requirement of the kind of you know constraints for instance here; there are three constraints and as a results we need a kind of you know unit matrix of order three as a result S 1, S 2, S 3 can give you three different unit matrix structure.

So, S 1 where you know the first component will be the unit a unit matrix the S 2, the second component will be the unit matrix and third S 3; that is the third component of the unit matrix; that means, technically the typical structure will be S 1, S 2, and S 3. So, these the structural way which we can follow is like this 100010 and 001 and; that is, what the all the diagonal elements will be we you know established and then since there are you know three constraints.

So, three a variables you know slack variable is used to have a unit matrix of n order three. As a result we apply the inverse structure through which a the first hand input matrix will convert into the unit matrix as a results we can able to use the optimum solution and accordingly we look for the solution and the kind of you know management requirement.

So, accordingly the symbolizing format is like this. So, it is the C 1 X 1, C 2 X 2 and these are all you know artificially introduced variables. So, since it is a artificially introduced variable to make the constraint into equality. So, as a result the objective function coefficients of the objective functions will not available and since it is not available by default the coefficients will be you know (Refer Time: 12:01) we can you know introduce 0 coefficient here by default and then we look for the kind of you know solution. So, the procedure of solution is like this.

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Comparison of Server Model and General Simplex Notation (Contd.)

| | | | | | | | |
|----|--------------------------------------|----------|----------|-------|-------|-------|----------------|
| a. | List of variables → | x_1 | x_2 | s_1 | s_2 | s_3 | |
| b. | Coefficients of objective function → | c_1 | c_2 | 0 | 0 | 0 | RP-RS values ↓ |
| | 1st constraint → | a_{11} | a_{12} | 1 | 0 | 0 | b_1 |
| | 2nd constraint → | a_{21} | a_{22} | 0 | 1 | 0 | b_2 |
| | 3rd constraint → | a_{31} | a_{32} | 0 | 0 | 1 | b_3 |
| c. | | 60 | 50 | 0 | 0 | 0 | ↓ |
| | x_1 | x_2 | s_1 | s_2 | s_3 | | |
| | | 4 | 10 | 1 | 0 | 0 | 100 |
| | | 2 | 1 | 0 | 1 | 0 | 22 |
| | | 3 | 3 | 0 | 0 | 1 | 39 |
| d. | C → | 60 | 50 | 0 | 0 | 0 | |
| | Basis ↓ | x_1 | x_2 | s_1 | s_2 | s_3 | Quantity |
| | s_1 | 0 | 4 | 10 | 1 | 0 | 0 |
| | s_2 | 0 | 2 | 1 | 0 | 1 | 0 |
| | s_3 | 0 | 3 | 3 | 0 | 0 | 1 |

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And so, the typical procedure is the a you know we like to prepare a matrix a like this and here. So, these are the initial input matrix and this is how the unit matrix and in totals if you simplify further; then this is what the actual matrix and this is what the unit matrix.

So, now we will introduce in such a way that you know uh. So, this variable will be transferred into a unit matrix format. So, as a result; So, the particular variables you know optimality can be you know reached so; that means, the usual structure is the this could be the unit matrix and by default this will be going through a kind of you know inverse matrix of you know original matrix.

So, as a result you know it is kind of you know (Refer Time: 13:00) you know procedure and we will move on one step to another steps and accordingly every stage we will get the improved objective function value and then finally, you reach the optimality where

you know the particular transformation will be unit matrix and then the a you know original matrix will be a going through a kind of you know inversed inverse matrix structure.

And a by default will get the you know optimality and the kind of you know the solutions as per the particular you know business requirement. So, accordingly; so, the procedure is the so, to set you know prepare the format like this. So, these are all the right hand constraint of the particular structure and these are all you know left hand side of the constraint these are the right hand side of the constraint and these are the kind of you know newly introduced (Refer Time: 13:54) not exactly they are artificial structure.

But it is the kind of you know slack variable structure through which actually look for the a kind of you know optimum solution and then the you know the procedure to move is like this and this is what the kind of you know structure.

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Completed Initial Tableau for the Server Problem

| Basis | C | 60 | 50 | 0 | 0 | 0 | Quantity |
|-------|----|-------|-------|-------|-------|-------|----------|
| | | x_1 | x_2 | s_1 | s_2 | s_3 | |
| x_1 | 0 | 4 | 10 | 0 | 0 | 0 | 100 |
| s_2 | 0 | 2 | 1 | 0 | 1 | 0 | 22 |
| s_3 | 0 | 3 | 3 | 0 | 0 | 1 | 39 |
| Z | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C-Z | 60 | 50 | 0 | 0 | 0 | 0 | |

Unit Vector

Each tableau represents a basic feasible solution to the problem

A simplex solution in a maximization problem is optimal if the C-Z row consists entirely of zeros and negative numbers (i.e., there are no positive values in the bottom row). When this has been achieved, there is no opportunity for improving the solution.

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So, what will you after introducing all these things so; that means, this is the original input structure and then.

So, this is the after the introduction of the slack variable and then finally, a it is the kind of you know firsthand uh optimum solutions, but it is not the final optimum solution; because you know we have not reached the optimality as per the you know constraints are you know the condition is a you know conditions are you know concerned.

So, typically the procedure is here we have a concept called as you know native evaluation and that is the difference between a the cost coefficient and the a Z value that is the objective functions and then we like to know what is the procedure through which you can get the optimum solution so; that means, typically the idea is that you know it is a kind of you know basic matrix through which you can get the values of the decision variables.

So, in the basis matrix it should be in a kind of you know unit matrix format so; that means, in a if it is not exactly the particular requirement. So, what will you do we like to you know move uh do the kind of you know movements. So, the kind of you know incoming structure and outgoing structures so; that means, initially in this particular format S 1, S 2, S 3 are the kind of you know firsthand entry, but actually S 1, S 2, S 3 is not the original values of the decision variable as a result this cannot be the final solution.

So, then what should be the final solution so; that means, the final solutions either X 1 will be there no X 2 or X 2 will be there no X 1 or both X 1 and X 2 can be there. So, as a result. So, the you know the kind of you know processing is the X 1 can be entered either in the place of. So, one or in the place of S 2 or in the place of S 3 or the move will be X 2 can be the incoming vector and the outgoing vector will be S 1, S 2, S 3 so; that means, X 2 can be in the place of either S 1 or S 2 or S 3.

So, likewise so; that means, this will be go like this or this will be go like this or this will be go like this and again. So, this may be the first move or if not the first move then X 1 can be the first move. So, X 1 can be replaced to S 1 X 1 can be equal to X 2 or X 1 can be go to the S 3.

So; that means, we have several kind of you know alternative structures, but which one will be the first hand priority and, then move go on and to reach the final optimum solution. So, there is a procedure and in the procedure of simplex method. So, it depends upon you know which particular; obviously, this cannot be the final optimum solution. So, either X 1 will go and S 1, S 2, S 3 will be out and, then X 2 will go again S 1, S 2, S 3 will be out then finally, we look for the optimum solution. So, it depends upon the net evaluation that is the C minus Z and that is the objective values of a cost coefficient of the objective function and the Z structures the kind of you knows net evaluations and on

the basis of that we can actually take a decisions. So, usually will stop where you know where you know the you know net evaluation will be negative or it is actually 0 with respect to unit matrix. So, if not then continue.

So, now in the first and this is this are all this is coming positive and this is coming 0. So, this is ok. So, for optimality this is final means fine and in the case of you know X 1 and X 2. So, this is coming actually positive so; that means, we have not reached the optimality. So, reach the optimality when the you know for any unit matrix. So, the net evaluation should be 0 and for other vectors. So, the net evaluation should be negative since you know X 1 and X 2 which is not which are not actually the unit matrix.

So, by default this could be the incoming vector through which you can move and finally, we reach the kind of you know optimum solution where both the vectors these are all called as non-basic variables right now. So, they will be coming into the basis matrix and finally, they will be the basic variables and again as usual structure this could be the negative sign. So, then we can reach the final optimum solutions

And, accordingly moving forwards so, by applying these rules.

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Determining the Entering and Exiting Variables

| | C | 60 | 50 | 0 | 0 | 0 | |
|-------|---|-------|-------|-------|-------|-------|--------------------------------|
| Basis | | x_1 | x_2 | s_1 | s_2 | s_3 | Quantity |
| s_1 | 0 | 4 | 10 | 1 | 0 | 0 | 100 $100/4 = 25$ Smallest |
| s_2 | 0 | 1 | 1 | 0 | 1 | 0 | 22 $22/2 = 11$ -nonnegative |
| s_3 | 0 | 3 | 3 | 0 | 0 | 1 | 39 $39/3 = 13$ ratio |
| Z | | 0 | 0 | 0 | 0 | 0 | 0 |
| C - Z | | 60 | 50 | 0 | 0 | 0 | |

Largest positive value s_3 leaves, x_1 enters

min $\left[\frac{100}{4}, \frac{22}{2}, \frac{39}{3} \right]$

Select the leaving variable as the one that has the smallest nonnegative ratio of quantity divided by substitution rate.

So, we can move a again a to this particular process and this is how the you know coming to the you know in the first hand choice corresponding to the previous kind of you know situation. So, now, we have we have actually calculate the net evaluation and;

that means, technically X_1 can go or X_2 can go in place of S_1, S_2, S_3 . So, in that case what will you do?

So, the procedure is the highest or largest positive value could be the first hand priority. So, as a result with these you know values. So, 60 is the highest one and that is (Refer Time: 19:32) actually X_1 so; that means, technically the indication is that you know X_1 is the incoming vector either in place of S_1 or S_2 or S_3 and; if X_1 is going in place of S_1 or S_2, S_3 and by default S_1 you know S_2, S_3 will be the leaving vector. So, now, with the with this particular you know indications a X_1 is the incoming vector.

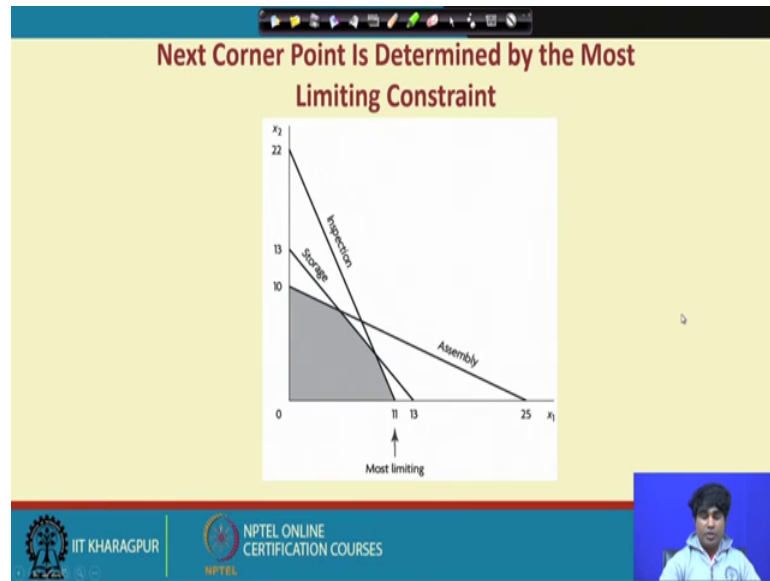
And which one is the outgoing vector out of S_1 and S_2, S_3 . So, we have to take another kind of you know indication in that case indication is the kind of you know minimum ratio between X_b and by X_k . So, X_b is the right hand side of the constraints and the X_k is the particular you know you know particular vector which can be the incoming one. So, as a result; so, the minimum ratio will be 100 by 4, 22 by 2 and 39 by 3 corresponding to the particular you know structure. So, as a result so, this will be the combinations. So, we like to find out the minimum of you know 100 by 4, then 22 by 2 and then 39 by 3. So, you know if you simplify then you will have a 25, 11 and 13 and as a result 11 is actually the minimum 1. So, by default; so, this will be the indications.

So, as a result so, this is the indication that you know the second vector is the outgoing vector corresponding to X_1 . So, X_1 is the incoming vector and then this particular indication is the S_2 so; that means, X_1 will go to S_2 and S_2 will be coming this place. So, so the accordingly the movement will be like this and the kind of you know intersection is called as you know pivot element so; that means, a in this (Refer Time: 21:23) this will be now unit matrix where the position of you know unit will be the middle one and the remaining place will be 00.

So, as a result so, the particular you know S_2 this is the S_2 format actually now this S_2 format will be move to this place where you know X_2 will be the a X_2 will be coming here and the S_2 will be coming here. So, as a result the particular transformation will be in a different structure all together.

And now in order to you know understand again. So, you know how is the kind of you know improvement. So, you can go to the a.

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Next (Refer Time: 22:05) and then the next this is how the graphically you can also cross check.

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Starting the Second Tableau

| Basis | C | 60 | 50 | 0 | 0 | 0 | Quantity |
|-------|----|-------|-------|-------|-------|-------|----------|
| | | x_1 | x_2 | s_1 | s_2 | s_3 | |
| s_1 | 0 | | | | | | |
| x_1 | 60 | | | | | | |
| s_3 | 0 | | | | | | |
| Z | | | | | | | |
| C - Z | | | | | | | |

| Basis | C | 60 | 50 | 0 | 0 | 0 | Quantity |
|-------|---|-------|-------|-------|-------|-------|----------|
| | | x_1 | x_2 | s_1 | s_2 | s_3 | |
| s_1 | 0 | 4 | 10 | 1 | 0 | 0 | 100 |
| s_2 | 0 | ② | 1 | 0 | 1 | 0 | 22 |
| s_3 | 0 | 3 | 3 | 0 | 0 | 1 | 39 |
| Z | | 0 | 0 | 0 | 0 | 0 | 0 |
| C - Z | | 60 | 50 | 0 | 0 | 0 | |

And again so, the particular you know structure is here and this is how this is how the kind of you know structure this is how the pivot element corresponding to the entry; that means, the X 1 will be coming here and X 2 will be coming here. So, accordingly we can actually a move forward and then finally, we look for the kinds of you know solution.

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The Pivot Row of the Second Tableau

| Basis | C | 60 | 50 | 0 | 0 | 0 | Quantity |
|-------|----|-------|---------------|-------|---------------|-------|----------|
| | | x_1 | x_2 | s_1 | s_2 | s_3 | |
| s_1 | 0 | | | | | | |
| x_1 | 60 | 1 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | 11 |
| s_3 | 0 | | | | | | |
| Z | | | | | | | |
| C - Z | | | | | | | |

| Basis | C | 60 | 50 | 0 | 0 | 0 | Quantity |
|-------|----|-------|---------------|-------|---------------|-------|----------|
| | | x_1 | x_2 | s_1 | s_2 | s_3 | |
| s_1 | 0 | 0 | 8 | 1 | -2 | 0 | 56 |
| x_1 | 60 | 1 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | 11 |
| s_3 | 0 | | | | | | |
| Z | | | | | | | |
| C - Z | | | | | | | |

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And in the second (Refer Time: 22:34) this is how the now the kind of you know x_1 is coming here as usual the indications and x_1 is coming here.

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Partially Completed Second Tableau

| Basis | C | 60 | 50 | 0 | 0 | 0 | Quantity |
|-------|----|-------|---------------|-------|----------------|-------|----------|
| | | x_1 | x_2 | s_1 | s_2 | s_3 | |
| s_1 | 0 | 0 | 8 | 1 | -2 | 0 | 56 |
| x_1 | 60 | 1 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | 11 |
| s_3 | 0 | 0 | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | 1 | 6 |
| Z | | | | | | | |
| C - Z | | | | | | | |

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And then the kind of you know x_2 will be now change the kind of you know position so; obviously,. So, the new improved solution will be the a the indication is like this you know here x_1 x_1 is coming in the place of you know a s_2 .

So, as a result so, now, x_2 will have actually unit matrix, because it is coming in the basis matrix and the particular s_2 is coming here in the case of you know x_1 . So, now,

this is the this is the kind of you know you know optimum solution after the first iteration and if you compare here the Z value is actually having improvement and compared to the previous one in the previous one the Z value is the simply 0, because this these are all the coefficient of you know Z value.

So, now, here actually a you know corresponding to objective function. So, 60 with respect to X 1 and; obviously, Z is having you know positive value. So, now, by the way whether you know means before you calculate the optimality of you know Z we like to check whether you know we have reached the optimum solution as per the simplex guideline the simplex guideline is that you know we will reach the optimum solution, when the values of the means the a net evaluation for unit matrix will be 0 and the particular you know non unit matrix the a net evaluation will be the negative one.

So, now for that we have to first check the net evaluation and then we think about the optimum value of Z.

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Completed Second Tableau

| Basis | C | 60 | 50 | 0 | 0 | 0 | Quantity |
|-------|----|-------|---------------|-------|----------------|-------|----------|
| | | x_1 | x_2 | s_1 | s_2 | s_3 | |
| s_1 | 0 | 0 | 8 | 1 | -2 | 0 | 56 |
| x_1 | 60 | 1 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | 11 |
| s_3 | 0 | 0 | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | 1 | |
| Z | 60 | 30 | 0 | 30 | 0 | 0 | 660 |
| C - Z | 0 | 0 | 0 | -30 | 0 | 0 | |

Interpreting the Second Tableau

At this point, variables s_1 , x_1 , and s_3 are in solution. Not only are they listed in the basis, they also have a 0 in row C - Z. The solution at this point is $s_1 = 56$, $x_1 = 11$, and $s_3 = 6$.

Note, too, that x_2 and s_2 are not in solution. Hence, they are each equal to zero. The profit at this point is \$660, which is read in the Quantity column in row Z. Also, note that each variable in solution has a unit vector in its column.

If not then we will we will move or we will move to the second iteration and then look for the kind of you know solution now after the first iteration the final table final table or final structure will be like this and this is how X 1 is entering in place of you know S 2 and as a result X 1 is now unit matrix and again, we calculate the net evaluations by default.

Since X_1 is unit matrix net evaluation will be 0 and this is coming negative and this fine and this is 0 this is fine this is also fines for optimum solution, but this is not fine; because this is a positive value for X_2 ah; that means, technically now the highest positive value will be the kind of you know new incoming vector and as a result here X_2 is the only variable only vector through which you know it can enter to the next iteration.

So; that means, now X_2 has already a you know you know coming to the kind of you know basic matrix or the kind of you know optimum structure and now X_2 is also having indication to come the basic matrix for the optimum solution now like the previous case a. So, X_2 is the indication of you know incoming vector. Now we are looking for which place X_1 , X_1 will move either it will go to the S_1 s or it will go to the X_1 X_1 or it will go to the S_3 . Since X_1 is already replaced with you know S_2 . So, there is a less chance that you know X_2 will go to X ones it may be possibility is there, but you know there is high chance that you know now X_2 may be replaced with S_1 or you know S_3 .

So, let us see you know how it can be actually you know having the kind of you know final solution. So, now, with the particular you know results. So, it is the indication that you know X_2 will be the incoming vector. Now, we are looking for you know outgoing vector through which you can do the kind of you know interchange X_2 with you know either S_1 or X_1 or S_3 .

So, now, for that we again go to the minimum ratio that is the X_b by X_k . So, which is nothing, but actually the right hand side of the constraint with respect to the input to the kind of you know second vectors that is X_2 . So, as a result so, we can get a you know clue that you know which one is the minimum ratio and the kind of you know vector which is having minimum ratio by default that will be the outgoing case.

(Refer Slide Time: 27:11)

Determining the Exiting Variable

| Basis | C | 60 | 50 | 0 | 0 | 0 | Quantity |
|-------|----|-------|---------------|-------|----------------|-------|----------|
| | | x_1 | x_2 | s_1 | s_2 | s_3 | |
| s_1 | 0 | 0 | B | 1 | -2 | 0 | 56 |
| s_2 | 60 | 1 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | 11 |
| s_3 | 0 | 0 | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | 1 | 6 |
| Z | 60 | 30 | 0 | 30 | 0 | 0 | 660 |
| C-Z | 0 | 20 | 0 | -30 | 0 | 0 | 0 |

Handwritten notes on the slide:
 - A red circle around the 'Quantity' column with arrows pointing to 56, 11, and 6.
 - Next to 56: $56/8 = 7$
 - Next to 11: $11/2 = 5.5$
 - Next to 6: $6/1.5 = 4$
 - A red arrow points to the value 4 with the text "Smallest non-negative ratio".
 - A red arrow points to the row for s_3 with the text "Exiting variable".

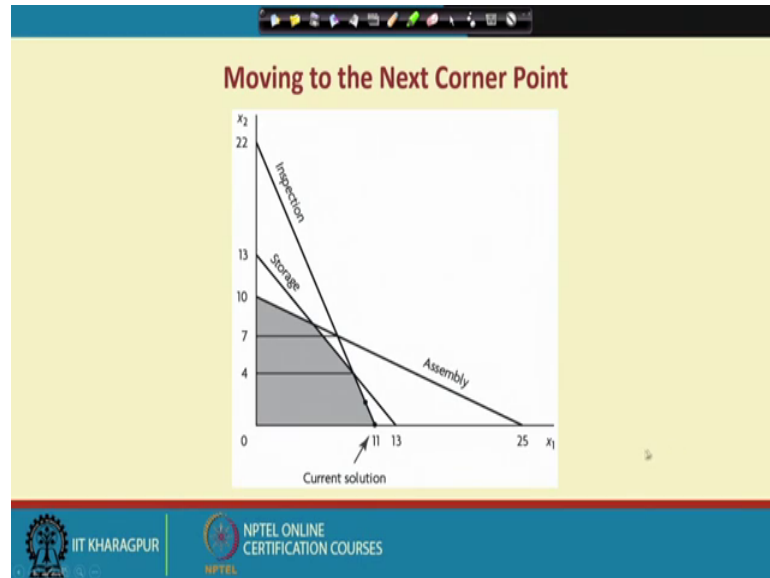
So, as a result to know the reality; So, we can move to the uh result part and here the if you simplify the minimum ratio which is actually here a 56, 11 and then you know 6 and against the income input X X k that is the input of you know X 2 is 81 by 21 by 2. So, now, if you will simplify here is this particular you know structure. So, you can get to know the minimum ratio.

So, out of these 3; so the minimum ratio is the 4 and as a result by default this will be the outgoing case so; that means, technically. So, this is the incoming indications because it is highest a positive a you know net evaluation and a here is the minimum ratio. So, as a result the final decision is the S 3 will be the outgoing vector and S 2 is the incoming vector.

So; that means, technically. So, the substitute will be like this X 2 will be coming to S 3 and S 3 will go to X 2. So, then while doing this you know we will again get the improved optimum solution that is called as improved basic feasible solution and, then again we check the kind of you know requirement if the particular requirement is satisfied; then that could be the final optimum solution if not then against we move on till you get the you know further improved you know optimum solutions and a corresponding to this.

So, it is clear cut indication that now the X_2 will be replaced and that too in place of you know S_3 and S_3 will go in place of you know X_2 so; that means, now in the basis matrix in the second iterations it will be X_1 , X_2 and S_1 .

(Refer Slide Time: 29:07)



So, then accordingly we move forward and this is how graphical again indication.

(Refer Slide Time: 29:10)

| Pivot Row Values for the Third Tableau | | | | | | | |
|--|----|-------|-------|-------|-------|---------------|----------|
| Basis | C | 60 | 50 | 0 | 0 | 0 | Quantity |
| | | x_1 | x_2 | s_1 | s_2 | s_3 | |
| s_1 | 0 | | | | | | |
| x_1 | 60 | | | | | | |
| x_2 | 50 | 0 | 1 | 0 | -1 | $\frac{1}{2}$ | 4 |
| Z | | | | | | | |
| C - Z | | | | | | | |

| Basis | C | 60 | 50 | 0 | 0 | 0 | Quantity |
|-------|----|-------|-------|-------|-------|-------------------|----------|
| | | x_1 | x_2 | s_1 | s_2 | s_3 | |
| s_1 | 0 | 0 | 0 | 1 | 6 | $-\frac{1}{2}s_3$ | 24 |
| x_1 | 60 | 1 | 0 | 0 | 1 | $-\frac{1}{2}s_3$ | 9 |
| x_2 | 50 | 0 | 1 | 0 | -1 | $\frac{1}{2}s_3$ | 4 |
| Z | | | | | | | |
| C - Z | | | | | | | |

So, against we are looking for the kind of, you know solution.

And here by default X 2 is coming to the coming in place of S 2 and again S 3 against S 2 place will be the unit matrix. Since the third row is the kind of you know interchange. So, by default a the unit matrix position will be the one here and as a result. So, this is 010 already and this will be 001 and. In fact, S 1 is now change corresponding to the a first iteration and second iteration.

So, as it is S 1 will be there. So, typically now basis matrix S 1, then X 1 then X 2 and by default S 2 and S 3 is already replaced with X 1 and X 2. So, now this could be the final optimum solution and the declaration is you know declaration is on the basis of the check of you know net evaluation and accordingly we will move to know the kind of you know net evaluation value through which you can you know take the decision whether we have reached the optimality or not.

(Refer Slide Time: 30:18)

Completed Third Tableau

| Basis | C | 60 | 50 | 0 | 0 | 0 | Quantity |
|-------|----|-------|-------|-------|-------|--------------------|----------|
| | | x_1 | x_2 | s_1 | s_2 | s_3 | |
| s_1 | 0 | 0 | 0 | 1 | 6 | $-\frac{4}{3}s_3$ | 24 |
| x_1 | 60 | 1 | 0 | 0 | 1 | $-\frac{1}{3}s_3$ | 9 |
| s_2 | 50 | 0 | 1 | 0 | -1 | $\frac{2}{3}s_3$ | 4 |
| Z | 60 | 50 | 0 | 0 | 10 | $\frac{40}{3}s_3$ | 740 |
| C-Z | | 0 | 0 | 0 | -10 | $-\frac{40}{3}s_3$ | |

Interpreting the Third Tableau

In this tableau, all of the values in the bottom row are either negative or zero, indicating that no additional potential for improvement exists. Hence, this tableau contains the optimal simplex solution, which is

$s_1 = 24; x_1 = 9; x_2 = 4$

$Z = 9 \times 60 + 4 \times 50$
 $Z = 618$
 $Z = 60x_1 + 50x_2$
 $Z = 60(9) + 50(4)$
 $Z = 540 + 200$
 $Z = 740$

So, now after the second iterations this is the final you know out come. So, now, X 1 is already replaced with the a S 2 and X 2 is already replaced with the X 2 as a result. So, this is now in the basis matrix and it is good for the optimum solutions as per the particular you know business requirement and a. In fact, S 1 is still in the basis matrix, but there is no way now you know out if it will be out then; obviously, either S 2 or S 3 will be in the process, because we need actually three a matrix a basis matrix through which actually you can streamline the process depending upon the kind of you know constraints.

And now to declare the optimality or the final solutions; we have to check the net evaluations and again for that we have to calculate the net evaluation and against by default net evaluation will be 0 for the unit matrix that is satisfied this is also 0; that is satisfied and this is also 0; that is satisfied and against net evaluation for non basic variable which is not unit matrix. Then it is coming negative minus 10 for S 2 and it is again minus 40 by 3 it is again for S 3.

So; that means, the rule is that you know once the net evaluation will be 0 or you know either you know 0 or negative; that means, we have reached the optimality. So, so technically in this for these problems we have reached now the optimum solutions as per the simplex a requirement is concerned. So, the uh the kind of you know net evaluation value is coming 0 for unit matrix and for non unit matrix it is coming actually negative.

So; that means, this is a clear cut signal that you know we have already reached the optimum solution; then in the optimum solution the combination is the S 1 and X 1 and X 2 and then the cost coefficient is actually coming 0 for S 1 by default and S 1 is the a this is 0, then 60 and this is 50 ok. So, now, we have reached the optimum solution now we like to know what should be the optimum you know kind of you know structure.

So, that is the cost coefficient and again. So, far as the values of the decision variable is concerned this is what the X b that is the values of the basic variable. So, now, this is actually coefficients of the objective functions and the corresponding values of the decision variable will be for S 1 it is 24 and then for X 1 it is 9 and then for X 2 it is actually 4.

Now, in the objective functions our representation is (Refer Time: 33:17) with respect to X 1 and X 2. So, as a result $C_1 X_1$ plus $C_2 X_2$, that is the linear combination between X 1 and X 2. So, now, C 1 is nothing, but actually 60 which is already given in the initial you know problem formulation and 50 is also C 2. So, now, with the operations we get the optimum values of the decision variable that too for X 1 it is 9 and for X 2 it is actually 4.

So, as a result final Z value will be 9 into 60. So, and plus you know 4 into you know 50 so; that means, technically 60 into 9 plus 50 into 4, that is $C_1 X_1$ plus $C_2 X_2$ which is actually coming the value called as you know 740. In fact, this particular you know

problem we have solved through graphical method already and we have already got the similar kind of you know results.

Now, simplex method is also giving thee similar kind of you know result; that means, this is somehow you know verified and the. In fact, the algebraic method and simplex method is much better technique; then the graphical structure where you know we can actually have a flexible kind of you know scenario; that means, in the graphical structures the first hand requirement is to find out all such corner points where the optimum solution you know can be a (Refer Time: 34:43) and then we actually check which one is the final optimum solution.

But the beauty of this particular you know simplex structure is the it is a iterative process and each iterations we will have a kind of you know improved feasible solution; that means, the similar kind of you know corner points are also here, but all the corner points corresponding to the Z value as it will be appearing like you know ascending to descending; that means, the minimum Z value will be at the origins where you know particularly for this problem with respect to X_1 and X_2 .

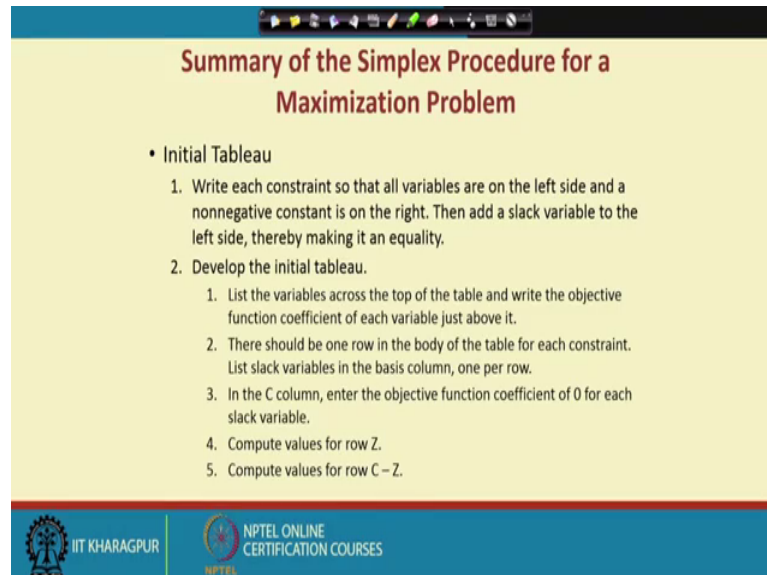
So, the if you arrange the values of the decision variable means values of the objective function corresponding to X_1 and X_2 . So, the minimum you know I mean say minimum value of Z is the where X_1 equal to 0; X_2 equal to 0, as a result Z equal to 0, then any improvement of you know either X_1 or X_2 or both then slowly the value of you know objective functions will move forwards.

And finally, (Refer Time: 35:48) stop at a particular point where the value of the objective function reach at the highest level and then other condition also satisfied and then we treat this solution as per the particular you know you know business requirement and the kind of you know management requirement. So, technically so, we have now these the optimum solution and as a result the values of the decision variable here X_1 equal to 9 and X_2 equal to 4 and S_1 is 24, but actually S_1 is not the variable in the original setup.

So, as a result the final optimum solution is with respect to X_1 and X_2 and objective function is already there. So, just you put the value and get the kind of you know value of the objective function which is actually coming 740 and; that means, technically Z optimum is 740 subject to X_1 equal to 9 and X_2 equal to 4.

And that is the a typical optimum solution corresponding to this a this problem and then if any kind of you know you know chain situation is there.

(Refer Slide Time: 36:56)



Summary of the Simplex Procedure for a Maximization Problem

- Initial Tableau
 1. Write each constraint so that all variables are on the left side and a nonnegative constant is on the right. Then add a slack variable to the left side, thereby making it an equality.
 2. Develop the initial tableau.
 1. List the variables across the top of the table and write the objective function coefficient of each variable just above it.
 2. There should be one row in the body of the table for each constraint. List slack variables in the basis column, one per row.
 3. In the C column, enter the objective function coefficient of 0 for each slack variable.
 4. Compute values for row Z.
 5. Compute values for row $C - Z$.

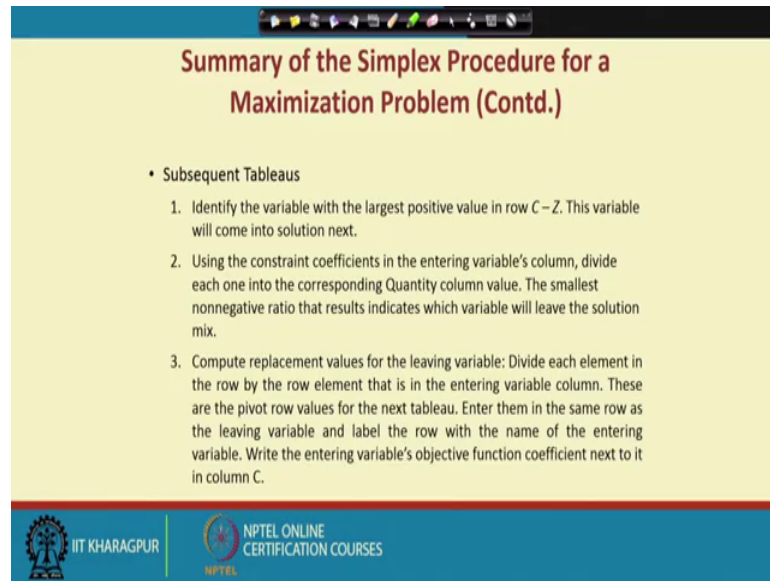
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So, you have to move accordingly as per the particular you know requirement. So, now, to sum up all the kind of, you know discussion. So, the summation is that you know algebraic method or simplex method is a kind of you knows structure where you know predictive analytics can solve.

So, prescriptive analytics can solve you know some of the problems you know business problems depending upon the particular you know objective functions structure and the kind of you know constraint where we are looking for the solution as per the typical business requirement and the kind of you know management requirement compared to graphical structure simplex method is a very standard structure and very systematic and very effective as the movement from you know uh one stage to another stage.

In each stage of you know movement. So, the values of the objective function has a kind of you know increasing structures that is with respect to maximization and (Refer Time: 37:54) the kind of you know minimization the particular structure will be also follow as per the particular you know structuring is concerned or the kind of you know requirement is concerned. So, in total this is very effective tool to solve some of the business problems as per the particular you know prescriptive analytics a environment is concerned.

(Refer Slide Time: 38:16)



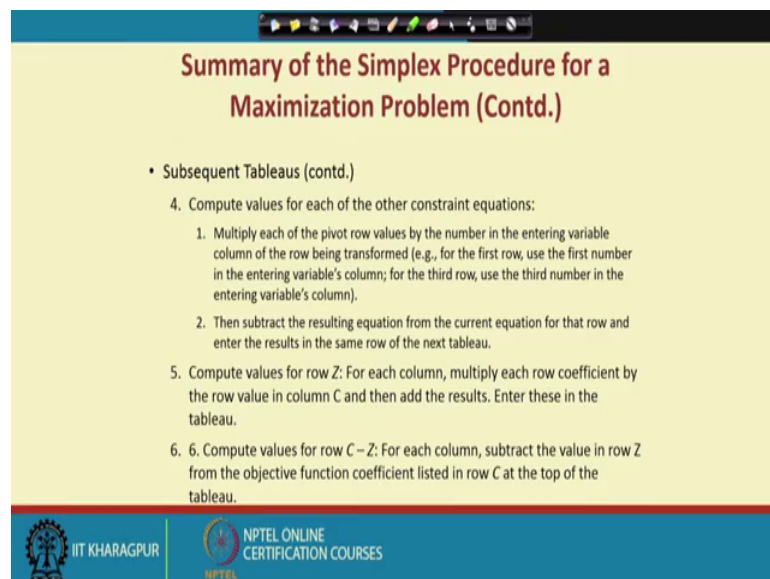
Summary of the Simplex Procedure for a Maximization Problem (Contd.)

- Subsequent Tableaus
 1. Identify the variable with the largest positive value in row $C-Z$. This variable will come into solution next.
 2. Using the constraint coefficients in the entering variable's column, divide each one into the corresponding Quantity column value. The smallest nonnegative ratio that results indicates which variable will leave the solution mix.
 3. Compute replacement values for the leaving variable: Divide each element in the row by the row element that is in the entering variable column. These are the pivot row values for the next tableau. Enter them in the same row as the leaving variable and label the row with the name of the entering variable. Write the entering variable's objective function coefficient next to it in column C.

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So, these are all you know you know kind of you know structuring.

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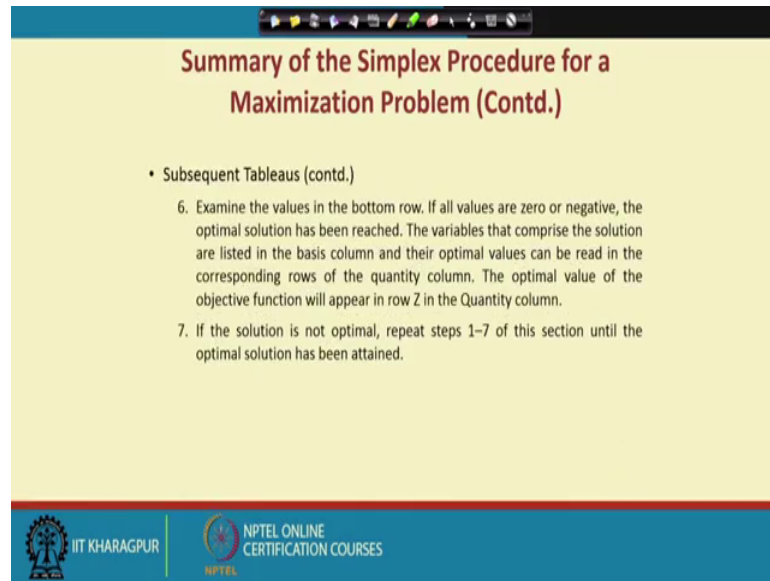


Summary of the Simplex Procedure for a Maximization Problem (Contd.)

- Subsequent Tableaus (contd.)
 4. Compute values for each of the other constraint equations:
 1. Multiply each of the pivot row values by the number in the entering variable column of the row being transformed (e.g., for the first row, use the first number in the entering variable's column; for the third row, use the third number in the entering variable's column).
 2. Then subtract the resulting equation from the current equation for that row and enter the results in the same row of the next tableau.
 5. Compute values for row Z : For each column, multiply each row coefficient by the row value in column C and then add the results. Enter these in the tableau.
 6. Compute values for row $C-Z$: For each column, subtract the value in row Z from the objective function coefficient listed in row C at the top of the tableau.

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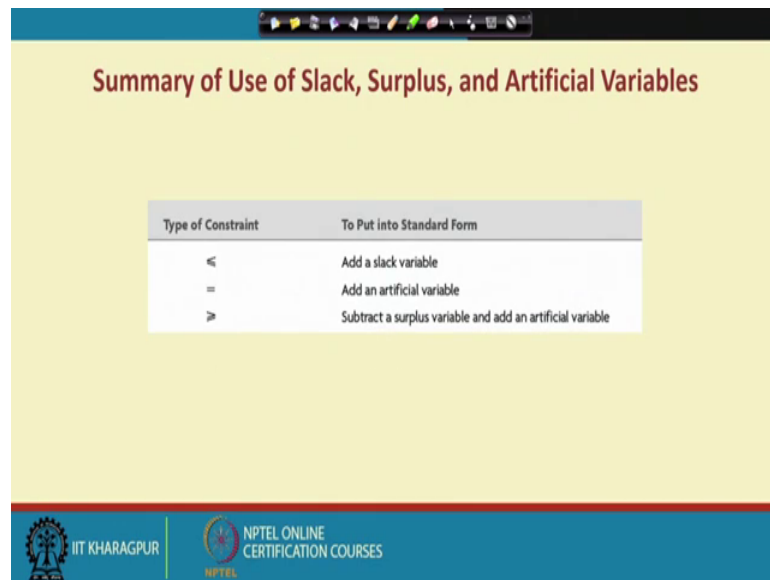
Summary of the Simplex Procedure for a Maximization Problem (Contd.)

- Subsequent Tableaus (contd.)
 6. Examine the values in the bottom row. If all values are zero or negative, the optimal solution has been reached. The variables that comprise the solution are listed in the basis column and their optimal values can be read in the corresponding rows of the quantity column. The optimal value of the objective function will appear in row Z in the Quantity column.
 7. If the solution is not optimal, repeat steps 1–7 of this section until the optimal solution has been attained.

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So, how we have to reach all the kind of you know solution; as per the particular you know business need or you know particular you know business requirement.

(Refer Slide Time: 38:25)



Summary of Use of Slack, Surplus, and Artificial Variables

| Type of Constraint | To Put into Standard Form |
|--------------------|--|
| \leq | Add a slack variable |
| $=$ | Add an artificial variable |
| \geq | Subtract a surplus variable and add an artificial variable |

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(Refer Slide Time: 38:28)

Solve this maximization problem using the simplex approach

maximize $Z = 6x_1 + 8x_2$



Subject to

Constraint 1 $x_2 \leq 4$

Constraint 2 $x_1 + x_2 = 9$

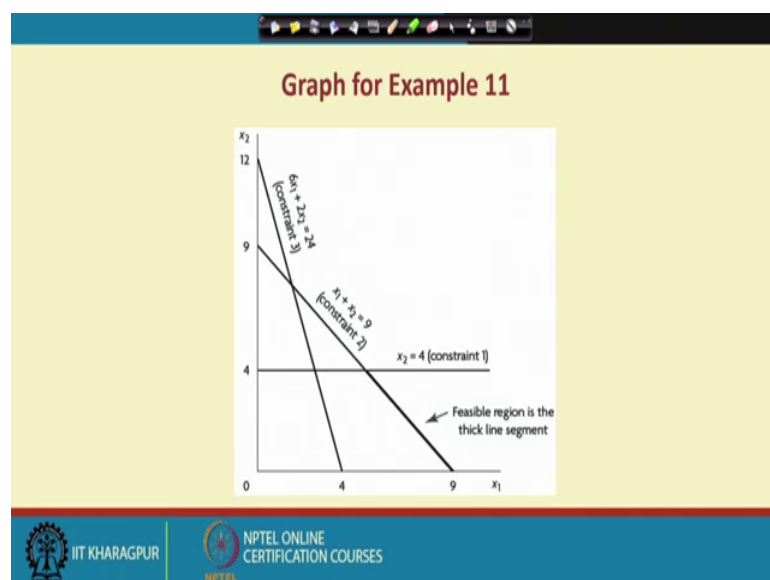
Constraint 3 $6x_1 + 2x_2 \geq 24$

x_1 and $x_2 \geq 0$

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And in fact, like the previous discussion you know we have normal case and we have actually complex case, where in the normal case corresponding to objective function the constraints means if it is you know maximization the normal case is the constraints should be less than type and if it is minimization you know objective function then the constraint will be greater than type.

(Refer Slide Time: 38:59)



(Refer Slide Time: 39:04)

Solve this maximization problem using the simplex approach

maximize $Z = 6x_1 + 8x_2$

Subject to

Constraint 1 $x_2 \leq 4$

Constraint 2 $x_1 + x_2 = 9$

Constraint 3 $6x_1 + 2x_2 \geq 24$

x_1 and $x_2 \geq 0$

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But we have solved some of the problems where the constraints are you know mixture type where greater than less than and again there may there are certain you know like this case here and a in totals against we may have a kind of you know situation where some constraints are you know conflicting and some structure like you know multiple solutions unbounded solutions in infeasible solutions.

So, again you know in the algebraic methods the kind of, you know indications starting with you know infeasible to unbounded then multiple. So, a it will be more interesting kind of you know structuring through which you can actually explore through simplex method and then you know take the decision about the particular you know business problems.

(Refer Slide Time: 39:52)

Initial Tableau for Example 11

| Basis | C | 6 | 8 | 0 | 0 | -M | -M | Quantity |
|-------|----|--------|--------|-------|-------|-------|-------|----------|
| | | x_1 | x_2 | s_1 | s_2 | A_2 | A_3 | |
| s_1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 4 |
| A_2 | -M | 1 | 1 | 0 | 0 | 1 | 0 | 9 |
| A_3 | -M | 6 | 2 | 0 | -1 | 0 | 1 | 24 |
| Z | | -7M | -3M | 0 | +M | -M | -M | -33M |
| C - Z | | 6 + 7M | 8 + 3M | 0 | +M | 0 | 0 | |

And then accordingly we will have a solution as per the particular you know business requirement and. In fact, if it is a maximization type situation then it is very easy to solve simplex method as usual procedure, but when we have a minimization problem where some of the constraints are you know greater than type or you know equal type then depending upon the particular you know constraint number of constraints.

So, we need to have a unit matrix format, but default we may not get a kind of you know unit matrix format; that means, technically a the problem which we have solved the earlier it is the less than type situation where there are three constraints and introducing slack variables we can easily get three unit matrix as per the a simplex requirement is concerned, but this is a kind of you know problem where less than type equality type and greater than type, but after introducing slack and surplus wherever there is requirement.

So, we may not in a position to get actually three unit matrix as per the particular requirement so; that means, technically in this kind of you know situation we have simplex method with different kind of you know mechanisms to look for the optimum solution and there are two different methods again under the simplex method then that took a situation where we have a constraint kind constraint like you know greater than type and equality type that is called as you know method one begin and the alternative method is called as you know two phase mechanisms.

In the (Refer Time: 41:26) method we like to introduce you know additional artificial variables which is represented as you know unit matrix as per the number of unit matrix is concerned so; that means, technically after introducing slack variable then you like to check actually how many unit matrix are (Refer Time: 41:45) corresponding to number of constraint available in a particular you know model.

So, for example, if there are three constraint in a particular model and we have a one kind of you know unit matrix by introducing slack and surplus variable, then by default we need two more kind of you know unit matrix to start the process and to look for the optimum solution. In that context we introduce two artificial variables and which is the actually kind of you know unit matrix format.

For example so, this is what the kind of you knows case here. So, we introduce two artificial variables and with two unit matrix because the original problem is up to this much where we have only one unit matrix and that too we have actually three different you know constraints. So, as a result simplex method will not work and in this context; so, we to introduce two artificial variable to start the process and so, far as (Refer Time: 42:41) mechanism is concerned.

So, the cost coefficient (Refer Time: 42:44) mechanism is you know we can start with you know introducing minus M and then yeah as usual you can proceed like this a. In fact, final optimum solution we can reach where you know the net evaluation will be 0 for unit matrix and a you know for non unit matrix it should be having actually a negative kind of you know situations.

And that too with respect to adjustment of you know M M is a kind of you know positive number here. So, then finally, you will reach the optimum solution and in the case of you know two phase mechanism. So, we start with similar kind of you know structure, but in the phase one we introduce artificial variable number of artificial variable as per the unit matrix requirement and then apply cost coefficient kind of you know unit you know minus 1.

Then again you solve once the you know phase one will end and then we start with the phase two where the actual cost coefficient will be there and we can remove artificial variables and then look for the solution and the final solution will be again judge on the basis of you know net evaluation criteria.

And or by default you will get the optimum solution as per the particular business requirement and the kind of you know management requirement.

(Refer Slide Time: 44:13)

The Second Tableau for Example 11

| | | Values for A_2 row | | | | | | |
|----------|----------------|----------------------|------------|---|----------------------|---|---------------|--|
| Column | Original Value | - | (Row Pivot | × | New Pivot Row Value) | = | New Row Value | |
| x_1 | 1 | - | 1 | × | 1 | = | 0 | |
| x_2 | 1 | - | 1 | × | $\frac{1}{3}$ | = | $\frac{2}{3}$ | |
| s_1 | 0 | - | 1 | × | 0 | = | 0 | |
| s_2 | 0 | - | 1 | × | $-\frac{1}{6}$ | = | $\frac{1}{6}$ | |
| A_2 | 1 | - | 1 | × | 0 | = | 1 | |
| Quantity | 9 | - | 1 | × | 4 | = | 5 | |

| Basis | x_1 | x_2 | s_1 | s_2 | $-M$ A_2 | Quantity | Ratio |
|-------|-------|--------------------|---------------|---------------------|----------------|----------|----------------------------------|
| s_1 | 0 | 1 | 1 | 0 | 0 | 4 | $4/1 = 4$ ← Minimum |
| A_2 | $-M$ | 0 | $\frac{2}{3}$ | 0 | $\frac{1}{6}$ | 5 | $5/(\frac{2}{3}) = 7\frac{1}{2}$ |
| s_2 | 6 | 1 | $\frac{1}{3}$ | 0 | $-\frac{1}{6}$ | 4 | $4/(\frac{1}{3}) = 12$ |
| Z | 6 | $2 - \frac{1}{2}M$ | 0 | $-1 - \frac{1}{6}M$ | $-M$ | 24 - 5M | |
| C - Z | 0 | $6 + \frac{1}{2}M$ | 0 | $1 + \frac{1}{6}M$ | 0 | | |

↑
Largest positive

So, looking forward actually; so, these problems you can actually move and then finally, we can see the kind of you know solution here. So, this is how the similar kind of you know this is how the largest positive value by default this could be the you know incoming vector a income incoming vector you know indication and then we by the kind of you know minimization criteria we look we look to find out which one is the outgoing.

So, in the case of you know earlier problems we like to actually uh means our target is to remove you know all these you know slack and surplus variable in the basis matrix and bring the actual decision variable to the basis matrix that is the kind of you know optimality or optimum solution.

But here we have a challenge again. So, it is not the removal of you know slack and surplus from the basis matrix. It is the target is the; to remove first the artificial variable from the basis matrix and then you know try to bring all the decision variables to the basis matrix as per the particular you know requirement and the particular business problem is concerned.

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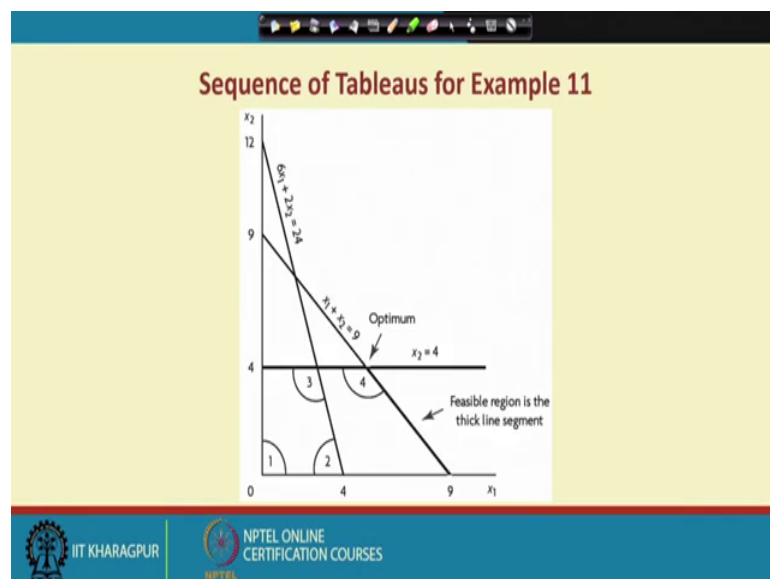
The Final Tableau for Example 11

| Column | Original Value | New Values for Row x_1 | | | | | New Row Value |
|----------|----------------|--------------------------|----------------|---|-----------------------|---|---------------|
| | | — | (Row Pivot) | × | (New Pivot Row Value) | = | |
| x_1 | 1 | — | $-\frac{1}{6}$ | × | 0 | = | 1 |
| x_2 | 0 | — | $-\frac{1}{6}$ | × | 0 | = | 0 |
| s_1 | $-\frac{1}{3}$ | — | $-\frac{1}{6}$ | × | -4 | = | -1 |
| s_3 | $-\frac{1}{6}$ | — | $-\frac{1}{6}$ | × | 1 | = | 0 |
| Quantity | $\frac{8}{3}$ | — | $-\frac{1}{6}$ | × | 14 | = | 5 |

| Basis | C | x_1 | x_2 | s_1 | s_3 | Quantity |
|-------|---|-------|-------|-------|-------|----------|
| x_2 | 8 | 0 | 1 | 1 | 0 | 4 |
| s_3 | 0 | 0 | 0 | -4 | 1 | 14 |
| x_1 | 6 | 1 | 0 | -1 | 0 | 5 |
| Z | | 6 | 8 | 2 | 0 | 62 |
| C - Z | | 0 | 0 | -2 | 0 | |

So, likewise if you move forwards then you know simple structure you know you will look for the optimality then finally will reach the optimality and then look for the optimum solution.

(Refer Slide Time: 45:21)



(Refer Slide Time: 45:23)

Example: minimization problem using the simplex method

minimize $Z = 7x_1 + 9x_2$
subject to
Constraint 1 $3x_1 + 6x_2 \geq 36$
Constraint 2 $8x_1 + 4x_2 \geq 64$
 x_1 and $x_2 \geq 0$

minimize $Z = 7x_1 + 9x_2 + 0s_1 + 0s_2 + MA_1 + MA_2$
subject to
Constraint 1 $3x_1 + 6x_2 - s_1 + A_1 = 36$
Constraint 2 $8x_1 + 4x_2 - s_2 + A_2 = 64$
All variables ≥ 0

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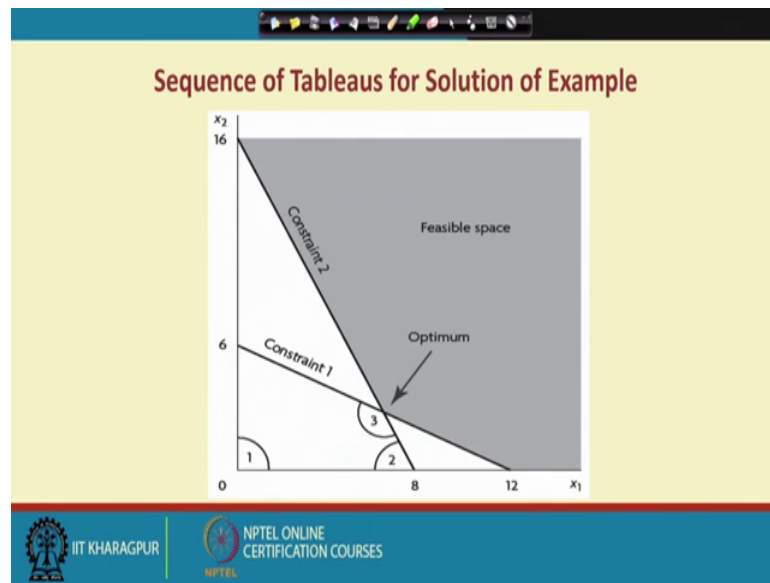
So; that means, corresponding to the particular problem. So, here now we have reached the optimum solutions because here the for unit matrix a; we have actually net evaluation 0 net evaluation 0 and against non unit matrix. So, we have actually net evaluation negative.

That means we have reached the optimality and in the basis matrix X 1 is the one kind of you know entry and X 2 is the another kind of you know entry and no artificial variable. In fact, you know the a slack and surplus variable is there that is S 3 and as a result. So, X 2 is equal to 4 here and X 1 is 5 here and corresponding objective function coefficient is 8 and 6 as a result.

So, the value of the objective function will be 8 into 4 plus 6 into 5 and; that is coming actually 62 and we satisfy all the condition and constraint and then finally, this is the optimum solution corresponding to this the particular you know l p model is concerned. So, likewise you know we have a different kind of you know structure through which we actually look the kind of you know solution and to verify this one since it is a two variable case you can also verify the case with respect to graphical and look for the solution.

And this is another kind of you know problem through which actually simplex method can be applied and for the kind of you know solution as per the business requirement.

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So; that means, technically a we have actually different kind of you know structure different kind of you know setups different kind of you know conditions through which actually the actual business problem can be treated and then we look for the optimum solution as per the business requirement subject to subject to you know satisfying all the conditions and the kind of you know constraints and whatever may be the shape of problems, whatever may be constraint and conditions.

The prescriptive analytics is a very helpful kind of you know structure through which you can have a solution which can solve the business problem, as per the particular you know managements management requirement is concerned and then we will be in a position to take you know to go for you know effective management decision with this we will stop here.

Thank you very much, have a nice day.