

Business Analytics for Management Decision
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Lecture – 44
Prescriptive Analytics (Contd.)

Hello, everybody. This is Rudra Pradhan here. Welcome to BMD lecture series and today, we will continue with prescriptive analytics that too coverage on linear programming problem. In the last couple of lectures we have already discussed this prescriptive analytics and too we have highlighted couple of issues like optimum solution and how to achieve optimum solutions a you know kind of you know maximization problem and minimization problems and the idea behind this particular structure is to get the values of the decision variables; corresponding to a particular business problem and the kind of you know management requirement.

So, technically we have solved some of the problems that is type of you know maximization case and the minimization case that is with respect to objective functions and the constants and the problems which we have solved for the management requirement or the business requirement where we have a you know normal case that too with respect to objective function maximization type the normal situations are the constraints must be in the less than type and in the case of minimization problem and that to minimizing the objective function and as usual the constraints will be greater than types.

So, accordingly we have solved cases for maximization problem where the constants are less than type and solve some of the problems less than type where the constraints are greater than types. So, usually in the normal linear programming problem corresponding to means maximization problem constraints should be less than type and corresponding to minimization problem constraints should be less than type.

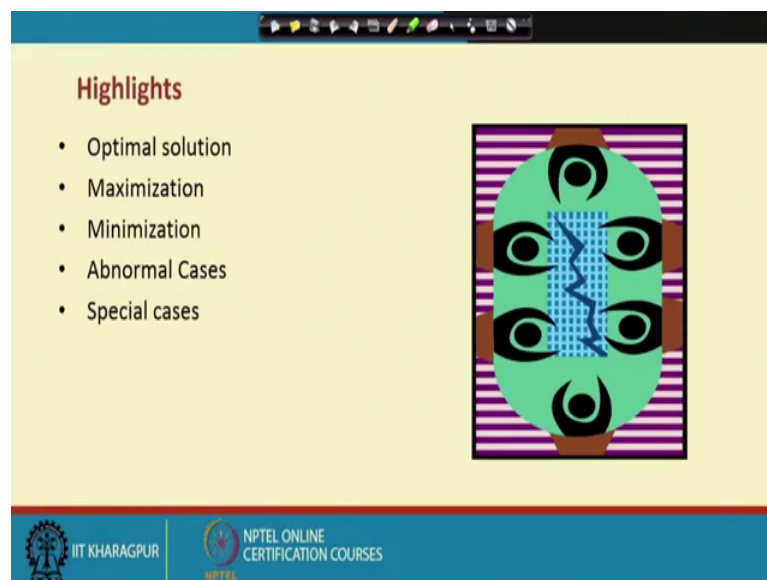
So, now, if the situations are other way around for instance having objective function maximization type the constraints are greater than type and with respect to minimization of objective function the constraints are less than type or in the third group with respect to maximizing objective function or minimizing objective function where the constraints are not in a kind of you know consistent structures like less typically less than type or

greater than type. So, that means, technically a irrespective of objective function maximization and minimizations and the constraints are you know kind of you know mixture kind of you know situations where we may have less than type we may have greater than type and we may have equality type.

So, that means, it actually depends upon a particular business problem and corresponding to business problems the idea is to formulate the LP is subject to you know the objective functions may be a maximization type and minimization type and then the constraints will be follow up as per the kind of you know business problem or the kind of you know management kind of you know requirement and so, once we design the models then we will look for the kind of you know solutions.

So, we have no issue to solve the problems irrespective of you know maximization of objective functions minimize[ation] or minimization of objective function subject to constraints which are not in a kind of you know typical format the less than type or greater than type where we may have actually mixture type. So, so the idea is that you know we may not have any problem.

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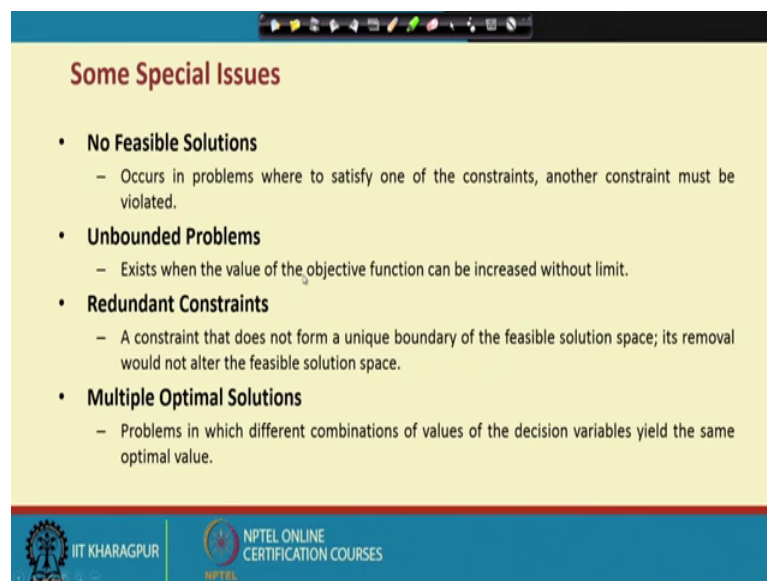


So, the first step of the particular process is we like to streamline the process depending upon the particular objective a may be maximization type or minimization type.

So, in total you know we have solved problems and looking for optimum solutions in the case of maximization and in the kind of you know minimizations where the problems are more or less consistent without any obstacles or without any kind of you know issues or some kind of you know complexity. But, in reality a you know irrespective of objective functions maximization and minimizations then the mixture kind of you know constraints we may find some kind of you know abnormal cases and you know and the kind of you know special cases where the obj[ective] the objective function and constraints may not give you some kind of you know unique you know optimum solution.

So, now, we like to address these issues in this particular you know lecture in reality there are certain business problems which may have such kind of you know indications then we should know how to solve these problems and look for the kind of you know solution or you know giving the kind of you know indication so that you know you know management decision can be taken into considerations.

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Some Special Issues

- **No Feasible Solutions**
 - Occurs in problems where to satisfy one of the constraints, another constraint must be violated.
- **Unbounded Problems**
 - Exists when the value of the objective function can be increased without limit.
- **Redundant Constraints**
 - A constraint that does not form a unique boundary of the feasible solution space; its removal would not alter the feasible solution space.
- **Multiple Optimal Solutions**
 - Problems in which different combinations of values of the decision variables yield the same optimal value.

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So, now you know to highlight all these things so, some of the special cases or you know issues like sometimes we may not have no feasible solutions. So, you know you know the kin[d] the kind of you know structure is like this. Sometimes we may have objective functions maximization or minimization and the constraints may be consistence maybe some kind of you know mixture still you we may not every time get the feasible solution. So, that means, technically we like to know what are this you know you know you know

specific cases where you may not have no feasible solution irrespective of you know objective function and the kind of you know constraints.

And, the second special issue is sometimes the problems is unbounded type, that means, say this may be the case where objective function and constants may not be consistent as per the particular you know business requirement or the kind of you know management requirement. In that case we may not be in a strong position to come with you a kind of you know management decision for a specific problem.

And, the third special case is the redundant constraints; that means, technically the usual structure of LPG we must have objective function and then subject to certain constraints maybe a typically less than types or typically greater than types or typically the mixture of both you know less than type or greater than ty[pe] and greater than type and you know sometimes may be equality type, a but widely you know doing the kind of you know solutions where you know the optimum solution is with respect to objective function the constraints must be consistent then in the process of you know you know optimality some constants may not be actually effective and as a result it can be a called as you know redundant.

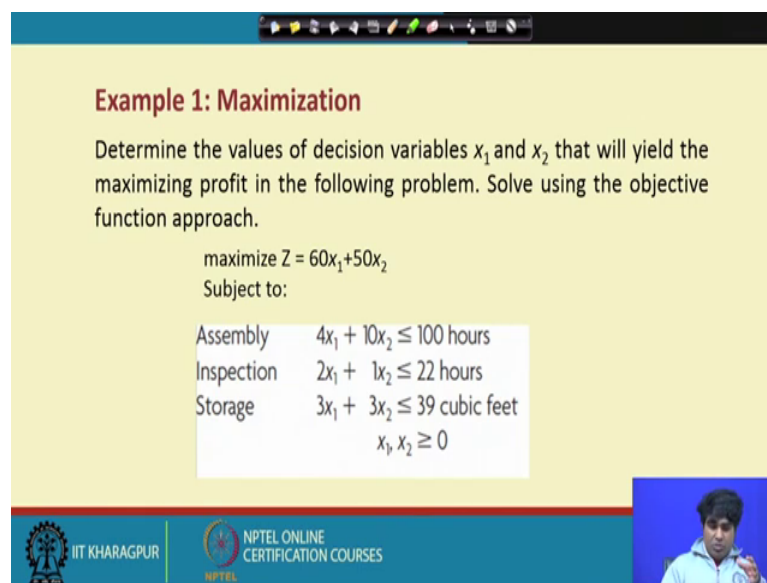
So, that means, in the final optimum solution this may not be you know the kind of you know the kind of you know linkage towards the optimum solutions. So, it is it is it may be the case of you know the dominance of you know other constraints to the objective function and the fourth special issue is the multiple solutions; that means, technically the usual structure of you know optimization or prescriptive analytics is that you know we like to find out the you know corner points; that means, every corner point may be in a position to give optimum solutions.

So, now, after knowing the corner points so, we had like to put all these corner points to the objective function and try to fix the particular point for optimality where the value of a objective function you know will be maximum for a maximization problem and you know extremely minimum for a minimization problem. But, we may find some special case where the values of the objective function are equal at two different corner points or three different corner points. So, in typical situations our you know structure is called as you know a problem having multiple optimum solutions.

So, these are all called as you know special cases and we like to see what are these you know special cases and how is this particular you know structure graphically and then we like to connect the LP structure with different you know application that too in the kind of you know management domain. So, that means, typically we like to you know how prescriptive structure is all about o and in a kind of you know problem like marketing, finance, operation research the kind of you know HR.

So, that means, typically in a kind of you know business environment prescriptive analytics is a kind of you know typical technique where you know all kinds of you know problems can be you know connected if provided it must be fitted with the linear programming format and then looking for the kind of you know solution and the kind of you know management decision.

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Example 1: Maximization

Determine the values of decision variables x_1 and x_2 that will yield the maximizing profit in the following problem. Solve using the objective function approach.

maximize $Z = 60x_1 + 50x_2$

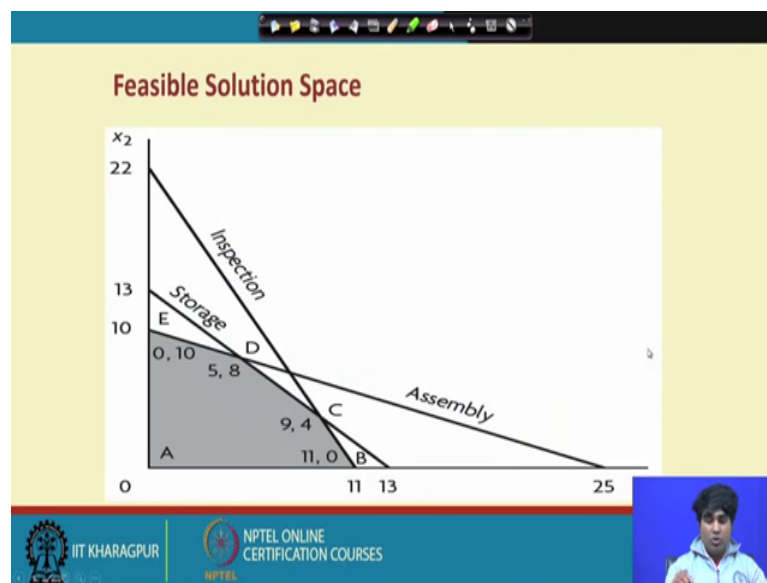
Subject to:

Assembly	$4x_1 + 10x_2 \leq 100$ hours
Inspection	$2x_1 + 1x_2 \leq 22$ hours
Storage	$3x_1 + 3x_2 \leq 39$ cubic feet
	$x_1, x_2 \geq 0$

The slide is part of an NPTEL presentation from IIT Kharagpur. It features a yellow background for the text and a small video inset of the presenter in the bottom right corner. The footer includes the IIT Kharagpur logo and the NPTEL Online Certification Courses logo.

And, by the way so, we will see first in the case of you know maximization slide this is this is strongly you know normal case for maximization problem where the objective function is the typically the kind of you know maximization type and then the constraints are typically less than type. So, this is the normal case of the kind of you know LP problem in that too the other to the case of you know maximizing the objective functions and this problem we have already discussed.

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And, in the counterpart the you know the kind of you know you know solution will be like this and we like to you know plot the constraints as per the particular you know structure and finally, we look for the corner point and then think about the value of the objective function corresponding to the these you know corner points and where the value is the highest that will be the final optimum solutions in response to maximizing the objective functions and.

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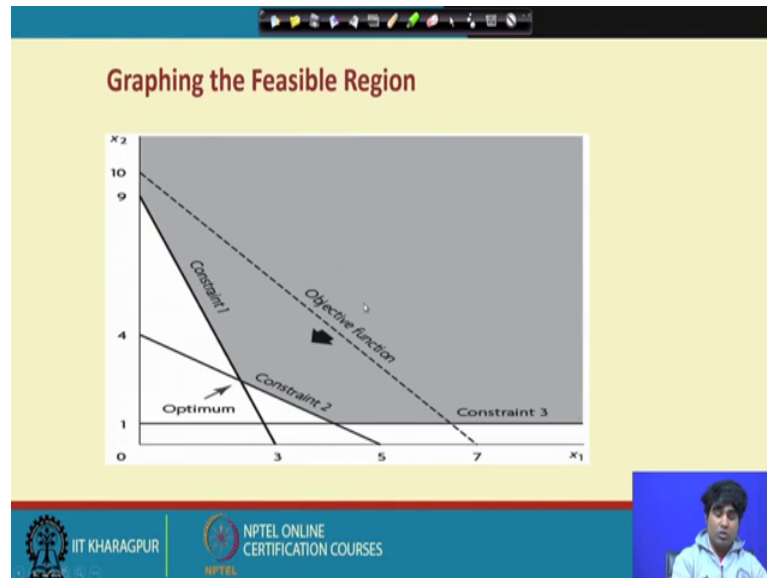
Example 2: Minimization

Determine the values of decision variables x_1 and x_2 that will yield the minimum cost in the following problem. Solve using the objective function approach.

minimize	$Z = .10x_1 + .07x_2$
subject to	
Constraint 1	$6x_1 + 2x_2 \geq 18$
Constraint 2	$8x_1 + 10x_2 \geq 40$
Constraint 3	$x_2 \geq 1$
	$x_1 \text{ and } x_2 \geq 0$

You know contrary to the maximization type the minimization problem structure will be like this and the objective function as usual minimizing a Z subject to constraints.

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And that too so, this is the minimization kind of you know structure where the Z is the minimization type and the kind of you know constraint is the typically greater than type.

So, this, that means, technically this is the normal case of you know minimizations corresponding to objective function the constraint should be a you know greater than type and the as usual there will be restriction you know for decision variable that should be positive in nature.

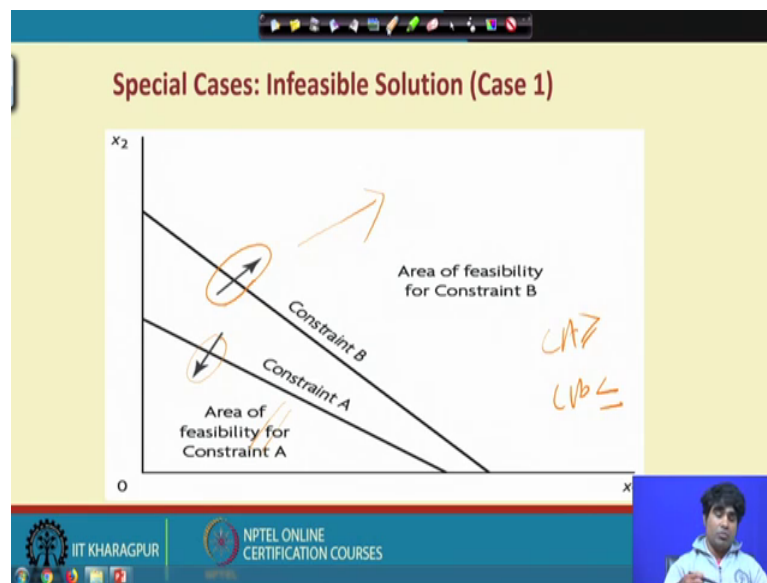
So, this is a as usual you know structure of LP problem and corresponding to the maximization problems so this is how the structure about the minimization problem and accordingly, we are looking for the solutions and the solution will be also we try to find out the feasible zone and then looking for the corner points and then looking for the kind of you know optimality by putting all these corner points value to the objective function and fixing the optimality.

So, this is the typical case of you know minimization problem. So, that means, technically. So, this is how the structure about the maximization type and this is how the structure of the you know minimization type; that means, the feasible zone will be in the upper side and we are looking for the bottom of the sides and in the case of you know

maximization type we like to fix the boundary and then the solution will be within the particular you know boundary.

So, that is the typical difference between the maximization structure and minimization structure and in the case of you know prescriptive analytics and that too in a kind of you know linear programming you know situations or the kind of you know management problem.

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And, corresponding to the kind of you know maximization case and minimization case that too in the normal situation. So, we have already sited two different examples and now, we come to the kind of you know special case the first special case is the infeasible solution. And, typically the objective function may not give you the kind of you know optimum solution as per the particular you know business requirement or the kind of you know management requirement and it may be the situation where corresponding to a particular objective functions may be maximization type may be minimization type, but the constraints are not consistent.

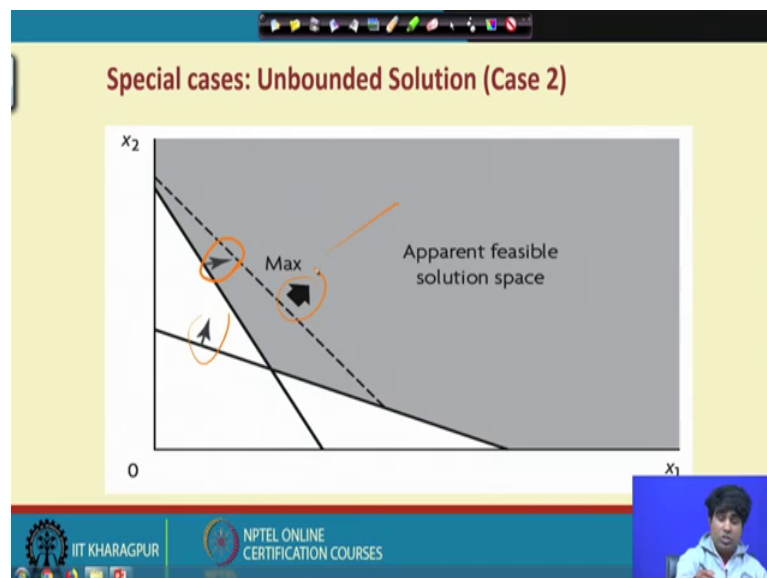
For instance, if there are two variables say x_1 and x_2 means since we are you know doing the graphical kind of you know indication to know the normal case and the kind of you know special case so, for the a objective functions may not reach the optimality if you know the constraints are not consistent. For instance if there are two constraint one will be greater than type and another one will be less than type and that is the typical

kind of you know situation where you may not get the kind of you know optimum solution.

The reason is that you know we may not in a position to find out the feasible regions through which you know we look for the corner points and look for the kind of you know optimum solution where depending upon the a kind of you know objective functions. So, that means, this is a typical case here and here so, the typical error indication represents that you know it is the greater than type of constraints and this error indicates that you know it is the less than type of you know constraint; that means, we have constraint A and constraint B constraint a is the kind of you know less than type of you know constraint and constraint A is the greater than type of you know constraints as a result.

So, to have a different kind of you know you know structural together as a result optimality may not you know possible and as a result we may not in a position to solve the business problem and may not get a kind of you know optimum solutions. So, this is a kind of you know inconsistent problem through which actually you know we may face in the case of you know management problems and as a result we may not get a optimum solution.

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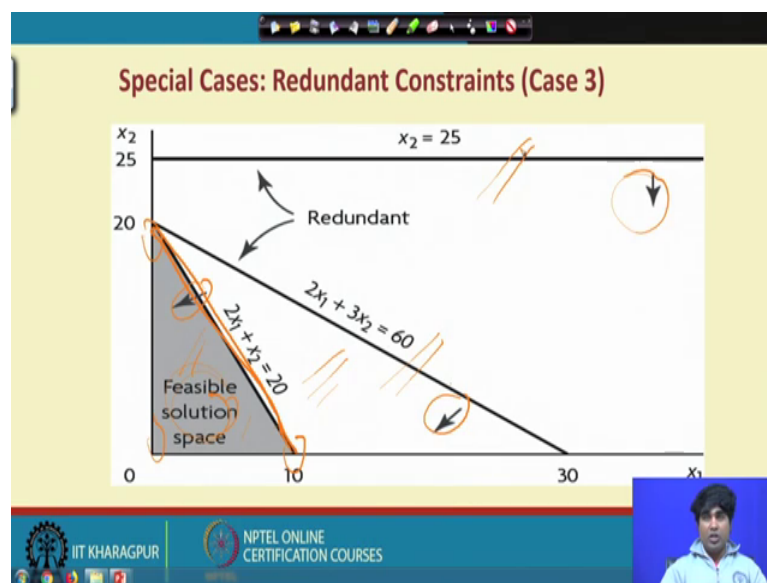


And, this is a typical case of you know another typical case that is the special case that that is called as you know unbounded case and a this is a this is a kind of you know situation where your objective function is the maximization type and the constraints are

typically greater than type. So, that means, technically so, you know looking here the indication of you know arrow so it gives the indication that it is the you know kind of you know greater than type of you know constraint and this is also greater than type of constraint and this is also greater than type of constraint. So, that means, technically the zone will be in the right side and we are looking for you know maximization of you know Z value.

So, in this case of you know in this case your boundary is not actually finite. So, it is it is actually unbounded typically. So, the feasible zone has no boundary. So, as a result so, you may not in a position to find out the all possible corner points and as a result you may not get the optimum solution. So, this is the second special case you may face in the case of you know optimality or in the kind of you know prescriptive analytic structure.

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And, the third special case is the redundant constraints. So, that means, what I have mentioned already. So, having a kind of you know you know problems maximization problem or minimization problem so, sometimes you know few constraints may dominate the kind of you know structure in such a way that you know you may get the objective you know values of the objective function and you may get the optimum solution, but all the constraints may not finally, in a kind of you know operative.

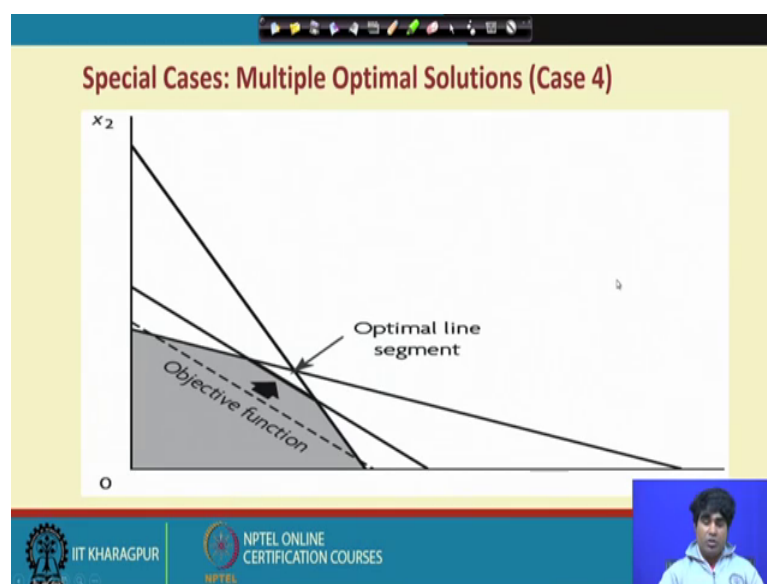
You know it may happen sometimes because you know as per the particular you know requirement so, the out of you know three constraints two constraints are you know

satisfying the case and as a result you may be in a position to get the optimum solution. For instance so, this is this is a kind of you know if you look the arrow so, it is the less than type and then the feasible zone will be accordingly this much and then this is also the case of you know second constraint the you know indication is also less than type. So, as a result so, your feasible zone will be also in the less than type.

Now, if you connect the first constraint and second constraint then technically the feasible zone will be only here. So, this will not be actually counted. So, that means, so, the first constraints will be you know dominate heavily and against coming to this particular you know constraint you know third constraint which is also less than type, but it is too much above. So, that means, the this particular constraints this particular constraints by default give you the kind of you know structure that you know optimal solution by default will be lying in these three corner points.

So, as a result this is not so effective and this is not so effective. So, but you know we are getting the optimum solution corresponding to the objective function, but these constraints are not so effective in the kind of you know optimum solution. Sometimes it may happens depending upon the kind of you know business problem and the kind of you know management problem. So, this is the third case a special case about the kind of you know prescriptive analytics.

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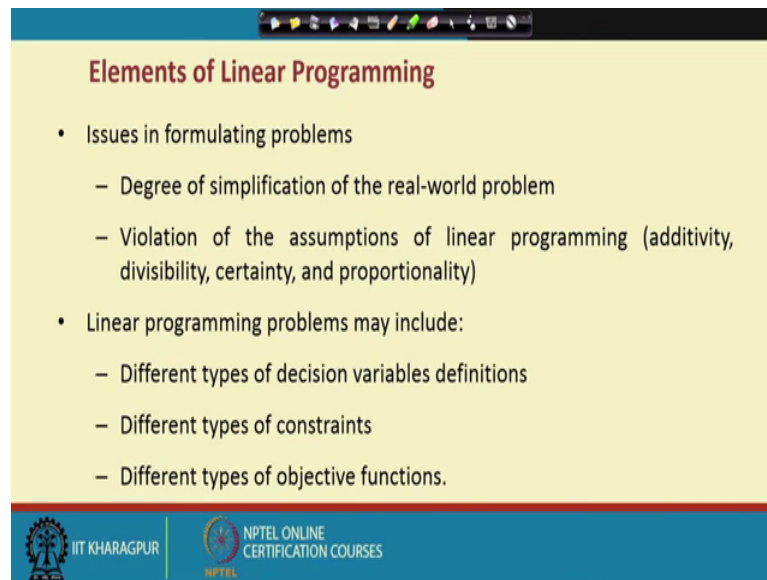
And, then the fourth case is the multiple optimal solutions and here. So, the constants are consistent corresponding to the objective function, but this is the case where at least you know two different corner point the value of the objective function is more or less similar. So, if that is the case then this will indicate that you know multiple optimum solutions or sometimes you know alternatives this is what I call called as you know alternative solution, that means, corresponding to a particular objective function. So, the there is another point where the objective values of the objective function you know is more or less same.

So, this is the case where we can call you can call a situation that you know the problem having multiple solutions. So, that means, it is you know you know flexible often flexible because when we are looking for the optimum solutions we must have a flexible points then we are we are you know checking and then declaring that this is the kind of you know optimality. Now, within the flexibility two different situation where the values of you know objective function will coincide and at least a two different situation then as a result so, this is the declaration about the multiple optimum solution.

So, now, if a particular problem is having indication about the multiple optimum solution; that means, it is good for the kind of you know business problem or the kind of you know you know management problem, that means, management decision will be more effective and efficient because we have the kind of you know alternative and that too it is called as a situation called as you know opportunity kind of based kind of you know situations where you can choose any of the two and as per the particular requirement.

Of course, the combination of the values of the decision variable will be different what you know the objective function value will be remain same sometimes you know depending upon the particular you know requirement for instance we have already discussed the case that you know economic feasibility and social feasibility. So, now, you know if you have a such kind of you know situation where the solution is multiple types then you can choose a particular of some which may not affect the objective function and then it may erase the social objective rather than you know economic objective.

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The slide is titled "Elements of Linear Programming" in a red font. It contains a bulleted list of issues and problem types. The first bullet point is "Issues in formulating problems", which has two sub-bullets: "Degree of simplification of the real-world problem" and "Violation of the assumptions of linear programming (additivity, divisibility, certainty, and proportionality)". The second bullet point is "Linear programming problems may include:", which has three sub-bullets: "Different types of decision variables definitions", "Different types of constraints", and "Different types of objective functions." The slide has a blue header and footer. The footer contains the IIT Kharagpur logo and the text "NPTEL ONLINE CERTIFICATION COURSES".

- Issues in formulating problems
 - Degree of simplification of the real-world problem
 - Violation of the assumptions of linear programming (additivity, divisibility, certainty, and proportionality)
- Linear programming problems may include:
 - Different types of decision variables definitions
 - Different types of constraints
 - Different types of objective functions.

That means technically in this kind of you know situations we have this concept called as you know sustainability kind of you know situation where we can take care both economic objective and social objective and sometimes it is a very rare, but of course, if you have the kind of you know situation this is good or you know plus point for the kind of you know management so that you know they can come with you know effective management decision.

And, some of the a other issues are there in the linear programming like you know issues in formulating problems. So, first thing is a degree of simplification of the real world problem and violation of the some of the LP assumptions like you know additivity, a certainty, proportionality, divisibility and the linear programming problems may include different type of you know decision variable definition and different types of you know constraints different types of you know objective functions. In fact, till now, whatever problems we have discussed it is with respect to single objective and in a real life scenario you may have a problem with you know multiple objectives and then we are looking for a kind of you know solutions which can satisfy all the objectives simultaneously.

So, in the that is the case called as you know goal programming and in the goal programming you may have a multiple objectives in the first end and then we are combining these objectives subject to constraint and looking for the kind of you know

solution and this is how the typical you know structure through which you can actually solve some of the problems and some of the some of these you know corresponding to this you know different you know kind of you know structure or the kind of you know complexity a I will just you know highlight couple of examples corresponding to kind of you know complex kind of you know scenario where, objective functions are not you know normally follow the kind of you know typical less than type of you know constraints or the greater than type of you know constraints.

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Example: Style of Comfort Furniture

x_1 = number of sofas produced
 x_2 = number of love seats produced
 x_3 = number of recliners produced
 x_4 = number of coffee tables produced

maximize $Z = 120x_1 + 80x_2 + 150x_3 + 40x_4$

subject to

$400x_1 + 300x_2 + 500x_3 + 150x_4$	$\leq 180,000$	(budget constraint)
x_1	≤ 250	(sales volume constraints)
x_2	≤ 300	
x_3	≤ 200	
x_4	≤ 600	
$2x_1 + 1x_2 + 2.5x_3 + .75x_4$	≤ 800	(machine hour constraint)
$2.5x_1 + 2x_2 + 3x_3 + 1x_4$	$\leq 1,200$	(labor hour constraint)
$240x_1 - 120x_2 - 200x_3 - 60x_4$	$= 0$	(sofa production cost)
$-100x_1 - 75x_2 + 375x_3 - 37.5x_4$	$= 0$	(recliner cost)
x_2	≥ 30	(minimum love seat production)
x_1, x_2, x_3, x_4	≥ 0	(nonnegativity constraint)

And, in this case first problem will be like this you see here this is a kind of you know problem where we have a maximization objective functions and we have a couple of constraints here like you know budget constraint, then sales volume constraint, machine hour constraint, labor hour constraint, then production cost other cost you know like you know lots of you know their typical you know constraints are there. So, we have a multiple constraints and then the a problem is also you know multivariate kind of you know structure having four different you know requirement that is the values of the decision variable that is typically x_1 , x_2 , x_3 and x_4 .

And, then some of the constraint mod number of constraint and out of you know or you know all constraints so, first couple of constraints are you know less than type and then we have a another set of you know constraint which are you know greater than type. So, that means it is a kind of you know mixture and then we look for the kind of you know

solution in this kind of you know problems usually graphically we cannot you know easily reach the optimality or optimum solution and in this kind of you know problems we like to solve or looking for the management decision by different you know approach or different methods which we called as you know simplex procedure or the kind of you know algebraic procedure.

So, that means, typically you have to write you know in fact, computer programming can be connected and then we are looking for the kind of you know solution. Of course, we will be connect with you know solver the case package and to solve this kind of you know problem to get the you know optimum results because when the problem is very big and complex it is not so easy to get quickly manually and in that context we like to use some of the sub tears may be or sub tears may be (Refer Time: 25:16) and the kind of you know solvers to look for the kind of you know solution.

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Example: Dietary Requirements for Diet Problem

	Wheat	Rice	Corn	Requirements per 12-Ounce Box
Protein (grams per ounce)	4	2	2	At least 27 grams
Carbohydrates (grams per ounce)	20	25	21	At least 240 grams
Calories per ounce	90	110	100	No more than 1,260 calories
Cost per ounce	\$.03	.05	.02	

Solution
Variable definitions
 x_1 = ounces of wheat per box
 x_2 = ounces of rice per box
 x_3 = ounces of corn per box

Formulate an LP model for this problem that will determine the optimal quantities of wheat, rice, and corn per box that will achieve the requirements at minimum cost.

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And, accordingly so, if you proceed I can you know give you more such examples where the situation is the you know kind of you know the kind of you know you know having you know inconsistent where some are you know less than type and the kind of you know or greater than type. So, this is another examples where we have objective functions and then we have the kind of you know the objective function is here cost per issues so; that means, technical it is a minimization type of you know problems subject to 3 constraint protein constraints, carbohydrates hydrates con constraints, then calories.

So, like you know these are all the at least means it is a greater than type kind of you know situation and a corresponding to this particular problem, so, that means, technically. So, this is the situation where we can have a less than type of you know situation and this is a profit functions and then and the kind of you know solution.

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Example 5: Diet Problem

minimize $Z = .03x_1 + .05x_2 + .02x_3$

subject to

Protein	$4x_1 + 2x_2 + 2x_3$	\geq	27 grams
Carbohydrates	$20x_1 + 25x_2 + 21x_3$	\geq	240 grams
Calories	$90x_1 + 110x_2 + 100x_3$	\leq	1,260 calories
Box size	$x_1 + x_2 + x_3$	$=$	12 ounces
	$x_1, x_2, \text{ and } x_3$	\geq	0

Solution to diet problem

$x_1 = 1.5$ (ounces of wheat per box)
 $x_3 = 10.5$ (ounces of corn per box)
 $z = \$0.26$ (minimum cost per box)

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So, that means, corresponding to this so, the particular you know formulation will be maximum minimizing Z and subject to all the constraint and a typically the (Refer Time: 26:32) here. So, this is actually minimization objective functions and we have a we have actually a four constraints so two are greater than type, one is a less then type and one is equality type and this is typically you know different you know you know kind of you know situation that is that that is actually it is completely mixture type of you know situation where we are looking for the you know optimality that too minimizing the objective functions.

And, obviously, a you know there is the optimum solution even if you know we have a different kind of you know constraints and according to corresponding to this problem. So, the values of the decision variable will be x 1 equal to 1.5, x 3 equal to 10.5 and in fact, x 2 is not coming here and Z is coming 0.26.

So, that means, you know it is not necessarily that you know all the values of the I mean decision variable will be active sometimes because of you know constraints and the kind of you know requirement or the with respect to objective function and some decision

variable may be finally, dropped. So, that is how we are looking for a kind of you know structure where you can take care the economic and social aspect simultaneously; that means, if you know consider the best structure through which you know business can be operative, if your objective is not so, you know commercial type of you know situation where you know you know we like to target you know more profit then, obviously, some of the recent variable may be dropped in the final you know solution, but in the kind of you know social kind of you know set up where all the decision variables should be a operative and then we look for the higher that could be the best way of you know kind of you know presenting the situation.

But, usually a we look for the solution as per the management requirement and business requirement and that too as per the specific objective you know a specific object is concerned you know. So, far as you know business requirement or the kind of you know management requirement.

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Example: Blending Problem

Formulate the appropriate model for the following blending problem: The sugar content of three juices—orange, banana, and pineapple—is 10, 15, and 20 percent, respectively. How many quarts of each must be mixed together to achieve one gallon (four quarts) that has a sugar content of at least 17 percent to minimize cost? The cost per quart is 20 cents for orange juice, 30 cents for banana juice, and 40 cents for pineapple juice.

Solution
Variable definitions
 O = quantity of orange juice in quarts
 B = quantity of banana juice in quarts
 P = quantity of pineapple juice in quarts

minimize $Z = 20O + 30B + 40P$

subject to

Sugar content $10O + 15B + 20P \geq 17(O + B + P)$

Total amount $O + B + P = 4$ quarts

Solution to blending problem

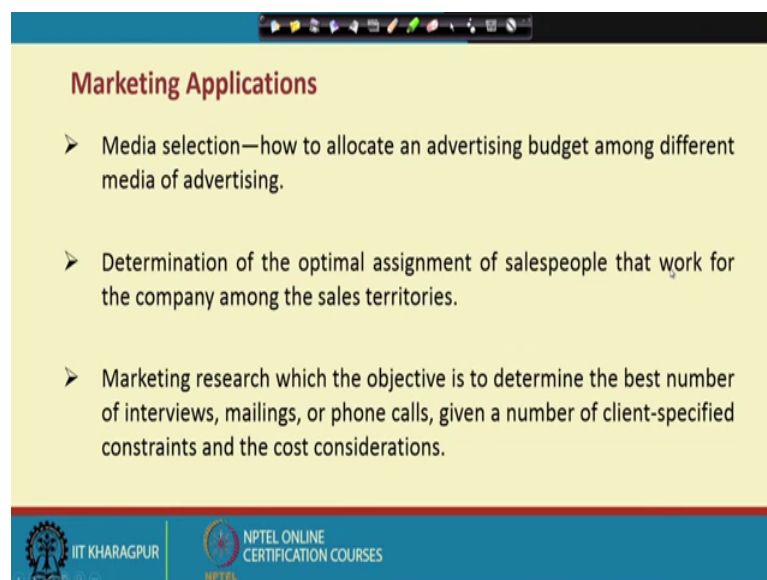
$x_1 = 12$ quarts (quantity of orange juice)
 $x_2 = 2.8$ quarts (quantity of pineapple juice)
 $Z = \$1.56$ (minimum total cost of the blended juice per gallon)

And, corresponding to the particular you know structure against I can highlight a couple of more problems and this is actually again a three decision variable case and O, B, P and this is the quantity of you know three different fruits and then we have actually kind of you know minimization problem minimize you know objective function subject to certain constraint here and so here the constraint is you first one is the greater than type and the second one is the equality type and still we have the optimum solution and

corresponding to this particular problem where we have actually three variables then finally, the optimum solution is coming with respect to two variable that too for x_1 and x_3 and the values of the objective function is also you know uniquely represented here.

So, that means, you know even if the problem is consistent and the constraints are you know not in a kind of you know unique format still we can get the optimum solution and the and there may be the situation where all the values of the decision variable may not be in a kind of you know positive structure. So, some may be dropped and finally, we like to fix the kind of you know optimality as per the specific objective or the management requirement.

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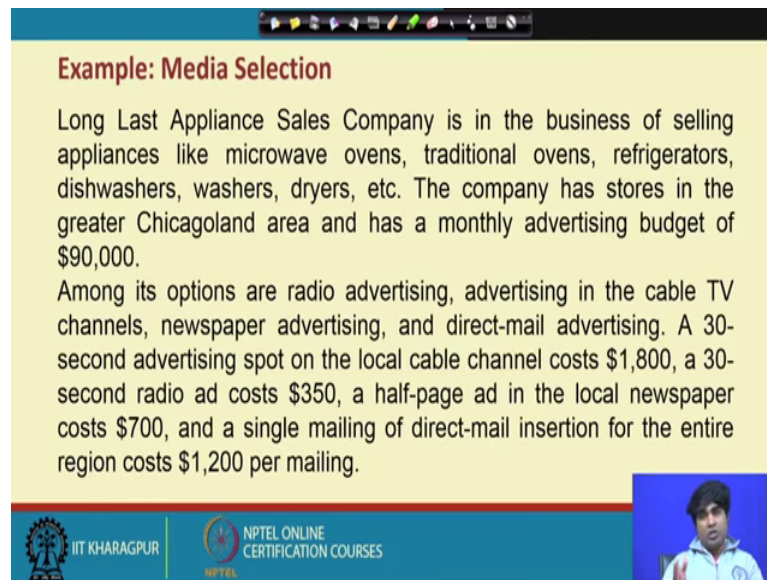
Marketing Applications

- Media selection—how to allocate an advertising budget among different media of advertising.
- Determination of the optimal assignment of salespeople that work for the company among the sales territories.
- Marketing research which the objective is to determine the best number of interviews, mailings, or phone calls, given a number of client-specified constraints and the cost considerations.

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And, accordingly so we can have a more such kind of you know examples and where we can actually highlight the particular you know issue special issues and there are also you know you know similar kind of you know application in the marketing area typically media selection, how to allocate an advertising expenditure among different media you know; for you know advertising and determination of the optimal assignment of say you know sales people that work for the company among the sales regions and marketing research which the objective is to determine the best number of you know interviews, mailings, phone calls like that you know in order to you know the you know cost considerations. So, likewise you know several kind of you know situation.

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Example: Media Selection

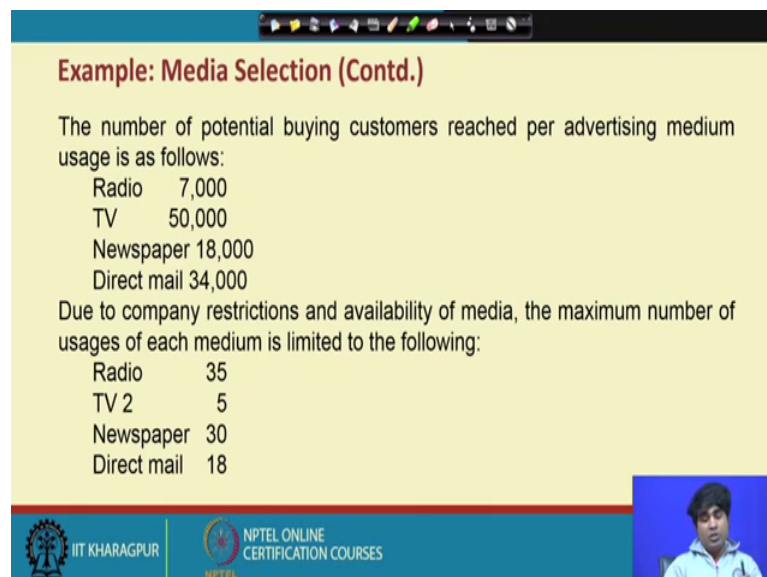
Long Last Appliance Sales Company is in the business of selling appliances like microwave ovens, traditional ovens, refrigerators, dishwashers, washers, dryers, etc. The company has stores in the greater Chicagoland area and has a monthly advertising budget of \$90,000.

Among its options are radio advertising, advertising in the cable TV channels, newspaper advertising, and direct-mail advertising. A 30-second advertising spot on the local cable channel costs \$1,800, a 30-second radio ad costs \$350, a half-page ad in the local newspaper costs \$700, and a single mailing of direct-mail insertion for the entire region costs \$1,200 per mailing.

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This is a kind of you know media selection problem and corresponding to this you know these are all actually business kind of you know effects and corresponding to the business effect and the kind of you know condition the kind of you know obstacles the kind of you know constraints.

(Refer Slide Time: 31:16)



Example: Media Selection (Contd.)

The number of potential buying customers reached per advertising medium usage is as follows:

Radio	7,000
TV	50,000
Newspaper	18,000
Direct mail	34,000

Due to company restrictions and availability of media, the maximum number of usages of each medium is limited to the following:

Radio	35
TV	5
Newspaper	30
Direct mail	18

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So, we have to transport the kind of you know structures to the kind of you know model and this is how the basic inputs to this you know particular problem.

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Example: Media Selection (Contd.)

The management of the company has met and decided that in order to ensure a balanced utilization of different types of media and to portray a positive image of the company, at least 10 percent of the advertisements must be on TV. No more than 40 percent of the advertisements must be on radio. The cost of advertising allocated to TV and direct mail cannot exceed 60 percent of the total advertising budget. What is the optimal allocation of the budget among the four media? What is the total maximum audience contact?



Formulation

Variable definitions

- x_1 = Number of radio ads
- x_2 = Number of television ads
- x_3 = Number of newspaper ads
- x_4 = Number of direct-mail ads

The objective function

Maximize $Z = 7,000x_1 + 50,000x_2 + 18,000x_3 + 34,000x_4$



And, finally, a again more such you know inputs and then finally, collect all such inputs and transferring into a kind of you know model. And, this is how the a for variable case that is the x_1, x_2, x_3, x_4 and depending upon you know four different I mean say you know four different ways to put the advertising that is with respect to radio and televisions newspaper and you know emails and accordingly the objective function is to maximize a Z equal to $7000 \times x_1 + 50000 \times x_2 + 18000 \times x_3 + 34000 \times x_4$ that is how the a problem structure is that you know basic requirement and then with respect to constraint we look for the kind of you know solution.

(Refer Slide Time: 32:09)



Example: Media Selection (Contd.)

The constraints

$$350x_1 + 1,800x_2 + 700x_3 + 1,200x_4 \leq 90,000 \quad (\text{budget constraint})$$
$$\begin{aligned} x_1 &\leq 35 \\ x_2 &\leq 25 \\ x_3 &\leq 30 \\ x_4 &\leq 18 \end{aligned} \quad (\text{maximum exposure constraints})$$
$$\begin{aligned} x_2 &\geq .10(x_1 + x_2 + x_3 + x_4) \quad (10\% \text{ minimum TV ads}) \\ x_1 &\leq .40(x_1 + x_2 + x_3 + x_4) \quad (40\% \text{ maximum radio ads}) \end{aligned}$$

Solution

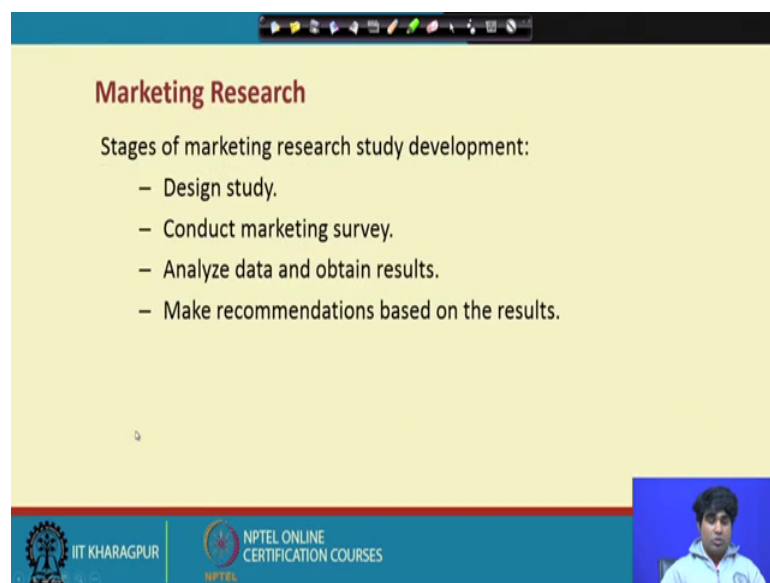
Solving this problem with Excel results in the following:

$$\begin{aligned} x_1 &= 25.71 && (\text{number of radio advertisements}) \\ x_2 &= 21.33 && (\text{number of TV advertisements}) \\ x_3 &= 30 && (\text{number of newspaper advertisements}) \\ x_4 &= 18 && (\text{number of direct-mail advertisements}) \end{aligned}$$
$$Z = 2,398,667 \quad (\text{total number of potential customers reached})$$


And, against this is a kind of you know problems where we have a less than type of a constraint greater than type of you know constraint and then still we are looking for the solution which can give you the kind of you know you know optimality. So, this is the less than type structure, this is the greater than type structure and this is again less than type of structure still we are getting the optimum solution and interestingly, in this particular you know structure here all the values of the decision variables are at the you know final you know kind of you know structure and then in the a then the objective function is also satisfactory.

So, this is again you know the kind of you know structure through which actually we are looking for the solutions and then you are looking for the kind of you know management decisions.

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Marketing Research

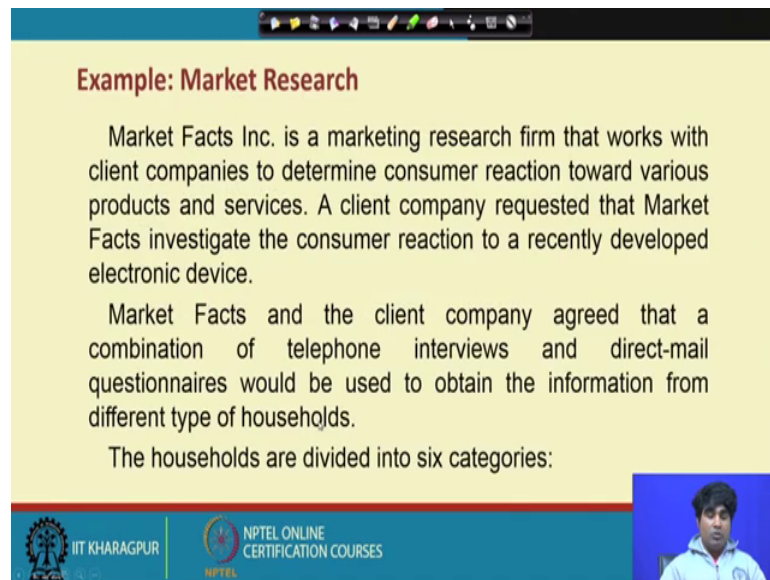
Stages of marketing research study development:

- Design study.
- Conduct marketing survey.
- Analyze data and obtain results.
- Make recommendations based on the results.

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And, this is again similar kind of you know problems and some marketing research you know kind of you know design study, conduct marketing survey, analyze data.

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Example: Market Research

Market Facts Inc. is a marketing research firm that works with client companies to determine consumer reaction toward various products and services. A client company requested that Market Facts investigate the consumer reaction to a recently developed electronic device.

Market Facts and the client company agreed that a combination of telephone interviews and direct-mail questionnaires would be used to obtain the information from different type of households.

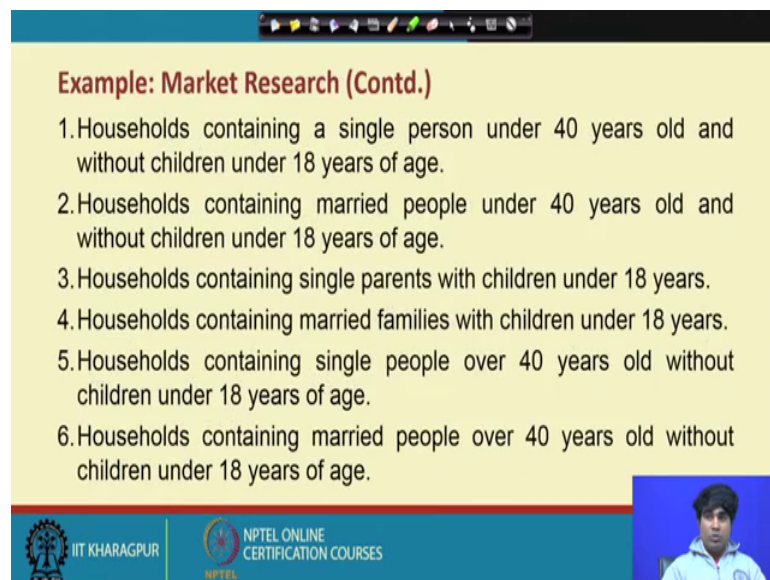
The households are divided into six categories:

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Like that you know we have another you know kind of interesting problem here again this is what the you know information's.


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Example: Market Research (Contd.)

1. Households containing a single person under 40 years old and without children under 18 years of age.
2. Households containing married people under 40 years old and without children under 18 years of age.
3. Households containing single parents with children under 18 years.
4. Households containing married families with children under 18 years.
5. Households containing single people over 40 years old without children under 18 years of age.
6. Households containing married people over 40 years old without children under 18 years of age.

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And, then the particular structure will be divided into different kind of you know inputs.

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Example: Market Research (Contd.)

Restrictions

1. At least 60 percent of the phone interviews must be conducted at households with children.
2. At least 50 percent of the direct-mail questionnaires must be mailed to households with children.
3. No more than 30 percent of the phone interviews and mail-in questionnaires must be conducted at households with single people.
4. At least 25 percent of the phone interviews and mail-in questionnaires must be conducted at households that contain married couples.

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And, corresponding to these inputs we can have a you know some kind of you know restrictions.

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Example: Market Research (Contd.)

x_{ij} = number of phone interviews or questionnaires used to reach household type j
 where
 $t = 1, 2$ 1 = phone interview, 2 = mail-in questionnaire
 $j = 1, 2, 3, 4, 5, 6$ (j represents the six household categories defined earlier)

Based on the above definitions of the decision variables, the objective function can be written as

minimize $Z = 10x_{11} + 11x_{12} + 12x_{13} + 15x_{14} + 7x_{15} + 9x_{16} + 1x_{21} + 1x_{22} + 1.5x_{23} + 1.5x_{24} + 1x_{25} + 1x_{26}$

We can begin to write the constraints for this problem by considering the target values of advertising. Since the target number of phone interviews is 2,000 and the target number of mail-in questionnaires is 48,000, the two constraints can be written as follows:

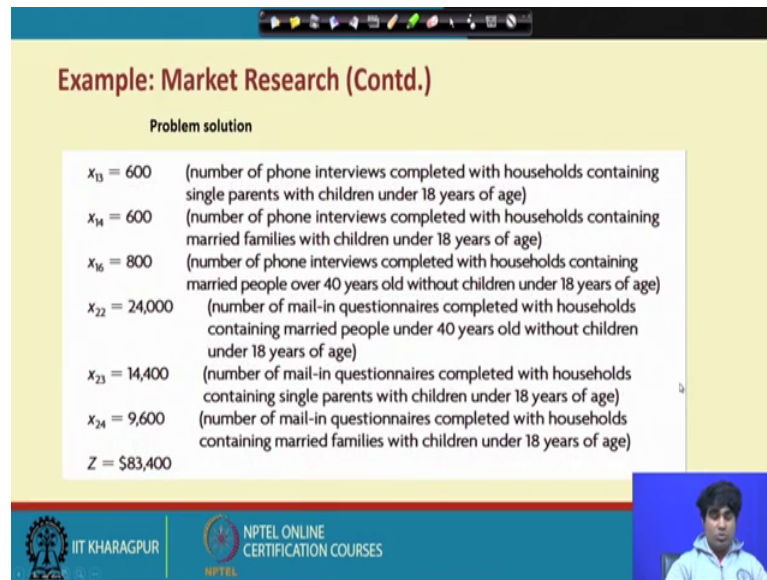
$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} = 2,000$ (target number of phone interviews)
 $x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} = 48,000$ (target number of mail-in questionnaires)

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And, finally we have a the kind of you know LP structures where you know we have the objective functions and the constraint and here typically the constraints are you know all are you know equality type still we are you know looking for the solutions and then we are you know actually you know looking for the kind of you know management decision. If the constraints are you know equality type by default then you may not have a kind of

you know big problem to get the solutions, but still you know we like to you know see what is the kind of you know reality and then we are looking for the kind of you know solution.

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Example: Market Research (Contd.)

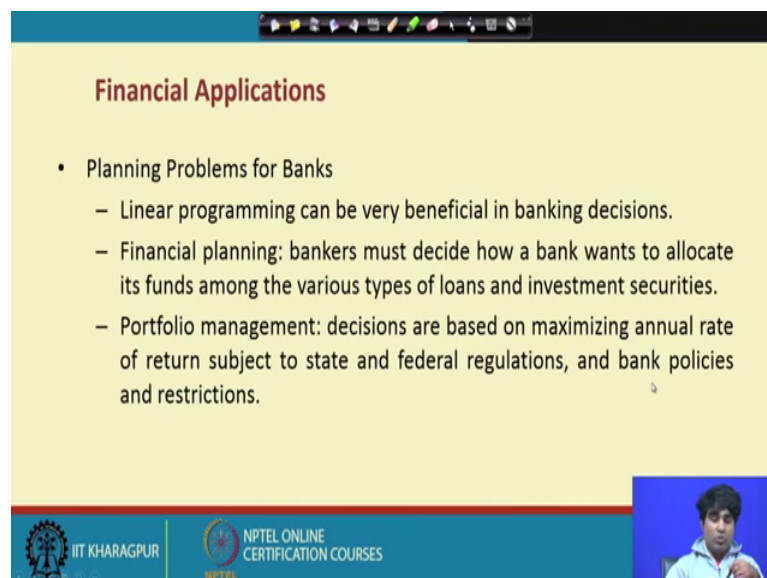
Problem solution

$x_{13} = 600$	(number of phone interviews completed with households containing single parents with children under 18 years of age)
$x_{14} = 600$	(number of phone interviews completed with households containing married families with children under 18 years of age)
$x_{16} = 800$	(number of phone interviews completed with households containing married people over 40 years old without children under 18 years of age)
$x_{22} = 24,000$	(number of mail-in questionnaires completed with households containing married people under 40 years old without children under 18 years of age)
$x_{23} = 14,400$	(number of mail-in questionnaires completed with households containing single parents with children under 18 years of age)
$x_{24} = 9,600$	(number of mail-in questionnaires completed with households containing married families with children under 18 years of age)
$Z = \$83,400$	

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So, in this problem so, we have still you know solutions and the values of the decision variable and the kind of you know optimality of you know, objective function.

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Financial Applications

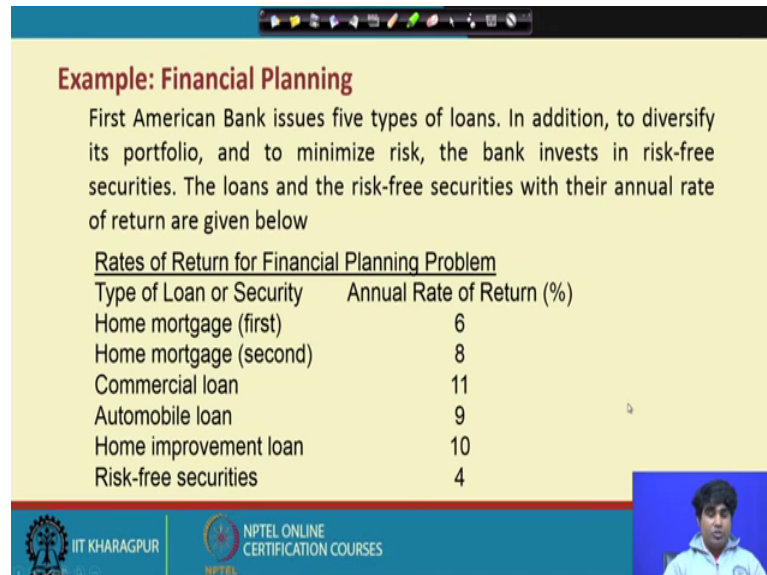
- Planning Problems for Banks
 - Linear programming can be very beneficial in banking decisions.
 - Financial planning: bankers must decide how a bank wants to allocate its funds among the various types of loans and investment securities.
 - Portfolio management: decisions are based on maximizing annual rate of return subject to state and federal regulations, and bank policies and restrictions.

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And, this particular problem again it can be applied to you know different financial you know problems and this is one typically know areas where a we can actually apply like

you know banking decisions then the kind of you know portfolio management decision, financial planning. So, so many ways you can actually apply.

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Example: Financial Planning

First American Bank issues five types of loans. In addition, to diversify its portfolio, and to minimize risk, the bank invests in risk-free securities. The loans and the risk-free securities with their annual rate of return are given below

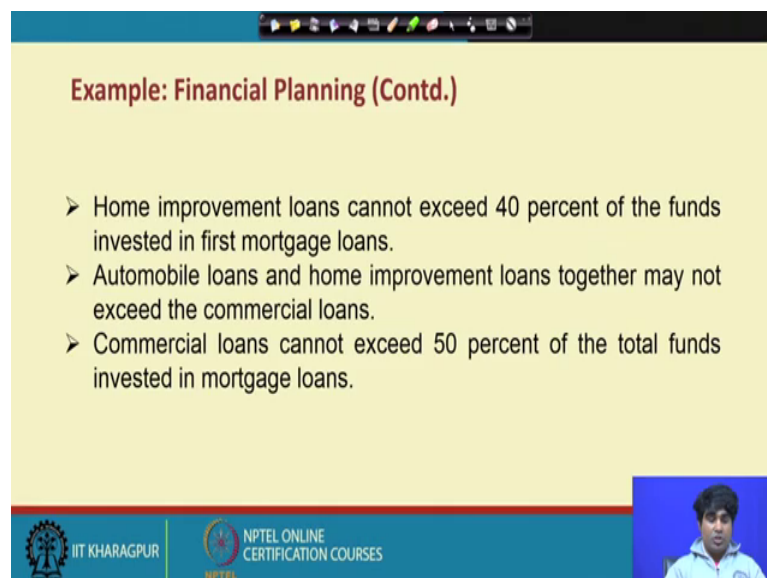
Rates of Return for Financial Planning Problem

Type of Loan or Security	Annual Rate of Return (%)
Home mortgage (first)	6
Home mortgage (second)	8
Commercial loan	11
Automobile loan	9
Home improvement loan	10
Risk-free securities	4

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And, one such example is here with respect to financial planning and issues about you know five different types of you know loans and then we are looking for the kind of you know optimum solution corresponding to these you know information's.

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Example: Financial Planning (Contd.)

- Home improvement loans cannot exceed 40 percent of the funds invested in first mortgage loans.
- Automobile loans and home improvement loans together may not exceed the commercial loans.
- Commercial loans cannot exceed 50 percent of the total funds invested in mortgage loans.

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Example: Financial Planning (Contd.)

Let

- x_1 = dollars invested in first home mortgage loans
- x_2 = dollars invested in second home mortgage loans
- x_3 = dollars invested in commercial loans
- x_4 = dollars invested in automobile loans
- x_5 = dollars invested in home improvement loans
- x_6 = dollars invested in risk free securities

maximize $Z = .06x_1 + .08x_2 + .11x_3 + .09x_4 + .10x_5 + .04x_6$

subject to:

- $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 90,000,000$ (budget constraint)
- $x_6 \leq 9,000,000$
- $x_3 \leq 8,000,000$
- $.40x_1 + .40x_2 - .60x_3 - .60x_4 - .60x_5 \geq 0$
- $x_1 - 2x_2 \geq 0$
- $-.4x_1 + x_5 \leq 0$
- $-x_3 + x_4 + x_5 \leq 0$
- $-.50x_1 - .50x_2 + x_3 \leq 0$
- $x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$ (nonnegativity constraint)

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And, we like to formulate a problem like heres and here the problem is with respect to six variables and by the way a by the way the objective function is the maximize maximization type and then we have been a couple of constraints and some are you know equality type some are greater than type some are in less than type and again we have a multiple kind of you know scenario and that to with again non negativity restrictions and still we are you know you know looking for the solutions and you know having the kind of you know best solution as per the particular you know requirement.

Now, corresponding to this a this particular you know situations so, the solution will be like this.


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Example: Financial Planning (Contd.)

Solution

$x_1 = 32,400,000$	(dollars invested in first home mortgage loans)
$x_2 = 16,200,000$	(dollars invested in second home mortgage loans)
$x_3 = 24,300,000$	(dollars invested in commercial loans)
$x_4 = 99,999.27$	(dollars invested in automobile loans)
$x_5 = 8,000,000$	(dollars invested in home improvement loans)
$x_6 = 9,000,000$	(dollars invested in risk-free securities)
$Z = \$7,082,000.00$	(profit over the investment of \$90,000,000)

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And, so the Solution is a against unique kind of you know situation where all the values of the decision variable are you know effective and the opti[mum] you know the values of the optimum function is also reached.


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Production Applications

- Linear programming in production management in manufacturing:
 - Multiperiod production scheduling
 - Workforce scheduling
 - Make-or-buy decisions.



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So, that means, technically we have actually kind of you know different structure and complexity still we have the optimality results and the kind of you know kind of you know values of the decision variable through which you can actually predict the business environment and then come for the kind of you know management decisions. And, it can

be also in the kind of you know production area and like you know multi-period production scheduling, workforce scheduling and buying decisions all these things will be there.

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Example: Multiperiod Production Scheduling

Morton and Monson Inc. is a small manufacturer of parts for the aerospace industry. The production capacity for the next four months is given as follows:

Month	Production Capacity in Units	
	Regular Production	Overtime Production
January	3,000	500
February	2,000	400
March	3,000	600
April	3,500	800

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So, this is another kind of you know structure; monthly requirement, regular production, overtime production.

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Example: Multiperiod Production Scheduling (Contd.)

The regular cost of production is \$500 per unit and the cost of overtime production is \$150 per unit in addition to the regular cost of production. The company can utilize inventories to reduce fluctuations in production, but carrying one unit of inventory costs the company \$40 per unit per month. Currently there are no units in inventory. However, the company wants to maintain a minimum safety stock of 100 units of inventory during the months of January, February, and March. The estimated demand for the next four months is as follows:

Month	Jan	Feb	Mar	Apr
Demand	2,800	3,000	3,500	3,000

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And, subject to typical you know constraints.

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Example: Multiperiod Production Scheduling (Contd.)

The production manager is in the process of preparing in four-month production schedule. What is the schedule that minimizes total cost, if the company wants to have 300 units in inventory at the end of April?


First, we define the decision variables as follows:

$i = 1, 2, 3, 4$ 1 = January, 2 = February, 3 = March, 4 = April,
 x_i = quantity produced in month i on a regular time basis for $i = 1, 2, 3, 4$
 y_i = quantity produced in month i on overtime for $i = 1, 2, 3, 4$
 I_i = quantity in inventory at the end of month i for $i = 1, 2, 3, 4$

The objective function and the constraints for this problem can be stated as follows:


minimize $Z = 500(x_1 + x_2 + x_3 + x_4) + 650(y_1 + y_2 + y_3 + y_4) + 40(I_1 + I_2 + I_3 + I_4)$

subject to:

$$\left. \begin{array}{l} x_1 \leq 3,000 \\ x_2 \leq 2,000 \\ x_3 \leq 3,000 \\ x_4 \leq 3,500 \end{array} \right\} \text{ (regular time production constraint for each month)}$$


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And, then finally, we have the objective function to minimize and that to with respect to different combination of you know inputs and against there are constraints which are less than type.

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Example: Multiperiod Production Scheduling (Contd.)

$$\left. \begin{array}{l} y_1 \leq 500 \\ y_2 \leq 400 \\ y_3 \leq 600 \\ y_4 \leq 800 \end{array} \right\} \text{ (overtime production constraint for each month)}$$

The total production (regular + overtime) minus what we carry in inventory must equal the demand for a particular month. These constraints result in the following transition equalities:


$$\begin{array}{l} x_1 + y_1 - I_1 = 2,800 \\ x_2 + y_2 - I_2 = 3,000 \\ x_3 + y_3 - I_3 = 3,500 \\ x_4 + y_4 - I_4 = 3,000 \end{array}$$

The inventory constraints can be formulated as follows:

$$\begin{array}{l} I_1 \geq 100 \\ I_2 \geq 100 \\ I_3 \geq 100 \\ I_4 = 300 \end{array}$$


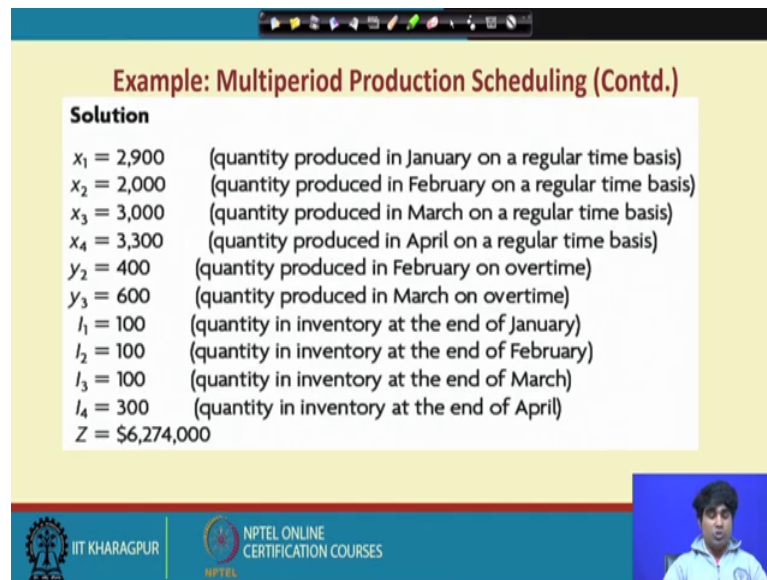
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And, the kind of you know greater than type and the equality type still we are you know this is a very complex problem.

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Example: Multiperiod Production Scheduling (Contd.)

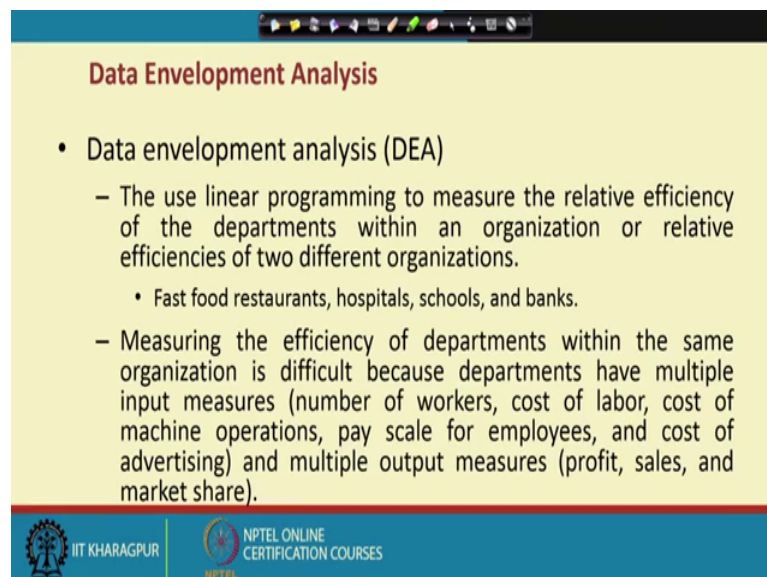
Solution

$x_1 = 2,900$	(quantity produced in January on a regular time basis)
$x_2 = 2,000$	(quantity produced in February on a regular time basis)
$x_3 = 3,000$	(quantity produced in March on a regular time basis)
$x_4 = 3,300$	(quantity produced in April on a regular time basis)
$y_1 = 400$	(quantity produced in February on overtime)
$y_2 = 600$	(quantity produced in March on overtime)
$I_1 = 100$	(quantity in inventory at the end of January)
$I_2 = 100$	(quantity in inventory at the end of February)
$I_3 = 100$	(quantity in inventory at the end of March)
$I_4 = 300$	(quantity in inventory at the end of April)
$Z = \$6,274,000$	

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Still we are you know getting the optimum solution and the values of the objective functions.

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Data Envelopment Analysis

- Data envelopment analysis (DEA)
 - The use linear programming to measure the relative efficiency of the departments within an organization or relative efficiencies of two different organizations.
 - Fast food restaurants, hospitals, schools, and banks.
 - Measuring the efficiency of departments within the same organization is difficult because departments have multiple input measures (number of workers, cost of labor, cost of machine operations, pay scale for employees, and cost of advertising) and multiple output measures (profit, sales, and market share).

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Likewise you know we can have also you know this particular you know predictive analytics that too linear programming in different a you know areas, that means, the application again with respect to data envelopment analysis where the linear programming is to measure the relative efficiency of the various departments within an organization or relative efficiency of two different organization. So, so, means this is a

kind of you know efficiency measurement technique, but typically again a linear programming and the predictive analytics can be used or to get the a efficient you know you know kind of you know results or the kind of you know environment which you can actually solve the business problem and come with a kind of you know effective management decision.


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Example: Airline Efficiency

A consumer advocacy group would like to evaluate the efficiency of three airline companies with respect to several criteria that are important to consumers. The three companies are Beta, Unified, and Southeast. The output measures are identified as average minutes late arriving, number of luggage loser per 10,000 customers, waiting time for check-in, friendliness, and quality of air service (result of a questionnaire that measured this output on a 100-point scale). The input measures were labor cost per day, number of flights per day, and over head cost per day. Table 4-9 summarizes the results.

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So, again so for as you know you know the application is concerned the we can actually look for the airline efficiency.


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Example: Airline Efficiency (Contd.)

	Airline Companies		
	Beta	Unified	Southeast
Input Measures			
Labor cost per day (i_1)	\$8,000,000	\$10,000,000	\$6,000,000
Overhead cost per day (i_2)	\$2,500,000	\$3,000,000	2,000,000
Number of flights per day (i_3)	300	400	200
Output Measures			
Average minutes late arriving (o_1)	-15	-20	-7
Number of luggage lost per 10,000 (o_2)	-10	-15	-5
Waiting time for check-in (o_3)	-10	-20	-5
Friendliness/quality of air service (o_4)	85	80	95

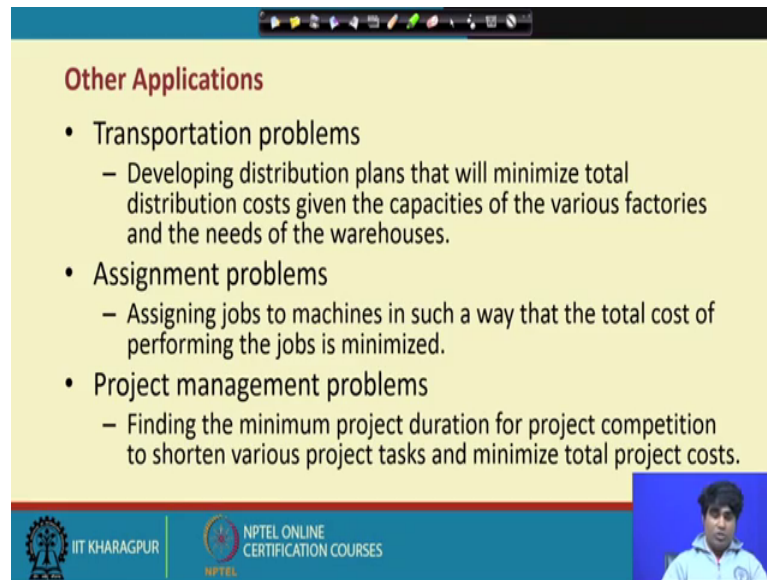



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So, the typical structure of the DIG you may have actually input cluster more number of inputs and more number of outputs and again linear programming problem can be fitted and then we can have the kind of you know efficient solutions.

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Other Applications

- Transportation problems
 - Developing distribution plans that will minimize total distribution costs given the capacities of the various factories and the needs of the warehouses.
- Assignment problems
 - Assigning jobs to machines in such a way that the total cost of performing the jobs is minimized.
- Project management problems
 - Finding the minimum project duration for project competition to shorten various project tasks and minimize total project costs.

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And, likewise we have also other application like you know we can apply the particular structure in transportation problem, we can apply in the assignment problems and the kind of you know project management problems. So, that means, you know whatever may be the kind of you know application area and if the particular problem is having you know typical you know objective means specify objective and the kind of you know constraints the kind of you know requirements.

Then the predictive analytics typically the kind of you know structure what we called as, you know the kind of you know linear programming structure through which you can look for the solution and that to optimum solutions and as a result we can address the problem, business problem more effectively and then we can you know come with you know kind of you know management decision as per the particularly you know business requirement.

So, with this we will stop here and thank you very much, have a nice day.